Weighted best first search
Anytime best first search
A* Search

- Computes optimal g-values for relevant states

at any point of time:

- the cost of a shortest path from $s_{start}$ to $s$ found so far
- $g(s)$

- an (under) estimate of the cost of a shortest path from $s$ to $s_{goal}$
- $h(s)$
A* Search

- **Heuristic function must be:**
  - admissible: for every state $s$, $h(s) \leq c^*(s, s_{goal})$
  - consistent (satisfy triangle inequality):
    \[ h(s_{goal}, s_{goal}) = 0 \text{ and for every } s \neq s_{goal}, h(s) \leq c(s, succ(s)) + h(succ(s)) \]
  - admissibility follows from consistency and often consistency follows from admissibility
A* Search

- Is guaranteed to return an optimal path (in fact, for every expanded state) — optimal in terms of the solution

- Performs provably minimal number of state expansions required to guarantee optimality — optimal in terms of the computations

- A* Search: expands states in the order of \( f = g + h \) values
**A* Search**

- Is guaranteed to return an optimal path (in fact, for every expanded state) — optimal in terms of the solution

- Performs provably minimal number of state expansions required to guarantee optimality — optimal in terms of the computations

- Weighted A*: expands states in the order of $f = g + \varepsilon h$ values, $\varepsilon > 1 = \text{bias towards states that are closer to goal}$
Effect of the Heuristic Function

- $A^*$ Search: expands states in the order of $f = g + h$ values

for large problems this results in $A^*$ quickly running out of memory ($\text{memory: } O(n)$)
Effect of the Heuristic Function

- **Weighted A* Search**: expands states in the order of $f = g + \varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal

  *what states are expanded?*

  *research question*
Effect of the Heuristic Function

- **Weighted A* Search**:
  - trades off optimality for speed
  - $\varepsilon$-suboptimal:
    \[
    \text{cost(solution)} \leq \varepsilon \text{cost(optimal solution)}
    \]
  - in many domains, it has been shown to be orders of magnitude faster than $A^*$
Effect of the Heuristic Function

- Weighted A* Search:
  - trades off optimality for speed
  - $\varepsilon$-suboptimal:
    \[ \text{cost(solution)} \leq \varepsilon \cdot \text{cost(optimal)} \]
  - in many domains, it has been faster than $A^*$

Is it guaranteed to expand no more states than $A^*$?
Effect of the Heuristic Function

• **Weighted A* Search**:  
  - trades off optimality for speed  
  - $\varepsilon$-suboptimal:  
    
    \[
    \text{cost(solution)} \leq \varepsilon \cdot \text{cost(optimal)}
    \]

  - in many domains, it has been faster than A*

  *Is it guaranteed to expand no more states than A*?*

  **NOPE!**
Experiments

- 16 pedigree networks from UAI 2008
- 20 protein instances.
- The pedigree instances were ran with i-bounds \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}, proteins with i-bounds \{2, 4, 6\}.
- The range of weights used in experiments is \{32, 16, 8, 4, 2, 1.8, 1.6, 1.4, 1.2, 1.1, 1\}.
- 4GB of RAM, 1 hour
Cost of the solution as a function of weight
exact heuristic (i-bound=6)

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)

weight w

- log(MPE)
Cost of the solution as a function of weight (i-bound=8)

-\log(MPE)

weight w

- pedigree38 (725, 5, 16, 52)
- pedigree13 (1078, 5, 30, 126)
- pedigree19 (793, 5, 21, 108)
Cost of the solution as a function of weight proteins (i-bound=4)
Number of nodes expanded as a function of weight

- pdb1a80, i=4
- pdb1a7w, i=4
- pdb1acf, i=6

Nodes expanded vs. \( w \)
pedigree1: cost vs weight, various i-bounds
(335, 5, 15, 59)
pedigree1: number of nodes expanded vs weight, various i-bounds
(335, 5, 15, 59)
pedigree41: cost vs weight, various i-bounds
(1063, 5, 29, 119)
pedigree41: number of nodes expanded vs weight, various i-bounds
(1063, 5, 29, 119)
Anytime Repairing A*

after each solution is found, decrease the weight before continuing the search.
use a technique to limit node reexpansions.
Anytime nonparametric A*

Change of the notation: $G = C$ (the cost of solution)

$$f(s) = g(s) + \varepsilon \cdot h(s),$$

**Main idea:** don’t expand the node with $\min f(s)$, instead expand the node with *maximal* $e(s)$:

The value of $e(s)$ is equal to the $e$ such that $f(s) \leq G$.

$$e(s) = \frac{G - g(s)}{h(s)}$$
ANA* vs. ARA*

Doesn’t require input parameters
Maximally greedy:
  finds initial solution faster (“usually”)

Spends less time between solution improvements (“usually”)
more gradually decreases the suboptimality bound of the current-best solution
 Doesn’t need to update evaluations of the nodes on OPEN as often