Facilitation of Competing Bids and the Price of a Takeover Target

David Hirshleifer
I. P. L. Png
University of California, Los Angeles

We present a model of corporate acquisitions in which initially uninformed bidders must incur costs to learn their (independent) valuations of a potential takeover target. The first bidder makes either a preemptive bid that will deter the second bidder from investigating or a lower bid that will induce the second bidder to investigate and possibly compete. We show that the expected price of the target may be higher when the first bidder makes a deterring bid than when there is competitive bidding. Hence, by weakening the first bidder's incentive to choose a preemptive bid, regulatory and management policies to assist competing bidders may reduce both the expected takeover price and social welfare.

Two means by which control of a publicly held firm may be acquired are tender offers and mergers. Until the passage of the (federal) Williams Act in 1968 and the state legislation that followed in its wake, cash tender offers were virtually unregulated in the United States. The Williams Act requires tender offers to remain open for a minimum period of 20 business days. The act


2 Securities and Exchange Commission Rule 14e-1.

3 Securities and Exchange Commission Rule 14d-6 and Schedule 14D-1.

© 1990 The Review of Financial Studies 0893-9454/90/$1.50
further requires a bidder making a tender offer to disclose, *inter alia*, the bidder's sources of funds and the purpose of the tender offer, including planned reorganization, sale of assets, or change of management. By providing competing bidders with more time and access to information, the act facilitates competitive bidding.

The Williams Act sparked an intense debate among legal commentators on its delay and disclosure provisions, and on the appropriate conduct of the management of a target of a tender offer. Easterbrook and Fischel (1981, 1982) argued that by reducing the gains from takeovers to first bidders, facilitation of competition in bidding would reduce first bidders' investigation of potential targets to a socially inefficient rate. They call for the repeal of the Williams Act, and for the management of targets to be bound by a "rule of managerial passivity" that would prohibit management of targets from soliciting competing bids.4

Bebchuk (1982a, 1982b, 1986) and Gilson (1981, 1982) contended that competitive bidding for a takeover target benefits society by permitting acquisition of the target by a second bidder who can increase the value of the target more than the first. They advocated, instead, a "rule of auctioneering," that "(1) provides, by regulating offerors, time for making competing bids; and (2) allows incumbent management to solicit such bids by providing information about the target to potential buyers" [Bebchuk (1982a, p. 1030)].

Although procedures for mergers are governed by corporation law of the individual states, bids to merge are indirectly subject to delay and disclosure provisions of federal securities law in the following way. A number of states require that a merger be approved by majority vote of the outstanding shares of the target.5 Under the (federal) Securities Exchange Act of 1934, however, most publicly held corporations seeking proxies for a shareholder vote must prepare a proxy statement,6 which must be filed with the Securities and Exchange Commission at least 10 days before distribution.7 By setting time limits and requiring disclosure of information, federal securities law facilitates the entry of competitors to a merger.9

A number of preceding authors have formally analyzed the role of the management of a takeover target in maximizing the price of the target. Giammarino and Heinke (1986) study the decision of management whether

---

4 See also Schwartz (1986).
5 See, for instance, California General Corporations Law, §1201(a); Delaware General Corporation Law, §251; and New York Business Corporation Law, §903. There are exceptions, for example, under California law, a "short-form merger" is available if the acquirer owns 90 percent of the shares of the target and plans to merge the target into the parent corporation.
6 §14A of 1934 Act.
7 Securities and Exchange Commission Rule 14a-6.
8 Even when a merger does not require a vote, the 1934 Act requires that a publicly held corporation issue an information statement, providing substantially the same information as the proxy statement (§14C of 1934 Act).
9 Management led buyouts can be affected by either tender offer or merger and hence are subject to federal and state regulation.
to accept a first bid when rejection allows a second bidder to enter. In Tiemann's (1988) model, one potential acquirer has better information about the possible improvement of the value of the target than the other. Tiemann shows that the management of the target can raise the expected price by disclosing information about itself only to the informationally disadvantaged bidder. Shleifer and Vishny (1986) demonstrate that the target management may pay greenmail to a low valuation acquirer in order to persuade another potential acquirer with higher valuation to investigate.

In contrast, we and others focus on the effects of government regulation on competition in bidding. In Fishman (1988), two potential acquirers can raise the value of the target by improvements of independent amount. Each acquirer must incur a cost to learn the size of his improvement. Having investigated, the first bidder must choose between a bid that leads a second bidder to investigate and a higher bid that deters investigation. If the second bidder should investigate and bid, he triggers an auction. In expectation, the price under an auction is higher than the price that would have been paid if the first bidder had deterred investigation. The lower the cost of investigation to the second bidder, the more inclined is the first bidder to choose a bid that leads to investigation. Hence, Fishman concluded that the expected price of the target will be maximized if the cost of investigation to the second bidder is minimized.

In an example drawn from Bhattacharya and generalized by Spatt (1989), each of several potential bidders must incur an entrance cost to participate in an auction of a target. This entrance cost would depend on delay and disclosure provisions of the law. Bidders may have identical or independent valuations of the target. In the symmetric mixed-strategy equilibrium, each participates with a probability that makes his expected revenue equal to the expected entrance cost, and the expected price of the target is decreasing in the number of potential bidders as long as there are at least two bidders. Hence exogenous elimination of potential bidders benefits the target. However, consistent with Fishman, they show that the expected price is decreasing in the entrance cost.10

All parties to the legal debate, as well as Tiemann (1988), Shleifer and Vishny (1986), Fishman (1988), and Spatt (1989) on Bhattacharya, assumed that competitive bidding in takeovers is equivalent to an English auction in which each bidder may costlessly revise his bid as the price rises. In reality, the cost of making and revising takeover bids is far from trivial. It includes fees to counsel, investment bankers, and other outside advisors, the opportunity cost of executive time, and the cost of obtaining financing for the bid. Bidding costs are also affected by federal and state laws governing takeovers. For instance, federal law specifies that shareholders who tender to one bidder must be allowed to withdraw their shares as long as the offer remains open.11 This allows shareholders to tender to a competing

10 For a detailed review of the literature on takeover bidding, see Spatt (1989).
bidders. The shorter the time available for shareholders to withdraw shares tendered to an earlier bidder, the higher the cost of making a counterbid.

We present the economic setting of the model in the following section. Our framework is similar to that of Fishman, differing in only one crucial respect—we assume that bidding is costly. In this setting, we show that the expected price of the target may be lower under competitive bidding than if the first bidder had bid to deter the potential competitor from investigating. In Section 2, each bidder may make at most one bid, so that if the second bidder competes, he needs only match the first bid in order to acquire the target. Section 3 provides an example in which each bidder may bid any number of times but must incur a cost each time he does so. If the second bidder should compete, the price of the target will not be bid up to the minimum of the two bidders’ valuations net of bidding cost, because once a bidder believes that his value is exceeded by that of his competitor, he prefers to quit rather than incur any further cost of bidding.

Since the price of the target need not be higher with competitive bidding than with a single deterring bid, lowering the cost of investigation may reduce the expected takeover price of the target. We show that by encouraging the first bidder to bid high to deter investigation, an increase in the second bidder’s cost of investigation from its minimum level may raise both the expected price of the target and social welfare. Section 4 presents the implications of the analysis for government regulation of takeover bidding and the conduct of the management of a target. We conclude with empirical implications and suggestions for future work.

1. Economic Setting

There are two potential acquirers of some target. The value of the acquisition to the first bidder (FB) is \( v_1 \) while the value to the second bidder (SB) is \( v_2 \). These values depend on such idiosyncratic factors as the quality of the bidder’s management, and the match between the bidder’s existing operations and assets with those of the target. To represent an extreme form of bidder-specific valuation effects, we assume that \( v_1 \) and \( v_2 \) have independent and identical discrete distributions with

\[
\Pr[v_1 = v] = \Pr[v_2 = v] = \lambda_v. \tag{1}
\]

Let \( v \) and \( \theta \) be the smallest and largest possible values of \( v_1 \), respectively, and assume that the expected value of the target to either bidder is negative, \(^13\)

\[
E(v_1) = E(v_2) = \sum v \lambda_v < 0
\]

\(^12\) In reality, there is likely to be an element of common value to the acquisition, and the management of the target may have private information about this common-value element. A bidder may wish to make a bid that is contingent on the value of the acquisition, say by offering payment in its securities rather than cash, to elicit the target’s information. See Hansen (1987) and Fishman (1989).

\(^13\) Hence it will not pay either bidder to bid without investigation.
The objective of the shareholders and the management of the target is to maximize the expected price of the shares. Accordingly, the shareholders or management will sell the target if they receive a bid above the value of the cash flows of the target absent a takeover, which, for simplicity, we normalize to zero.\footnote{We abstract from the free-rider problem in tendering described by Grossman and Hart (1980). However, as they note, this problem may be mitigated by the power of a successful bidder to dilute the minority holding.} If there are competing bids, we assume that the target will accept the higher bid. If the two bids are of equal amount, we break the tie by assuming that the target will be sold to the last bidder.

FB's decision to investigate occurs before the analysis proper begins. The events under consideration take the following sequence. Initially, at date 1, FB knows $v_1$, while SB does not yet know $v_2$. FB makes a bid, $b_1 \in [0, v_1]$, and thereby alerts the potential competing bidder to the possibility of a profitable takeover. At date 2, SB must decide whether to investigate and learn $v_2$ at a cost of $c \geq 0$. Since, by Equation (1), $E(v_2) < 0$, SB will not bid unless he first investigates. Hence, if SB does not investigate, FB will acquire the target. If SB investigates, he must decide whether to bid.

SB's bid depends on the ensuing structure of the game: in Section 2, we assume that each bidder may bid at most once, while in Section 3, each may bid any number of times at a fixed cost per bid. Throughout the analysis, we adopt the convention that SB will investigate only if his expected return from investigation is strictly positive.\footnote{This ensures the existence of an optimal bid in the class of first bids by FB that deter investigation.}

As in the previous literature, we assume that bids cannot be withdrawn. Let the value of $c$ and the prior distribution of $v_1$ and $v_2$ be common knowledge, and let the realized values of $v_1$ and $v_2$ be private information of the respective bidders if they should investigate. All parties are risk-neutral.

2. Facilitation of Competing Bids

In this section, we assume that each bidder may make at most one bid and consider the effect on the expected price of facilitating competition by reducing the second bidder's cost of investigation. We solve this game recursively. Beginning with the second bidder's (SB's) bid, we then consider SB's decision whether to investigate, and finally turn to the initial bid by the first bidder (FB). FB must consider that his bid will set a floor price for the target and, in this way, will affect SB's decision whether to investigate, and if SB should investigate, SB's decision whether to bid for the target.

Since FB can bid only once, SB will acquire the target by matching FB's bid, $b_1$. If SB finds $v_2 \leq b_1$, he cannot gain from bidding. We assume that SB will match the first bid to acquire the target and realize a gain of $v_2 -$
only if \( v_2 > b_1 \). As \( v_1 \) and \( v_2 \) are independent, the first bid conveys no information to SB about \( v_2 \). Hence, given a first bid of \( b_1 \), the expected return from investigation to SB,

\[
R_2(b_1) \overset{\text{def}}{=} -c + \mathbb{E}_2[v_2 \leq b_1] + \mathbb{E}_2(v_2 - b_1 \mid v_2 > b_1) \cdot \mathbb{P}[v_2 > b_1]
\]

where \( \mathbb{E}_2(\cdot) \) denotes the expectation with respect to the distribution of \( v_2 \). By our convention, SB will investigate only if \( R_2(b_1) > 0 \)

To find FB's equilibrium bidding strategy, we distinguish two classes of bids—those that deter SB from investigating and those that accommodate investigation. Suppose that there exists a bid \( b_1 \in [0, v_1] \) such that SB does not investigate. With such a bid, FB will acquire the target with certainty and realize a gain of \( v_1 - b_1 \). Hence, the optimal deterring bid for FB is

\[
b_1^*(c) \overset{\text{def}}{=} \arg\max \left\{ v_1 - b_1 \mid R_2(b_1) \leq 0, b_1 \geq 0 \right\} = \min \left\{ b_1 \mid R_2(b_1) \leq 0, b_1 \geq 0 \right\}
\]

which is the minimum nonnegative value of \( b_1 \) sufficient to deter investigation.\(^{17}\) If such a bid exists, it does not vary with \( v_1 \), but is weakly decreasing in \( c \).

Suppose that there exists some bid \( b_1 \geq 0 \) that leads SB to investigate. With this bid, FB's profit depends on whether SB finds \( v_2 > b_1 \). If \( v_2 > b_1 \), SB will acquire the target, and FB will gain nothing. If \( v_2 \leq b_1 \), FB will acquire the target for a profit of \( v_1 - b_1 \). Hence, the expected return to FB is \( (v_1 - b_1) \cdot \mathbb{P}[v_2 \leq b_1] \), since \( v_1 \) and \( v_2 \) are independent. Thus, the optimal accommodating bid is

\[
b_1^*(v_1, c) \overset{\text{def}}{=} \arg\max \left\{ (v_1 - b_1) \cdot \mathbb{P}[v_2 \leq b_1] \mid R_2(b_1) > 0, b_1 \geq 0 \right\}
\]

Consider those pairs \((v_1, c)\) for which there exist both \( b_1^*(c) \), the optimal deterring bid, and \( b_1^*(v_1, c) \), the optimal accommodating bid. By Equation (2), SB's expected return from investigation, \( R_2(b_1) \), is weakly decreasing in \( b_1 \). Hence, any deterring bid must be higher than any accommodating one, so in particular

\(^{16}\) Similar results may be proved with the alternative assumptions that if \( v_1 = b_1 \), SB (1) does bid, or (2) randomizes between bidding and not.

\(^{17}\) This bid may be viewed as the lowest exercise price such that a risk-neutral call option premium on \( v_2 \) does not exceed \( c \) (aside from the constraint \( b_1 \geq 0 \)).

\(^{18}\) It will be shown in Lemma 1 that the optimal bid \( b_1^*(v_1, c) \) exists for any \( v_1 \) in which FB chooses to accommodate investigation.
as illustrated in Figure 1.

To summarize, FB has two alternatives. First, he can bid high and acquire the target with certainty. Alternatively, he can bid low knowing that SB will investigate, but hoping that SB will draw a low $v_i$. If both of these alternatives are profitable, he will choose the bid that yields the larger expected profit. Assuming that FB will deter if these alternatives yield equal profit,\(^\text{19}\) he will bid high if

$$ v_i - b^\pi(v_i, c) \geq [v_i - b^\pi(v_i, c)] \Pr[v_i \leq b^\pi(v_i, c)] $$

If all profitable bids deter investigation, FB will choose the minimal such bid, and similarly, if all profitable bids lead SB to investigate, FB will choose the optimal accommodating bid.

Let $A(c)$ be the set of values $v_i$ such that FB makes accommodating bids, and $D(c)$ be the set of values such that FB makes a deterring bid. These sets are mutually exclusive and exhaustive. One of the two sets may be empty; for instance, if SB's cost of investigation exceeds the largest possible realization of $v_i$, that is, $c > b_i$, then SB will never investigate, and $A(c)$ is empty.

To characterize FB's strategy, it is helpful to define $\hat{b}_i(v_i) \overset{\text{def}}{=} b^\pi(v_i, 0)$, the optimal accommodating bid when the cost of investigation is zero. Now if FB accommodates, SB investigates and bids if $v_i > b_i$. Hence, given that the bid is accommodating, FB's profit will not be affected by SB's cost of investigation. Accordingly, for each $v_i$, FB's optimal accommodating bid will be identical for all values of $c$. We then have

**Lemma 1.** The optimal accommodating bid by the first bidder is independent of $c$,

$$ b^\pi(v_i, c) = \hat{b}_i(v_i) $$

for all $v_i \in A(c)$.

To show that FB will deter for high realizations of $v_i$ and accommodate for lower realizations, consider $v_i \in A(c)$: by Equation (6) and Lemma 1,

$$ v_i - b^\pi(c) < [v_i - \hat{b}_i(v_i)] \Pr[v_i \leq \hat{b}_i(v_i)] $$

\(^{19}\) The same results may be proved with the alternative assumptions that when expected profits are equal, FB accommodates, or randomizes between accommodating and deterring.
For any \( v'_i < v_i \), FB’s profit from deterring will be lower than in state \( v_i \) by an amount \( v_i - v'_i \). FB’s profit from accommodation, however, will be lower by less than \( v_i - v'_i \), because FB can do no worse than hold his accommodating bid unchanged at \( \hat{b}_i(v_i) \). Thus, FB with valuation \( v'_i \) will also accommodate investigation. This proves that FB either deters in all states \( v_i \), accommodates in all states \( v_i \), or there exists a critical realization \( v'_i(c) \), at or above which FB deters and below which FB accommodates.

Consider FB with valuation \( v'_i(c) \): by Equation (3), if \( c \) were lowered, the deterring bid would weakly increase, thus weakly reducing FB’s profit from deterring. By Lemma 1, however, the reduction in \( c \) will not affect FB’s profit from accommodating investigation. Therefore, a reduction in \( c \) will weakly reduce the set of \( v_i \) in which FB will deter investigation. We summarize the last two findings in

**Lemma 2.** Either the first bidder deters investigation in all states \( v_i \geq 0 \), accommodates in all states \( v_i \geq 0 \), or there exists some cut-off, \( v'_i(c) \), such that \( A(c) = \{ v_i \mid 0 \leq v_i < v'_i(c) \} \) and \( D(c) = \{ v_i \mid v_i \geq v'_i(c) \} \). Furthermore, \( D(c) \) is weakly increasing in \( c \).

Since each bidder may bid at most once, SB need only match the first bid to acquire the target, so that the price at which the target is acquired will be FB’s bid—regardless of which bidder acquires the target. Therefore, the expected takeover price of the target is

\[
P(c) \overset{\text{def}}{=} \sum_{v_i \in A(c)} \hat{b}_i(v_i)\lambda_{v_i} + \sum_{v_i \in D(c)} b'_i(c)\lambda_{v_i}
\]

SB’s cost of investigation, \( c \), affects the expected takeover price in two ways. By Equation (3), a lower cost will weakly increase the deterring bid \( b'_i(c) \). By Lemma 2, however, a lower cost will lead FB to deter investigation in fewer states \( v_i \).

For sufficiently large \( c \), \( E_{SB}[\max(v_2, 0)] \leq c \), so any nonnegative bid will deter investigation, and hence \( p(c) = 0 \). If \( c = 0 \), it is impossible to deter investigation, hence \( A(0) = \{ v_i \mid v_i \geq 0 \} \) and \( D(0) \) is empty. By Lemma 2, the set \( D(c) \) is weakly increasing in \( c \). Let \( \zeta \) denote the smallest cost of investigation that will induce FB with the highest valuation \( \hat{v} \) to make a deterring bid. Then for all \( c < \zeta \), FB with any valuation will accommodate.

Recall that the price received by the target is the first bid, and consider a small reduction in \( c \) below \( \zeta \). By Lemma 1, the change in \( c \) does not affect the accommodating bid of FB with \( v_i < \hat{v} \). So the only effect on the price of the target arises from the shift of FB with \( v_i = \hat{v} \) from deterring to accommodating investigation. Since FB’s deterring bid strictly exceeds his accommodating bid [by Equation (5)], the reduction in the cost of investigation lowers the expected price of the target. This intuition is proved in the following result:
Proposition. The expected takeover price is maximized at a positive value of the second bidder's cost of investigation.

Proof. See Appendix A.

In this setting, social welfare may be measured as the expectation of the sum of the price received by the target, and the profits of FB and SB. Essentially, social welfare is the expectation of the valuation of the bidder who acquires the target less SB's cost of investigation (if incurred). In Appendix A, we show that in the following example, an increase in \( c \) from zero raises social welfare.

Let \( \lambda_2 = 0.39, \lambda_0 = 0.05, \lambda_1 = 0.38, \) and \( \lambda_2 = 0.18 \). With \( c = 0 \), FB accommodates in all states \( v_0 \geq 0 \). In particular, when FB with valuation \( \bar{v} \) bids \( \bar{b}_1(\bar{v}) \), SB investigates and, if \( v_2 \in (\bar{b}_1(\bar{v}), \bar{v}) \), acquires the target even though the target is socially more valuable in the hands of FB. A sufficient increase in \( c \) will induce FB with valuation \( \bar{v} \) to switch to a deterring bid, and hence eliminate this source of inefficiency. The rise in \( c \), however, means that SB will incur larger deadweight costs of investigation whenever FB has \( v_1 \in A(c) \) and accommodates. On balance, in our example, an increase in \( c \) from 0 to 0.1584 raises expected social welfare.

In sharp contrast to the proposition, Fishman (1988) showed that if each bidder can revise his bid costlessly as in an auction, then the expected takeover price will be maximized when SB's cost of investigation is minimized. Clearly, both Fishman's assumption of costless counterbidding and our assumption that each bidder may bid at most once are polar cases. Accordingly, in the next section, we analyze the more realistic intermediate situation.

3. Costly Counterbidding with Voluntary Termination: An Example

In this section, we provide an example to confirm that the main results of the previous section are consistent with a setting in which each bidder may bid any number of times at a fixed cost per bid, and bidding ends only when one bidder quits voluntarily. We modify the assumptions of the preceding section in the following respects. First, each bidder must incur a fixed cost of \( \gamma \) to place a bid or to revise his bid. If the second bidder (SB) bids, the first bidder (FB) may counterbid, then SB may counterbid, and bidding continues in turn until one bidder quits. Whereas each bidder must incur the cost \( \gamma \) for each bid or counterbid, SB need bear the cost of investigation, \( c \), only once to learn \( v_0 \). Second, we assume that the marginal distribution of valuations, \( \lambda_{eq} > 0 \) only for \( v_1 \in (0, 1, 2) \), with \( \lambda_0 = \lambda_1 + \lambda_2 = 1 \).

In an English auction, the auctioneer calls out an increasing price and the price rises until it reaches the valuation of the bidder with the lower

---

20 We show below that when bidding is costly, our earlier assumption that \( E(v_0) < 0 \) is not necessary to ensure that SB bids only after investigating.
valuation. To bias our results toward those of analyses that model takeover bidding as English auctions, we adopt the convention that if, at any stage, a bidder can raise the price of the target \textit{without sacrificing expected profit}, then he will "close the gap" by bidding to raise the price.

3.1 Equilibrium

In order to explore the effect of changes in $c$ on the expected takeover price and social welfare, we specify ranges for the parameters such that, in equilibrium, FB both accommodates and deters with positive probability. We use the concept of perfect bayesian equilibrium.\footnote{This essentially is Nash equilibrium with the further requirements that beliefs be revised according to Bayes' rule wherever possible and that strategies continue to be a Nash equilibrium at all later stages of the game. See, for instance, Rasmusen (1989).} In this setting, however, there potentially exist multiple distinct classes of equilibria. Cho and Kreps (1987) developed the intuitive criterion for removing implausible equilibria in the context of a game of asymmetric information in which each player had only one move. In the present setting, however, following a defection by FB, each bidder may bid more than once. In Appendix B, we extend the intuitive criterion in a simple way to allow examination of such defections.\footnote{See appendix 2 of Hirschleifer and Png (1989) for a description of how our extension of the intuitive criterion may be used to narrow the set of equilibria. We show there that the equilibrium described below satisfies this criterion.}

The class of equilibria of interest is characterized as follows:

I. First bidder:
   (i) $v_1 = 0$: Never bid.
   (ii) $v_1 = 1$: Bid 0 with probability $1 - \mu$ or bid $b^*_1 > 0$ with probability $\mu$.
   (iii) After a bid of zero, if SB bids, infer $v_2 = 1$ or 2, and quit.
   (iv) $v_1 = 2$: Bid $b^*_2$.

II. Second bidder:
   (v) If first bid is at least $b^*_2$, infer $v_1 = 1$ or 2; do not investigate.
   (vi) If first bid is below $b^*_2$, infer $v_1 = 1$, and investigate. In any later round, if FB makes a bid below $b^*_2$, infer $v_1 = 1$, while if FB makes a bid at or above $b^*_2$, infer $v_1 = 2$.
   (vii) $v_2 = 0$: Quit.
   (viii) $v_2 = 1$ or 2: Match the first bid.

In the following, we describe a range of parameters under which the stated strategies and beliefs are an equilibrium. The reader not interested in the technical details may skip directly to the next subsection, where we show that the expected price of the target and social welfare may increase with the cost of investigation.

(i) and (vii) Since bidding is costly, a bidder with valuation 0 will never make a bid.
(ii) We next require that FB with \( v_1 = 1 \) be indifferent between bids of 0 and \( b_0^p \). If he bids zero, he incurs a cost \( \gamma \), and SB investigates. With probability \( \lambda_0 \), SB draws \( v_2 = 0 \) and quits, so that FB gets \( 1 - \gamma \). With probability \( \lambda_1 + \lambda_2 \), SB matches, from which FB infers that \( v_2 = 1 \) or 2. If FB were then to counterbid, he would at best break even, but with positive probability would lose an additional \( \gamma \), so he prefers instead to quit with a loss of \( \gamma \). Hence the expected profit to FB with \( v_1 = 1 \) from a bid of zero is

\[
\lambda_0 (1 - \gamma) + (\lambda_1 + \lambda_2)(-\gamma) = \lambda_0 - \gamma
\]

If FB with \( v_1 = 1 \) bids \( b_1^p \), he incurs a cost of \( \gamma \), and in equilibrium SB does not investigate. Hence, FB’s profit from bidding \( b_1^p \) is \( 1 - b_1^p - \gamma \). In a mixed-strategy equilibrium, FB must receive equal expected profit from bidding 0 and \( b_1^p \), so

\[
b_1^p = 1 - \lambda_0 \tag{8}
\]

and this profit must exceed that from not bidding, \( 1 - b_0^p - \gamma > 0 \), or

\[
\lambda_0 > \gamma \tag{9}
\]

(iii) In equilibrium, if FB bids zero, SB will investigate and bid only if \( v_2 = 1 \) or 2, hence FB should then quit.

(iv) We require that FB with \( v_1 = 2 \) prefer the deterring bid \( b_1^p = 1 - \lambda_0 \) to bidding zero. For analytic simplicity, we impose the slightly stronger constraint that even if, contrary to the proposed equilibrium, SB infers from a first bid of 0 some probability that \( v_1 = 2 \), FB would still prefer to deter. Suppose that FB with \( v_1 = 2 \) were to bid 0. Since SB with \( v_2 = 1 \) believes that he faces \( v_1 = 1 \) or 2, he cannot make a positive profit, and has a chance of losing money, hence he quits. SB with \( v_2 = 2 \) will bid, at which point FB will bid \( 2 - \gamma \) to close the gap and acquire the target. Thus the expected profit to FB with \( v_1 = 2 \) from bidding 0 is

\[
(\lambda_0 + \lambda_1)(2 - \gamma) + \lambda_2[2\gamma + 2 - (2 - \gamma)] = 2(\lambda_0 + \lambda_1) - \gamma
\]

Alternatively, by deterring, FB can gain \( 2 - b_1^p - \gamma = 1 + \lambda_0 - \gamma > 0 \), by Equations (8) and (9). It follows that FB will bid to deter if

\[
\lambda_1 < \lambda_2 \tag{10}
\]

(v) and (vi) We require that SB bids only after investigating. If SB bids without investigating, he could do no worse than if he acquired the target with a bid of 0. Then his expected profit would be his expected valuation less the cost of bidding. We therefore assume \( \lambda_1 + 2\lambda_2 - \gamma < 0 \), or

\[
1 - \lambda_0 + \lambda_2 < \gamma \tag{11}
\]

It is convenient to derive the condition for (vi) before the conditions for (v). For (vi), we require that if \( b_1 < b_0^p \), then SB investigates. Since \( b_1 < b_0^p \) leads SB to infer that \( v_1 = 1 \), if SB investigates, he will bid if \( v_2 = 1 \) or 2. If SB should bid, FB with \( v_1 = 1 \) will infer that \( v_2 = 1 \) or 2, and will
quit, hence SB with $v_2 = 1$ or 2 can acquire the target simply by matching the first bid.\textsuperscript{23} So SB's expected return from investigation,

$$R_2(b_1) = -c + \lambda_2(-\gamma + 1 - b_1) + \lambda_2(-\gamma + 2 - b_1)$$

$$= -c + (\lambda_1 + \lambda_2)(-\gamma + 1 - b_1) + \lambda_2$$

But $b_1 < b_1^p$, so by Equations (8) and (9), $b_1 < 1 - \lambda_0 \leq 1 - \gamma$. Hence $R_2(b_1) > -c + \lambda_2$, and SB will investigate if

$$\lambda_2 > c$$

(12)

Turning to (v), we seek a condition that ensures that a bid of $b_1^p$ or higher deters investigation. To simplify the analysis, we will impose conditions that imply, in addition, that conditional on a first bid of $b_1^p$ or higher, if SB (contrary to the proposed equilibrium) were to investigate, he would find it profitable to bid only if $v_2 = 2$. Suppose SB with $v_2 = 1$ bids $b_1 \geq b_1$. In the proposed equilibrium, FB will believe $v_2 = 2$, so if $v_1 = 1$, FB will quit, and SB will realize $1 - \gamma - b_2$, while if $v_1 = 2$, FB will bid $2 - \gamma$ to close the gap and acquire the target, and SB will lose $\gamma$. By Bayes' rule, the likelihood of $v_1 = 1$ conditional on a first bid $b_1^p$ is $\mu \lambda_1 / (\mu \lambda_1 + \lambda_2)$. So SB with $v_2 = 1$ will quit if

$$-\gamma + \frac{\mu \lambda_1}{\mu \lambda_1 + \lambda_2} (1 - b_2) < 0$$

By Equation (8), $b_2 \geq b_1 \geq b_1^p$ implies $b_2 \geq 1 - \lambda_0$, hence the above will hold for all $b_2 \geq b_1^p$ and $\mu \leq 1$ if

$$\lambda_1 (1 - \gamma) < \gamma \lambda_2$$

(13)

Consider SB with $v_2 = 2$. Since he faces FB with $v_1 = 1$ or 2, he will match $b_1$, and FB will infer that $v_2 = 2$. Hence if $v_1 = 1$, FB will quit and SB will gain $2 - b_1 - \gamma$. If $v_1 = 2$, FB will counterbid $2 - \gamma$ to close the gap, and SB will quit with a loss of $\gamma$. By Bayes' rule, SB's expected return from investigation is

$$R_2(b_1) = -c + \lambda_2 \left[ -\gamma + \frac{\mu \lambda_1}{\mu \lambda_1 + \lambda_2} (2 - b_1) \right]$$

(14)

which is increasing in $\mu$ and decreasing in $b_1$. To ensure that a first bid of $b_1^p$ or higher deters investigation, we require that $R_2(b_1^p) = 0$ for some

\textsuperscript{23} At this stage, if FB with $v_1 = 1$ counterbids $b_1 < b_1^p$, SB will continue to believe that $v_1 = 1$, hence FB will have incurred $\gamma$ without changing SB's beliefs. If FB were to counterbid $b_1 \geq b_1^p$, SB will infer that $v_1 = 2$ and will quit if $v_2 = 1$ or close the gap to $2 - \gamma$ if $v_2 = 2$. Hence, FB's expected profit from counterbidding would be

$$-\gamma + \frac{\lambda_1}{\lambda_1 + \lambda_2} - (1 - b_1) \leq -\gamma + \frac{\lambda_1}{\lambda_1 + \lambda_2} \lambda_0 < 0$$

by Equation (13). Thus, FB with $v_1 = 1$ will not counterbid.
value $\mu \leq 1$. Substituting from Equation (8) into Equation (14), this condition is

$$\hat{\mu}(c) = \frac{\lambda_2}{\lambda_1[(1 + \lambda_0)/(c/\lambda_2) + \gamma] - 1} \leq 1 \quad (15)$$

By Equations (9) and (12), $\hat{\mu}(c) > 0$. Hence, a first bid of $b_1 \geq b_0^p$ deters investigation for all $0 \leq \mu \leq \hat{\mu}$.24

(viii) Along or off the equilibrium path, if at any stage FB bids less than $b_0^o$, SB will infer that $v_1 = 1$, hence SB with $v_2 = 1$ or 2 can acquire the target by matching the FB's bid.

Given the constraints (9)–(13) and (15), a family of equilibria is described by the strategies and beliefs I and II above with $0 \leq \mu \leq \hat{\mu}$. The constraints are satisfied by the values $\lambda_0 = 0.62$, $\lambda_1 = 0.18$, $\lambda_2 = 0.2$, $\gamma = 0.6$, and $c = 0.03$ implying $\hat{\mu} = 0.958$.

### 3.2 Effect of reducing cost of investigation

Three features of the equilibrium are worthy of note. First, FB deters investigation by simultaneously setting a floor price for the target, and credibly signaling to SB that $v_1$ is high.25 Second, when FB with $v_1 = 1$ bids 0, SB investigates and bids 0 if $v_2 = 1$ or 2. SB's bid of 0 suffices to convince FB that $v_2 \geq v_1$, hence FB quits rather than incur further bidding costs. So the "bidding war" ends after one round and the target is sold at a price of 0! Therefore, when $v_1 = 1$, the deterring bid of $1 - \lambda_0$ exceeds the price of the target under competitive bidding. Finally, the expected price of the target,

$$[\lambda_1(1 - \mu) - 0 + (\lambda_2\mu + \lambda_0)(1 - \lambda_0)] = (\lambda_2\mu + \lambda_0)(1 - \lambda_0) \quad (16)$$

increases with $\mu$, that is, the more frequently FB bids to deter, the higher the expected price of the target.

Having derived the equilibrium, we now consider the effect of lowering the cost of investigation, $c$, on the expected price of the target. Since a reduction in $c$ does not affect constraints (9), (10), (11), (13), and weakens constraints (12) and (15), the strategies and beliefs stated in I and II continue to be an equilibrium. Consider first the equilibrium with $\mu = \hat{\mu}$. When $c$ is reduced, this equilibrium no longer exists because at a bid of $b_0^p$ investigation is now strictly profitable. Hence the surviving equilibria have $\mu < \hat{\mu}$. By Equation (16), this implies that the expected price of the target will be lower.

This reasoning extends to equilibria with $0 < \mu < \hat{\mu}$ in the following way. We assume that given some equilibrium characterized by $\mu$, as $c$ is

---

24 If $\hat{\mu} > 1$, then $\hat{\mu} b_0^p < \lambda_0$, hence FB can deter investigation with a bid of $b_0^p$ for any $\mu \leq 0$. This equilibrium is weakly dominated for FB by an equilibrium with a slightly lower bid that still deters.

25 By contrast, in the model in which each bidder was restricted to at most one bid, FB bid to deter solely to set a floor price. We believe that, in general, both concerns will motivate deterring bids, as illustrated in the present equilibrium.
reduced, the same equilibrium continues to obtain so long as it exists. By
Equation (14), \( R_0(b') \) rises as \( c \) falls. Let \( \mu^* \) be the value of \( \mu \) such that
\( R_0(b') = R_0(1 - \lambda_0) = 0 \) when \( c = 0 \). Then for each \( \mu \in (\mu^*, \hat{\mu}) \), there exists
some value of \( c > 0 \) such that \( R_0(1 - \lambda_0) = 0 \), hence if \( c \) is reduced below
that level only equilibria with probabilities \( \mu' < \mu \) remain. Thus reducing
the cost of investigation successively removes equilibria in which the
expected price of the target is higher, leaving equilibria with lower expected
price.

For \( \mu \in [0, \mu^*] \), SB will not investigate for all \( c \geq 0 \). Therefore, the price
of the target declines only weakly with \( c \). If, however, we restrict attention
to reductions in investigation cost that do promote investigation (by ruling
out equilibria with higher \( \mu \)), then the expected price increases strictly.25

Expected social welfare is the expected valuation of the acquiring bidder
less SB’s cost of investigation and less the bidding costs of both bidders,
that is,

\[
\lambda_3 \mu + \lambda_1 (1 - \mu) [-c + \lambda_0 + \lambda_1 (1 - \gamma) + \lambda_3 (2 - \gamma)] + 2 \lambda_2
\]

In the example with \( \lambda_0 = 0.62, \lambda_1 = 0.18, \lambda_2 = 0.2 \), and \( \gamma = 0.6 \), beginning
with \( c = 0.03 \) and \( \mu = \hat{\mu}(c) = 0.958 \), a reduction in \( c \) will lower expected
social welfare.

4. Implications for Regulation of Takeover Bids

In the legal debate on the delay and disclosure provisions of the Williams
Act and on the proper conduct of the management of a takeover target, it
was assumed that competitive bidding for a takeover target would always
lead to a higher price than a single bid that deterred competition [Easter-
brook and Fischel (1981, 1982), Bebchuk (1982a, 1982b, 1986), and Gilson
(1981, 1982)]. Indeed, this premise was confirmed by Fishman (1988),
who modeled competitive takeover bidding as an English auction.

Our analysis shows that if, as is realistic, bidding is costly, competitive
bidding may yield a price lower than that from a single deterring bid. This
implies that the policy prescriptions based on the English-auction model
must be qualified. For instance, Bebchuk (1982a, 1982b, 1986) argued that
from the standpoint of shareholders of the target, the optimal degree of
facilitation of competitive bidding balances the increase in the expected
price of a target from competition against the reduction in the expected
price resulting from the diminished incentive for first bidders to search
for targets ex ante. At a minimum, our results suggest that the balance in
Bebchuk’s analysis must be tilted in the direction of less facilitation.27

---

25 By our convention, SB will not investigate if \( R_0(b) = 0 \). Similar results may be proved with the alternative
convention that, when \( R_0(b) = 0 \), SB investigates with positive probability less than 1. See appendix 2 of
Hirsheifer and Png (1989).

27 We have not analyzed the ex ante investigation decision of the first bidder. By increasing the first bidder’s
expected profit, a higher cost of investigation for competing bidders will encourage first bidders to investi-
gate. Our arguments against facilitation to the maximum apply, a fortiori, when this factor is also taken
into account.
Facilitation of Competing Bids

From the standpoint of social welfare, we should consider the post-acquisition value of the target less the deadweight costs of investigation and bidding. An increase in the second bidder's cost of investigation will lead the first bidder to switch from accommodation to deterrence in some states of the world, thereby tending to raise social welfare in three ways: (1) there will be fewer instances in which a second bidder with valuation greater than the amount of the first bid but less than the first bidder's valuation will acquire the target, (2) the cost of investigation will be incurred less frequently, and (3) less investigation by the second bidder implies fewer bidding wars and a possible saving in costs of bidding.\(^{28}\)

The increase in the cost of investigation, however, may reduce social welfare: First, the second bidder will incur a higher cost whenever the first bidder accommodates, and second, there will be more instances in which a first bidder with a lower valuation than the second bidder will acquire the target by a deterring bid. We have shown that, on balance, an increase in the cost of investigation may raise expected social welfare.

As discussed in the introduction, the cost of investigation for a competing bidder is affected by the delay and disclosure provisions of federal and state laws and regulations. The more information that a first bidder discloses, the more will be freely available to a potential competitor. The shorter the time that a bid need remain open, the less time available for a potential competitor to investigate, and hence the greater his reliance on more costly methods of information collection and processing.\(^{29}\)

The cost of investigation to competing bidders may also be influenced by the management of the target. At one extreme, the management of the target may institute a policy to provide free access to information about itself through regular briefings to analysts and published reports, while at the other, it may adopt a rule not to release any information other than that required by law and regulation.

There is, however, a problem of commitment for the management of a target in fixing the degree of assistance to competing bidders. Once a first bidder has announced his bid, the management of the target has an incentive to assist potential competitors, for that could only increase the price of the target. One solution is to set the cost of investigation by law and regulation, and then restrain the management of targets through a "rule of managerial passivity." The drawback of this approach is that the cost of investigation cannot be tailored to the situation of individual targets and their potential bidders.

If the first bidder is the management of the target seeking a buyout, the target will not have this ex post incentive to assist a competing bidder. Surprisingly, this would be in the shareholders' interest insofar as it encour-

\(^{28}\) The first effect is strong in the basic model and weak in the example of costly bidding. We conjecture, however, that it will arise in a model of costly bidding in which at least two types of first bidder accommodate.

\(^{29}\) Consistent with our analysis, Jarrell and Bradley (1980) observed that on average, premia in tender offers increased with the enactment of the Williams Act.
ages the first bidder to make a high preemptive bid. The commitment problem thus leads to a bias in favor of management buyouts.

5. Concluding Remarks

There is a clear empirical distinction between our analysis and the English-auction models of takeover bidding. Under the auction model, the first bidder bids at a premium only if he seeks to deter investigation by a potential competitor. If the first bidder chooses to let the second bidder investigate, he will bid with zero premium. But in reality, as Spatt (1989) has noted, bids at a substantial premium to market are frequently followed by the entry of competing bidders.

In contrast, our basic model is consistent with this empirical observation. Under the assumption that each bidder is allowed at most one bid, the first bidder will bid at a premium when accommodating investigation in order to increase the likelihood that the second bidder will draw a valuation below the first bid. In the example of costly bidding with voluntary termination, the first bidder with intermediate valuation does bid initially with zero premium when accommodating investigation. However, we conjecture that this is an artifact of the example; if at least two types of first bidder accommodate investigation, then the highest type among these may bid higher to distinguish himself and so avoid the cost of successive rounds of counterbidding.

On a broader level, our analysis indicates that costly bidding leads to an equilibrium expected price that may differ markedly from that of the standard English-auction model in which bids are made and revised costlessly. The latter model suggests that if both potential bidders enter, so that competitive bidding occurs, the price of a takeover target will be the minimum of the bidders' valuations. In our example with costly bidding, we show that a bidder's willingness to incur the bidding cost per se may signal a valuation high enough to persuade the competing bidder to quit. Thus, even if both potential bidders make offers, the price at which the target is sold may be well below the minimum of the bidders' valuations less the bidding cost.\(^{31}\)

We have shown that the English auction is in some ways incomplete as a model of takeover bidding. The costs of bidding and revising bids may also be important in other settings, such as in the market for real property. Future work should be directed toward a general analysis of competitive

---

\(^9\) To see this, suppose that the second bidder investigates. Then the price will be bid up to the minimum of the two bidders' valuations. So the probability that the first bidder acquires the target depends only on the relative magnitudes of the first and second bidders' valuations. To bid initially with positive premium would merely reduce the first bidder's gain from acquisition if he should acquire the target without providing any countervailing benefit.

\(^{31}\) In related work, Bhattacharya (1988) generalizes the English auction model to allow for a once-and-for-all cost of entry for each bidder prior to the commencement of competitive bidding. He also shows that, consistent with a conjecture of Spatt (1989), unsuccessful bids at a premium do occur.
bidding in which bidders may counterbid indefinitely at a fixed cost per bid.

In the context of takeover bidding such an analysis will provide a more general insight into the relation between the expected takeover price, the cost of investigation, and the cost of bidding. Presumably the cost of bidding is a motivation to bid higher and so avoid the cost of successive rounds of counterbidding. On the other hand, by discouraging competing bidders, it may allow lower bids to succeed. Furthermore, the cost of bidding is dissipated, reducing the total improvement in value that the target may hope to appropriate. A general analysis should yield a clearer picture of the net outcome of these effects.

**Appendix A: Basic Model—Proof and Example**

**Proof of proposition**

We first show that a reduction of SB's cost of investigation from \( c \) to \( c - \Delta c \) will reduce the expected price \( p \). By construction of \( c \), the set \( D(c - \Delta c) \) will be empty, hence by Equation (7), the effect on \( p \) will be

\[
\Delta p = \sum_{(v_i, c - \Delta c) \in A} b^*_i(v_i, c - \Delta c) \lambda_{v_i} - \sum_{(v_i, c) \in A} b^*_i(v_i, c) \lambda_{v_i} - \sum_{(v_i, \infty) \in A} b^*_i(\infty) \lambda_{v_i}
\]

\[
= \sum_{(v_i, c) \in A} [b^*_i(v_i, c - \Delta c) - b^*_i(v_i, c)] \lambda_{v_i}
\]

\[
+ \sum_{(v_i, \infty) \in A} [b^*_i(v_i, c - \Delta c) - b^*_i(\infty)] \lambda_{v_i}
\]  \hspace{1cm} (A1)

By Lemma 2, if \( v_i \in A(c) \), then \( v_i \in A(c - \Delta c) \) also, and by Lemma 1, for such \( v_i \), \( b^*_i(v_i, c - \Delta c) = \hat{b}_i(v_i) = b^*_i(v_i, c) \). Substituting in Equation (A1),

\[
\Delta p = \sum_{(v_i, \infty) \in A} [b^*_i(v_i, c) - b^*_i(\infty)] \lambda_{v_i}
\]

But by Equation (5), \( b^*_i(v_i, c) < b^*_i(\infty) \), thus, \( \Delta p < 0 \).

Next, if \( c < \xi \), all first bids will be accommodating, hence, by Lemma 1, any change in \( c \) will not affect the first bid and hence not affect the expected price. Finally, we show that \( p(c) > 0 \). For \( c < \xi \), \( A(c) = \{ v_i | v_i \geq 0 \} \). Hence by Equation (7) and Lemma 1,

\[
p(c) = \sum_{(v_i, c) \in A} b^*_i(v_i, c) \lambda_{v_i} = \sum_{v_i \in A(c)} \hat{b}_i(v_i) \lambda_{v_i} \geq 0
\]

Therefore, by the first and second steps of this proof, \( p(c) > 0 \). \( \square \)

**Example: Increase in \( c \) raises social welfare.** Let \( \lambda_{v_i} > 0 \), for \( i = -2, 0, 1, 2 \) only, with \( \lambda_{-2} + \lambda_0 + \lambda_1 + \lambda_2 = 1 \). By Equation (1), we require that \( E(v_2) < 0 \), that is,

\[
2\lambda_{-2} > \lambda_1 + 2\lambda_2
\]  \hspace{1cm} (A2)
If $c = 0$, FB accommodates in all states $v_1 \geq 0$, hence expected social welfare as a function of $c$ is

$$W(0) = \sum_{r_1 \geq 0} \left[ v_1 \Pr[v_2 \leq \hat{b}_1(v_1)] + \sum_{r_2 > \hat{b}_1(v_1)} v_2 \lambda_{r_2} \right] \lambda_{r_1}$$

Consider an increase to $c > 0$ just sufficient to induce FB with $v_1 = 2$ to deter. For such $c$,

$$W(c) = \bar{v} \lambda_{\bar{v}} + \sum_{r_1, r_2 \geq 0} \left[ -c + v_1 \Pr[v_2 \leq \hat{b}_1(v_1)] + \sum_{r_2 > \hat{b}_1(v_1)} v_2 \lambda_{r_2} \right] \lambda_{r_1}$$

The increase in $c$ results in a change in expected welfare of

$$\Delta W \overset{\text{def}}{=} W(c) - W(0) = -c \Pr[\bar{v} > v_1 \geq 0] + \lambda_{\bar{v}} \left( \sum_{r_2 > \hat{b}_1(v_1)} (\bar{v} - v_2) \lambda_{r_2} \right) \quad (A3)$$

Assume that

$$\lambda_{-2} + \lambda_0 > \lambda_1 \quad (A4)$$

then the accommodating bid for FB with $v_1 = 2$ is

$$\hat{b}_1(2) = 0 \quad (A5)$$

by Equation (6), since FB with $v_1 = 2$ just prefers to deter, $2 - b_0^p = [2 - \hat{b}_1(2)] \Pr[v_2 \leq \hat{b}_1(2)]$, that is, $b_0^p = 2(\lambda_1 + \lambda_2)$. Let

$$\lambda_1 + \lambda_2 > \frac{1}{2} \quad (A6)$$

then $b_0^p > 1$.

By construction, $c$ is just large enough to deter investigation, that is, by Equation (2),

$$R_0(b_0^p) = -c + \sum_{v > b_0^p} (v_2 - b_0^p) \lambda_{v_2} = -c + [2 - 2(\lambda_1 + \lambda_2)] \lambda_2 = 0$$

so $c = 2\lambda_{2}(\lambda_{-2} + \lambda_0)$. Substituting for $c$ and from Equation (A5) in Equation (A3),

$$\Delta W = \lambda_2 [-2(\lambda_{-2} + \lambda_0)(\lambda_0 + \lambda_1) + \lambda_1]$$

Therefore expected social welfare will increase if

$$\lambda_1 > 2(\lambda_{-2} + \lambda_0)(\lambda_0 + \lambda_1) \quad (A7)$$

Conditions (A2), (A4), (A6), and (A7) are satisfied by $\lambda_{-2} = 0.39$, $\lambda_0 = 0.05$, $\lambda_1 = 0.38$, and $\lambda_2 = 0.18$. 

604
Appendix B: Extension of Intuitive Criterion

In Section 3, we described a family of mixed-strategy equilibria in which FB (first bidder) with $v_1 = 1$ randomizes between a low accommodating bid of 0 and a high deterring bid of $b^*_2 = 1 - \lambda_o$. In this appendix, we define a refinement that directly extends the intuitive criterion of Cho and Kreps (1987) to our setting in which each bidder potentially can bid any number of times. In appendix 2 of Hirshleifer and Png (1989), we show that the equilibrium described in the main text survives this refinement and describe three other families of equilibria that arise within the parameter ranges defined by conditions (9)–(13) and (15). We show there that (1) two families are eliminated by the intuitive criterion and (2) the main results of the article also hold in the third.

Essentially, the intuitive criterion, as applied to some equilibrium of a game of one move by each player, requires that if a defection from the equilibrium is not profitable (compared to the equilibrium payoff) for a first player of type $t'$ under any inference by the next player, then the second player who observes such a defection should infer that the first player is not of type $t'$. If the defection is profitable for the first player of some type $t$ under some inference by the second player that does not place any weight on $t'$, then the equilibrium outcome must be ruled out.

In our setting, we begin by defining a defection to be the first move taken by a bidder that has zero probability of being made by that bidder (regardless of his type) at that stage of the game along any equilibrium path. Consider some defection by a given bidder (the defecting bidder), $i$: let the posterior beliefs of the next bidder, $j$, at that point be that the defecting bidder has $v_i = 0, 1, 2$ with likelihood $\beta = (\beta_0, \beta_1, \beta_2)$ with $\beta_0 + \beta_1 + \beta_2 = 1$. Given the defection and these beliefs, we consider subsequent strategies and beliefs of the two bidders that form a perfect bayesian equilibrium from the defection onward.

A perfect bayesian equilibrium for the entire game satisfies the extended intuitive criterion if it is not ruled out by the following procedure. If for one or more types $t'$ of the defecting bidder there does not exist any belief and subgame equilibrium such that the defection is preferable to the equilibrium payoff, then the next bidder must infer that the defecting bidder is not of type $t'$. If for some type $t$ there exists a belief $\beta$ that puts zero weight on type(s) $t'$ and a subsequent subgame equilibrium such that the defection is preferable, then the equilibrium must be ruled out.\textsuperscript{32} The procedure must be applied to every possible defection.\textsuperscript{33}

\textsuperscript{32} The original Cho and Kreps analysis allows iterated pruning of beliefs and message pairs. Since iteration is not essential for our purposes, we simplify the definition by allowing only a single stage of pruning.

\textsuperscript{33} For purposes of this definition, an SB will be treated as a given type even if he has not yet learned his valuation. In consequence, the definition allows for defections by investigating and bidding when not called for in equilibrium.
References


