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Feedback and the success of irrational investors ☆

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Abstract

We provide a model in which irrational investors trade based upon considerations that have no inherent connection to fundamentals. However, trading activity affects market prices, and because of feedback from security prices to cash flows, the irrational trades influence underlying cash flows. As a result, irrational investors can, in some situations, earn abnormal (i.e., risk-adjusted) profits that can exceed the abnormal profits of rational informed investors. Although the trading of irrational investors cause prices to deviate from fundamental values, stock prices follow a random walk.

JEL classification: G14; G12; G19

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1. Introduction

Investors often share common misconceptions and errors of analysis. For example, a substantial number of investors employ technical rules that are supported by neither

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conceptual considerations nor empirical evidence. Fads of investment in industry sectors, methods of security analysis, and simplistic theories of the stock market tend to proliferate through the mass media and word of mouth. (Shiller, 2000, discusses such phenomena.) Groups of investors who have fallen prey to common elementary errors, such as confusing the company Telecommunications Incorporated with the firm with ticker symbol TCI, have caused large price movements in one stock based upon news arrival in another unrelated stock (see Rashes, 2001). As another indication of the commonality of trading errors, investors and prices sometimes react to the republication of information that is already public (see Ho and Michaely, 1988 and Huberman and Regev, 2001).

Anecdotally, during the late 1990s it became increasingly popular to value stocks based upon ad hoc heuristics. For example, many analysts (and presumably the investors who listened to them) began to value tech firms based upon revenue instead of earnings; and to value e-commerce firms based upon eyeballs instead of revenue. These valuation methods allegedly were inappropriate, a criticism that, at least with the benefit of hindsight, seems to carry some weight.[1] With the rise of the Internet, there has been increased opportunity for investors to gain improved information about stocks. However, questionable stock market theories can also be spread more rapidly and widely.

A growing literature explores the effects of irrational trading on market prices and the profitability of such trading (Hirshleifer, 2001, and Barberis and Thaler, 2003, review this literature.). While our paper contributes to this literature, our focus is different. In contrast to the existing literature, irrational trading in our model does not provide profit opportunities to uninformed rational investors, even if they are aware that traders with psychological biases are in the market. From the perspective of the rational, but uninformed, investors, the market is informationally efficient. Nevertheless, irrational trading affects prices and, thereby, affects firms' fundamental values. Moreover, the irrational investors in our model can, under some conditions, earn positive expected profits that can even exceed the profits of rational informed traders.

We are not the first to consider conditions under which irrational traders can earn higher expected profits than fully rational ones. However, in previous work, the irrational traders either earn high average profits by accepting greater exposure to systematic risk or achieve higher average risk-adjusted profits by more aggressively exploiting private information.[2,3]

[1]See, for example, "Eyeballs, Bah! Figuring Dot-Coms' Real Worth," *Business Week Online*, October 30, 2000, or www.fvginternational.com/industries/industries_internet.html, that respectively discuss the invalidity and validity of these approaches. Ofek and Richardson (2003) provide evidence suggesting that the internet bubble was driven by the naïvely optimistic trading of some investors.

[2]In DeLong et al. (1991), investors with fundamental information underestimate risk, and therefore take larger long positions in the risky asset. Therefore these investors take fuller advantage of the asset's risk premium than their rational counterparts. Thus, in DeLong et al., high irrational returns reflect a premium for market risk. Several other papers have examined how in an imperfectly competitive securities market overconfident informed traders can benefit by trading more aggressively on private information (In an imperfectly competitive securities market, such aggressiveness can intimidate other informed traders; see Kyle and Wang (1997), Wang (1998) and Fischer and Verrecchia (1999). In a competitive securities market, Hirshleifer and Luo (2001) show that overconfident investors who trade aggressively in response to their private information signals can exploit liquidity traders more profitably than rational investors.). All of these papers, however, require that irrational investors have direct information about fundamentals.

[3]Another strand of literature goes beyond the analysis of trading profitability to consider long-run consumption and wealth accumulation when individuals make intertemporal consumption/investment decisions. In Blume and Easley (1990) and Kogan et al. (2003), irrational investors who have utility functions that are closer to logarithmic than those of rational investors can accumulate more wealth in the long-run than their rational counterparts.

In our setting prices are set by risk-neutral uninformed market makers, so that the expected profits of irrational investors in our model do not derive from exposure to priced risk. Moreover, the irrational investors in our model have no inherent private information relating to the cash flows that firms generate. However, the misperceptions of irrational investors affect these underlying cash flows because of feedback from stock prices to cash flows. As a result, in equilibrium, irrational trading is correlated with cash flows.

Feedback, which plays a central role in our model, can arise in practice for a variety of reasons. For example, a higher stock price can help firms attract customers and employees, can reduce the firm's cost of capital, and may provide a cheap currency for making acquisitions.[4] Higher stock prices also can encourage increased investment in complementary technologies.[5] In our basic model, feedback arises because the stakeholders of the firm (such as suppliers, customers, and employees) choose to make greater firm-specific investments (e.g., the workers exert more effort) when the firm's prospects are better. It is rational for them to do so because the firm provides better opportunities to its stakeholders when it is doing well. We also consider an extended model in which feedback arises because stock prices affect the amount of capital raised in a security issue.[6]

Another central feature of our model is that irrational traders are all eventually infused with the same sentiment. Some, however, are either infused with the sentiment earlier or simply are able to act on the sentiment and submit their trades before others. This gives an advantage to the irrational traders who submit their orders early, relative to those who submit their orders later. Furthermore, as we show, this can allow the early irrational investors to outperform rational informed investors, who observe a private signal about future cash flows but detect the irrational sentiment only after the early irrationals trade.[7]

Within this setting, if irrational investors are bullish, prices tend to increase first when the early irrationals buy and then again when the late irrationals buy. Irrational investors earn positive expected profits when they trade early but earn negative expected profits

(*footnote continued*)
Kogan et al. also show that irrational traders can sometimes have a long-term price impact even if their relative wealth converges to zero over time.

[4]Subrahmanyam and Titman (2001) discuss feedback in a related context. Feedback of the type examined in this paper is also related to what Soros (2003) refers to as his "theory of reflexivity," which holds that stock prices affect fundamentals.

[5]Existing evidence suggests that irrational mispricing affects investment choices (e.g., Baker et al., 2003; Polk and Sapienza, 2004), including takeovers (e.g., Dong et al., 2006).

[6]Anecdotally, feedback from stock prices to the behavior of potential stakeholders was evident during the dot-com bubble of the 1990s. Master of Business Administration students from top programs such as Stanford University and the Wharton School were leaving school without their degrees to pursue employment with new dot-coms, spurning once coveted investment banking positions (see *Fortune*, "MBAs Get Dot.com Fever," August 2, 1999). After stock prices collapsed, so did the willingness of MBAs to work for dot-coms (see, e.g., *Duke Magazine*, May–June, 2003). Similarly, high level executives were lured from secure positions at bricks-and-mortar firms for the promise implicit in dot-com firms with high stock prices.

[7]The idea that rational investors cannot directly observe the trades of irrational investors is standard in this literature. For example, in DeLong et al. (1990), the inability of rational traders to ascertain the trades of the noise traders causes risk averse rational traders to avoid noise-prone securities. The classical paradigms of Glosten and Milgrom (1985) and Kyle (1985) also preclude rational agents from observing the orders of noise (or liquidity) traders. To understand why it could be difficult to predict irrational sentiment, suppose that among a hundred irrelevant information items that can potentially influence irrational investors, one turns out to be especially salient to these individuals. A rational trader who dismisses irrelevant information items could have trouble identifying which of the hundred items is the salient one that tempts the irrational traders into error. We discuss this issue in more depth in Section 6.

when they trade late. In the absence of feedback, because of the adverse market impact of their trades, the average profit of the irrational investor is negative. However, with feedback from prices to cash flows, the price effect of the irrational trades is not completely reversed, because the sentiment induced buying has a positive effect on fundamentals. As we show, when this feedback effect is sufficiently strong, the expected gains from early irrational trading outweighs the expected loss from late irrational trading, so that the combined profits can on average be positive and can even exceed the profits of traders with fundamental information.

Feedback allows early irrational investors to, in effect, exploit a kind of second order private information, not about fundamentals per se, but about the future order flow of investors with similar psychological biases and the feedback to fundamentals that results from these later trades. However, this argument does not require that the irrational investors be sophisticated enough to anticipate the feedback effect. Indeed, we assume that irrational traders naïvely ignore the causality from prices to cash flows. The sequential arrival of correlated irrational trades automatically positions the early irrational investors to profit.

Because prices are set by rational risk-neutral market makers, prices are efficient in the sense that there are no profitable trading strategies based upon publicly observable information. Nevertheless, irrational trading causes prices to deviate from fundamental value, and distorts real decisions. Thus, the analysis illustrates that market efficiency in the conventional sense does not preclude real effects of investor irrationality.

Our analysis is closely related to some earlier models of securities trading. Subrahmanyam and Titman (2001) and Khanna and Sonti (2004) analyze feedback from the stock market to cash flows in a setting with fully rational investors. Here we examine the consequences of imperfectly rational trading. The relevance of having investors receive information at different times was explored by Froot et al. (1992) and Hirshleifer et al. (1994), both of which considered settings with rational investors and no feedback.[8]

The remainder of the paper is structured as follows. Section 2 describes the economic setting. Section 3 derives asset demands, expected profits to different kinds of traders, and numerical comparisons of the profitability of different kinds of traders. Section 4 provides a simplified version of the model and analytic results about the profitability of different trader classes. Section 5 explores equity issuance as an alternative channel of feedback. Section 6 provides an additional discussion of our assumptions and results and Section 7 concludes. Except where otherwise noted, all proofs are in the Appendix.

2. The economic setting

In this section we describe our assumptions about the firm and its investors.

[8]The main focus of these papers is on information and herding rather than the profitability of irrational trading. However, in the Froot, Scharfstein and Stein model, investors can potentially realize abnormal profits by trading on noise. In their model, the first investors who trade on the noise can profit from the price pressure of later-arriving investors who trade on the same noise. Noise investors (early and late) on average make money if they can reverse their trades at a favorable price in the final trading round. The ability of noise investors to reverse at a favorable price derives from the assumption that all traders reverse their positions in the last round. Hirshleifer et al. (1994) examine a setting that is similar except that the reversal of trades is endogenous. Owing to their risk aversion, investors wish to unwind their trades once their information (or noise) is revealed to the market. In this setting, which assumes no feedback, it is not possible to profit by trading on noise.

2.1. The firm

Consider a firm that generates a single random terminal payoff. Equity claims on the firm are traded at each of dates 1 and 2, at prices P_1 and P_2. At date 3 the firm pays out the amount $F = \theta + \varepsilon + \delta$ per share, where θ and ε are independent normal random variables with mean zero. The random variable δ captures the effect of feedback. Specifically, we assume that the prices generated at dates 1 and 2 have an effect on cash flows generated at date 3.

We allow stock prices to influence how much the firm's stakeholders invest in their relationship with the firm. For example, workers may perceive the employment opportunities at a firm more favorably if its stock price is high, enabling it to attract more productive employees for a given wage. In the model, we assume that the gains to resource suppliers such as employees investing specific human or physical capital in the firm are based on the supplier's expected gain from transacting with the firm. Stakeholders draw rational inferences about the expected size of this gain, which is an increasing function of the firm's future cash flows. In other words, firms that do well provide better future opportunities for their stakeholders.

We consider a single representative stakeholder (thereby abstracting from the free-rider problem among stakeholders in contributing to the firm) who decides upon his investment at date 2 after observing market prices P_1 and P_2. We further assume that the marginal benefit to the stakeholder from the investment is increasing in the stakeholder's expectation of θ (the inherent, non-feedback component of the firm's cash flow), $\mu_\theta = \mathrm{E}(\theta | P_1, P_2)$.

Let X be the stakeholder's investment. It is convenient to capture the effect of μ_θ on the marginal benefits to the stakeholder of investing in its relation with the firm using a specific functional form. We specify the profit to the stakeholder of investing X as

$$\Pi(X; \mu_\theta) = C_0 + C_1\mu_\theta + K_0 X - \frac{K_1 X^2}{2\mu_\theta}, \tag{1}$$

where constants $C_0, C_1, K_0, K_1 > 0$.

Setting the derivative with respect to X equal to zero, the optimal investment is

$$X^*(\mu_\theta) = \left(\frac{K_0}{K_1}\right)\mu_\theta, \tag{2}$$

which is proportional to μ_θ. This solution describes the stakeholder's investment indirectly as a function of the market prices P_1 and P_2 observed at date 2.

We assume that the firm is more profitable if stakeholders invest in their relationship (e.g., profits are higher if the workers choose to be better trained). Specifically, the component of profitability δ is assumed to be proportional to the stakeholder investment,

$$\delta(X) = C_2 X. \tag{3}$$

So in equilibrium, profit is

$$\delta(X^*) = \left(\frac{C_2 K_0}{K_1}\right)\mu_\theta, \tag{4}$$

which is proportional to μ_θ. The date 3 cash flow of the firm can then be expressed in the form

$$F = \theta + \varepsilon + k\mathrm{E}(\theta|P_1, P_2), \tag{5}$$

where $k \equiv C_2K_0/K_1$. Thus, the final form of the feedback effect incorporates the effect of prices on underlying cash flows through the effect of price on stakeholder behavior. Section 5 explores the consequences of an alternative channel of feedback in which a higher stock price allows the firm to raise more capital for profitable investments, thus increasing the profitability of the firm.

2.2. The investors

All traders behave competitively. We assume that there are two types of rational informed investors. The *early informed* learn precisely the realization of θ when the market opens at date 1, while the *late informed* do not learn the realization of θ until the market opens at date 2. The error term ε remains unknown at both trading dates.

There is also a group of utility-maximizing irrational traders who mistakenly believe that the security pays off $\eta + \varepsilon$, where η is independent of all other exogenous random variables. Thus, η has no inherent relation to fundamentals but endogenously becomes related through the feedback effect. We assume that η is normally distributed with mean zero and variance v_η. The early irrational traders observe the realization of η at date 1 and the late irrational traders observe the realization at date 2. We assume that the rational traders do not observe η at date 1. The rational traders may or may not observe η at date 2; as we discuss later, in equilibrium the rational traders are not influenced by η if it is observed at date 2. For simplicity, we assume that the irrational traders do not anticipate the feedback effect.

The mass (or measure) of the early irrational traders is denoted by M, while the total mass of early and late trading irrational investors is normalized to unity, so that the mass of late trading irrational investors is $1 - M$. Similarly, the mass of early informed traders is N, and their total mass also is normalized to unity. Informed and irrational investors have negative exponential utility over terminal wealth with a common absolute risk aversion coefficient R.

Liquidity demand shocks for the claim in amounts of z_1 and z_2 arrive at dates 1 and 2, respectively. These shocks are normally distributed with mean zero and are independent of each other and of θ and ε. The common variance of the liquidity shocks is denoted as v_z. These shocks can also be interpreted as uncorrelated irrational trades, in contrast to the correlated trades made by agents who observe η sequentially.

In our model, prices are set by a group of competitive, risk-neutral 'market makers,' who possess no information about the fundamental value of the risky security. Specifically, they do not observe the information of the rational traders or the sentiment of the irrational traders. These agents represent a competitive fringe of risk-neutral traders (e.g., floor brokers, scalpers, or institutions that monitor trading floor activities) who are willing to absorb the net demands of the other traders at competitive prices. Because of the risk-neutrality and competitiveness assumption, prices are set equal to expected terminal value conditional on prices (or, equivalently, total order flows) to date. Thus, $P_2 = \mathrm{E}(F|P_1, P_2) = (1 + k)\mathrm{E}(\theta|P_1, P_2)$.

Competitive risk-neutral market makers are concerned about order flow only if some traders are truly informed. So our assumption that there are informed traders is important. We further assume that rational investors (whether uninformed market makers or informed traders) do not directly observe the trades of the irrational investors. As we explain later, our results are not affected if the rational informed investors observe the irrational signal along with the late irrational investors at the later date.

3. Analysis

We begin by analyzing optimal demands by investors. We then define and solve for equilibrium prices.

3.1. The demands of investors

To derive the linear equilibria, we begin by postulating that prices are linear functions of the private information variable θ and the current and past liquidity demand shocks, such that

$$P_1 = a_1\theta + a_2\eta + a_3 z_1, \tag{6}$$

and

$$P_2 = b_1\theta + b_2\eta + b_3 z_1 + b_4 z_2. \tag{7}$$

Let x_1 and x_2 represent the demands of the early informed (rational) investors at dates 1 and 2, respectively. Because date 2 wealth is conditionally normally distributed, one can use the mean–variance framework and standard methodology to show that the optimal risky holdings of each early and late trading rational investor at the end of date 2 are identical and are

$$x_2 = \frac{\theta + k\mathrm{E}(\theta|P_1, P_2) - P_2}{Rv_\varepsilon}. \tag{8}$$

Let $\mathrm{E}_r(P_2)$ and $v_r(P_2)$ denote, respectively, the mean and variance of P_2 conditional on the information set of the early informed at date 1. This information set consists of θ and the market price P_1. Although the rational informed traders do not know η precisely, they infer it partially from market prices. The Appendix shows that the optimal date 1 demand of an early informed trader is

$$x_1 = \left[\frac{\mathrm{E}_r(P_2) - P_1}{R}\right]\left[\frac{1}{v_r(P_2)} + \frac{1}{v_\varepsilon}\right] + \frac{\theta - \mathrm{E}_r(P_2)/(1+k)}{Rv_\varepsilon}. \tag{9}$$

The demand represented by Eq. (9) consists of two components. The first exploits the expected price appreciation between dates 1 and 2. The second locks in at the current price the expected quantity to be demanded at date 2.[9]

It can easily be shown that the date 1 demand of the late-trading informed investors equals zero in equilibrium. Intuitively, this occurs for two reasons. First, the equilibrium date 1 price does not offer a risk premium because of the presence of risk-neutral market makers. Second, the late-trading investors cannot hedge their date 2 demand in advance

[9]Since $P_2 = \mathrm{E}(F|P_1, P_2) = (1+k)\mathrm{E}(\theta|P_1, P_2)$, demand in Eq. (8) can be written as $x_2 = [\theta - P_2/(1+k)]/(Rv_\varepsilon)$.

because, conditional on their date 1 information set (which does not contain the informational variable θ), the date 2 price is an unbiased forecaster of the terminal cash flow, so that their expected date 2 trade is zero (proof available on request).

Informed investors may be able to observe the irrational signal η with a lag, perhaps by talking to other investors or studying irrational analyzes disseminated in the business media. However, whether or not an informed investor can observe η at date 2 has no consequence for his date 2 trading decision. The relevance of η for firm value derives solely from its effect on P_2, and the feedback to the expected terminal cash flow F is a linear function of P_2. Thus, P_2 is a sufficient statistic for η in forecasting F. Because at date 2 informed traders directly observe P_2, η provides no incremental information about expected payoffs.

Let y_1 and y_2 represent the demands of the early irrational traders at dates 1 and 2. Again the date 2 demands (i.e., holdings) of the early and late irrational traders are identical,

$$y_2 = \frac{\eta - P_2}{Rv_\varepsilon}. \tag{10}$$

Let $\mathrm{E}_n(P_2)$ and $v_n(P_2)$ denote the mean and variance of P_2 conditional on the information set of the early irrational traders at date 1; this information set includes P_1 and their signal η. Analogous to Eq. (9), the date 1 demand of an early irrational trader is

$$y_1 = \left[\frac{\mathrm{E}_n(P_2) - P_1}{R}\right]\left[\frac{1}{v_n(P_2)} + \frac{1}{v_\varepsilon}\right] + \frac{\eta - \mathrm{E}_n(P_2)}{Rv_\varepsilon}, \tag{11}$$

whereas the date 1 demand of a late irrational trader equals zero for the same reason as for the late informed trader (proof available upon request).

3.2. Definition of equilibrium

Our equilibrium concept closely parallels those of Vives (1995) and Hirshleifer et al. (1994). Specifically, we assume that at dates 1 and 2, informed investors submit demand schedules (limit orders) that are functions of their information and the market prices. The risk-neutral market makers observe the combined demand schedules of the informed and liquidity traders and set competitive prices at each date. Let γ_1, γ_2 denote the aggregate demand schedules (total quantities held) at dates $t = 1, 2$, where $\gamma_1 = Nx_1 + My_1 + z_1$ and $\gamma_2 = x_2 + y_2 + z_1 + z_2$.

Because market makers are risk-neutral and competitive, they set prices that are semi-strong form efficient. Thus, at each date the security's price is equal to the expectation of the terminal cash flow of the security, conditional on the information set of the market makers, i.e.,

$$P_1 = \mathrm{E}\left[F|\gamma_1(\cdot)\right]$$

and

$$P_2 = \mathrm{E}[F|\gamma_1(\cdot), \gamma_2(\cdot)]. \tag{12}$$

The date 3 price, P_3, is equal to the final value of the claim, F. We consider linear equilibria, wherein pricing functions are linear in the random variables θ, η, z_1, and z_2. Given such functions, it can easily be shown that the demand schedules can be written as

$$\gamma_1(P_1) = f(\tau_1) + f_p(P_1)$$

and

$$\gamma_2(P_2) = g(\tau_2) + g_p(P_2), \tag{13}$$

where $f(\cdot), f_p(\cdot)$, $g(\cdot)$, $g_p(\cdot)$ are linear functions with non-stochastic coefficients, while τ_1 and τ_2 are linear combinations of the information variable θ, the irrational noise variable η, and the liquidity trades z_1 and z_2. Because prices are common knowledge, the total demand at date 1 is informationally equivalent to τ_2, and total demand at date 2 is informationally equivalent to τ_2. Therefore,

$$P_1 = \mathrm{E}[F|D_1(\cdot)] = \mathrm{E}[F|\tau_1]$$

and

$$P_2 = \mathrm{E}[F|D_1(\cdot), D_2(\cdot)] = \mathrm{E}[F|\tau_1, \tau_2]. \tag{14}$$

3.3. Equilibrium prices

After substituting for the individuals' demands in $\gamma_1 = Nx_1 + My_1 + z_1$ and dropping the terms associated with P_1, we have

$$\tau_1 = N\left\{\left[\frac{\mathrm{E}_r(P_2)}{R}\right]\left[\frac{1}{v_r(P_2)} + \frac{k}{(1+k)v_\varepsilon}\right] + \frac{\theta}{Rv_\varepsilon}\right\} + M\left[\frac{\mathrm{E}_n(P_2)}{Rv_n(P_2)} + \frac{\eta}{Rv_\varepsilon}\right] + z_1. \tag{15}$$

Similarly, dropping the terms involving P_2 in $\gamma_2 = x_2 + y_2 + z_1 + z_2$ and multiplying by Rv_ε results in

$$\tau_2 = \theta + \eta + Rv_\varepsilon(z_1 + z_2). \tag{16}$$

Observe that $\mathrm{E}_r(P_2) = \mathrm{E}(P_2|P_1, \theta)$, and can be written as $r_1\theta + r_2\eta + r_3z_1$. Further, $\mathrm{E}_n(P_2) = \mathrm{E}(P_2|P_1, \eta)$ and can be written as $h_1\theta + h_2\eta + h_3z_1$. In addition, $v_r(P_2)$ and $v_n(P_2)$ are not functions of the realizations of the random variables but are well-known functions of the price coefficients and variances of the random variables. We assume that irrational traders accurately understand the coefficients of the market makers' pricing function, in order to show that a single irrationality can, when combined with feedback, generate excess profits for irrational traders.

Given the expressions for the early and late irrational trader demands, it can be shown that the expected profits of the late irrational traders are

$$\begin{aligned}\pi_{\mathrm{nl}} = \mathrm{E}[x_2(F - P_2)] &= v_\theta\left[\frac{-b_1}{Rv_\varepsilon}\left(1 - \frac{b_1}{1+k}\right)\right] \\ &\quad + v_\eta\left[\frac{1-b_2}{Rv_\varepsilon}\left(-\frac{b_2}{1+k}\right)\right] + v_z\frac{b_3^2 + b_4^2}{(1+k)Rv_\varepsilon},\end{aligned} \tag{17}$$

and the expected profits of the early irrational traders are

$$\begin{aligned}\pi_{\mathrm{ne}} &= \mathrm{E}[x_2F - (x_2 - x_1)P_2 - x_1P_1] \\ &= R^{-1}v_\theta\left\{\frac{h_1}{v_n(P_2)} - a_1\left[\frac{1}{v_n(P_2)} + \frac{1}{v_\varepsilon}\right]\right\}(b_1 - a_1)\end{aligned}$$

$$+ R^{-1}v_\eta \left\{ \frac{h_2}{v_n(P_2)} - a_2 \left[\frac{1}{v_n(P_2)} + \frac{1}{v_\varepsilon} \right] \right\} (b_2 - a_2)$$
$$+ R^{-1}v_z \left\{ \frac{h_3}{v_n(P_2)} - a_3 \left[\frac{1}{v_n(P_2)} + \frac{1}{v_\varepsilon} \right] \right\} (b_3 - a_3) + \pi_{\mathrm{nl}}. \tag{18}$$

The above expressions indicate that the early irrational traders can profit from feedback because the date 2 price is correlated both with their irrational assessment η as well as with the fundamental θ. The stronger the feedback, the more the price moves at date 2. The profit term for the early informed is $x_2(F - P_2) + x_1(P_2 - P_1)$, which is increasing in $P_2 - P_1$, and for a positive irrational shock, P_2 is positively related to k, as is F.

Details of the solution process to obtain the equilibrium value of coefficients a_t and b_t, $t = 1, 2, 3$ in the price functions postulated in Eqs. (7) and (6) are provided in the Appendix. Unfortunately, the model cannot be solved in closed form. However, Proposition 1, which partially describes the equilibrium, can be demonstrated (the second part obtains through numerical analysis).

Proposition 1. *In the general model*

(1) *Price changes are serially uncorrelated.*
(2) *There exists a non-empty set of exogenous parameter values under which the ex ante expected profits of the irrational traders are positive and exceed those of the rational informed traders.*

Part 1 of Proposition 1 follows from the fact that market makers are risk-neutral and set prices to be expectations of final value conditional on all public information. Hence prices follow a martingale.

Part 2 of Proposition 1 is demonstrated by means of numerical comparative statics on irrational trader profits with respect to changes in the strength of the feedback effect, which we summarize by k.[10] We examine the effects of k on the expected profits of the irrational traders (Fig. 1), and on the expected profit differential between irrational and rational traders (Fig. 2).

In Fig. 1, the late irrational expected profits are always negative. The early irrational expected profits are negative when $k = 0$, but they increase as k increases. The total expected profits also start negative and increase with k.

Further numerical analysis (not reported) indicates that the threshold level of k above which the total irrational expected profits are positive is increasing in the ratio v_θ/v_η, the ratio of the variances of the inherent information, θ, and of the irrational belief, η, and are decreasing in the variance of liquidity trades v_z; these are intuitive results. Irrational traders who think they are informed tend to be too willing to accommodate the opposite side of rationally informed trades. In consequence, the greater relative variability of informed trading tends to reduce the profits of irrational traders and consequently increases the threshold k. In contrast, irrational investors profit by absorbing the opposite side of liquidity traders, so higher v_z tends to reduce k.

[10] From Eq. (5), the sensitivity of cash flows to stock prices (i.e., the strength of the feedback effect) depends on k, which in turn is optimally determined by the parameters of the stakeholder profit function as in (1).

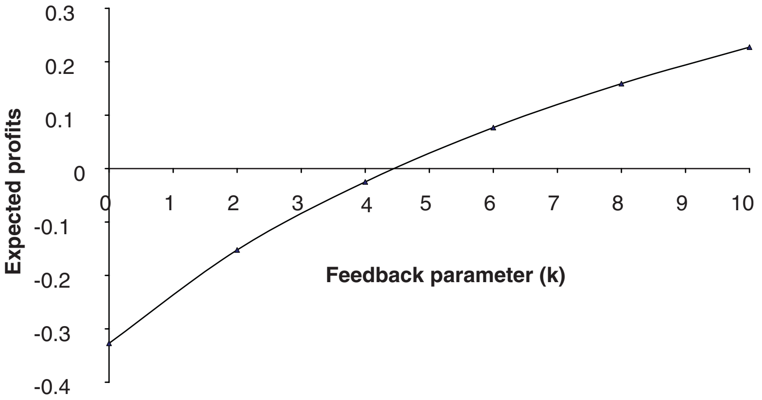

Fig. 1. Expected profits of irrational traders, as a function of the parameter (k) that captures the feedback from stock prices to cash flows. It is assumed that the variance of the information variable (θ) is 0.4, the variance of the irrational signal (η) is 1.9, the risk aversion coefficient R is 0.18, the variance of liquidity trades z is 0.6, the variance of the noise in the information signal (ε) is 4.2, the mass of informed who receive information early M is 0.05, and the mass of the irrational traders who receive the signal early N is 0.12.

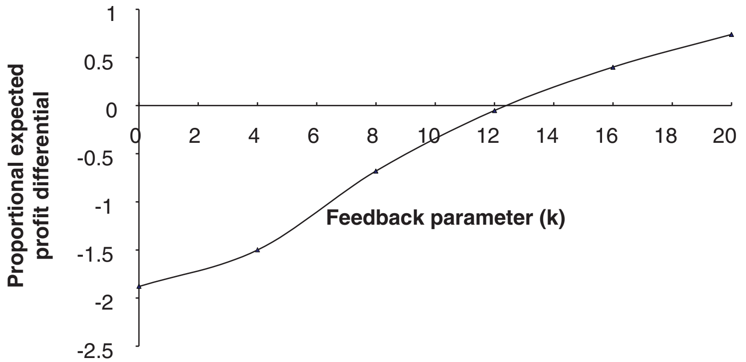

Fig. 2. Difference between the expected profits of irrational and informed traders, as a proportion of the expected profits of the informed traders. It is assumed that the variance of the information variable (θ) is 0.1, the variance of the irrational signal (η) is 3.14, the risk aversion coefficient R is 0.21, the variance of liquidity trades z is 0.99, the variance of the noise in the information signal (ε) is 3.18, the mass of informed who receive information early M is 0.08, and the mass of the irrational traders who receive the signal early N is 0.06.

We also find, somewhat surprisingly, that the threshold level of k needed for positive expected profits for irrational investors is decreasing in the risk v_ε that is not resolved prior to the terminal date by the signal received by the rational informed. This occurs because increasing v_ε tends to decrease the size of the position held by the irrational traders, which mitigates the late irrationals' expected losses as well as the early irrationals' expected gains. Under the parameter values considered, the former effect dominates. Consequently, weaker feedback suffices for the expected profits of the irrational traders to be positive.

Fig. 2 compares the total expected profits earned by irrational traders with the total expected profits earned by rational informed traders as a function of the feedback parameter. The vertical axis shows the difference in profits scaled by the informed profit (which is always positive) in the denominator. When the feedback parameter k is low, this scaled difference is negative, indicating that irrational traders do worse than rational informed traders. However, irrational traders do better in relative terms as k increases and earn higher profits than rational informed traders when k is greater than approximately 12.

4. A simplified model

The general model considered in Section 3 provides intuitive numerical comparative statics but does not lend itself to analytical results. The difficulties in finding a closed-form solution are caused by the intertemporal hedging demands of the informed and irrational agents. As demonstrated by Eqs. (9) and (11), these demands depend non-linearly on both the date 1 and date 2 price coefficients, and this presents technical difficulties in solving the fixed-point problem inherent in the rational expectations equilibrium.

In the simplified model, we assume that rational informed investors receive information only at date 2, which makes it easy to determine the date 1 price and implies that rational informed investors trade only at date 2. In addition, we assume that irrational traders are myopic in the sense that at each date they reverse out their positions from the previous date. Even though this assumption requires the early irrational investors to trade myopically (which is suboptimal from their own perspective), we show that irrational traders can still generate higher expected profits than rational traders.

Each early irrational investor trades at date 1 and reverses his trade at date 2. Further, he forms his date 1 demand under the belief that the date 2 payoff is $\eta + \varepsilon$. The total mass of early irrational traders is specified to equal M. We assume that there is no informed trading at date 1, but at date 2 a unit mass of informed agents enter the market. In addition a mass $1 - M$ of late irrational traders enter the market at date 2. Under these assumptions, it is easy to show that the irrational investor's trade equals

$$y_1 = \frac{\eta - P_1}{Rv_\varepsilon}. \tag{19}$$

The date 2 trades of the rational and irrational informed agents as a function of the price remain unchanged in this framework.

The market makers observe the variables $\tau_1 \equiv M\eta + Rv_\varepsilon z_1$ at date 1 and $\tau_2 \equiv \theta + (1 - M)\eta + Rv_\varepsilon(z_1 + z_2)$ at date 2. The date 2 price is given by $P_2 = \mathrm{E}(F|\tau_1, \tau_2) = (1 + k)\mathrm{E}(\theta|\tau_1, \tau_2)$. Let $\mathrm{E}(\theta|\tau_1, \tau_2) = b'_1\theta + b'_2\eta + b'_3 z_1 + b'_4 z_2$. Then, using standard properties of normal distributions, the equilibrium values of the coefficients in this linear function are

$$b'_1 = v_\theta(M^2 v_\eta + R^2 v_\varepsilon^2 v_z)/D, \tag{20}$$

$$b'_2 = R^2 v_\varepsilon^2 v_\theta v_z(1 - 2M)/D, \tag{21}$$

$$b'_3 = -MRv_\varepsilon v_\eta v_\theta(1 - 2M)/D, \tag{22}$$

$$b'_4 = Rv_\varepsilon v_\theta(M^2 v_\eta + R^2 v_\varepsilon^2 v_z)/D, \tag{23}$$

where

$$D \equiv M^2 v_\eta (5R^2 v_\varepsilon^2 v_z + v_\theta) - 4MR^2 v_\varepsilon^2 v_\eta v_z + R^2 v_\varepsilon^2 v_z (R^2 v_\varepsilon^2 v_z + v_\eta + v_\theta) > 0. \tag{24}$$

The coefficients on η and z_1 in the date 1 price are zero. This again follows from the fact that the date 1 conditional expectation of market makers,

$$\mathrm{E}(F|\tau_1) = \mathrm{E}[\{\theta + k\mathrm{E}(\theta|\tau_1, \tau_2)\}|\tau_1] \tag{25}$$

is equal to zero because $\mathrm{E}(\theta|\tau_1) = 0$ and

$$\mathrm{E}[\{\mathrm{E}(\theta|\tau_1, \tau_2)\}|\tau_1] = [Mv_\eta b_2' + Rv_\varepsilon v_z b_3']\tau_1/\mathrm{var}(\tau_1) \tag{26}$$

is also equal to zero by the law of iterated expectations.

It is intuitively clear that the early irrational traders cannot profit unless the mass of late irrational traders is sufficiently large. Indeed, the expected profits of the early irrational traders are positive if and only if $M < 0.5$. For the remaining part of this section, we explore in more detail the implications that arise when early irrational traders earn expected profits, and we thus assume that the condition $0 < M < 0.5$ holds. Given this assumption, and the equilibrium price coefficients described above, it is straightforward to derive Proposition 2.[11]

Proposition 2. (1) *The irrational signal η, and the trades of the early irrational traders are both positively correlated with the fundamental value of the firm's claim so long as $k > 0$.*

(2) *The correlation between the trades of the late irrational traders and the firm's date* 3 *fundamental value is of ambiguous sign. However, for M close to zero, this correlation is positive so long as $k > 0$ and v_η is sufficiently large.*

Proposition 2 indicates that irrational trading affects the firm's fundamental value. Indeed, the trades of the early irrational investors are positively correlated with fundamental value so long as there is feedback from prices to cash flows. For small enough mass of the early irrational M, the correlation between the trades of the late irrational investors and the fundamental value is positive so long as the variance of the irrational signal is sufficiently large.

To understand the latter result further, note that the price impact of the late irrational trades can cause a same-direction shift in price. Owing to feedback, this can in turn cause a same-direction shift in fundamentals. However, there is another effect which promotes a negative correlation with fundamentals. Rational informed traders buy based on the inherent fundamental θ, which drives the price high (low) when the inherent fundamental is high (low). Because the late irrationals think they have information, they tend to be too willing to accommodate the opposite side of these trades, which induces a negative correlation of their trades with fundamentals. To get the positive correlation effect to dominate, the variance of the irrational signal η (and the resulting feedback) need to be sufficiently large relative to the variance of the fundamental θ.

Proposition 3 describes the correlation between irrational trades and price moves.

Proposition 3. (1) *The trades of the early irrational traders are positively correlated with the date* 2 *price move, $P_2 - P_1$.*

[11] All propositions in this section are proved in the Appendix.

(2) *The trades of the late irrational traders are positively correlated with the date* 2 *price move*, $P_2 - P_1$ *if and only if*

$$v_\theta(1+k)(R^2 v_\varepsilon^2 v_z + M^2 v_\eta) < R^2 v_\varepsilon^2 v_\eta v_z (1-2M). \tag{27}$$

(3) *The trades of the late irrational traders are positively correlated with the trades of the early irrational traders if*

$$\frac{D}{R^2(1+k)v_\varepsilon^2 v_\theta v_z (1-2M)} > 1. \tag{28}$$

Part 1 of Proposition 3 indicates that irrational traders do indeed affect prices. Intuitively, market makers think that their trades might have come from informed investors. Together, Parts 2 and 3 provide conditions under which the late irrational investors tend to trade in the same direction as early irrational investors and in the same direction as the date 2 price move. Late irrational trades can be negatively correlated with the date 2 price move when the inequality in Eq. (27) is reversed. To see why, suppose that there is strong feedback, so that the rational informed buy very aggressively on a positive signal. The late irrationals, who think they are informed, are too willing to accommodate the rational trades. So the late irrationals end up trading against the price. As inequality Eq. (28) indicates, when the feedback k is sufficiently large, the late irrationals trade in the same direction as the early irrationals owing to their shared beliefs based upon η.

We now describe autocorrelation patterns in prices and order flows, where the latter are defined as the total (signed) quantities of the stock absorbed by the market makers at each date.

Proposition 4. (1) *Equilibrium order flows are positively autocorrelated.*

(2) *Unconditional price changes are serially uncorrelated in equilibrium*; *i.e.*,

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1) = 0. \tag{29}$$

(3) *Equilibrium prices exhibit positive autocorrelation conditional on the irrational signal and negative autocorrelation conditional on the rational signal. Specifically*,

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1 | \eta) > 0$$

and

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1 | \theta) < 0. \tag{30}$$

Part 1 of Proposition 4 indicates that order flows are serially dependent because of the sequential arrival of irrational traders. Part 2 again confirms that the results are not driven by market inefficiency; prices at each date are equal to the expected value of an equity claim on the firm conditional on all publicly available information. Together Parts 1 and 2 show that serial dependence in order flow is not inconsistent with serial independence in price movements. These results are thus consistent with the positive autocorrelation in order imbalances but virtually zero autocorrelation in daily stock returns shown by Chordia et al. (2002).

Part 3 of Proposition 4 shows that prices exhibit persistence after controlling for the irrational signal. This happens because order flows are noisy transformations of the rational investors' trades, so that prices underreact to the valid information signal θ. However, because the trades of the irrational investors get mixed in with those of the informed traders, prices overreact to irrational trades and consequently exhibit reversals after controlling for the rational signal.

The expected profits of the early irrational traders in terms of the price coefficients are

$$\pi_{\mathrm{ne}} = \mathrm{E}[x_1(P_2 - P_1)] = \frac{(1+k)b_2' v_\eta}{R v_\varepsilon}, \tag{31}$$

and those of the late irrational traders are

$$\begin{aligned}\pi_{\mathrm{nl}} &= \mathrm{E}[x_2(F - P_2)] \\ &= \frac{v_\theta[-b_1'(1+k)](1-b_1') + v_\eta[1 - b_2'(1+k)](-b_2') + (1+k)v_z(b_3'^2 + b_4'^2)}{R v_\varepsilon}.\end{aligned} \tag{32}$$

The ex ante expected profits of the irrational traders are $\pi = M\pi_{\mathrm{ne}} + (1-M)\pi_{\mathrm{nl}}$. Substituting for the price coefficients, the ex ante expected profits are

$$\pi = \frac{R v_\varepsilon v_\eta v_\theta v_z (1-2M)[(k+2)M - 1]}{D}, \tag{33}$$

where D is as given in Eq. (24). This leads to Proposition 5.

Proposition 5. (1) *If there is no feedback, i.e.,* $k = 0$, *then the ex ante expected profits of irrational traders are always negative.*

(2) *The ex ante expected profits of irrational traders are positive so long as* $k > (1-2M)/M$.

Part 1 of Proposition 5 confirms that when there is no feedback, irrational trading is unprofitable. Part 2 indicates that, in contrast, sufficient feedback makes irrational trading profitable. The intuition is the same as that outlined in Section 3. Again, the expected profits are not a consequence of sophisticated exploitation of the feedback effect; indeed irrational investors are naïve and simply ignore this effect. They inadvertently profit from feedback because their trades are correlated with those of later irrational investors.

Proposition 4 also illustrates the critical role of the assumption that irrational investors do not all receive their signals at the same time. Part 2 of Proposition 5 demonstrates that as the mass of early irrational traders, M, goes to zero, the bound on k goes to infinity. This indicates that for any finite level of feedback, a strictly positive mass of early irrational traders is necessary for the irrational traders to earn positive expected profits. Further, for any finite k, M has to lie in the open interval $(0, 0.5)$ for irrational traders to earn positive expected profits.

The expected profits of the informed traders in terms of the coefficients b_i' are

$$[v_\theta(1-b_1')^2 + v_\eta b_2'^2 + v_z(b_3'^2 + b_4'^2)]/[R v_\varepsilon]. \tag{34}$$

Again, substituting for the price coefficients, expected informed profits, denoted by π_i, are

$$\pi_i = \frac{R v_\varepsilon v_\theta v_z [v_\eta\{M^2 + (2M-1)^2\} + R^2 v_\varepsilon^2 v_z]}{D}. \tag{35}$$

A comparison of Eqs. (33) and (35), results in Proposition 6.

Proposition 6. *A sufficient condition for the ex ante expected profits of irrational traders to be greater than the ex ante expected profits of informed traders is that*

$$k > \frac{R^2 v_\varepsilon^2 v_z + v_\eta (9M^2 - 8M + 2)}{M v_\eta (1 - 2M)}. \tag{36}$$

Proposition 6 indicates that, consistent with the numerical results of the general model, a sufficiently high feedback parameter causes the expected profitability of irrational traders to be greater than that of rational informed traders.

We next describe comparative statics on the expected profit difference between the irrational and the rationally informed traders.

Proposition 7. (1) *The expected profit difference between irrational traders and informed traders is increasing in the feedback parameter, k.*

(2) *The expected profit difference between irrational traders and informed traders is increasing in the variance of the signal observed by irrational traders, v_η, so long as*

$$k > \frac{R^2 v_\varepsilon^2 v_z (1 - 2M)}{M(R^2 v_\varepsilon^2 v_z + v_\theta)}. \tag{37}$$

(3) *The expected profit difference between irrational traders and informed traders is increasing in the variance of the signal observed by informed traders, v_θ, so long as*

$$k > \frac{v_\eta (9M^2 - 8M + 2) + R^2 v_\varepsilon^2 v_z}{M v_\eta (1 - 2M)}. \tag{38}$$

Part 1 of Proposition 7 indicates, consistent with intuition, that the performance of the irrational traders relative to the rational informed ones increases with the feedback parameter. Furthermore, Part 2 indicates that the expected profits of the irrational traders relative to those of informed traders increase in the variance of the irrational signal so long as the feedback effect is strong. Intuitively, the coordinating signal and feedback drive trades and profits. Finally, as indicated by Part 3, the expected profit difference increases in the variance of information v_θ so long as the feedback effect is strong. This is because as the ex ante variance of private information increases, the signal to noise ratio in the net demand increases, which causes the market maker to adjust prices more to irrational (as well as other) trades, thereby strengthening the feedback resulting from irrational trades.

Our final proposition in this section describes how irrational trading affects resource allocation.

Proposition 8. (1) *The optimal stakeholder contribution $X^*(z)$ is positively correlated with the irrational signal η.*

(2) *The ex ante volatility of stakeholder contributions is increasing in the variance of the irrational signal, v_η.*

Proposition 8 shows that irrational investor optimism increases the commitment of stakeholders to the firm and that greater variability of investor sentiment causes greater variability in stakeholder inputs to the firm. Thus, feedback has real economic consequences.

5. Equity issuance

In this section, we explore an alternative channel by which feedback from stock prices to cash flows can occur. Instead of stakeholder contributions to the firm, we examine how feedback can arise when firms issue new equity.

To illustrate how equity issuance can create feedback, we extend the model by assuming that the firm issues an exogenous number of shares, N_m, at date 2 and invests the proceeds in an investment opportunity that provides a gross payoff of vI for an investment of I, with $v \geqslant 1$. The gross cash flow derived from this investment is therefore $vI = vP_2N_m$.[12] Feedback arises in this setting because the final payoff is increasing in the dollar amount raised by way of the equity issue, which, in turn is increasing with the price at date 2.

To obtain analytic solutions, we consider the simplified model of Section 4. Our analysis must now account for an additional complication, that equity issuance dilutes the claim on the firm's existing assets, which pays $\theta + \varepsilon$. Letting N denote the number of shares prior to the equity issuance, the total per share payoff inclusive of both the existing project and the new one is

$$F = \frac{N(\theta + \varepsilon + k\mu_\theta) + vN_mP_2}{N + N_m}, \tag{39}$$

where $\mu_\theta \equiv \mathrm{E}(\theta|P_1, P_2)$. Because $P_2 = \mathrm{E}(F|P_1, P_2)$,

$$P_2 = \frac{N(1 + k\mu_\theta) + vN_mP_2}{N + N_m}. \tag{40}$$

So, solving for P_2 gives

$$P_2 = \frac{N\mu_\theta(1 + k)}{N + N_m(1 - v)}. \tag{41}$$

The trades of the early and late irrational investors as a function of η, P_1, and P_2 are unchanged from the base case (as in Section 4, the irrational investors ignore feedback, which, in the present case, includes the consequences for cash flow per share of equity issuance). Let $\rho \equiv N/(N + N_m)$. Then the demand of the informed investors (who receive the signal at date 2) is

$$x_2 = \frac{\mathrm{E}(F|\theta, P_1, P_2) - P_2}{R\rho^2 v_\varepsilon}. \tag{42}$$

As in Section 4, we can construct the variables $\tau_1 = M_\eta + Rv_\varepsilon z_1$, and $\tau_2 = (1 - M)\rho\eta + \theta + R\rho v_\varepsilon(z_1 + z_2)$ from the demands submitted to the market makers, and then $\mu_\theta = \mathrm{E}(\theta|\tau_1, \tau_2)$. Let the coefficients on the variables θ, η, z_1, and z_2 in this expectation be b'_1, b'_2, b'_3, and b'_4, respectively. These coefficients can easily be obtained by using standard methods of projecting one normally distributed vector on another.

[12] An alternative, conceptually very similar approach would be to assume that the firm raises exactly enough equity to fund an investment of I. This would generate a similar feedback effect, since fewer shares would be issued if the stock price were higher, resulting in higher terminal cash flow per share. However, such an alternative specification is less tractable because, with the number of shares issued being a function of price, there is non-linearity in per-share cash flows as a function of state variables.

Let

$$k_1 \equiv \frac{N v \mu_\theta (1+k)}{N + N_m(1-v)} \tag{43}$$

and $k_2 \equiv \rho k + k_1(1-\rho-1/v)$. The expected profits of the early irrational investors are $\pi_{\text{ne}} = \mathrm{E}[y_1(P_2 - P_1)] = b_2' k_1 v_\eta/(kRv_\varepsilon)$. The expected profits of the late irrational investors are

$$\begin{aligned}\pi_{\text{nl}} &= \mathrm{E}[y_2(F-P_2)] \\ &= (Rv_\varepsilon)^{-1}[k_2 b_2' v_\eta\{1-(k_1/v)b_2'\}\rho k_1 b_1' v_\eta/v - k_1 k_2/v\{b_1'^2 v_\theta + (b_3'^2 + b_4'^2)v_z\}].\end{aligned} \tag{44}$$

The ex ante expected profits of irrational investors are given by the expression

$$\pi = M\pi_{\text{ne}} + (1-M)\pi_{\text{nl}}. \tag{45}$$

Calculating the right-hand side of Eq. (45), we find that the ex ante expected profits of the irrational investors are positive if and only if

$$\frac{vN_m(M-1) - (N+N_m)[(k+2)M-1]}{vN_m - (N_m+N)} > 0. \tag{46}$$

Define $\xi \equiv (1-M)/[1-(k+2)M]$. Then the above condition reduces to the simple form $N_m > N/(\xi v - 1)$. Hence, issuance of a sufficiently large number of shares allows the irrationals to make positive expected profits.

This finding does not require that the investment project have a positive net present value or that there be any feedback derived from stakeholder contributions. If $v = 1$ and $k = 0$, the inequality in Eq. (46) reduces to the condition $MN_m - N(1-2M) > 0$. If this condition holds, the ex ante expected profit of irrational investors are positive even if the project has zero net present value and there is no feedback from stakeholders (as modeled in Section 2). Thus, the irrationals are able to earn positive expected profits at the expense of the liquidity or noise traders solely because of the feedback created by equity issuance.

Essentially, the trades of the early irrationals reflect information not just about the future trades of the late irrationals, but also about the resulting feedback induced by equity issuance. This feedback occurs because the trades of the late irrationals affect the price at which the firm issues equity, and therefore the terminal per share cash value of the firm. The information about future feedback provides a source of expected profit to the irrationals as a group.

A similar lower bound on N_m, denoted N_m^*, can be derived to describe the condition under which the expected profits of the irrationals exceed those of the rational investors. The expected profits of the informed investors are

$$\pi_i = \mathrm{E}[x_2(F-P_2)] = \frac{v_\theta(\rho + k_2 b_1')^2 + k_2^2 b_2'^2 v_\eta + k_2^2(b_3'^2 + b_4'^2)v_z}{R\rho^2 v_\varepsilon}. \tag{47}$$

Taking the difference between the expected profits of the irrational and rational investors using Eqs. (45) and (47), we obtain Proposition 9.

Proposition 9. *In the setting with equity issuance, if the maximum number of shares that can be issued N_m exceeds N_m^*, where N_m^* is a function of the exogenous parameters, then irrational investors earn higher expected profits than rational investors.*

The specific expression for N_m^* is provided in the proof of Proposition 9 in the Appendix. It follows from Proposition 9 that irrational investors can earn higher expected profits than rational investors when there is no stakeholder contribution (i.e., when $k = 0$) as long as the firm is issuing a sufficient number of shares. Intuitively, the more shares the firm issues, the greater is the feedback from market price to future cash flows.

6. Discussion

Irrational investors are able to generate profits in this setting because of two relatively novel features of our model. The first is the feedback mechanism, which allows irrational trades to influence fundamental values. The second has to do with the nature of irrational sentiment, which affects some irrational traders before it affects others. In combination, these assumptions imply that irrational investors that are affected by sentiment early make money, because they in a sense have information about the future order flow that comes from their late-trading irrational counterparts. In addition, because of the feedback effect, the order flow affects fundamental values and can make the expected profits of an irrational investor (who could turn out to trade either early or late) be positive as well.

Irrational investors earn profits in this setting that can potentially exceed the profits of the rationally informed investors because their actions reflect information that their more rational counterparts do not have. Specifically, because the early irrational investors are susceptible to the same environmental cues that induce trades by the late irrational investors, the trades of the early irrationals forecast the trades of the late irrationals.

In contrast, we assume that the rational investors do not observe the irrational signal at date 1. As we mentioned earlier, the irrational signal does not provide useful information to the rational informed investor at date 2. This is because the feedback to cash flows is a linear function of the date 2 price, which already is public knowledge at date 2.[13] The expected profits of the irrational investors will of course decline if some rational investors are able to identify the irrational signal at date 1 (i.e., without any delay). When this is the case, their trades compete with those of the early irrational investors, driving down the profits of the early irrational investors. Continuity arguments imply that our qualitative results still obtain if the mass of early rational investors that observe the irrational signal is sufficiently small; details are available from the authors. Nevertheless, the competitive effect suggests that our analysis requires that the irrational investors have a comparative advantage associated with trading on irrational sentiment without delay.

To understand why this is likely to be the case, consider the possibility that there are a number of publicly observable variables, denoted $\psi \equiv (\psi_1, \psi_2, \psi_3, \ldots, \psi_N)$, which are irrelevant for inherent fundamental value but can influence irrational sentiment. We assume that with probability $1/N$ all the irrational investors respond to a single variable $\psi_{i'}$, where i' is a given value of the index $i = 1, \ldots, N$.[14] Some of the irrational investors are influenced early, while others are influenced with some lag. Rational investors understand that this is the case and observe all the public signals, but they cannot

[13] The firm's cash flow does not directly depend on sentiment, only indirectly through the stock price. At date 2 the rational informed can directly observe the stock price, so η provides them with no incremental information.

[14] The ψ_i's could be characteristics of the stock (growth versus value, high versus low earnings or return momentum, market value), whether a 'guru' has recommended the stock, or whether the stock has been discussed a lot in an internet chat room.

initially detect which of them influence the irrational investors. Hence, if N is sufficiently large, the rational investors acquire virtually no information from ψ about irrational trading at date 1.

The idea that irrational triggers can influence the behavior of some investors but not others is consistent with psychological evidence that suggests that groups of individuals with different personalities, decision styles, education, and intelligence are prone to different kinds of or degrees of bias (see, e.g., Stanovich and West, 2000, and Toplak and Stanovich, 2003). Those groups that are not affected by an irrelevant trigger that influences others (the rational individuals in our model) do not necessarily know what public signals irrational individuals react to. For example, it might not even occur to a finance professor who adheres to the efficient markets hypothesis (rationally, for the sake of this argument) to buy a stock today just because the firm had a positive earnings surprise on the previous day. Such an individual perhaps does not realize that other individuals who do not understand the efficient markets hypothesis are tempted to do so. Compelling evidence exists that humans have limited access to the unconscious cognitive processing that feeds into judgment and decision making (see, e.g., the review of Camerer et al., 2004). Thus, an individual who refrains from reacting to an irrelevant trigger perhaps does not know what is special about his thought processes that allowed him to do so or how the thought processes of others differ with respect to that signal.

7. Concluding remarks

An efficient financial market is often defined as a market in which all publicly available information is fully reflected in the prices of securities. This concept is important, in part, because of the link between the information conveyed by market prices and the allocation of resources (See, e.g., Hayek, 1945, and recent models by Fishman and Hagerty, 1989; Khanna et al., 1994; Subrahmanyam and Titman, 1999). In addition to the allocation of capital, the efforts of workers and other stakeholders may be allocated more efficiently when the actions of rational informed agents make financial market prices more informative. Our model considers the flip side of the coin and examines how irrational trading can affect resource allocation by way of its influence on market prices. As we show, when there is feedback from prices to real decisions, irrational trading strategies can generate positive expected profits. This suggests that irrational strategies can persist and have long-term effects on both market prices and the allocation of resources.

Although irrational trading affects market prices and distorts resource allocation, it does not necessarily make markets informationally inefficient in the conventional sense. Indeed, in our model stock prices follow a random walk, and investors who receive no private information (real or imagined) are unable to realize abnormal returns.[15] As a result,

[15]Thus, in our setting standard tests would not identify any market efficiency. Nevertheless, in our setting it is possible to detect the effects of irrational trading indirectly by examining cross-sectional relationships. In particular, the model suggests that irrational investors will be more active in sectors such as high-tech, where feedback is likely to be strong because of the interdependence of firms within this sector. Weston (2001) estimates a microstructure model and finds evidence that noise traders are especially active in the technology-heavy Nasdaq market.

standard tests of market efficiency do not provide a gauge of the extent to which irrational investors distort market prices. Even if risk-adjusted returns are unpredictable, irrational trading can induce substantial deviations of prices from fundamentals, and can cause substantial shifts in resource allocation.

While we analyze feedback-related phenomena at the firm level, our analysis also applies to industries that become susceptible to waves of investor sentiment. Indeed, the turn-of-the-millennium tech boom was consistent with sentiment influencing stakeholders as well as investment within the Internet sector. High stock prices encouraged executives and programmers to leave secure high-level positions to join Internet startups (see footnote 6), and also allowed Internet companies to raise capital and increase investment rapidly.[16] Furthermore, owing to positive externalities between the investments of different dot-com firms, favorable investor sentiment may have fed back into higher long-run firm profitability.[17] During this period, individuals and firms were being educated about the benefits of shopping on the Internet. The high stock prices and resulting activities of startups such as Amazon, e-Bay, and Yahoo helped develop a population of regular Web users, which probably contributed to the profitability of these firms.

At the aggregate macroeconomic level, the irrationality in our model can be viewed as Keynesian animal spirits in the stock market. Irrational optimism or pessimism can affect aggregate corporate profitability through the feedback effect, and business activity could respond in a confirming way to irrational swings in sentiment. Such effects can be magnified when there are positive investment externalities across firms, as in Shleifer (2000) and Cooper and John (1988). Our approach suggests that even when aggregate stock price movements are motivated by irrational beliefs, the investors who drive these price movements could be making profitable investments. This in turn suggests that animal spirits can have a continuing influence on stock prices, investment, and business cycles.

Appendix A

A.1. Derivation of Eq. (9)

The wealth of the early-informed trader, denoted by W^{E}, is

$$W^{\mathrm{E}} = x_2 F - (x_2 - x_1)P_2 - x_1 P_1. \tag{48}$$

[16] Hendershott (2004) reports that dot-com firms invested \$21 billion raised from equity investors. Similarly, in the late 1990s, the vast investments in optical cable bandwidth capacity by Worldcom and other telecommunications companies were, according to commentators, induced by the booming valuation of telecom stocks (see, e.g., Frieden, 2003).

[17] The failure of many Internet startups makes the low profitability of the losers highly salient. However, Wall Street Journal (2000) argues that irrational exuberance in Internet stocks helped remedy what would otherwise have been sub-optimal development of the Internet. Further, Hendershott (2004) reports that investment in dot-com firms led to \$39 billion in value at end of 2001, for an annualized return-on-equity capital invested of 19%. He concludes that "Despite the distorted price signals and contrary to popular perception, wealth was created during the dot-com investment boom."

Let $\mu \equiv \theta + k\mathrm{E}(\theta|P_1, P_2)$. Substituting for x_2 from Eq. (8) into the expression for W^{E} results in

$$\begin{aligned} W^{\mathrm{E}} &= \frac{(\mu - P_2)}{Rv_\varepsilon}(\mu + \varepsilon) - \frac{(\mu - P_2)}{Rv_\varepsilon} P_2 - x_1(P_1 - P_2) \\ &= \frac{(\mu - P_2)^2}{Rv_\varepsilon} + \frac{(\mu - P_2)\varepsilon}{Rv_\varepsilon} - x_1(P_1 - P_2). \end{aligned} \tag{49}$$

From the formula for the characteristic function of a normal distribution, if $u \sim \mathcal{N}(\mu, \sigma^2)$, then $\mathrm{E}(\exp(vu)) = \exp(\mu v + (1/2)\sigma^2 v^2)$. In our case, setting $u = W^{\mathrm{E}}$, $v = -R$, and using the fact that, from the perspective of the early informed, the only unknown at date 2 is the random variable ε results in

$$\mathrm{E}(-\exp(-RW^{\mathrm{E}})|\phi_2) = -\exp\{-R[x_1 P_2 - x_1 P_1 + (\mu - P_2)^2/(2Rv_\varepsilon)]\}. \tag{50}$$

It follows that at date 1, the early informed traders maximize the derived expected utility of their date 2 wealth

$$\mathrm{E}[[-\exp\{-R[x_1 P_2 - x_1 P_1 + (\mu - P_2)^2/(2Rv_\varepsilon)]\}]|\phi_1]. \tag{51}$$

Eq. (51) can be written as

$$\begin{aligned} &-[2\pi v_r(P_2)]^{-1/2} \int_{-\infty}^{\infty} \exp\left\{-R\left[x_1 P_2 - x_1 P_1 + \frac{(\mu - P_2)^2}{(2Rv_\varepsilon)}\right]\right. \\ &\quad \left. -\frac{1}{2}\frac{(P_2 - \mathrm{E}_r(P_2))^2}{v_r(P_2)}\right\} \mathrm{d}(P_2 - \mathrm{E}_r(P_2)). \end{aligned} \tag{52}$$

Completing squares, the expression within the exponential above can be written as

$$-\left[\frac{1}{2} w^2 s + hw + l\right], \tag{53}$$

where

$$w = P_2 - \mathrm{E}_r(P_2), \tag{54}$$

$$h = Rx_1 - \frac{(\mu - \mathrm{E}_r(P_2))}{v_\varepsilon}, \tag{55}$$

$$s = \frac{1}{v_r(P_2)} + \frac{1}{v_\varepsilon}, \tag{56}$$

and

$$l = Rx_1(\mathrm{E}_r(P_2) - P_1) + \frac{(\mu - \mathrm{E}_r(P_2))^2}{2v_\varepsilon}. \tag{57}$$

Define $u \equiv \sqrt{s}w + h/\sqrt{s}$. Then, Eq. (53) becomes $-(1/2)u^2 + (1/2)h^2/s - l$. The Jacobian of the transformation from w to u is $s^{-1/2}$, and thus the integral Eq. (52) becomes

$$\begin{aligned} &-[2\pi v_r(P_2)s]^{-1/2} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}u^2 + \frac{1}{2}\frac{h^2}{s} - l\right) \mathrm{d}u \\ &\quad = -\frac{1}{[v_r(P_2)s]^{1/2}} \exp\left(\frac{1}{2}\frac{h^2}{s} - l\right). \end{aligned} \tag{58}$$

Solving for the optimal x_1 by maximizing the above objective, we obtain (9). An identical technique allows us to derive Eq. (11).

A.2. Derivation of the equilibrium price coefficients in (7) and (6)

First, simple application of the formulas for the conditional moments of multivariate normal distributions allow us to obtain expressions for the coefficients r_1–r_3 and h_1–h_3 in terms of the price coefficients. Specifically, r_1, r_2, and r_3 are given, respectively, by the coefficients of θ, η, and z_1 in the scalar quantity $A_r B_r^{-1} C_r'$, where

$$A_r \equiv [a_1 b_1 v_\theta + a_2 b_2 v_\eta + a_3 b_3 v_z, b_1 v_\theta], \tag{59}$$

$$B_r \equiv \begin{bmatrix} a_1^2 v_\theta + a_2^2 v_\eta + a_3^2 v_z & a_1 v_\theta \\ a_1 v_\theta & v_\theta \end{bmatrix}, \tag{60}$$

and

$$C_r \equiv [a_1\theta + a_2\eta + a_3 z_1, \theta], \tag{61}$$

whereas $v_r(P_2)$ is given by $A_r B_r^{-1} A_r'$.

Similarly, the coefficients h_1, h_2, and h_3 are given, respectively, by the coefficients of θ, η, and z_1 in $A_n B_n^{-1} C_n'$, where

$$A_n \equiv [a_1 b_1 v_\theta + a_2 b_2 v_\eta + a_3 b_3 v_z, b_2 v_\eta], \tag{62}$$

$$B_n \equiv \begin{bmatrix} a_1^2 v_\theta + a_2^2 v_\eta + a_3^2 v_z & a_2 v_\eta \\ a_2 v_\eta & v_\eta \end{bmatrix}, \tag{63}$$

and

$$C_n \equiv [a_1\theta + a_2\eta + a_3 z_1, \eta], \tag{64}$$

while $v_n(P_2)$ is given by $A_n B_n^{-1} A_n'$.

Substituting for $\mathrm{E}_r(P_2)$ and $\mathrm{E}_n(P_2)$ into the expressions for τ_1 and τ_2 in Eq. (15) and Eq. (16) results in

$$\begin{aligned}\tau_1 = {} & \theta\left[\frac{Nr_1}{R}\left\{\frac{1}{v_r(P_2)} + \frac{k}{(1+k)v_\varepsilon}\right\} + \frac{N}{Rv_\varepsilon} + \frac{Mh_1}{Rv_n(P_2)}\right] \\ & + \eta\left[\frac{Nr_2}{R}\left\{\frac{1}{v_r(P_2)} + \frac{k}{(1+k)v_\varepsilon}\right\} + \frac{Mh_2}{Rv_n(P_2)} + \frac{M}{Rv_\varepsilon}\right] \\ & + z_1\left[\frac{Nr_3}{R}\left\{\frac{1}{v_r(P_2)} + \frac{k}{(1+k)v_\varepsilon}\right\} + \frac{Mh_3}{Rv_n(P_2)} + 1\right],\end{aligned} \tag{65}$$

which can be written as

$$\tau_1 = k_1\theta + k_2\eta + k_3 z_1. \tag{66}$$

We then have $\mathrm{E}(\theta|\tau_1, \tau_2) = g_1\tau_1 + g_2\tau_2$. Because $P_2 = (1+k)\mathrm{E}(\theta|\tau_1, \tau_2)$, by equating coefficients, we have

$$b_1 = (1+k)g_1 k_1 + (1+k)g_2, \tag{67}$$

$$b_2 = (1+k)g_1 k_2 + (1+k)g_2, \tag{68}$$

$$b_3 = (1+k)g_1k_3 + (1+k)Rv_\varepsilon g_2, \tag{69}$$

and

$$b_4 = (1+k)g_2Rv_\varepsilon. \tag{70}$$

We solve for P_1 as

$$P_1 = \mathrm{E}(\theta|\tau_1) + k\mathrm{E}[\mathrm{E}(\theta|\tau_1,\tau_2)|\tau_1] = \mathrm{E}(\theta|\tau_1) + \left(\frac{k}{1+k}\right)\mathrm{E}(P_2|\tau_1). \tag{71}$$

Define $D_1 = k_1^2v_\theta + k_2^2v_\eta + k_3^2v_z$ and $D_2 = k_1b_1v_\theta + k_2b_2v_\eta + k_3b_2v_z$. Then it follows from a simple application of the projection theorem that

$$a_1 = \frac{k_1^2v_\theta + kk_1D_2/(1+k)}{D_1}, \tag{72}$$

$$a_2 = \frac{k_1k_2v_\theta + kk_2D_2/(1+k)}{D_1}, \tag{73}$$

and

$$a_3 = \frac{k_1k_3v_\theta + kk_3D_2/(1+k)}{D_1}. \tag{74}$$

Eqs. (67)–(70) and (72)–(74) define a system of seven non-linear equations in the seven price coefficients a_1–a_3 and b_1–b_4. This completes the solution procedure for the price coefficients.

Proof of Proposition 1. Part 1 follows from the fact that the sequence of prices P_1, P_2 and F form a martingale, increments to which are serially uncorrelated. Part 2 is demonstrated by direct calculation, as illustrated in Figs. 1 and 2. □

Proof of Proposition 2. The covariance between the trades of the early-irrational investors and the terminal value is

$$\mathrm{cov}[(\eta - P_1)/(Rv_\varepsilon), \theta + k\mathrm{E}(\theta|P_1,P_2)] = \frac{kb_2'v_\eta}{Rv_\varepsilon}. \tag{75}$$

Because, from Eq. (21), b_2' is positive if and only if $M<0.5$, Part 1 follows.

To show Part 2, note that the covariance between the trades of the late irrational investors and the terminal value is

$$\mathrm{cov}[(\eta - P_2)/(Rv_\varepsilon), \theta + k\mathrm{E}(\theta|P_1,P_2)]. \tag{76}$$

This covariance can be expressed in terms of the price coefficients as

$$-b_1'(1+k)(1+ka_1')v_\theta + b_2'[1-(1+k)b_2']v_\eta - (1+k)(b_3'^2 + b_4'^2)v_z. \tag{77}$$

Substituting for the coefficients b_1'–b_4' from Eqs. (20)–(23), the covariance becomes

$$\begin{aligned} &-v_\theta D^{-1}[k^2v_\theta(M^2v_\eta + R^2v_\varepsilon^2v_z) + k\{2M^2v_\eta v_\theta + 2MR^2v_\varepsilon^2v_\eta v_z \\ &\quad - R^2v_\varepsilon^2v_z(v_\eta - 2v_\theta)\} + v_\theta(M^2v_\eta + R^2v_\varepsilon^2v_z]. \end{aligned} \tag{78}$$

As $M \to 0$, the expression in square brackets approaches

$$\frac{-v_\theta[k^2 v_\theta + k(2v_\theta - v_\eta) + v_\theta]}{R^2 v_\varepsilon^2 v_z + v_\theta + v_\eta}, \tag{79}$$

which is positive so long as v_η is sufficiently large. □

Proof of Proposition 3. The covariance in Part 1 is

$$\mathrm{cov}[(\eta - P_1)/(Rv_\varepsilon), P_2 - P_1] = (1+k)b_2' v_\eta/(Rv_\varepsilon), \tag{80}$$

which is positive so long as $b_2' > 0$. By Eq. (21), this is the case if and only if $M < 0.5$.

Similarly, the covariance in Part 2 is

$$\mathrm{cov}[(\eta - P_2)/(Rv_\varepsilon), P_2 - P_1], \tag{81}$$

which is positive if and only if

$$b_2' v_\eta > (1+k)(a_1'^2 v_\theta + b_2'^2 v_\eta + (b_3'^2 + b_4'^2) v_z. \tag{82}$$

By Eqs. (20)–(23), the above condition is true if and only if Eq. (27) holds.

Finally, the covariance in Part 3 is

$$\mathrm{cov}[(\eta - P_2)/(Rv_\varepsilon), (\eta - P_1)/(Rv_\varepsilon)], \tag{83}$$

which is positive if and only if

$$1 > 1 - b_2'(1+k), \tag{84}$$

so that Eq. (28) follows from Eq. (21). □

Proof of Proposition 4. To prove Part 1, we observe that prices form a martingale (as in Part 1 of Proposition 1) or perform an explicit calculation. In particular,

$$\mathrm{cov}(F - P_2, P_2 - P_1) = b_1'(1 - b_1')v_\theta - b_2'^2 v_\eta - (b_3'^2 + b_4'^2)v_z. \tag{85}$$

Substituting for the equilibrium values of b_1'–b_4', Part 1 follows.

To prove Part 2, note that the date 1 order flow, denoted by Q_1, is $Q_1 = M\eta/(Rv_\varepsilon) + z_1$, whereas the date 2 order flow, Q_2, is

$$Q_2 = (1-M)(\eta - P_2)/(Rv_\varepsilon) + (\theta - P_2)/(Rv_\varepsilon) + z_2 - M\eta/(Rv_\varepsilon). \tag{86}$$

Because $P_2 = (1+k)(b_1'\theta + b_2'\eta + b_3' z_1 + b_4' z_2)$,

$$\begin{aligned}\mathrm{cov}(Q_1, Q_2) = {} & Mv_\eta[\{1-M\}\{1-(1+k)b_2'\} - (1+k)b_2' - M] \\ & + Rv_\varepsilon v_z[-(1+k)(1-M)b_3' - (1+k)b_3'].\end{aligned} \tag{87}$$

Substituting for b_2' and b_3' from Eqs. (21)–(22), the above covariance reduces to $Mv_\eta(1-2M)$, and Part 2 follows.

Consider Part 3. The standard formula for a conditional variance–covariance matrix of a normal vector $X_1 \sim \mathcal{N}(0, \Sigma_1)$ conditional on another normal vector $X_2 \sim \mathcal{N}(0, \Sigma_2)$ is

$$\mathrm{var}(X_1|X_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}, \tag{88}$$

where Σ_{ij} represents the covariance matrix between X_i and X_j. Now, $P_3 - P_2 = \theta - \mathrm{E}(\theta|\tau_1, \tau_2)$ and $P_2 - P_1 = (1+k)\mathrm{E}(\theta|\tau_1, \tau_2)$. Letting $X_1 = [P_3 - P_2, P_2 - P_1]$ and $X_2 = \eta$, we find that

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1|\eta) = \frac{R^4 v_\varepsilon^4 v_\theta^2 v_z^2 v_\eta (2M-1)^2(1+k)}{D^2}, \tag{89}$$

which is always positive. Similarly, letting $X_1 = [P_3 - P_2, P_2 - P_1]$ and $X_2 = \theta$, we find that

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1|\theta) = -[1+k][(b_2'^2 v_\eta + (b_3'^2 + b_4'^2)v_z], \tag{90}$$

which is always negative. □

Proof of Proposition 5. This proposition follows directly from an examination of the expression on the right-hand side of Eq. (33). □

Proof of Proposition 6. Because D is a common denominator in both Eqs. (33) and (35), comparing the expected profits reduces to comparison of the numerators on the RHS of these equations. So the expected profits of the irrational traders exceed those of the rational traders if and only if

$$Rv_\varepsilon v_\eta v_\theta v_z(1-2M)[(k+2)M-1] > R^2 v_\varepsilon^2 v_\theta v_z[v_\eta\{(2M-1)^2 + M^2\} + R^2 v_\varepsilon^2 v_z]. \tag{91}$$

Straightforward algebra shows the equivalence of the above condition to Eq. (36), so long as $M < 0.5$. □

Proof of Proposition 7. From Eq. (33) to Eq. (35), the expression for the profit differential, denoted by $\Delta\pi$, is

$$\Delta\pi = \frac{Rv_\varepsilon v_\theta v_z[kMv_\eta(1-2M) - v_\eta(9M^2 - 8M + 2) - R^2 v_\varepsilon^2 v_z]}{D}. \tag{92}$$

The derivative of the above expression with respect to k is

$$\frac{MRv_\varepsilon v_\eta v_\theta v_z(1-2M)}{D}. \tag{93}$$

This is positive if and only if $M < 0.5$.

Similarly, the derivative of the right-hand side of Eq. (92) with respect to v_η is

$$\frac{R^3 v_\varepsilon^3 v_z^2 v_\theta(1-2M)[kM(R^2 v_\varepsilon^2 v_z + v_\theta) - (R^2 v_\varepsilon^2 v_z + 2v_\theta)(1-2M)]}{D^2}. \tag{94}$$

This is positive so long as $M < 0.5$ and Eq. (37) is satisfied.

Finally, differentiating the right-hand side of Eq. (92) with respect to v_θ yields

$$\begin{aligned} &R^3 v_\varepsilon^3 v_z^2[v_\eta\{(2M-1)^2 + M^2\} + R^2 v_\varepsilon^2 v_z] \\ &\quad \times [kMv_\eta(1-2M) - (9M^2 - 8M + 2)v_\eta - R^2 v_\varepsilon^2 v_z], \end{aligned} \tag{95}$$

divided by D^2, and the above expression is positive so long as $M < 0.5$ and Eq. (38) holds. □

Proof of Proposition 8. Substituting for $X^*(z)$ from Eq. (2) into Eq. (1) indicates that the optimized stakeholder profit is given by

$$C_0 + \left(C_1 + \frac{K_0^2}{2K_1}\right)z \tag{96}$$

and the stakeholder investment is linear in $z \equiv \mathrm{E}(\theta|P_1, P_2)$. The correlation between η and z is positive if and only if b_2' is positive, and this is true if and only if $M < 0.5$ (see Eq. (21)), proving Part 1.

It can be shown from Eqs. (20)–(23) that the ex ante variance of z is

$$D^{-1}[v_\theta^2(M^2 v_\eta + R^2 v_\varepsilon^2 v_z]. \tag{97}$$

The derivative of this quantity with respect to v_η is

$$-D^{-2}[R^4 v_\varepsilon^4 v_\theta^2 v_z^2 (2M-1)^2], \tag{98}$$

which is always negative, proving Part 2. □

Proof of Proposition 9. The coefficients b_1'–b_4' are given by

$$b_1' = \frac{v_\theta(M^2 v_\eta + R^2 v_\varepsilon^2 v_z)}{D}, \tag{99}$$

$$b_2' = \frac{\rho R^2 v_\varepsilon^2 v_\theta v_z (1-2M)}{D}, \tag{100}$$

$$b_3' = \frac{M\rho R v_\theta v_\varepsilon v_\eta (2M-1)}{D}, \tag{101}$$

and

$$b_4' = \frac{\rho R v_\varepsilon v_\theta (M^2 v_\eta + R^2 v_\varepsilon^2 v_z)}{D}, \tag{102}$$

where

$$D \equiv M^2 v_\eta (5\rho^2 R^2 v_\varepsilon^2 v_z + v_\theta) - 4M\rho^2 R^2 v_\varepsilon^2 v_\eta v_z + R^2 v_\varepsilon^2 v_z[\rho^2(R^2 v_\varepsilon^2 v_z + v_\eta) + v_\theta]. \tag{103}$$

Substituting for the above coefficients into the expressions on the right-hand sides of Eqs. (45) and (47) and simplifying, the expression $\pi - \pi_i$ is positive if and only if

$$N_m > N_m^* = N/[\chi v - 1], \tag{104}$$

where

$$\chi \equiv \frac{(7M^2 - 7M + 2)v_\eta + R^2 v_\varepsilon^2 v_z}{(9M^2 - 8M + 2)v_\eta + R^2 v_\varepsilon^2 v_z - kMv_\eta(1-2M)}. \quad \square \tag{105}$$

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