This paper examines the effect of memory loss on the continuity of behavior. We consider a player (individual or firm) who remembers previous actions but not underlying rationales. In a stable environment, relative to a full-recall scenario, memory loss increases the probability of following old policies (inertia). In a volatile environment, memory loss can decrease this probability (impulsiveness). The model provides a memory-loss explanation for some documented psychological biases, implies that inertia and organizational routines should be more important in stable environments than in volatile ones, and provides empirical implications relating memory and environmental variables to economic decisions.

... the most radical revolutionary will become a conservative on the day after the revolution.

Hannah Arendt, *The New Yorker*, September 12, 1970

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Be not the first by whom the new are tried,
Nor yet the last to lay the old aside.

Alexander Pope, *An Essay on Criticism*, 1711

When I was younger I always conceived of a room where all these [strategic] concepts were worked out for the whole company. Later I didn’t find any such room. . . . The strategy [of the company] may not even exist in the mind of one man. I certainly don’t know where it is written down. It is simply transmitted in the series of decisions made.

General Motors executive, quoted in Quinn (1980)

1. Introduction

The problem of memory loss permeates human choices, from the shopper trying to remember which detergent cleans better to the newly arrived manager trying to learn the relevant aspects of his firm’s history. Yet there has been remarkably little research on the consequences of such amnesia for economic decisions. This paper offers an economic model of how memory loss affects the continuity of behavior.

An individual’s habits, an organization’s existing policies and routines, and a society’s traditions are often firmly entrenched even when the rationales are not evident. Policies are often maintained and even escalated despite opposing information (see, e.g., Arkes and Blumer, 1985; Ruef, 1997). Similarly, the nonadoption of a potential activity often continues despite the arrival of favorable information. Several authors have argued that firms commonly use hurdle rates that exceed the cost of capital, thereby discouraging new projects [see references in Dixit (1992)]. There is also strong evidence of inertia in employees’ decisions about whether to participate in, and how much to contribute to, 401(k) retirement plans (Kusko et al., 1998; Madrian and Shea, 2000).

At the level of the firm, our paper offers a new theory of the determination of organizational continuity vs. change in the face of memory loss.\(^1\) It seems plausible that the behavior of a firm is most likely to change when a new manager arrives. However, a new manager is likely to be hampered by lack of information about the source

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1. Rumelt (1995) argues that the issue of the optimal degree of organizational inertia is central to corporate strategy research, that managers should take into account a firm’s own inertia in evaluating actions, and that strategic success is often due to the inertia of competitors as much as to the cleverness of the innovator.
of past decisions. In some cases, a new manager may infer that there is a good rationale for past policies, even if he or she is not sure what that rationale is. If so, it may make good sense to maintain, or at least carefully consider, the status quo.²

Even executives hired specifically to effect change, and ardent social reformers, usually perpetuate many of the policies of the regime they replace, limiting change to a small set of high-profile issues. However, our primary focus is not upon the rare traumatic events in which managers are replaced in order to effect change and restructuring. In such situations, the forces described here are likely to apply, but other effects may be at least as important. Instead, we examine behavioral continuity vs. change in relation to the problems of memory loss regular faced by individuals and firms in the ordinary course of life. Policy continuity is prevalent (if less striking) after routine losses of memory that arise from imperfect communication, incomplete records, and routine managerial transitions (for retirement, promotion, illness, or other personal reasons).

At the individual level, experimental psychologists have identified action-induced shifts in attitudes or beliefs, which has motivated the theory that individuals adjust their beliefs to reduce cognitive dissonance. Our analysis is related to psychological theories of self-attribution, the process by which people inferentially attribute reasons or motives to themselves based upon observation or recall of their own actions and experiences (Bem, 1965). In our approach, such inferences determine whether past behaviors continue. An individual who remembers his old actions and presumes that there must have been a good reason for them will, under conditions that we delineate, exhibit excess inertia.³ We discuss applications of the model to behaviors identified in experimental psychology studies, such as

² For example, when Louis Gerstner, the RJR Nabisco CEO, replaced John Akers as CEO of IBM following a period of dramatically poor performance, Gerstner surprised observers by deciding not to embark on a radical change of course. As an executive with a low-tech marketing background, Gerstner may have initially lacked the expertise needed to critically evaluate existing IBM policies, and thus may have been compelled to rely on these established policies: “Three months into the job, Gerstner has made it clear that he has no intention of reconstructing IBM. Instead, the man everyone saw as the Great Changemeister is determined, for the moment, to carry out a set of policies put in place by none other than the much-maligned Akers.” [See “At IBM, More of the Same—Only Better? In Sales and Strategy, Louis Gerstner Is Following John Akers’ Path,” Business Week, July 26, 1993. According to this article, Gerstner was “…still following through on Akers’ two-year-old restructuring.”

³ A notable example of faulty reconstruction of reasons based on an action is provided by split-brain experiments. When an experimenter directs a command to a single brain hemisphere of a split-brain patient to get up, the other hemisphere (the verbal one) commonly invents and believes an imaginary explanation for why the person has left his seat (Gazzaniga, 1992).
action-induced belief changes, escalation and foot-in-the-door biases, and the endowment effect in Section 5.4. Our basic approach views individuals as optimally responsive to past memory loss.

Although inertia is common, it sometimes seems that individuals or organizations are oversensitive to new information. Private individuals and managers are sometimes criticized for mercurial or “weather-vane” decision styles, or of having a “grasshopper mentality.” Several competing theories of organizational inertia are discussed in Section 6, but very few of these theories examine the opposite phenomenon, which we term impulsiveness. We offer a model to explore the conditions under which firms will be more responsive or less responsive to new information, i.e., when inertia vs. impulsiveness will result. In our setting, patterns of inertia vs. impulsiveness can arise without any difference in tastes or characteristics of decision makers. Rather, these are consequences of the characteristics of the decision environment.

The degree of behavioral continuity then depends on the balance between newly arrived information and the pool of old information. A natural presumption is that memory loss reduces the weight of old information in determining later actions. We show that sometimes the opposite will be the case.

Our approach is based on the premise that past actions (policies, routines) are remembered well, while past information signals are remembered poorly. Actions are more visible, salient, and memorable than underlying justifications. Information transmission, absorption, and retention is costly, so the full reasons for past decisions are often forgotten. Furthermore, organizations can lose access to old information even if individual managers, like elephants, never forget. A decision maker with such differential recall must infer past signals from the coarse summary provided by past actions. Our conclusions derive from the fact that this coarseness distorts later decisions—even when the decision maker optimally adjusts for the resulting information loss.

The basic argument of the paper has two parts. First, suppose that an initial player has adopted a policy for some time and that the new player—without access to his predecessor’s information—must decide whether to adopt a similar action as he receives a sequence of further signal realizations. The new player does not know how strongly his predecessor’s information favored the original decision. Consequently, even if the new player receives an opposing signal, he will optimally tend to continue the old behavior if the presumed favorable information of his predecessor outweighs the new signal. So at first, he never switches. In contrast, in a benchmark regime of perfect recall, a continuing player whose information happens to barely favor the initial project will switch after even a single opposing signal.
So in this situation a player with perfect memory sometimes switches, while one who recalls only past decisions never switches—memory loss increases the stability of behavior.

More generally, as more signals accumulate, the new player will sometimes switch. However, it is still the case, in view of the initial player’s adoption decisions, that when a new player starts out he is not near the borderline of rejecting. In contrast, a continuing initial player who, despite his past series of adoptions, happens to have a relatively adverse signal sequence starts out after the transition date very close to the borderline of rejecting. Thus, even though beliefs are unbiased, there is a higher probability of an action switch when there is no memory loss. (See also the discussion in footnote 19.) This situation—where the probability of an action change is lower when the individual remembers only past actions than when he has full recall—is called \textit{excess inertia} under amnesia relative to a full-recall regime.

The second part of the argument concerns forces that can weaken the inference that should be drawn from the predecessor’s action. For example, if the gains to adopting a project change stochastically, then an old choice that was correct may become incorrect later. A continuing player with perfect recall would in some cases know that the old information was quite strong, and in such cases would follow old policies rather than new signals. In contrast, a new replacement player may find it optimal to follow his own signal regardless of past actions. In this changing-environment scenario, amnesia causes \textit{excess impulsiveness} rather than inertia—an individual with poor memory of past signals is more likely to switch behaviors than one with good memory.

In sum, our model of choice under memory loss provides implications about several determinants of inertia vs. impulsiveness. These include the information load an individual must process, volatility of the decision environment, how long policies and executives have been in place, and the quality of a firm’s information systems.

In modeling approach, the analysis is related to the informational cascades literature of Bikhchandani et al. (1992), Banerjee (1992), and Welch (1992). Both approaches assume a discrete action set, which coarsens the information available to a decision maker who observes (or recalls) earlier decisions. The cascades outcome is an extreme
limiting case of what we call inertia: the individual is certain to follow past actions and never accumulates enough information to change course. A key difference is that here we examine not just the formation and dissolution of cascades, but the more general issue of inertia vs. impulsiveness—whether the probability that a player follows past actions is increased or decreased by memory loss. This definition of inertia and impulsiveness provides a way to analyze the continuity of behavior even when individuals continue to use their own information.

Even in circumstances in our model where a cascade is impossible, inertia and impulsiveness occur. For example, when there are many periods in our setting, all mistaken cascades must eventually break. Nevertheless, we find medium- and long-run inertia. In contrast, in the original cascades model, in a stable environment with identical individuals a cascade once started lasts forever; the issue of inertia is not addressed.

The remainder of the paper is structured as follows. Section 2 discusses why actions are likely to be recalled better than signals. Section 3 provides a basic five-period model that examines the effect of environmental volatility. Section 4 considers many periods to examine the effects of longer action histories and whether inertia or impulsiveness can persist in the long run. Section 5 discusses implications of the model based on the value and sources of memory. Section 6 discusses alternative and related approaches. Section 7 concludes. Proofs are in the appendix.

2. Are Actions Recalled Better Than Signals?

There is strong and consistent experimental evidence that individuals recall actions they have performed (subject-performed tasks) better than verbal phrases. Better recall of actions than reasons makes sense. Actions lead to more tangible consequences than discussion or thinking, and hence are more salient. Salience increases memorability. Taking actions often requires physical motion and personal interaction.

5. We have also developed a simple two-period example with continuous signals and unbounded likelihood ratios in which cascades never form, yet there is inertia. This highlights the fact that inertia is distinct from (though related to) informational cascades. For a general analysis of informational cascades when signals are continuous, see Smith and Sorensen (2000).

6. Relevant studies are surveyed by Engelkamp (1998), Engelkamp and Zimmer (1994), and Cohen (1989). In addition, Zimmer and Helstrup (2000) argue that an automatic “pop-out” retrieval mechanism (as opposed to directed memory search) is more effective for physical tasks than for verbal tasks.
Such activity causes arousal. Not surprisingly, there is evidence that arousal at the time of a stimulus facilitates later recall.

Lingle and Ostrom (1979) discuss evidence that when people reach a conclusion about another individual, their subsequent decisions are based upon this conclusion, whereas the original data that underlay this judgment is not used. Thus, the ‘action’ of arriving at a judgment is preserved accessibly in memory better than the underlying reasons. Furthermore, Arkes and Harkness (1980) describe how physicians tend to find it much easier to remember a diagnosis that they arrived at than to recall the symptoms that led to the diagnosis.

Furthermore, memory is triggered by associations (see, e.g., Anderson and Bower, 1973). A cue tends to trigger memories that are associated with similar cues. Each association is a “hook” that can trigger recall. Taking an action rather than just thinking about or discussing it may create a wider set of associations. Rehearsal also promotes recall; retrieving a memory increases the likelihood that the memory will be retrieved again in the future. We argue that actions are often more likely to be retrieved than reasons.

For example, consider the purchase by a consumer of a brand of detergent based on a friend’s claim that it is good at removing stains. The act of purchase creates arousal: at the market the consumer actively searches for and pays for the product. This activity also creates associations: with the distinctive color of the box, the location in the store, and the sight of the product with each use. Repeated purchase of the product provides opportunities for rehearsal, making it easier to remember the product.

On the other hand, there can also be some recall of the reason for purchasing the product. However, if the product is reasonably effective, there is no need for repeated retrieval of the reason for purchase. A year later it may be hard to remember whether the reason was stain-removing ability, low price, or gentleness on clothes.

At the organizational level, a similar phenomenon is likely to occur. Organizational action requires arousal and attention by coordinating individuals. When an action is taken and repeated, it leads to effects, and these effects provide repeated reminders of what the action was that led to these effects. Actions are therefore salient, and involve a richness of arousal, associations, and rehearsal. A final reason for superior recall of actions is that it is often easier to record and communicate clearly records of actual actions than reasons for these actions—as indicated in the General Motors executive’s description of

7. In some cases the reasons for actions are rehearsed as well. But such rehearsal will tend to decline if the action becomes a routine, continuing policy.
corporate strategy at the head of this text: “It is simply transmitted in the series of decisions made.”

3. The Shadow of History: The Basic Model

We now describe a stylized model of the shadow that the action history casts on future behavior after a memory loss, and the factors that can deepen or dispel this shadow. With imperfect memory a player cannot attune his policies perfectly to old information. A new player observes only a coarse summary of what the previous player knew, as reflected in past actions. We will show that if past actions are highly informative, a player with memory loss optimally exhibits excess inertia; if past actions are less informative, the player may optimally exhibit excess impulsiveness; and if past actions are very uninformative, there is neither inertia nor impulsiveness.

To illustrate simply, consider a player (individual or firm) who faces a sequence of five identical decisions, or projects. The adoption value of each project, which is constant through time, can be either good (value state \( \phi = G \)) or bad (value state \( \phi = B \)), with equal ex ante probability. Payoffs are perfectly correlated across projects, but are not observed until the end of the game. A player observes a single new signal, \( H \) or \( L \), each period. If the underlying state is good, the player observes signal \( H \) with probability \( p \) (\( p > 0.5 \)), and \( L \) with probability \( 1 - p \). If the underlying state is bad, these probabilities are reversed (see Table I). Conditional upon the value state, the successive signals are assumed to be statistically independent.\(^8\) Note that information arrives each period regardless of whether the project was accepted or rejected in the previous period.

In each period, the player chooses an action, either adopt \( A \) or reject \( R \). Net payoffs are such that the player adopts (rejects) the project if state \( G \) is more (less) likely than state \( B \), and flips a fair coin when \( G \) and \( B \) are equally likely.\(^9\) To benchmark behavior, we compare a full recall scenario, in which both actions and signals are retained (i.e., the initial player \( I \) stays in place for all five periods and retains access to his old information) with an amnesiac scenario, in which only actions are remembered (i.e., the new player \( N \) can recall

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\(^8\) An alternative interpretation of the model would define actions \( A \) and \( R \) as expanding or shrinking a project that is already in place, states \( G \) and \( B \) as higher and lower probabilities of getting a high payoff per period, and signals \( H \) or \( L \) as immediately observed high or low payoffs.

\(^9\) For expositional simplicity we examine a symmetric setting, which leads to the possibility of indifference and randomization. Qualitatively similar results apply when distributions are asymmetric.
the two actions taken prior to the transition date (period 3), but not the signals of his predecessor.\textsuperscript{10} Just before date 3, the underlying value of adopting vs. rejecting may change. Specifically, with probability $\sigma$ the state is redrawn. If it is redrawn, with equal probability the new state is $G$ or $B$. Thus, $\Pr(\phi' = G | \phi = G) = 1 - (\sigma/2)$.

We first describe the transitional behavior of $N$ versus $I$ at date 3. There are four possible action patterns for the first two dates. $AA$ can arise from either $HH$ or $HL$, and $RR$ from either $LL$ or $LH$. In contrast, $AR$ can arise only from the single signal sequence $HL$, and $RA$ only from $LH$. So opposing actions allow $N$ to infer perfectly that the first two signals were opposed ($HL$ or $LH$), and that a coin flip led to the second action choice. Thus, under either full recall or amnesia, posterior beliefs are based on the cancellation of the first two signals, and the date-3 player $N$ follows his latest signal. Since this is true under both full recall and amnesia, such realizations do not contribute to inertia or impulsiveness. Thus, we focus on realizations where $I$ follows like actions for two dates, $AA$ or $RR$. Like actions are likely to arise from like signals. $AA$ could arise either from $HH$, or from $HL$ with a coin flip. $RR$ could arise either from $LL$, or from $LH$ with a coin flip.

When volatility is low, the probability of an action switch under amnesia is zero, whereas there is a positive probability of switch under full recall. Consider, for example, the case of a fully stable environment ($\sigma = 0$). In an amnesia scenario where the decision maker observes the past two actions but not signals, the new player draws

\begin{table}
\centering
\caption{Conditional Signal Distribution Probabilities}
\begin{tabular}{lll}
\hline
\textbf{Signal Value} & \textbf{Value} & \textbf{State $G$} & \textbf{B} \\
\hline
$H$ & $p$ & $1-p$ & \\
$L$ & $1-p$ & $p$ & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{10} The model’s assumption that memory loss occurs at a single transition point fits the example of an organization facing managerial turnover. Individual memory loss is also not completely steady. Individuals experience discrete events, such as relocations and changes in jobs or mates, which reduce the frequency of memory-triggering cues about pretransition signals, and opportunities for memory-enhancing rehearsal. However, the main motivation for the discrete-memory-loss assumption is tractability. We conjecture that settings with steady memory decay would lead to similar effects.
### TABLE II.

equilibrium date-3 behavior with amnesia or full recall in the basic model (given a matched-action history)

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Low $\sigma$ ($\sigma &lt; \sigma_N$)</th>
<th>Medium $\sigma$ ($\sigma_N &lt; \sigma &lt; \sigma^0$)</th>
<th>High $\sigma$ ($\sigma^0 &lt; \sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amnesia</td>
<td>Follow matched pretransition actions</td>
<td>Follow latest signal</td>
<td>Follow latest signal</td>
</tr>
<tr>
<td>Full recall</td>
<td>Follow matched pretransition signals</td>
<td>Follow matched signals</td>
<td>Follow latest signal</td>
</tr>
<tr>
<td>Interpretation:</td>
<td>Inertia</td>
<td>Impulsiveness</td>
<td>First best</td>
</tr>
</tbody>
</table>

**Description:** Under amnesia the player observes only the preceding two actions. Under full recall the player observes the preceding two signals. Matched actions are a pair of like actions at dates 1 and 2, i.e., (adopt, adopt) or (reject, reject). Matched signals are a pair of like signals, i.e., $HH$ or $LL$. The table shows excess inertia for low $\sigma$, because under amnesia past matched actions are always continued, whereas under full recall they are only sometimes continued. For medium $\sigma$, there is excess impulsiveness, because under amnesia at date 3 the player always follows the latest signal without any regard to past events, whereas under full recall the action is related to past information.

A favorable inference from $AA$ (that the signals were $HH$ or $HL$). Because of a high likelihood that $AA$ came from $HH$, even if he observes $L$, he still adopts. He knows, based on the first $A$, that the first signal was $H$. This offsets his latest $L$ signal. The second $A$ came from either an $H$ or an $L$ followed by a coin flip. The former is more likely, since a coin flip could lead to $R$ instead of $A$. Therefore, on balance it is more likely that the state is $G$ than $B$. Thus, under amnesia, $N$ always adopts. Similarly, after $RR$ he always rejects. In these cases, $N$ is in an informational cascade (i.e., his action is independent of his signal). In contrast, if the two signals were indeed $HL$ or $LH$, a continuing player $I$ with an opposing signal in the third period reverses course.

This outcome of excess inertia when environmental volatility is low is shown in the second column in Table II. Intuitively, in a stable environment (low $\sigma$), old policies are very informative relative to new signals. It is therefore optimal for an amnesiac to place heavy weight on the past, leading to inertia.

We now show that in a more rapidly changing environment, a full-recall player sometimes follows his old signal. (The full-recall player is more willing to put some weight on the past, because he knows past signals accurately instead of having to infer them from action.) Thus, $N$ tends to overreact to new information, and is

\[\Pr(HH|AA) = \frac{(1 - 2p + 2p^2)}{(1 - p + p^2)} > \Pr(HL|AA) = \frac{p(1 - p)}{(1 - p + p^2)} .\]
### TABLE III.
**EQUILIBRIUM DATE-4 BEHAVIOR WITH AMNESIA OR FULL RECALL IN THE BASIC MODEL (GIVEN A MATCHED-ACTION HISTORY)**

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Low $\sigma$ ($\sigma = 0$)</th>
<th>Medium $\sigma$ ($0 &lt; \sigma &lt; 1$)</th>
<th>High $\sigma$ ($\sigma = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amnesia</td>
<td>Follow matched pretransition actions unless both signals oppose.</td>
<td>Follow matched latest signals (past actions if offsetting).</td>
<td>Follow matched latest signals (coin flip if offsetting).</td>
</tr>
<tr>
<td>Full recall</td>
<td>Follow matched pretransition signals (but coin flip if later signals both oppose).</td>
<td>Follow matched latest signals (past signals if latest offset, coin flip if all signals offset).</td>
<td>Follow matched latest signals (coin flip if offsetting).</td>
</tr>
</tbody>
</table>

Interpretation: Strong inertia

**Description:** Under amnesia the player observes only the preceding two actions. Under full recall the player observes the preceding two signals. *Matched actions* are a pair of like actions at dates 1 and 2, i.e., (adopt, adopt) or (reject, reject). *Matched signals* are a pair of like signals, i.e., HH or LL. The table shows excess inertia for low and medium $\sigma$, because past matched actions are more likely to be influential under amnesia than under full recall.

### TABLE IV.
**EQUILIBRIUM DATE-5 BEHAVIOR WITH AMNESIA OR FULL RECALL IN THE BASIC MODEL (GIVEN A MATCHED-ACTION HISTORY)**

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Behavior</th>
<th>Low $\sigma$</th>
<th>Medium $\sigma$</th>
<th>High $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amnesia</td>
<td>Follow matched pretransition actions (unless three opposing new signals)</td>
<td>Follow preponderance of three recent signals</td>
<td>Follow preponderance of three recent signals</td>
<td></td>
</tr>
<tr>
<td>Full recall</td>
<td>Follow preponderance of all five signals</td>
<td>Follow preponderance of all five signals</td>
<td>Follow preponderance of three recent signals</td>
<td></td>
</tr>
</tbody>
</table>

Interpretation: Inertia

**Description:** Under amnesia the player observes only the preceding two actions. Under full recall the player observes the preceding two signals. *Matched actions* are a pair of like actions at dates 1 and 2, i.e., (adopt, adopt) or (reject, reject). *Matched signals* are a pair of like signals, i.e., HH or LL. The table shows excess inertia for low $\sigma$, because under amnesia past matched actions are likely to be continued unless there are three opposing signals, whereas under full recall several less extreme signal outcomes cause an action change. For medium $\sigma$, there is excess impulsiveness, because under amnesia at date 3 the player only follows recent signals, whereas under full recall past information also carries weight.
too quick to reverse old policies. In such an environment, memory improvements tend to make the player more willing to retain old policies.

As environmental volatility increases (medium $\sigma$ in Table II), players place relatively little weight on the past (under either full recall or amnesia). At some point, the relevance of old policies to current decisions weakens just enough that an amnesiac player always follows his new signal. In contrast, at this point a continuing $I$ does not always follow his latest signal. (Specifically, if prior signals have overwhelmingly favored a project, $I$ sticks with that project.) Therefore memory loss causes impulsiveness. As volatility increases, a point is reached for $N$ ($\sigma = \sigma^N$) where a single $L$ signal outweighs the information implicit in $AA$ (which could have arisen from $HH$ or $HL$). $N$ then always follows his own signal without regard to the past. In contrast, since $I$ has more refined information about the past, when he observes strongly favorable past information $HH$, he adopts even if his third signal is $L$. Thus, in this range of $\sigma$ amnesia causes impulsiveness.

Finally, for a highly unstable environment (high $\sigma$ in Table II), even past signals have weak relevance for the future, so even a full-recall player will follow the new signal. In other words, an $L$ signal outweighs not only two $AA$ signals but also two $HH$ signals. Since $I$ and $N$ behave identically, there is neither excess inertia nor impulsiveness. This reasoning suggests the following result:

**Proposition 1**: In the model with possible environmental shifts, the critical value for $\sigma$ above which the player always follows the date-3 signal is larger for the full-recall player than for the amnesiac player: $\sigma^N < \sigma^I$. Thus, if the probability of a value shift is:

1. low ($\sigma < \sigma^N$), then under memory loss the player exhibits excess inertia;
2. intermediate ($\sigma^N < \sigma < \sigma^I$), then the player exhibits excess impulsiveness;
3. high ($\sigma^I < \sigma$), then there is neither excess impulsiveness nor inertia.

At date 4, there is inertia for all $\sigma < 1$, and first-best behavior if $\sigma = 1$. At date 5, as at date 3, there is inertia for low $\sigma$, impulsiveness for all intermediate values of $\sigma$, and first-best behavior for high $\sigma$.

This proposition describes the conditions that promote impulsiveness vs. inertia. Firms in relatively stable industries are predicted to be excessively inert, and those in more rapidly changing industries such as computing and telecommunications to exhibit less inertia, and
perhaps impulsiveness. This is especially the case when employee job mobility (and loss of firm-specific memory) is high, e.g., as in Silicon Valley during the rise of personal computing and the Internet. Part (3) of the proposition seems to be an extreme case; it is unlikely that many individuals or industries are in environments so volatile that past signals are uniformly useless.

Environmental instability causes the value of information to decay, because information about the past value state becomes less relevant for the current adoption decision. Thus, even with full recall it is no surprise that instability leads to greater change. The interesting point is that amnesia exaggerates the importance of old signals in a stable environment, and can exaggerate the importance of new signals in a fairly volatile one.

Intuitively, amnesia coarsens information about the past, thereby rendering the player less aware of unusual circumstances. If the information reflected in past actions is quite informative for the future, deviation from past actions is seldom needed. Past actions are followed frequently; the risk that this is a mistake is worth taking. Similarly, if past actions are not very informative, it is optimal to follow past actions seldom, and bear the risk that this is a mistake. Thus, coarse information causes a kind of overshooting. Conditions that, under full recall, favor low (high) weight upon past information can cause excess impulsiveness (excess inertia) when memory is lost.

Reasoning broadly similar to that for date-3 decisions applies at dates 4 and 5 as well. At date 5, there is still a progression as environmental volatility rises from inertia to impulsiveness to first-best decisions. However, it is no longer the case that inertia is associated with a certainty of following past actions; it is just that the probability of doing so is increased by memory loss. At date 4, the progression is

12. Thus, Henry Ford’s view that “History is more or less bunk…. We don’t want tradition…. The only history that is worth a tinker’s damn is the history we make today” is more appropriate to the Ford of 1916 than the Ford of today.

13. Bikhchandani et al. (1992) provided a numerical example of informational cascades with stochastic environmental shifts, to show the possibility of what they called “fads” (somewhat analogous to impulsiveness here). Our analysis goes further to analyze systematically how continuity of behavior changes as volatility changes. We show that there is sometimes a nonmonotonic pattern (inertia, impulsiveness, first best) as volatility rises; and that sometimes higher volatility reduces inertia but does not actually cause impulsiveness. Bikhchandani et al.’s example fixed the environmental change probability, which precluded such analysis. Also, their calculation examined the probability of a long-run shift from one cascade to another cascade. Immediately after the environmental shock, in their example a cascade is impossible. So their calculation says nothing whatsoever about short-term impulsiveness vs. inertia immediately after the shock. Here we examine impulsiveness vs. inertia both in the short term and at longer lags. Moscarini et al. (1998) find that when the probability of environmental change is sufficiently large, cascades never form.
from strong inertia to weaker inertia to first best; the region of actual impulsiveness vanishes.\textsuperscript{14}

These findings suggest why certain factors affect the shadow of the past. We have just seen that since environmental volatility weakens the informativeness of old actions for the future, high volatility tends to cause greater impulsiveness. Furthermore, we have shown in an analysis similar to the basic model that if there is only a probability each period that a player receives a meaningful signal, then even in a stable environment there can be impulsiveness. The possibility that past actions are based on relatively little information reduces the weight on them relative to a new signal (if a new signal is received). On the other hand, the next section examines a factor that can intensify the weight placed on the past: a longer pretransition action history.

4. Long-Term Effects of Amnesia

So far we have assumed a fairly short history prior to memory loss (two dates), and have examined only one decision immediately after memory loss. We have therefore allowed history to cast only a short shadow (a few periods). We now examine whether such behavior can persist in the long term. Furthermore, we examine how the length of the project adoption history (how well established the existing policy is) affects long-term inertia vs. impulsiveness. We therefore extend the model to many periods before and after the memory loss, but simplify by assuming a stable environment. We will show that the longer an action is established by the initial player, the greater the inertia; but that there is excess inertia even for fairly recently established policies.

4.1 A Multiperiod Decision Setting

This section differs from the basic model in three ways. First, it allows for any number of periods either before or after the memory-loss (player transition) date. Second, the environment is stable ($\sigma = 0$). Third, for analytical convenience we allow a third action choice (see below).

\textsuperscript{14} Intuitively, at date 4 posttransition information is either powerful (two like signals) or else completely uninformative (two opposing signals). The case of precisely offsetting signals contributes toward inertia by encouraging someone with partial recall to follow past actions, but someone with full recall to flip a coin if the pretransition signals were equally split. But when the number of posttransition periods becomes large, the case of a precisely equally split set of posttransition signals becomes very unlikely. This reasoning suggests that an impulsiveness region will be present for odd numbers of periods and when the number of periods is reasonably large.
In each of a number of discrete periods, the current decision maker (individual or firm) has to choose publicly an action, either to adopt the project (A), to reject the project (R), or to abstain from decision (∅). There are two possible value states, G and B, which are constant through time. Players adopt if G is more likely than B, reject if B is more likely than G, and abstain if G and B are equally likely. The appendix describes a simple payoff structure such that this behavior is optimal.

As before, the player’s prior belief is that G and B are equally likely, and each period the player receives one private signal, H or L. Payoffs are revealed only when the game ends. The signal structure is summarized in Table I of the previous section. Define the signal state of I at time t as the difference between the numbers of H and L signals that he observes, st = nh − nl (in t draws). Thus, conditional on the unknown value state, the player’s signal state follows a Markov process. The tie value st = 0 is the point where the player switches from either A or R to ∅. The appendix derives the probability of reaching st = 0 given an information set.

Suppose I has been in place for M periods. For N, observing I’s past actions (adopt, reject, or abstain) is equivalent to observing only whether the signal states s1, . . . , sM were above, below, or at 0. We call abstention an action switch, since the player shifts from preferring one action to being indifferent. Whenever an action switch occurs, N perfectly infers I’s signal state to be zero at the switch date.

Since adopt and reject are symmetric, we focus on the consequences of past adoption. We wish to determine whether past actions are continued more in the different scenarios. We therefore calculate the probability of actions conditional upon the individual having observed a sequence of exactly M uninterrupted adopt decisions immediately preceding the memory loss event. It is useful to generalize the notion of signal state to apply to either I and N.

15. As in Section 3 (see footnote 9), our simplifying assumption of symmetry leads to the possibility of exact indifference. Introducing the (somewhat artificial) option of abstaining simplifies the algebra and discussion by eliminating randomization, tie-breaking rules, or assumed asymmetry. We have derived similar results when there is no option of abstention and distributions are asymmetric.

16. For example, if I has observed HHLHL, he is in signal state +1 at date 5. Given the value state (G or B), the transition probabilities from signal state s to s + 1 and to s − 1 are just the conditional probabilities of another H or L signal being observed.

17. This is without loss of generality, because the signal state is zero whenever there is an act of abstaining. For example, suppose there have been 100 past actions, ending with the string ∅AAAA. The first 95 actions are irrelevant (the abstain demonstrates 48 H’s and 48 L’s, which jointly are uninformative about the value state) for N, so this history is equivalent to one where N = 4 and a string of 4 A’s has occurred.
**Definition:** The equivalent state $\bar{e}$ for a new or continuing player is the minimum number of consecutive opposing a signals ($L$-signals for an adopting player, $H$-signals for a rejecting player) required for the player to switch the action to abstaining or beyond.

For a continuing $I$, the equivalent state is the same as the signal state.

### 4.2 Transitional, Medium, and Long-Term Inertia

We now compare the likelihood of action changes under amnesia (a change in players) with the full-recall scenario where $I$ remains in place. Just after a transition, $N$ with amnesia has an equivalent state that depends only on the action history. We denote the equivalent state by $\bar{e}[M, p]$ as a function of the number of past adoptions and the signal precision. The next proposition shows that a player with amnesia initially is inert (relative to a full-recall player):

**Proposition 2:** Conditional on $M$ prior adopt decisions, a new player never switches before time $M + \bar{e}(M, p)$ periods, whereas there is a strictly positive probability that a continuing player switches by time $M + 2$, where $\bar{e}(M, p) \geq 2$ for $M > 2$.

(For $M \leq 2$, $N$ makes a perfect inference and consequently behaves like $I$.) Intuitively, after a sequence of $M$ adopts by $I$, $N$ does not know how strong the evidence was in favor of adopting. At best there could have been $M$ consecutive $H$ signals; at worst there could have been close to equal numbers of $H$ and $L$ signals. $N$ will infer statistically that the signal state is likely to be intermediate between these extremes. If $M$ is sufficiently large, $N$ believes it is unlikely $I$ was very near the borderline, so he does not switch actions after observing just a single $L$ signal. But a continuing $I$ who has observed the actual sequence might have been right on the borderline. So if there was only very weak evidence supporting the current action, a continuing $I$ switches action in response to one or only a few opposing signals with positive probability. Since $N$ is more likely to switch than a continuing $I$, there is excess inertia. This intuition suggests that the result of transitional inertia (which was the focus of Sec. 3) is quite robust with respect to assumptions about signal structure and value distribution.

Does inertia persist beyond the first few decisions? Sequences of signal realizations exist such that $N$ switches when $I$ does not, and vice versa.$^{18}$ We therefore compare the probability of an action switch

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18. $N$ may switch when a continuing $I$ would have maintained, if $N$ thinks that $I$'s signals were only moderately favorable to the old policy, whereas the actual signal
over a given period of time for N versus I. Asymptotically, N exhibits excess inertia:

**Proposition 3:**

(i) **Holding constant the number** $M > 3$ **of immediately consecutive prior adoptions, as** $t \to \infty$, **the probability that the new player reverses action at least once is less than or equal to the probability that the continuing player does so.**

(ii) **Holding constant the number** $M' > 3$ **of periods before the transition (which may include A or R), as** $t \to \infty$, **the probability that the new player reverses action at least once is strictly less than the probability that a continuing (full-recall) player does so.**

Intuitively, if the player begins with the wrong action, then with either full recall or amnesia (I or N), as information accumulates, eventually there will be a switch to the right action. In this case there is no difference. Suppose instead that the player begins with the right action. Then, under full recall, often the player begins close to the decision boundary, and switches temporarily to the wrong action along the way. This is less common under amnesia, because forgetting of past signals compresses N’s beliefs away from the edge of the decision boundary. Thus, we have shown that for a period immediately after the transition, and also in the asymptotic long run many periods after the transition, memory loss causes excess inertia.\(^{19}\)

Although we have no general proof, for an intermediate time period $T$ after the switch, extensive simulations uniformly indicate that the probability that N switches is strictly below the probability that I switches for any $M \geq 3$. (For $M < 3$, N perfectly infers I’s information and so behaves identically to a continuing I.) Thus, inertia obtains uniformly. The cumulative switch probabilities in the two scenarios are graphed for different parameter values in Figures 1 and 2.\(^{20}\)

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19. The opposite case—where, under full recall, the player begins far from the decision boundary—contributes to a higher frequency of a temporary switch to a wrong action. However, the two cases do not offset. It is very easy starting near a decision boundary to cross it by chance—even if the drift is away from the boundary. This is relatively unlikely starting far from the boundary. So the case in which the full-recall player is close to the decision boundary is a more likely contributor to action changes than the case in which he is far from the boundary.

20. Because of the noisiness of the signal, the cumulative probabilities are close to, but do not asymptote to, 1.0 in Figures 1 and 2. Excess inertia holds uniformly for the union of $M = 4, 32, 64$ and $p = 0.5001, 0.51, 0.6, 0.75, 0.9$. All results have been both computed from the described formulas and confirmed in simulations (using the computed equivalent state decision rule).
The uniformly higher probability of switch under full recall illustrates inertia in the short, medium, and long terms.

4.3 Strategic Signaling

So far, we have assumed that I maximizes expected profits each period. However, it may pay for a player to sacrifice short-run profit to improve future decisions. If a transition is foreseen, then I can adjust his policies as a function of his information in a sort of code to communicate with his later self, who lacks access to I’s information signals. Such behavior can be termed strategic signaling. One appealing such code would require I, right before the transition, to act in opposition to the preponderance of evidence if he is close to indifference. The expected cost is low, since incorrect decision-making occurs only when the choice is almost a toss-up. We have shown by example that the benefit from such strategic signaling sometimes outweighs the cost. However, in practice it may be hard to motivate
Amnesia, Inertia, and Impulsiveness

At the level of individual decisions, it is a commonplace that people do not always allow sufficiently for memory loss: misplaced keys and circuitous automobile searches for previously visited locations are obvious examples. Casual observation suggests that people seldom intentionally take the wrong action to signal to their later selves. Furthermore, there is psychological evidence that in making current decisions, individuals sometimes ignore predictable future changes in their mental states.22

21. According to Huber (1991), “Everyday observations make clear (1) that personnel turnover creates great loss for the human components of an organization’s memory,” and “(2) that nonanticipation of future needs for certain information causes great amounts of information not to be stored . . . or not to be stored such that it can be easily retrieved, and (3) that organizational members with information needs frequently do not know of the existence or whereabouts of information possessed or stored by other members.”

22. People exaggerate the resemblance of their prospective future feelings and preferences in different situations to their current feelings and preferences (Loewenstein
5. Sources and Value of Memory

Because economic analysis usually assumes perfect memory, we discuss a range of phenomena in which memory-loss effects are likely to be important. We do not think that memory loss is the unique consideration in these examples—agency and other issues are clearly important. However, since these alternative explanations are more familiar, we generally omit them here. A comparison of the range of empirical applicability of alternative theories is offered in Section 6.

The inefficiencies in our model can be eliminated if I accurately records the signal state and communicates it to N. Thus, our model is most applicable for policies for which information is hard to record, store, retrieve, and transmit. Several studies describe factors leading to imperfect individual and organizational memory.23 Fear of legal liability and “paper trails” discourages record keeping, implying that industries that are vulnerable to litigation should be more prone to problems of amnesia.

Amnesia in organizations is likely to be more severe when decision-making capacity is under a severe information load, when there is a long time lag between related decisions, for decisions that depend on confidential or legally sensitive information, and for decisions that are hard to foresee. The theory implies that it is these firms in which inertia and impulsiveness are likely to be most important (inertia for venerable policies in stable environments, impulsiveness for recent policies in volatile environments).

5.1 Information Technology and Record Keeping

Owing to a high turnover rate, reverse-engineering the software developed by departed programmers is a growing problem (Peterson, 1993). Although it is easy to communicate the actions of individual programs (or major subroutines) to a new programmer, it is usually difficult to understand the detailed rationale for specific pieces of code. This limits the ability of a new programmer to make major changes in old code.24 This effect can lock a firm into an unwieldy information system.

et al. (2000) analyze this projection bias. Furthermore, “People underappreciate how their own behavior and exogenous factors affect their future utility.” Although memory and preferences are different, this evidence is suggestive that people may also fail to engage in sophisticated forms of signaling in order to influence their own later selves.

23 For example, individual: Kahneman et al. (1982), Nisbett and Ross (1980), and Starbuck and Milliken (1988); organizational: Huber (1991), Han (1997).

24 Subroutines and lines of code can have hidden rationales and side effects that make them difficult for a later programmer to understand, or even for the original
The introduction of groupware and knowledge management systems at many firms provides natural experiments on the effects of improvements in organizational memory. Such software facilitates the recording, retrieval, and transmission of transactions and decisions. If effective, groupware should reduce inertia and impulsiveness. Our analysis therefore predicts that introduction of groupware and other knowledge management procedures will reduce inertia for firms in stable industries, but in more volatile environments will increase stability (by reducing impulsiveness).

Several authors in the strategy literature have argued that small firms tend to be more flexible in production, swift, and willing to innovate than large firms. It has been argued that larger firms may be insulated from changing competitive conditions by a greater stock of slack resources (Cyert and March, 1963), that bureaucracy and organizational complexity can make a large firm less responsive to environmental change (March, 1981), and that smaller firms have a greater need to compete aggressively (Aldrich and Auster, 1986).

Empirically, firm size is related to the probability of change in core characteristics of firms, R&D spending, and innovation. Acs and Audretsch (1988) find that in most of the industries they examined, most innovations (as tabulated by the Small Business Administration) came from larger firms, but that in some industries the contrary was the case. Chen and Hambrick (1995) find that smaller airlines have greater speed and readiness to initiate competitive challenges, but are slower to respond to a competitive initiative than are larger competitors.

From the perspective of our approach, managerial specialization and the spreading of decision making in large firms probably leads to a relatively steady rate of forgetting of decision rationales in large firms as managers depart. In contrast, small firms run by small entrepreneurial teams are likely to have very good memory when no turnover occurs, and very poor memory when an executive leaves. Thus, our approach predicts that small entrepreneurial firms should usually be more agile than large bureaucratic firms, but with occasional bursts of extreme inertia or impulsiveness. In this respect our

programmer to revisit. Indeed, the movement first to procedural languages and now to object-oriented programming (see, e.g., Forbes ASAP, December 2, 1996, p. 187) encourages modularization to reduce the need to make any changes to old code.

25. The economic importance of electronic knowledge management is highlighted by IBM’s 1995 $3.3 billion acquisition of Lotus in order to acquire Lotus’s Notes software.
approach emphasizes variability and differences in small-firm behavior more than does past literature.

Furthermore, scale economies in memory systems create an advantage for larger firms. These arise from the fixed costs of computerizing, introducing sophisticated internal accounting systems, and hiring specialized managers. This memory advantage should increase the nimbleness of large firms relative to small firms in reacting to change. The possibility that large firms can be nimbler contrasts with the theoretical arguments in much of the strategy literature.28

5.2 Executive Mobility and Succession

Our focus is not on traumatic involuntary replacements of managers designed to effect change. Such cases obviously involve effects outside our model. However, organizational theorists have emphasized the disruptive effects even of routine managerial succession (e.g., Grusky, 1960). Furthermore, the announcement of voluntary CEO departures without pressure by the board is on average associated with negative abnormal stock returns (Furtado, 1989). This evidence is consistent with memory loss being costly to firms. Two trends suggest that the problems that arise from memory loss may be of great importance: the increasing tendency of large US firms to hire outsider CEOs and directors, and the movement toward elimination of middle-management levels through downsizing.29 The high rates of environmental change and management turnover in high-tech industries imply severe problems of memory loss, and high impulsiveness.

Corporate boards of directors are reservoirs of memory when there is turnover. This provides a rationale for insider presence on the board. Opposed to this memory advantage is the value of disinterested external oversight. Not surprisingly, the board’s memory is

28. Empirical study of these issues requires attention to a possible postselection bias. Small firms that suffer drastic memory loss will often die. There is a tendency to compare large firms only with surviving small competitors, meaning those that were successful ex post. This may contribute to the popular view that small firms track environmental shifts more adeptly than large firms.

29. According to The Economist (April 20, 1996, pp. 51–52) in an article entitled “Fire and Forget?”: “Having spent the 1990s in the throes of restructuring, re-engineering and downsizing, American companies are worrying about corporate amnesia.” The article provides several anecdotal examples of firms that achieved leaner workforces but failed to improve performance, apparently because of loss of the information possessed by key employees. Similarly, according to Forbes ASAP, December 4, 1995, “Companies that went overboard ‘rightsizing’ are now desperately trying to keep experienced hands to steady the ship.” An article in Forbes (February 10, 1997, p. 64) entitled “Ford’s boss-a-year plan” attributes poor performance at Ford Europe to its treatment of the top Europe position as a temporary stepping stone (ten bosses in 18 years) and consequent inertia.
imperfect. The notorious reluctance of directors to remove poorly performing CEOs is consistent with excess inertia. Our analysis implies that in volatile environments, inertia vanishes and impulsiveness may result. Thus, entrepreneurs financed by venture capitalists may be removed more often than would be optimal under full recall, in contrast to the frequent entrenchment of managers of large publicly held firms.

Our theory implies that firms whose managers are routinely more mobile, retire earlier, and are hired from the outside have stronger inertia or impulsiveness than firms that provide lifetime employment, groom managers for promotion, and have late retirement. [Agency approaches offer an alternative path to this prediction (see Sec. 6.) While employee mobility allows firms to adjust labor usage easily, it may cause undesirable memory loss and lead to inertia or impulsiveness in decision making. Factors that might seem to entrench the status quo—low turnover, internal promotion, and late retirement—can, by improving memory, therefore sometimes improve the firm’s ability to adapt.

A more distinct implication of our approach concerns organizational decision styles. Consensus management distributes memory about reasons for decisions (not just the decisions themselves) among more participants. This reduces the memory loss associated with turnover. Similarly, the benefits of functional specialization within a management team must be balanced against the memory loss when a key member departs.

Turnover should impose lower average amnesia costs on large than on small firms, owing both to steadier proportional turnover and scale economies in information management systems. This analysis therefore implies that large firms should adopt policies more conducive to employee turnover. Grusky (1961) finds that turnover rates are indeed higher in larger firms.

5.3 The Value of Memory

The importance of memory loss is evidenced by the resources devoted to information preservation by many means.30 Our approach implies that the extent to which individuals and firms use costly devices for

30. These include files, personal information managers, minutes, memos, secretaries, accounting systems, expert systems, groupware (e.g., email, document management, workflow management, and knowledge sharing), establishment of knowledge management systems and chief-knowledge-officer positions for preservation of organizational memory (see, e.g., Business Week, June 10, 1996, p. 6, and Forbes ASAP, August 24, 1998, p. 92), and deferral of employee retirement. An impressive example is the extensive cataloging activity of Ernst and Young’s 200-person Center for Business Knowledge.
improving memory depends on the value of full recall over recall of actions but not signals. Consider a firm that faces repeated environmental shifts and repeatedly loses memory of past signals. If the probability of a value shift is quite low, then the past history of actions is highly informative about the value state. Thus, the value of full recall of signals in addition to actions is modest. If volatility is high, past history of any sort has little relevance for the future, so again the gain to preserving past signals is low. Thus, the value of full recall is greatest for players who face intermediate environmental volatility. The general trend (cited earlier) among US firms toward hiring outsider CEOs since the 1960s could, in part, be related to reduced value of memory in a more volatile, globalizing business environment.

When a firm does poorly, executives and employees are tempted to leave. Departures carry away memory, which causes the firm to make more mistakes. Thus, memory-loss effects can accelerate the collapse of distressed firms. Consistent with such a “death spiral” of memory loss, in a sample of distressed firms, Hambrick and D’Aveni (1992) found rapid and accelerating divergences in managerial team characteristics (such as team size, average managerial tenure in the firm, and functional expertise mix) between failing firms and matched survivors in the years preceding bankruptcy. These variables are possible memory proxies. The authors state that “The results thus suggest that deterioration of the top management team is a central element of the downward spiral of large corporate failures.” Our approach implies that such problems of amnesia are likely to be especially important for service firms, for which a greater part of organizational memory resides in individuals’ minds rather than written records.

5.4 Applications to Individual Behavior

There is extensive evidence that people have imperfect access to and recall of the reasons for their actions, and thus that a process of inference is required to form an imperfect reconstruction. Corner

31. Consistent with this argument, Moorman and Miner (1997) report evidence from 92 new-product development projects that survey-based proxies for better and more dispersed organizational memory are associated with good short-term financial performance of new products, but that under conditions of high environmental turbulence the positive effect of high memory dispersion disappears.

32. The analysis also suggests that preservation of memory is relevant for optimal term lengths and limits for public officials. Memory preservation provides one rationale for a civil service in which a newly elected executive cannot replace bureaucrats at will. It also provides a reason to reelect incumbents (“Don’t change horses in mid-stream”). Our theory implies that the desirability of incumbency advantage and long tenure for officials is greatest in environments with intermediate volatility.

33. Wilson et al. (1995) discuss this literature; Koriat and Goldsmith (1996) review the large general literature on memory as a reconstructive process.
et al. (1994) describe the parallels between organizational-level and individual-level information storage and retrieval mechanisms. In this context, our theory can help explain (and offer alternative reasons for) such effects as action-induced belief changes [in contrast with the cognitive-dissonance-reduction theory for such phenomena; see Festinger (1957); see also Akerlof and Dickens (1982)], escalation bias, i.e., excessive continuation and extension of old beliefs and policies (Arkes and Blumer, 1985) and foot-in-the-door effects (Gorassini and Olson, 1995), and endowment effects wherein the assignment of ownership of an object to an individual increases his valuation of the object (Kahneman et al. 1990).

Common to these effects is an excessive adherence to past states of mind or behaviors. The endowment effect can operate even if there is no time delay, which casts doubt on a memory-loss explanation. However, we would argue that effective memory loss often occurs immediately after an action. So long as the individual finds it difficult or tiresome to access the reasons for his decision, his future decisions reflect incomplete access to old reasons. In any case, by relating such behaviors to (measurable) proxies for recall and to environmental volatility, our explanation can be distinguished empirically from alternative hypotheses.

6. Other Theories

This section compares our theory with several alternative and complementary theories. Several theories of individual and organizational inertia and conservatism are based on other deviations from perfect rationality; see, e.g., Thaler (1980) and the reviews of Kuran (1988) and Rumelt (1995). Such invalid use of information that the individual has ample opportunity to absorb (as in theories of irrational aversion to change) should be distinguished from a response of reasonable individuals to their own inability to broadcast or absorb information (implicit in a failure of organizational memory).

The effects of limited memory were previously studied by Dow (1991) in a paper on the decision problem of a consumer looking for the lowest price. His focus was on the endogenous choice of what to remember about past prices. In independent work, Mullainathan

34 Furthermore, at an evolutionary level, approach suggests that past problems of imperfect recall have caused natural selection for emotional mechanisms that influence choices between continuation and change. Thus, our approach indirectly can help explain the endowment effect discussed in Kahneman, Knetsch, and Thaler (1991). Arkes and Ayton (1999) develop an evolutionary account of mechanisms for continuation or change.
(2000) also applies memory loss to economic issues. He examines individuals who apply Bayes’s rule to a selectively recalled signal history as if it were the true one. Our approach differs in two main respects. First, we examine inertia vs. impulsiveness. Second, in our model, decision makers apply Bayes’s rule properly, subject to the single cognitive limitation of amnesia. In a recent paper, Sarafidis (2001) examines the strategic timing of information disclosures in order to manipulate the memories of others. In contrast, we examine a single-agent model.

Actual behavior is of course influenced by a combination of factors. Managers are sometimes hired specifically to bring about policy changes, which raises issues outside our model. To control for such effects, organizational tests of our model should focus on more common and routine forms of managerial turnover (e.g., retirements, voluntary departures, or death rather than firing).

In an overview, Rumelt (1995) opposes organizational inertia (persistence) to plasticity, a readiness to respond appropriately to environmental change. Thus, inertia (or lack of plasticity) is an obstacle to optimal behavior. This categorization implicitly excludes the possibility that firms sometimes place undue weight on new information (impulsiveness) and hence adjust behavior too readily.

The organizational-ecology literature generally assumes that firms do not change, and focuses on how populations of firms are modified through processes of creation and selection. This approach contrasts with an adaptationist perspective that emphasizes organizational change in response to the environment (see, e.g., Nelson and Winter 1982). Our theory offers a middle path by predicting a degree of inertia.

Some theories of inertia are based on divergence of interests between different decision-makers (see, e.g., Rasmusen, 1994). For example, a manager may take actions in order to persuade observers that he has high ability. Our analysis is based on information loss rather than influencing the beliefs of others, and therefore does not require that the decision maker be observed by others. Our theory shares with Prendergast and Stole’s the implication that the

35. Hannan and Freeman (1984) base this inertia assumption on the tendency of organizations to institutionalize procedures to maintain reliability. In our approach, only in a stable environment is high stability an optimal response to memory loss.

36. See Kanodia, Bushman, and Dickhaut (1989), Prendergast and Stole (1996), Zwiebel (1995), and Hirshleifer and Thakor (1992). In Zwiebel, a manager sometimes avoids a superior innovative project. In Prendergast and Stole, a manager initially uses information too aggressively, and later adheres too strongly to past decisions. In Hirshleifer and Thakor, managers refrain from adopting new risky projects to avoid the risk of conspicuous early failure.
old should be more conservative. In our setting this is because the old have poorer memory and a longer action history, rather than a reputation-based attachment to positions they have staked out. Duhaime and Grant (1984) find only weak (statistically insignificant) support for their hypothesis that managers of divested units had low managerial “attachment” to (past responsibility for) their divisions.\textsuperscript{37}

Dixit (1992) offers an options theory of inertia based upon the benefit of deferring costly changes until further information arrives. His theory implies that inertia is strongest in a highly volatile environment. In our theory, volatile environmental change opposes inertia and can lead to its opposite, impulsiveness.

As this discussion indicates, our theory differs from alternative theories in its prediction of which variables favor inertia or impulsiveness. Indeed, past research has devoted little attention to the possibility of impulsiveness. The variables suggested by our model include information load, managerial mobility, managerial overlap, and the quality of information storage/retrieval systems (see, e.g., the predicted effects of introduction of groupware in Sec. 5.1). In contrast with several reputational theories, our approach implies that high managerial attachment to a project can lead to impulsiveness, not inertia, depending on the memory and environmental variables mentioned earlier. Furthermore, the analysis offers implications about which firms should invest more in retaining organizational memory (see Sec. 5.3). The contrasting predictions of our theory and the real-options theory for the effects of volatility on inertia vs. impulsiveness were discussed earlier.

7. Summary and Extensions

Casual observation and extensive evidence from psychology suggest that memory loss frequently affects decision outcomes. Our paper examines the implications of memory loss for the continuity of behavior over time. We find that excess inertia and impulsiveness are optimal responses to memory loss. Thus, intervention or legislation designed to oppose inertia or impulsiveness, such as requiring or blocking change, would, in the absence of extra information, reduce decision quality. Behavior that may seem to reflect faulty cognitive

\textsuperscript{37} This discussion does not exhaust the list of significant alternative approaches. In Kuran (1987), individual preference or social pressure causes conformity. This results in periods of stability interrupted by occasional large shifts. Our model focuses on a problem of inference rather than one of social pressure. Rasmusen (1992) provides a statistical model in which a manager should be conservative in undertaking new projects because of measurement error in project quality and the regression phenomenon.
strategies (e.g., escalation bias, action-induced belief changes) can alternatively be understood as reasonable responses to memory constraints. The model can be viewed as an examination of what happens when the traditional economic assumption of full rationality is relaxed only slightly, in order to focus on some fairly direct effects of memory loss.

Memory loss occurs at both the individual and organizational levels. Firms lose information from individual managerial amnesia, from turnover, and from failures in communication and record keeping. How information retention affects organizational policies is of growing importance as modern economies shift to services and information management, and as restructuring causes managerial and employee turnover.

Our model is based on the premise that a “new player” (an individual after memory loss, or a firm’s new manager) can observe/recall previous actions but not the rationale for these actions. We show that memory loss not only leads to poorer decisions, but causes sensible individuals (who adjust optimally for past memory loss) to adjust their proclivity to follow or deviate from past policies. When the environment is stable, and if a player has followed an old policy a long enough time, there is excess inertia: as new information arrives, the player optimally tends to maintain old policies more under amnesia (e.g., a new player) than with full recall (e.g., a continuing initial player). Thus, the model implies that individual habits, organizational routines, and cultural norms that institutionalize inertia should be more extensive and effective in stable than in volatile environments. Under the opposite set of circumstances, i.e., when the value of old information is weak and/or decays over time, there is excess impulsiveness: an amnesiac player optimally shifts policies more often than would a full-recall player.

The analysis identifies several forces that can intensify or dispel the shadow of the past: the age (duration) of the activity or policy; the volatility of the decision environment; the quality of information (both new and old); the information load; the amount of managerial and employee turnover; and the quality of record-keeping and information systems. Some of the resulting empirical implications are diametrically opposed to competing theories (see Sec. 6).

Although the focus of the model is on loss of memory about past signals, sometimes amnesia is so severe that past actions and signals are both lost. This possibility creates a benefit to institutionalizing past decisions through traditions, taboos, rituals, conventions, organizational routines, and cultural norms. It could be argued that such institutions are designed to protect against hostile, unpredictable
environments. However, our approach implies that such traditions will thrive in highly stable environments, where inertia is an optimal response to memory loss. In such environments, the institutionalization of inertia (and consequent loss of flexibility) has relatively low cost.

Explicit analysis of how memory loss affects economic choices promises to be fruitful for a wide range of applications. We close by mentioning four:

1. The determination of brand loyalty of consumers. Our approach suggests that it is in the interests of sellers to exploit imperfect memory and inertia. Obviously, advertisers repeatedly remind people of the brand to encourage later recall. More interestingly, the theory implies that it is useful to remind the consumer that he chose the brand.38

2. Project recommendation within organizations. Loss of information between hierarchical levels can be viewed as a kind of memory loss.

3. The formation of individual habits and social traditions when there are repeated rounds of memory loss. Finally,

4. The feedback from the behavior of many individuals and firms to the business environment. Our finding that environmental stability promotes inertia and instability promotes impulsiveness suggests that self-reinforcing effects can occur. A slight increase in environmental volatility may increase the optimal impulsiveness of individuals’ and firms’ policies. This in turn will tend to make the environment even less stable. Thus, aggregate shifts in the volatility of economic choices such as investment may be disproportionate to observable causes.

Appendix

A.1 Proofs for Section 3

In the model of Section 3, the algebraic details of the proof of the following lemma are routine, and are omitted.

Lemma 1:

(1) Let \( G' \) denote the event \( \phi' = G \). Then \( \Pr(G'|G, L_3) > \Pr(G'|B, L_3) \).

(2) \( \Pr(G|H_1H_2L_3) > \Pr(G|A_1A_2L_3) \).

38 Some airlines have their flight attendants read a scripted statement, ”Thank you for flying [name of airline]. We realize you have a choice of carriers, and we appreciate your choosing [name of airline].”
(3) \( \Pr(G'|G, H_3L_4L_5) > \Pr(G'|B, H_3L_4L_5) \).
(4) \( \Pr(G|H_1H_2H_3L_4L_5) > \Pr(G|A_1A_2H_3L_4L_5) \).

Intuitively, conditioning on the \( G \) rather than \( B \) state increases
the probability that the later state is \( G' \); and conditioning on \( H_1H_2 \) is
more favorable than conditioning on \( A_1A_2 \), which may or may not
have come from \( H_1H_2 \).

**Proof of Proposition 1.** Date-3 decisions: We will show that \( \sigma^N < \sigma^O \), so
that the region of excess impulsiveness of the amnesiac player actually
does exist. We first outline the argument. At the end of period 2, the
difference between \( H \) and \( L \) signals after \( HH \) or \( LL \) is 2 or \(-2\). For
the amnesiac player, the subequivalent state after \( AA \) or \( RR \) is less
than 2 in absolute value, because the second action may have been
the result of a coin flip. By the definition of \( \sigma^N \), the amnesiac player
is virtually indifferent after \( AA \) followed by \( L \) (or \( RR \) followed by \( H \)).
\( HH \) is more favorable than \( AA \), so it follows that the full-recall player
strongly prefers to follow the old signals after \( HH \) (or \( LL \):

\[
\Pr(G'|AAL) = \Pr(G'|AAL, G)\Pr(G|AAL) + \Pr(G'|AAL, B)\Pr(B|AAL),
\]
\[
\Pr(G'|HHL) = \Pr(G'|HHL, G)\Pr(G|HHL) + \Pr(G'|HHL, B)\Pr(B|HHL).
\]

So

\[
\Pr(G'|AAL) = \Pr(G'|L, G)\Pr(G|AAL) + \Pr(G'|L, B)[1 - \Pr(G|AAL)]
= [\Pr(G'|L, G) - \Pr(G'|L, B)]\Pr(G|AAL) + \Pr(G'|L, B),
\]
\[
\Pr(G'|HHL) = \Pr(G'|L, G)\Pr(G|HHL) + \Pr(G'|L, B)[1 - \Pr(G|HHL)]
= [\Pr(G'|L, G) - \Pr(G'|L, B)]\times \Pr(G|HHL) + \Pr(G'|L, B).
\]

By Lemma 1, \( \Pr(G'|AAL) < \Pr(G'|HHL) \). If \( \sigma \approx 0 \), then

\[
\Pr(G|HHL) \approx \Pr(G|HHL)
= \frac{\Pr(HHL|G)\Pr(G)}{\Pr(HHL|G)\Pr(G) + \Pr(HHL|B)\Pr(B)}
= p.
\]

So for \( p > \frac{1}{2} \) and \( \sigma > 0 \) small, the continuing I will always adopt after
\( HHL \). On the other hand, if \( \sigma \approx 1 \), then history is irrelevant, so I will
always reject.
Similarly, consider \( \sigma \) close to 1. Then

\[
\Pr(G'|AAL) \approx \frac{\Pr(L|G')\Pr(G')}{\Pr(L|G')\Pr(G') + \Pr(L|B')\Pr(B')}
\]

\( = 1 - p. \)

So for \( p > \frac{1}{2} \) and \( \sigma \approx 1 \), \( N \) always rejects after \( AAL \).

If \( \sigma \approx 0 \), then the probability that the state is \( G \) given observation of \( AAL \) is

\[
\Pr(G'|AAL) \approx \frac{\Pr(G, AAL)}{\Pr(AAL)}
\]

\( = \frac{1}{2} \left[ \Pr(HHL|G) + \frac{1}{2} \Pr(HLL|G) \right] \)

\( = \frac{1 + p}{3}. \)

So if \( p > \frac{1}{2} \), then \( N \) always accepts after \( AAL \).

To summarize, as \( \sigma \) increases from 0 to 1, both \( \Pr(G'|AAL) \) \( \Pr(G'|AAL) \) and \( \Pr(G'|HHL) \), decrease from greater than \( \frac{1}{2} \) to less than \( \frac{1}{2} \). Since \( \Pr(G'|AAL) < \Pr(G'|HHL) \) for all \( \sigma \in [0,1) \), it follows by continuity of these probabilities in \( \sigma \) that for all \( p > \frac{1}{2} \) there exists a value of \( \sigma \in [0,1) \) such that \( \Pr(G'|AAL) < \frac{1}{2} \) and \( \Pr(G'|HHL) > \frac{1}{2} \). Thus, for some \( \sigma \), \( N \) rejects after \( AAL \) while \( I \) accepts after \( HHL \).

**Date-4 decisions:** After two pretransition adoptions, so long as \( \sigma > 0 \), both \( I \) and \( N \) will always follow identical date-3 and -4 signals. (If \( \sigma = 0 \), then \( I \) who observed \( H_1H_2 \) will flip a coin after \( L_3L_4 \), which contributes to inertia.) After \( H_3L_4 \) or \( L_3H_4 \), \( I \) who observed pretransition \( H_1L_2 \) tosses a coin and may reject. In contrast, so long as \( \sigma < 1 \), \( N \) always adopts, since the pretransition signals could be either \( HH \) or \( HL \). Thus, there is inertia if \( \sigma < 1 \), and first-best behavior if \( \sigma = 1 \).

**Date-5 decisions:** We compare the behavior of \( N \) vs. \( I \) when \( I \) follows like actions for two dates, \( A_1A_2 \) (the results are symmetric for \( R_1R_2 \), \( A_1A_2 \) could arise either from \( H_1H_2 \), or from \( H_1L_2 \) with a coin flip. At date 5, when at least two of the next three signals after the transition date are \( H \) (\( H_3H_4L_5, H_3L_4H_5, H_4L_3H_5 \), and \( L_3H_4H_5 \), both \( N \) and \( I \) adopt. When all three of them are \( L \) (\( L_3L_4L_5 \), both reject. When one of them is \( H \) and the other two are \( L \) (\( H_3L_4L_5, L_3H_4L_5 \), and
As $\sigma$ increases, the relevance of old policies to the date-5 decision weakens, and at some point, at some point, $N$ switches from adoption to rejection. At that point, $I$ who observed $H_1H_2$ in the first two periods still adopts, since he knows $A_1A_2$ came from $H_1H_2$ and puts higher probability on the state being $G'$ than $N$ does.

High $\sigma$: As $\sigma$ increases further, $I$ also rejects, since the weight placed on the first two $H$-signals becomes low. Both $I$ and $N$ behave identically in a highly unstable environment. We will show that there exists $\sigma \in [0, 1)$ such that

$$\Pr(G|A_1A_2H_3L_4L_5) < \frac{1}{2} < \Pr(G'|H_1H_2H_3L_4L_5).$$

(1) (the cases of $L_3H_4L_5$ and $L_3L_4H_5$ are similar). Expanding the conditional probabilities gives

$$\Pr(G'|A_1A_2H_3L_4L_5) = \Pr(G'|A_1A_2H_3L_4L_5, G) \Pr(G|A_1A_2H_3L_4L_5)$$

$$+ \Pr(G'|A_1A_2H_3L_4L_5, B) \Pr(B|A_1A_2H_3L_4L_5),$$

$$\Pr(G'|H_1H_2H_3L_4L_5) = \Pr(G'|H_1H_2H_3L_4L_5, G) \Pr(G|H_1H_2H_3L_4L_5)$$

$$+ \Pr(G'|H_1H_2H_3L_4L_5, B) \Pr(B|H_1H_2H_3L_4L_5).$$

So

$$\Pr(G'|A_1A_2H_3L_4L_5) = \Pr(G'|H_3L_4L_5, G) \Pr(G|A_1A_2H_3L_4L_5)$$

$$+ \Pr(G'|H_3L_4L_5, B)[1 - \Pr(G|A_1A_2H_3L_4L_5)]$$

$$= [\Pr(G'|H_3L_4L_5, G) - \Pr(G'|H_3L_4L_5, B)]$$

$$\times \Pr(G|A_1A_2H_3L_4L_5) + \Pr(G'|H_3L_4L_5, B),$$

$$\Pr(G'|H_1H_2H_3L_4L_5) = \Pr(G'|H_3L_4L_5, G) \Pr(G|H_1H_2H_3L_4L_5)$$

$$+ \Pr(G'|H_3L_4L_5, B)[1 - \Pr(G|H_1H_2H_3L_4L_5)]$$

$$= [\Pr(G'|H_3L_4L_5, G) - \Pr(G'|H_3L_4L_5, B)]$$

$$\times \Pr(G|H_1H_2H_3L_4L_5) + \Pr(G'|H_3L_4L_5, B).$$
Consider now some $\sigma$ arbitrarily close to zero. Then
\[
\Pr(G|H_1H_2H_3L_4L_5) \\
\approx \frac{\Pr(G|H_1H_2H_3L_4L_5)}{\Pr(H_1H_2H_3L_4L_5|G) \Pr(G) + \Pr(H_1H_2H_3L_4|B) \Pr(B)} \\
= p.
\]
So for $p > \frac{1}{2}$ and $\sigma > 0$ small, the continuing I will always adopt after $H_1H_2H_3L_4L_5$. N also adopts after $A_1A_2H_3L_4L_5$ in this case, since
\[
\Pr(G|A_1A_2H_3L_4L_5) \\
\approx \frac{\Pr(G, A_1A_2H_3L_4L_5)}{\Pr(A_1A_2H_3L_4L_5)} \\
= \frac{1}{2} \left[ \Pr(H_1H_2H_3L_4L_5|G) + \frac{1}{2} \Pr(H_1L_2H_3L_4L_5|G) \right] \\
\times \frac{1}{2} \left[ \Pr(H_1H_2H_3L_4L_5|G) + \frac{1}{2} \Pr(H_1L_2H_3L_4L_5|G) \right] \\
+ \frac{1}{2} \left[ \Pr(H_1H_2H_3L_4L_5|B) + \frac{1}{2} \Pr(H_1L_2H_3L_4L_5|B) \right]^{-1} \\
= \frac{1 + p}{3}.
\]
On the other hand, if $\sigma \approx 1$, then history $H_1H_2$ is irrelevant, so I will always reject.
N also rejects after $A_1A_2H_3L_4L_5$, because
\[
\Pr(G|A_1A_2H_3L_4L_5) \approx \Pr(G'|H_3L_4L_5) \\
= \frac{\Pr(H_3L_4L_5|G') \Pr(G')}{\Pr(H_3L_4L_5|G') \Pr(G') + \Pr(H_3L_4L_5|B') \Pr(B')} \\
= 1 - p.
\]
To summarize, as $\sigma$ increases from 0 to 1, both $\Pr(G'|A_1A_2H_3L_4L_5)$ and $\Pr(G'|H_1H_2H_3L_4L_5)$ decrease from more than $\frac{1}{2}$ to less than $\frac{1}{2}$. Since, by Lemma 1, $\Pr(G'|A_1A_2H_3L_4L_5) < \Pr(G'|H_1H_2H_3L_4L_5)$ for all $\sigma \in [0, 1)$, it follows by continuity of these probabilities in $\sigma$ that for all $p > \frac{1}{2}$ there exists a value of $\sigma \in [0, 1)$ such that $\Pr(G'|A_1A_2H_3L_4L_5) < \frac{1}{2}$ and $\Pr(G'|H_1H_2H_3L_4L_5) > \frac{1}{2}$. Thus, for some $\sigma$, N rejects after $A_1A_2H_3L_4L_5$ while I accepts after $H_1H_2H_3L_4L_5$. \qed
A.2 Section 4: The Multiperiod Model

In the multiperiod model of Section 4, without loss of generality we compute the probability that a continuing I with full recall and a new N with amnesia switch actions within $t$ periods of the date of potential memory loss, after just having adopted for $M$ periods. The analysis is symmetric for past rejections.

A.2.1 Payoff Structure. The net gains each period from adopting or rejecting the current project are summarized in Table V. The net discounted value of the project is unknown, either $v = 1$ (G value state) or $v = -1$ (B value state), equally probable and unknown to the players. These are the payoffs obtained by adopting (undertaking the project). Rejecting the project yields a payoff of 0 in both states. Abstaining from decision generates net payoffs of $0$ or $\epsilon$ (if $s_0 > 0$ small) in states $G$ and $B$, respectively, which are halfway between that from adopting and rejecting (plus $\epsilon$). With $\epsilon$ small, the player abstains if and only if he is (arbitrarily close to) indifferent between adopting and rejecting.

A.2.2 Preliminary Lemmas. The probability that I’s action changes is that of reaching signal state $s_t = 0$. The following two well-known lemmas about Markov processes are used in the subsequent analysis. Let $\theta$ be the probability of an up move. For notational simplicity, we focus on the case of $s_0 > 0$.

**Lemma 2:** Starting at position $s_0 > 0$, the probability of reaching state $s_t = 0$ exactly once at (and not before) time $t$ is

$$a(s_0, t, \theta) = \begin{cases} 0 & \text{if } t < s_0, \text{ or } s_0 + t \text{ is odd} \\ \frac{s_0}{t} \left( \frac{t}{t + s_0} \right) \theta^{(t-s_0)/2} (1-\theta)^{(t+s_0)/2} & \text{otherwise.} \end{cases}$$
**Lemma 3:** Starting at position $s_0 > 0$, the probability of reaching state $s_t = 0$ at least once ("absorption") by time $t$ is

\[
A(s_0, t, \theta) = \begin{cases} 
0 & \text{if } t < s_0 \\
A(s_0, t-1, \theta) & \text{if } t + s_0 \text{ is odd, and } t > s_0 \\
B\left(\frac{t - s_0}{2}, t, \theta\right) + \left(1 - \frac{1}{\theta}\right)^{s_0} B\left(\frac{t - s_0}{2} - 1, t, 1 - \theta\right) & \text{otherwise}, 
\end{cases}
\]

where $B(\cdot)$ is the cumulative binomial distribution, $B(k, t, \theta) = \sum_{i=0}^{k} \binom{t}{i} \theta^i (1 - \theta)^{t-i}$.

The recursion implicit in the middle entry of $A(\cdot)$ must "bottom out," since moving backward from an odd value for $t + s_0$ must lead to an even value.

Given a sequence of $M$ adopt decisions, the probability that the value state is $G$ is

\[
\Pr(G|M, p) = \frac{\Pr(M|G) \Pr(G)}{\Pr(M|G) \Pr(G) + \Pr(M|B) \Pr(B)}.
\]

Lemmas 2 and 3 can be applied to compute $\Pr(M|G)$ and $\Pr(M|B)$. Any qualifying signal sequence must lead at the start to two adopts, and thus must start with two $H$ signals. Then, starting at signal state 2, it must be followed by a signal sequence that does not touch state $s = 0$ (observable abstention) within $M - 2$ periods. Thus,

\[
\begin{align*}
\Pr(M|G, p) &= p^2[1 - A(2, M - 2, p)]. \\
\Pr(M|B, p) &= (1 - p)^2[1 - A(2, M - 2, 1 - p)],
\end{align*}
\]

and $\Pr(G|M, p)$ becomes

\[
\Pr(G|M, p) = \frac{p^2[1 - A(2, M - 2, p)]}{p^2[1 - A(2, M - 2, p)] + (1 - p)^2[1 - A(2, M - 2, 1 - p)]}.
\]
A.2.3 Proofs of Propositions 2 and 3. Let $A(k, t, p)$ (calculated earlier) be the probability that I or N who starts in equivalent state $k$ receives enough low signals to make an action switch (abstain or beyond) at least once within $t$ periods.

I’s optimal decision is based on the difference in the number of H- and L-signals. The probability of an action switch by time $t$ is the probability that the number of L-signals equals the number of H-signals by time $t$. In Markov terms, this happens when the particle reaches the (absorbing) boundary 0. Let

$$c(k; M) ≡ \frac{a(k, M, 1 - p)}{p'[1 - A(2, M - 2, p)]} \quad (k ≤ M). \tag{5}$$

We now verify that $c(k; M, p)$ is the conditional probability of starting in state $k$, given $M$ prior adopts. We first compute the posterior probability of a player being in state $k$ given $M$ adopts if the probability of an H signal is $p$ (i.e., assuming for the moment that the value state is $G$).

The numerator of equation (5) gives the probability of being in state $k$ and having $M$ prior adopts. The denominator normalizes the probabilities. It exploits the fact that $\sum_k a(k, M, 1 - p) = p'[1 - A(2, M - 2, p)]$, because the first two signals must be H, and no absorption may take place within the remaining $M - 2$ periods. It follows that the probability that the initial player, having made exactly $M$ successive adopt decisions, switches action at least once within $t$ periods is

$$r_1(M, t, p) ≡ \Pr(G|M) \sum_{k=1}^{M} c(k; M, p)A(k, t, p)$$

$$+ \Pr(B|M) \sum_{k=1}^{M} c(k; M, 1 - p)A(k, t, 1 - p), \tag{6}$$

where $c(k; M, p)$ is the probability that he is in state $k$ given exactly $M$ prior adopts.

The first term on the right-hand side conditions on the value state being good. If the value state is good, I can be in state $k$, for which the probability is $c(k; M, p)$. If he starts in state $k$, the probability that he will reach state 0 (observable action reversal) is $A(k, t, p)$. The second term conditions on the value state being bad, and consequently, the probability of observing a low signal is not $p$, but $1 - p$.

Since the equivalent state must be an integer, for continuous calculations it is useful to define what we call the subequivalent state.
Definition. Let $\bar{e}$ be the equivalent state. A subequivalent state is defined as any real number on the interval $(\bar{e} - 1, \bar{e}]$ if $\bar{e} > 0$, and on the interval $[\bar{e}, \bar{e} + 1)$ if $\bar{e} < 0$.

Thus, $I$'s equivalent state is equal to the subequivalent state if the latter is an integer, and to the integer next larger in absolute value if there is a fractional part.

The higher in absolute value is a player’s equivalent state, the less likely he is to switch his action at least once within a given time period. At a pretransition date at which he abstains, there are an equal number of $H$ and $L$ signals, which is uninformative. Thus, later decisions can be based solely on the signals or actions observed since the latest pretransition abstention. So the relevant summary of pretransition actions is just the number of immediately preceding consecutive all-adopt (or all-reject) decisions. We therefore need only show, starting with an arbitrary number $M$ of all-adopt decisions, that $N$ has a lower probability of switching than $I$ within an arbitrary $t$ periods after the switch.

Let $I$ be the information set the player has about events prior to the date of possible player change. Let $N$ be the signal sequence received by the player after the transition. By Bayes’s rule, the decision rule for a player is to abstain if

$$\frac{\Pr(N|G) \Pr(I|G)}{\Pr(N|G) \Pr(I|G) + \Pr(N|B) \Pr(I|B)} = \frac{1}{2}, \quad (7)$$

reject if expectation is less than $\frac{1}{2}$, and adopt in the reverse case. Simplifying this expression, the condition for abstaining becomes $\Pr(N|G) \Pr(I|G) = \Pr(N|B) \Pr(I|B)$.

Let $e$ be the subequivalent state. The equivalent state is the excess number of $L$-signals over $H$-signals required to bring a player who has observed $I$ back to indifference (or beyond). In our Markov structure, $\Pr(N|G) = (1 - p)^e$ and $\Pr(N|B) = p^e$, so

$$(1 - p)^e \Pr(I|G) = p^e \Pr(I|B), \quad (8)$$

or

$$e = \frac{\log[\Pr(I|G)/\Pr(I|B)]}{\log[p/(1 - p)]}. \quad (9)$$

Noninteger solutions for $e$ give subequivalent states. Adding one more $H$ signal to $I$ increases $e$ by 1 regardless of the information in $I$. After

39. Proof: the set of possible signal realizations that lead the player in a higher state to switch is a subset of the set of possible signal realizations that lead the player in a lower state to switch.
one additional $H$ signal, $\Pr(\mathcal{I}|G)$ is replaced by $\Pr(\mathcal{I}|G)p; \Pr(\mathcal{I}|B)$, by $\Pr(\mathcal{I}|B)(1 - p)$. Thus, a player can act by computing his state (or state inference) based on the last abstain decision—when $\mathcal{I}$ was publicly known to be in state 0.

Finally, $\Pr(\mathcal{I}|G)$ and $\Pr(\mathcal{I}|B)$ are given by (3). These are the sums of the conditional probabilities of all sequences that lead to uniform adopt decisions, i.e., that start with two $H$-signals, and are followed by sequences that do not lead to absorption within $M - 2$ periods. Therefore, the subequivalent state for $N$ who has observed $M$ successive adopt decisions by $I$ (and no signals of his own) is

$$e[M, p] = 2 + \frac{\log\left[\frac{1 - A(2, M - 2, p)}{1 - A(2, M - 2, 1 - p)}\right]}{\log[p/(1 - p)]}. \quad (10)$$

An action switch only occurs when the boundary of 0 is crossed. So if the subequivalent state $e$ is not an integer, the equivalent state is found by rounding up in absolute value.\(^{40}\)

By standard probability calculus, the probability that a new player, having observed $M$ successive adopt decisions, switches action within $t$ periods after the transition is

$$r_N(M, t, p) \equiv \Pr(G|M) A(\bar{e}[M, p], t, p)$$

$$+ \Pr(B|M) A(\bar{e}[M, p], t, 1 - p). \quad (11)$$

The first term again conditions on the probability that the value state is good. A reversal is observed if $\bar{e}[M, p]$ low signals are observed [which happens with probability $A(\bar{e}, t, p)$]. If the value state is bad, the probability of a low signal is $1 - p$ instead of $p$.

**Proof of Proposition 2.** This follows directly from equations (5) and (10). □

**Proof of Proposition 3.**\(^{41}\) We begin with two lemmas.

**Lemma 4:** The subequivalent state $e$ and the probability $\pi$ that the value state is $G$ are related by

$$e = \frac{\log[\pi/(1 - \pi)]}{\log[p/(1 - p)]}. \quad (12)$$

40. The (sub)equivalent state is not equal to the player’s expectation of the signal state, because of the player’s uncertainty about the values of previous signals. For example, when the expected signal state is greater than +1, a single additional $L$ can sometimes cause the player to revise his inference about pretransition signals in favor of $L$ over $H$ so much that the player switches from adopt to reject.

41. This proof was pointed out to us by Larry Glosten.
Proof. Let \( \pi_e \) be the probability that the value state is \( G \) given the subequivalent-state expression given above. (It is easy to show that there is a one-to-one map between the two.) Taking the odds ratio of \( \pi \) and assuming that \( \Pr(G) = \Pr(B) \), we find that

\[
\frac{\pi_e}{1 - \pi_e} = \frac{\Pr(G|e)}{\Pr(B|e)} = \left( \frac{p}{1 - p} \right) ^ e,
\]

by equation (9) for the subequivalent state, with \( I \) representing \( e \) pretransition adoptions. Solving for \( e \) gives (12).

Let \( C \) denote the event that no reversal ever occurs, let \( C' \) be the complementary event that reversal occurs, and let \( \pi_e \) be the probability that the value state is \( G \) if the player starts at subequivalent state \( e \). Then:

**Lemma 5:** The cumulative asymptotic probability of ever observing a reversal given an arbitrary number of past adoptions is \( \Pr(C'|e) = 2 - 2\pi_e \).

**Proof.** The probability of observing a reversal as \( T \to \infty \) if \( I \) took the wrong action is 1, so the probability of \( C \) is the probability \( \pi_e \) that the action is correct given the (unknown) value state (so that there is a drift away from zero) times the probability of (temporarily) not reaching a state of 0. Consequently, the probability of no reversal is

\[
\Pr(C|e) = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right) ^ e \right],
\]

By (12), substituting for \( e \) and using the identity \( x^a \log y = y^a \log x \) gives

\[
\Pr(C|e) = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right) ^ {\log \left( \frac{\pi_e}{1 - \pi_e} \right)} \right]
= \pi_e \left[ 1 - \left( \frac{\pi_e}{1 - \pi_e} \right) \right] = 2\pi_e - 1.
\]

We now complete the proof of Proposition 3.
Part (i): I’s reversal behavior is the weighted sum of reversal probabilities:

\[
\Pr_I(C|A^M) = \sum_{s=1}^{M} \Pr(C|s) \Pr(s|A^M),
\]

where \(A^M\) denotes a sequence of \(M\) adots. By Lemma 5, this can be simplified to

\[
\sum_{s=1}^{M} (2\pi_i - 1) \Pr(s|A^M) = \sum_{s=1}^{M} [2\Pr(G|s) \Pr(s|A^M) - \Pr(s|A^M)]
\]

\[
= 2\hat{\pi}_M - 1,
\]

where \(\hat{\pi}_M\) is the probability that the value state is \(G\) given \(M\) pretransition adots.

\(N\): The equivalent state \(\bar{e}\) is the sum of the subequivalent state \(e\) and a nonnegative fraction \((0 \leq f < 1)\). By Lemma 5,

\[
\Pr_N(C|\bar{e}) = \pi_e \left[1 - \left(\frac{1-p}{p}\right)^{e/f}\right] = \pi_e \left[1 - \left(\frac{1-p}{p}\right)^{e+f}\right]
\]

\[
= \pi_e \left[1 - \left(\frac{1-p}{p}\right)^{\pi_e / \pi_e} \left(\frac{1-p}{p}\right)^{f}\right]
\]

\[
= \left(2\pi_e - 1\right) + (1 - \pi_e) \left[1 - \left(\frac{1-p}{p}\right)^{f}\right],
\]

which is greater than \(2\pi_e - 1\) when \(f > 0\).

Part (ii): If there are \(M'\) periods prior to the memory transition, there could any number from 0 through \(M'\) of consecutive immediate pretransition adopt decisions. This number is all that is relevant in determining future actions after memory loss. Conditional on \(M'\), in some cases, both \(N\) and \(I\) will switch with equal probability; in other cases, \(N\) will switch after \(I\). Therefore the unconditional probability that \(I\) switches at least once is greater than the probability that \(N\) switches at least once. \(\square\)

References

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