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Investor Overconfidence and the Forward Premium Puzzle

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ABSTRACT

We offer an explanation for the forward premium puzzle in foreign exchange markets based upon investor overconfidence. In the model, overconfident individuals overreact to their information about future inflation, which causes greater overshooting in the forward rate than in the spot rate. Thus, when agents observe a signal of higher future inflation, the consequent rise in the forward premium predicts a subsequent downward correction of the spot rate. The model can explain the magnitude of the forward premium bias and several other stylized facts related to the joint behavior of forward and spot exchange rates. Our approach is also consistent with the availability of profitable carry trade strategies.

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1 Introduction

Nominal interest rates reflect investor expectations about future inflation. If investors rationally forecast inflation, then (assuming perfect markets and risk-neutrality) currencies in which bonds offer high nominal interest rates should on average depreciate relative to low-nominal-interest-rate currencies.A strong empirical finding, however, is that at times when short-term nominal interest rates are high in one currency relative to another, that currency subsequently appreciates on average (see, e.g., surveys of Hodrick 1987, Lewis 1995, and Engel 1996). An equivalent finding is that the forward premium (defined as the difference between the forward and spot exchange rates) negatively forecasts subsequent exchange rate changes, a pattern known as the forward premium puzzle.[1]

The most extensively explored explanation for the forward premium puzzle is that it reflects time-varying rational premia for systematic risk (e.g., Fama 1984). However, the survey of Hodrick (1987) concludes that "we do not yet have a model of expected returns that fits the data" in foreign exchange markets; Engel (1996) similarly concludes that models of equilibrium risk premia do not explain the strong negative relation between the forward premium and the future exchange rate change for any degree of risk aversion, even when nonstandard utility functions are employed.[2] He therefore suggests that an approach based upon imperfect rationality can potentially offer new insights into the puzzle.

We propose an explanation for the forward premium puzzle based upon investor overconfidence, a well-documented psychological bias wherein people believe that they are better than they really are on various dimensions. According to DeBondt and Thaler (1995), overconfidence is "perhaps the most robust finding in the psychology of judgement." Our approach is based upon a large body of evidence from cognitive psychological experiments and surveys and from experimental markets indicating that people, including those from various professional fields, tend to overestimate the accuracy of their judgments in various domains. As summarized by Rabin (1998), "... there is a mass of psychological research that finds

[1] The average slope coefficient in regressions of future changes in the log spot exchange rate on the forward premium across some 75 published estimates surveyed by Froot and Thaler (1990) is -0.88.

[2] For example, Bekaert (1996) finds that his habit formation model would require unrealistically volatile exchange rates to deliver exchange rate risk premia that are variable enough to explain the forward premium puzzle. Verdelhan (2010) proposes a consumption-based model that can generate negative covariance between exchange rates and interest rate differentials. Burnside et al. (2010), however, provide new evidence suggesting that conventional models of time-varying exchange rate risk premia do not explain the forward premium puzzle. Carlson, Dahl, and Osler (2008) and Burnside, Eichenbaum and Rebelo (2009) show that a market microstructure approach can potentially shed light on the puzzle.

people are prone toward overconfidence in their judgments. The vast majority of researchers argue that such overconfidence is pervasive ...".[3] Biais et al. (2005) find that individuals who have greater judgmental overconfidence experience poorer trading performance in an experimental financial market. Consistent with the importance of judgmental overconfidence, Froot and Frankel (1989) provide evidence of overreaction in currency traders' expectations about future exchange rate depreciations. Furthermore, survey evidence indicates that currency market professionals tend to overestimate the precision of their information signals (Oberlechner and Osler, 2008).

A growing analytical and empirical literature has argued that investor overconfidence explains puzzling patterns in stock markets of return predictability, return volatility, volume of trading, and individual trading losses; Hirshleifer (2001) and Barberis and Thaler (2003) provide recent reviews. If a systematic bias such as overconfidence causes anomalies in stock markets, it should also leave footprints in bond and foreign exchange markets. An explanation for anomalies is more credible if it explains a wide range of patterns, rather than being tailored to just one puzzle in one type of market.

In our model, overconfident individuals think that their information signal about the future money growth differential is more precise than it actually is. As a result, investor expectations overreact to the signal. This causes both the forward and spot exchange rates to overshoot their average long run levels in the same direction. The consumption price level and the spot exchange rate are influenced by a transactions demand for money, whereas forward rates are additionally influenced by speculative considerations, i.e., the expected return from holding domestic or foreign bonds. In our monetary framework, which is conventional except for the presence of overconfident investors, these considerations cause the forward rate to overshoot more than the spot rate, which implies that the forward premium rises in response to a positive signal. Later, the overreaction in the spot rate is, on average, reversed. The rise in the forward premium is a predictor of this correction, and hence, on average (under reasonable parameter values), is a negative predictor of future exchange rate changes.

Another feature of foreign exchange markets is that professional forecast errors, defined as the difference between the exchange rate realization and the exchange rate forecast, are negatively correlated with the forward premium (Froot and Frankel, 1989; Bacchetta, Mertens and van Wincoop, 2009). Our model is consistent with this finding if we interpret the profes-

[3] There are, however, exceptions; see, for example, Clark and Friesen (2009).

sional forecasts as matching the expectations of investors in our model, because the forward rate reflects investor expectations of the future spot exchange rate. Since these expectations and the forward rate overreact to information more strongly than the current spot rate, a rise in the forward premium is associated with a negative forecast error.

The sign of the slope coefficient in a regression of the future spot rate change on the forward premium reflects two opposing effects. Overreaction to signals, as described above, favors a negative coefficient. On the other hand, any foreseeable component in the money growth differential that is not subject to overreaction favors the forward premium positively predicting future spot rate changes. This is the conventional effect that makes the empirical findings a puzzle.

Consistent with the data, we show that over short horizons the overreaction-correction effect dominates, but over long horizons the positive conventional effect eventually dominates. Intuitively, over time mispricing in the spot exchange rate attenuates, whereas the effects of foreseeable differences in expected money growth and inflation rates across countries accumulate. Thus, a distinctive feature of our model is that it explains evidence that the forward premium regression coefficients switch from negative to positive at very long horizons (Chinn and Meredith, 2004).

There is a tendency for countries with high average interest rates relative to the U.S. over long periods of time also to have high average depreciation relative to the dollar (e.g., Cochrane, 1999). Consequently, if average rates of depreciation against the dollar are regressed on average interest rate differentials, the slope coefficient in this cross-sectional regression is typically positive. Our model is consistent with this contrasting pattern in cross-sectional versus time-series regressions. In our model, the long-run averages of the interest rate differentials and rates of currency depreciation between countries reflect average money growth differentials, and tend to average out the transitory effects of mispricing. So the cross-sectional regression behaves conventionally. In contrast, as we have discussed above, mispricing plays a crucial role in the behavior of the time-series regression.

Our benchmark model assumes purchasing power parity (PPP), but the qualitative and quantitative implications do not rely upon this assumption. In Section 5, we modify the model to allow for deviations from PPP at the level of the aggregate consumer prices by incorporating nontraded goods and sticky prices as in Calvo (1983). We find that the magnitude of the forward premium bias remains about the same for reasonable values of the

Calvo price stickiness parameter. The modified model, however, has the desirable feature that the price level does not overshoot its average long-run level in response to a signal about future money growth differentials.

Our benchmark model also assumes that monetary policy is characterized by exogenous money growth. This allows us to capture the basic insight in closed-form. In reality, policy makers adjust short-term interest rates in response to economic conditions. In Section 6, we therefore characterize monetary policy as an interest rate rule. We show that if this rule is incorporated in our sticky price model we still obtain downward forward premium bias so long as there is overconfidence, where the magnitude of the bias is increasing in the degree of overconfidence.

Several recent papers have provided insightful analyses of how investor irrationality can potentially explain the forward premium puzzle.[4] An early application of irrationality to foreign exchange markets is provided by Frankel and Froot (1990a). In the model of Mark and Wu (1998), distortion in investors' beliefs is exogenously specified to occur in the first moment of exchange returns: noise traders overweight the forward premium when predicting future changes in the exchange rate. Gourinchas and Tornell (2004) offer an explanation of the forward premium puzzle based upon a distortion in investors' beliefs about the dynamics of the forward premium, but are agnostic as to the source of the distorted beliefs.[5]

Our paper differs from past behavioral explanations for the forward premium puzzle in possessing a combination of features: assumptions about belief formation based upon evidence from psychology, explicit modeling of the belief formation process, and explicit modeling of the equilibrium forward premium without making exogenous assumptions about its dynamics.[6] Furthermore, our approach provides a distinctive additional set of predictions about the forward premium bias, and the psychological bias that we assume has been shown to have realistic implications for security markets in general, not just the foreign exchange market.

[4] Bacchetta and Wincoop (2007, 2009) propose a middle ground between behavioral and fully rational risk-premium explanations for the forward discount puzzle. In their approach, the forward premium puzzle can result from a combination of infrequent and partial information processing.

[5] Gourinchas and Tornell (2004) assume that the forward premium follows a persistent process, but investors mistakenly perceive an additional transitory component in its dynamics. This distorted belief leads the nominal exchange rate to underreact to interest rate innovations, which is opposite to the overconfidence-induced overreaction in our model. Thus, the mechanism used by Gourinchas and Tornell (2004) to explain the forward discount puzzle is different from that studied here.

[6] McCallum (1994) also emphasizes the need for behavioral approaches to provide an underlying motivation for their assumptions about the form of irrationality or noise trading.

Specifically, we show that the average negative relationship between the forward premium and future exchange rate changes is a natural consequence of a well-documented cognitive bias—overconfidence. We derive price relationships from investor beliefs, rather than directly making assumptions about trading behavior. Furthermore, we do not assume that belief errors have a particular correlation with the forward premium, but rather derive this correlation from the psychological premise.

Overconfidence is not an ex post explanation chosen specifically to fit the forward premium puzzle. Investor overconfidence has been used to explain a range of other cross-sectional and time-series patterns of return predictability in securities markets as well as patterns in volume, volatility, and investor trading profits.[7] Thus, our approach offers a parsimonious explanation for a range of anomalies in asset markets, which helps avoid possible concerns about overfitting the theoretical model to the anomaly being explained.

A common challenge to psychology-based approaches to securities markets anomalies is to explain how irrational investors can have an important effect on market prices if there are smart arbitrageurs. In our setting, there is an opportunity for rational investors to profit from the currency carry trade, a strategy that exploits the forward premium bias. This involves borrowing money in a country with a low interest rate, and investing in another country with a higher interest rate. However, the risk inherent in carry trades limits the extent to which risk averse investors will engage in arbitrage.[8] Uncertainty about a country's inflation rate is a systematic risk, so that even if the market prices reflect incorrect expectations, rational investors are not presented with a risk-free arbitrage opportunity (on imperfect arbitrage of systematic mispricing, see Daniel, Hirshleifer, and Subrahmanyam, 2001).

Furthermore, the behavioral finance literature offers several reasons why irrational investors do not necessarily lose money competing with the rational ones, and why even if irrational investors are prone to losing money, imperfect rationality can still influence price.[9]

[7]Individual investors trade actively and on average lose money on their trades, which is consistent with overconfidence (e.g., DeBondt and Thaler, 1985, Barber and Odean, 2000). Investor overconfidence has been proposed as an explanation for several patterns in stock markets, such as aggressive trading and high return volatility (e.g., Odean 1998), price momentum, long-term reversals, and underreactions to corporate events (e.g., Daniel, Hirshleifer and Subrahmanyam, 1998; 2001), return comovements (Peng and Xiong, 2006), and speculative price bubbles (Scheinkman and Xiong, 2003).

[8]Predictability in excess currency returns implied by the forward premium puzzle is low (with R^2 typically less than 0.05) and largely overshadowed by uncertainty about future exchange rates (Bacchetta and Wincoop, 2006). The carry trade entails substantial risk. For example, the carry trade of Goldman Sachs' Global Alpha Fund between Japanese yen and Australian dollar led to major losses in August 2007.

[9]Reasons why imperfectly rational investors may earn high expected profits and/or remain important include a possible greater willingness of overconfident investors to bear risk or to exploit information ag-

For example, Biais and Shadur (2000) show that Darwinian selection does not eliminate irrational traders.

In the foreign exchange context, even if less sophisticated currency users on average lose relative to a set of smart speculators, less sophisticated individuals will still need to hold money balances, so their money demands will still play a role in determining equilibrium price levels and therefore spot and forward exchange rates. Hence, we do not expect complete elimination of the forward premium bias. Froot and Thaler (1990) and Burnside et al. (2006) provide evidence that market frictions and other practical constraints limit the profitability of trading strategies designed to take advantage of the forward premium anomaly.

2 The Basic Idea

In foreign exchange markets, there is a need for subjective judgment in forecasting future inflation, which creates scope for overconfidence.[10] To illustrate how overconfidence affects the forward premium regression,

$$s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \xi_{t+1}, \tag{1}$$

we now present a simple intuitive discussion that relies on risk neutrality and price dynamics that are typical in monetary models of the exchange rate. Figure 1 plots the path of movement for the spot and forward exchange rates from date -1 to date 1, conditional on a positive date-0 signal about the date-1 inflation differential. For ease of presentation, we assume that at date -1 the economy is in a steady state in which the expected future inflation differential is zero, so that the spot exchange rate s coincides with the forward rate f.

If there were no overreaction to the date-0 signal in the spot market, the spot exchange rate would rise to s_0^R at date 0, and would be expected to rise further to the point labeled $E_0^R(s_1)$ at date 1; the R superscript indicates values under rational beliefs. The two-step increase in the exchange rate in response to news about future inflation is a standard feature

gressively, limited investment horizons of the arbitrageurs, wealth reshuffling across generations, and the existence of market frictions. See Hirshleifer (2001) and Barberis and Thaler (2003) for discussions of these issues.

[10] The existence of an active industry selling macroeconomic forecasts is consistent with our assumption that at least some investors believe they can obtain high quality signals about future inflation. Previous studies share our assumption that individuals believe they possess meaningful information signals about aggregate macro-factors (e.g., Subrahmanyam, 1991).

of rational-expectations-based monetary models. Under risk neutrality, if there were no overreaction in the forward rate, it would be $f_0^R = E_0^R(s_1)$. The forward premium at date 0 would be exactly equal to the depreciation expected between date 0 and date 1, $f_0^R - s_0^R = E_0^R(s_1) - s_0^R > 0$. In this environment the coefficient in (1) would be one.

How can the coefficient be negative? The answer lies in the fact that both the spot and the forward exchange rates are market prices, each subject to its own misreaction.[11] Overconfidence causes investors to overreact to their date-0 signal about the future inflation differential, which drives the forward rate above the level predicted by rational expectations. We illustrate this overreaction in Figure 1, by assuming that the date-0 forward rate rises from $f = s$ to $f_0 > E_0^R(s_1)$. Meanwhile, such overreaction also causes the spot rate to overshoot its rational level in the same direction, so that $s_0 > s_0^R$. If investors are sufficiently overconfident in the quality of their signals, the spot rate can overshoot its long-run average level, $s_0 > E_0^R(s_1)$, this being the case illustrated in Figure 1.[12]

In summary, there are two possibilities. In both cases, the monetary model predicts that the forward rate overshoots more than the spot rate does in response to the positive inflationary signal; i.e. $f_0 - s_0 > 0$. In the first case, where investors have a modest degree of overconfidence, s_0 overshoots the rational-expectations equilibrium price, s_0^R, but not the long-run average level of the exchange rate, so that $E_0^R(s_1) - s_0 > 0$. In this scenario the coefficient β_1 in (1) is positive but less than one. In the second case, where investors have a stronger degree of overconfidence, s_0 overshoots its average long-run level, $E_0^R(s_1)$, so that $E_0^R(s_1) - s_0 < 0$. In this case, the coefficient β_1 in (1) is negative. In the latter scenario, the rise in the forward premium coincides with the overconfidence-induced overreaction in the spot exchange rate, but also predicts the subsequent correction in the spot rate. It is this effect that results in a negative slope coefficient.

[11] The asset market approach to exchange rate determination has long recognized that exchange rate movements are primarily driven by news that changes expectations (e.g., Obstfeld and Rogoff, 1996, p. 529).

[12] It is evident from this argument that if the spot rate did not overreact, the forward premium regression coefficient would be less than one, but could not be negative. This case can be likened to behavioral stock market models in which the market-to-book ratio is a negative predictor of future returns. In this analogy, the stock's market price corresponds to the forward rate (which overreacts strongly), and its book value corresponds to the spot rate (which does not overreact). This comparison indicates that we cannot fully explain the forward premium puzzle unless we take into account a distinctive feature of the monetary setting, the fact that the spot exchange rate (unlike book value) is a market price that itself can overreact.

3 The Model

In the basic model we consider a general equilibrium setting in which representative individuals in two countries hold their respective national monies because domestic real money balances directly provide utility.[13]

3.1 The Individual's Problem

We assume that the representative individual in the home country has additively separable preferences over consumption and real balances, with instantaneous utility function

$$U(c_t, m_t) = \ln(c_t) + \varrho \frac{m_t^{1-\nu}}{1-\nu}, \tag{2}$$

where c_t is aggregate consumption, m_t is the date t stock of real money balances, and $\varrho > 0$ and $\nu > 0$ are constants.

We assume that a complete set of state contingent securities are available for sequential trade. Let the individual's net purchases of a claim to one unit of consumption at date $t+1$ if state of the world z_{t+1} is realized be denoted $a_t(z_{t+1})$, and let the date t price of this claim be denoted $q_t(z_{t+1})$. Then the individual's date t budget constraint is

$$c_t + \int_{z_{t+1}} q_t(z_{t+1}) a_t(z_{t+1}) dz_{t+1} + m_t = y + \tau_t + a_{t-1} + m_{t-1}\Pi_t^{-1}, \tag{3}$$

where Π_t is the gross domestic inflation rate, y is a constant endowment of the single good, and τ_t is lump sum transfers received from the government. In what follows, we denote the history of events up to date t by $z^t \equiv (\ldots, z_{t-1,} z_t)$.

The stock of domestic money at date t is M_t. At date t, the domestic government creates new money, $M_t - M_{t-1}$, that it transfers to the representative individual. The real value of this transfer is $\tau_t = (M_t - M_{t-1})/P_t$, where P_t is the domestic price level. Since $\Pi_t \equiv P_t/P_{t-1}$, it follows that $\tau_t = m_t - m_{t-1}\Pi_t^{-1}$.

At each date t, the individual chooses c_t, m_t, a function $a_t(\cdot)$, and contingency plans for future values of these variables, to maximize lifetime expected utility, subject to the budget constraint, (3), and no-Ponzi scheme condition, $a_t(z_{t+1}) \geq -y/(1-\beta)$, at all dates, where β is the time discount parameter. The individual's problem can be represented by the Bellman

[13] Qualitatively similar results could be derived using a setting in which money has value because it reduces the transaction cost of making consumption purchases.

equation:

$$V\left(a_{t-1}, m_{t-1}; z^t\right) = \max_{c_t, m_t, a_t(\cdot)} \ln(c_t) + \varrho \frac{m_t^{1-\nu}}{1-\nu} + \beta \int_{z_{t+1}} V\left[a_t(z_{t+1}), m_t; z^{t+1}\right] \psi_t(z_{t+1}|z^t) dz_{t+1}$$

subject to (3), where $\psi_t(z_{t+1}|z^t)$ is the individual's subjective probability density function over z_{t+1} given z^t. The first order conditions for the individual's problem are

$$\begin{aligned}
c_t &: \quad c_t^{-1} = \lambda_t \\
m_t &: \quad \varrho m_t^{-\nu} + \beta \int_{z_{t+1}} V_m\left[a_t(z_{t+1}), m_t; z^{t+1}\right] \psi_t(z_{t+1}|z^t) dz_{t+1} = \lambda_t \\
a_t(z_{t+1}) &: \quad \beta V_a\left[a_t(z_{t+1}), m_t; z^{t+1}\right] \psi_t(z_{t+1}|z^t) = \lambda_t q_t(z_{t+1}),
\end{aligned}$$

where λ_t is the Lagrange multiplier on the budget constraint. The envelope conditions are

$$\begin{aligned}
a_{t-1} &: \quad V_a\left(a_{t-1}, m_{t-1}; z^t\right) = \lambda_t \\
m_{t-1} &: \quad V_m\left(a_{t-1}, m_{t-1}; z^t\right) = \lambda_t / \Pi_t.
\end{aligned}$$

Combining conditions to eliminate V_a, V_m and λ_t we have

$$c_t^{-1} = \varrho m_t^{-\nu} + \beta \int_{z_{t+1}} \left[c_{t+1}(z_{t+1}) \Pi_{t+1}(z_{t+1})\right]^{-1} \psi_t(z_{t+1}|z^t) dz_{t+1} \tag{4}$$

$$c_t^{-1} q_t(z_{t+1}) = \beta c_{t+1}(z_{t+1})^{-1} \psi_t(z_{t+1}|z^t) \tag{5}$$

We write (4) more compactly as

$$c_t^{-1} = \varrho m_t^{-\nu} + \beta E_t^C c_{t+1}^{-1} \Pi_{t+1}^{-1}, \tag{6}$$

where the expectations operator E_t^C denotes the individual's subjective expectation given time-t information.

A domestic nominal bond purchased at date t pays one unit of domestic money in all states at date $t+1$, or equivalently pays $1/P_{t+1}(z_{t+1})$ units of consumption if event z_{t+1} is realized. Therefore, its price in units of consumption at date t is $Q_t = \int_{z_{t+1}} P_{t+1}(z_{t+1})^{-1} q_t(z_{t+1}) dz_{t+1}$. Using (5) to substitute out $q_t(z_{t+1})$ from this expression, the price of the bond in domestic currency units (DCUs) at date t is

$$Q_t = \int_{z_{t+1}} \beta \left[c_{t+1}(z_{t+1})/c_t\right]^{-1} \Pi_{t+1}(z_{t+1})^{-1} \psi_t(z_{t+1}|z^t) dz_{t+1} = \beta E_t^C \left(c_{t+1}/c_t\right)^{-1} \Pi_{t+1}^{-1}. \tag{7}$$

The foreign economy is symmetric to the domestic economy. In what follows, foreign variables are denoted by an asterisk (*). Individuals in the two economies have the same preferences, except that foreign individuals derive utility from holding real balances in foreign currency. Individuals in the two economies also have the same information sets and subjective probability distributions over events. First order conditions symmetric to (6) and (7) hold in the foreign economy. We define the growth rates of domestic and foreign money as $\mu_t = \ln(M_t/M_{t-1})$ and $\mu_t^* = \ln(M_t^*/M_{t-1}^*)$, where μ_t and μ_t^* are stationary stochastic processes to be specified in more detail below.

3.2 Competitive Equilibrium

The world level of output is constant and given by $y + y^*$, so the market clearing condition for goods is

$$c_t + c_t^* = y + y^*. \tag{8}$$

We assume that there are no trade frictions, so PPP holds,

$$P_t = S_t P_t^*. \tag{9}$$

Since the world endowment of goods is constant, equilibrium consumption in the two countries is constant across time and states of the world, $c_t = c$ and $c_t^* = c^*$ for all t.

We simplify the first order conditions (6) and (7) using the condition that $c_t = c$ for all t, and the fact that $\Pi_t = m_{t-1}e^{\mu_t}/m_t$:

$$1 = \varrho m_t^{-\nu} c + \beta E_t^C e^{-\mu_{t+1}}(m_{t+1}/m_t), \tag{10}$$

$$Q_t = 1 - \varrho m_t^{-\nu} c. \tag{11}$$

The nominal interest rate in the domestic economy is $i_t = Q_t^{-1} - 1$. Given a law of motion for the money growth rate, μ_t, (10) is solved for the equilibrium value of m_t. Given this solution, (11) is solved for the equilibrium price of the domestic bond. Analogs of (10) and (11) in the foreign economy are solved for m_t^* and Q_t^*. Finally, the PPP condition is used to solve for the nominal exchange rate, which gives

$$S_t = \frac{P_t}{P_t^*} = \left(\frac{M_t/m_t}{M_t^*/m_t^*}\right). \tag{12}$$

3.3 Information and Expectations

For simplicity, we assume that the only source of random variation in the two economies is monetary policy. Specifically, we assume that money growth at date $t+1$ in the home country is

$$\mu_{t+1} = \mu + \eta_{t+1} + u_t, \tag{13}$$

where $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ and $u_t \sim N(0, \sigma_u^2)$ are i.i.d. processes. At date t individuals observe only a noisy signal about u_t, $\zeta_t = u_t + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is an i.i.d. process. We assume that money growth in the foreign country has a similar law of motion. Individuals' date-t information set contains (μ_t, η_t, ζ_t), all lagged information, $\{\mu_{t-j}, \eta_{t-j}, \zeta_{t-j}, u_{t-j}, \epsilon_{t-j}\}_{j=1}^{\infty}$, and the foreign analogs of these processes. We assume that u_t, ϵ_t, η_t, u_t^*, ϵ_t^* and η_t^* are mutually independent, although covariance between the shocks in the two countries could easily be accommodated.

As in several previous models of overconfidence in securities markets, we model overconfidence as overestimation of signal precision. Overestimation of the precision of the signal ζ_t means that individuals underestimate the variance of ϵ_t, assuming that it is equal to $\sigma_C^2 < \sigma_\epsilon^2$. Thus, overconfident individuals take the noisy signal as more informative than it actually is.

After receiving the signal ζ_t, individuals update their expectations about future money growth in a Bayesian fashion, subject to their misperceptions of signal precision. We describe the degree of overconfidence using the parameter γ, where

$$\gamma \equiv 1 - \frac{\lambda^R}{\lambda^C}, \quad \lambda^C \equiv \frac{\sigma_u^2}{\sigma_C^2 + \sigma_u^2}, \quad \lambda^R \equiv \frac{\sigma_u^2}{\sigma_\epsilon^2 + \sigma_u^2}.$$

The superscripts C and R denote overconfident and rational perceptions. Since individuals in the model are overconfident, λ^C describes their belief updating, i.e., $E_t^C u_t = \lambda^C \zeta_t$. Similarly, λ^R describes how a rational individual would update, i.e., $E_t^R u_t = \lambda^R \zeta_t$. Since $\sigma_C^2 < \sigma_\epsilon^2$, it follows that $\lambda^C > \lambda^R$ and thus $E_t^C u_t > E_t^R u_t$. In other words, overconfident individuals overreact to the date-t signal about future money growth. The parameter γ measures the degree of overconfidence. Since $\lambda^C > \lambda^R$, $\gamma > 0$. The lower is σ_C^2, the greater is the degree of overconfidence and the larger is γ. At date $t+1$, the true value of u_t is revealed, and individuals correct their date-t expectational errors.

We allow for differences between σ_u^2, σ_ϵ^2, σ_η^2 and their foreign counterparts so that λ^R varies by country. However, we assume that γ is common to the two countries, since overconfidence is a property of individual perceptions, not of the underlying processes for μ_t and μ_t^*.

3.4 Prices, Interest Rates and Exchange Rates

We solve for the non-stochastic steady state value of real money balances, m, by setting $m_{t+1} = m_t = m$ and $\mu_{t+1} = \mu$ in (10), which gives $m = [\varrho c / (1 - \beta e^{-\mu})]^{1/\nu}$. To solve the model, we use a log linear approximation in the neighborhood of the nonstochastic steady state:

$$E_t^C \hat{m}_{t+1} - \alpha^{-1} (1+\alpha) \hat{m}_t = E_t^C \hat{\mu}_{t+1}, \tag{14}$$

where $\hat{m}_t \equiv (m_t - m)/m$, $\hat{\mu}_t \equiv \mu_t - \mu$, $i \equiv \beta^{-1} e^{\mu} - 1$ is the steady state interest rate, and $\alpha \equiv 1/(\nu i)$ (details are provided in the Appendix). A similar condition holds in the foreign country, where the steady state interest rate is i^* is $\beta^{-1} e^{\mu^*} - 1$, and $\alpha^* \equiv 1/(\nu i^*)$.

The unique stationary solution for $\hat{m}_t$ is given by

$$\hat{m}_t = -\sum_{j=1}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^j E_t^C \hat{\mu}_{t+j}. \tag{15}$$

Given the information structure and overconfident expectations described above, $E_t^C \hat{\mu}_{t+1} = \lambda^C \zeta_t$ and $E_t^C \hat{\mu}_{t+j} = 0$ for $j > 1$. Hence

$$\hat{m}_t = -\kappa \zeta_t, \tag{16}$$

where $\kappa = \alpha (1+\alpha)^{-1} \lambda^C$. Similarly, in the foreign country

$$\hat{m}_t^* = -\kappa^* \zeta_t^*, \tag{17}$$

where ζ_t^* is the signal of future foreign money growth, and $\kappa^* = \alpha^* (1+\alpha^*)^{-1} \lambda^{C*}$.

We can write (11), once linearized, in terms of $\hat{\imath}_t = (i_t - i)/(1+i)$:

$$\hat{\imath}_t = -\hat{m}_t / \alpha = (\kappa/\alpha) \zeta_t. \tag{18}$$

>From (18) we see that α can be interpreted as the semi-elasticity of the demand for real money balances with respect to the nominal interest rate. Similarly, the foreign interest rate is

$$\hat{\imath}_t^* = (\kappa^*/\alpha^*) \zeta_t^*. \tag{19}$$

With these solutions in hand, a log-linear approximation of the spot exchange rate can be obtained from (12). Letting $s_t \equiv \ln S_t$, and using (18) and (19), to first order,

$$s_t = \ln(m^*/m) + \kappa \zeta_t - \kappa^* \zeta_t^* + \ln(M_t / M_t^*). \tag{20}$$

This implies that the change in the spot rate is

$$\begin{aligned} \Delta s_{t+1} &= \kappa \Delta \zeta_{t+1} - \kappa^* \Delta \zeta_{t+1}^* + \mu_{t+1} - \mu_{t+1}^* \\ &= \bar{\mu} + \kappa \Delta \zeta_{t+1} - \kappa^* \Delta \zeta_{t+1}^* + \bar{\eta}_{t+1} + \bar{u}_t, \end{aligned} \tag{21}$$

where $\bar{\eta}_t \equiv \eta_t - \eta_t^*$, $\bar{u}_t \equiv u_t - u_t^*$ and $\bar{\mu} \equiv \mu - \mu^*$.

Finally, covered interest parity holds in this economy, $1 + i_t = (1 + i_t^*)F_t/S_t$, where F_t is the forward exchange rate. Letting $f_t \equiv \ln F_t$, to first order,

$$f_t - s_t = \bar{\mu} + \hat{\imath}_t - \hat{\imath}_t^* = \bar{\mu} + (\kappa/\alpha)\zeta_t - (\kappa^*/\alpha^*)\zeta_t^*. \tag{22}$$

4 Empirical Predictions of the Model

We now explore several empirical predictions of the basic model. The central result is that individuals' overconfidence causes the forward rate to overshoot more than the spot rate, making the forward premium a measure of overreaction and a predictor of the subsequent correction in the spot rate. We show that with sufficient overconfidence, the model is consistent with a wide variety of empirical findings found in the literature.

4.1 Forward Premium Regressions

Proposition 1 *In the model, the probability limits of the estimated coefficients in the forward premium regression, (1), are*

$$\beta_0 = (1 - \beta_1)\bar{\mu}$$

$$\beta_1 = 1 - \gamma \frac{(1+\alpha)\left(\frac{\kappa}{\alpha}\right)^2 \mathrm{var}(\zeta_t) + (1+\alpha^*)\left(\frac{\kappa^*}{\alpha^*}\right)^2 \mathrm{var}(\zeta_t^*)}{\left(\frac{\kappa}{\alpha}\right)^2 \mathrm{var}(\zeta_t) + \left(\frac{\kappa^*}{\alpha^*}\right)^2 \mathrm{var}(\zeta_t^*)}.$$

If the two countries have the same average money growth rates, $\bar{\mu} = 0$, then

$$\beta_0 = 0 \text{ and } \beta_1 = 1 - \gamma(1+\alpha).$$

Proof: See the appendix.■

Proposition 1 shows that overconfidence can explain the forward premium puzzle. When there is no overconfidence (i.e., $\gamma = 0$), $\beta_1 = 1$, $\beta_0 = 0$, and uncovered interest rate parity (UIP) holds. When there is overconfidence (i.e., $\gamma > 0$), β_1 is less than unity and becomes negative with a sufficiently high level of investor overconfidence.

To understand the result, it is helpful to impose symmetry across the two money growth processes. Assume, therefore, that $\bar{\mu} = 0$, and that the processes u_t, u_t^*, ϵ_t, and ϵ_t^* are such that $\lambda^R = \lambda^{R*}$. These assumptions imply that $\alpha = \alpha^*$, $\lambda^C = \lambda^{C*}$ and $\kappa = \kappa^*$. As the proposition states, they also imply that $\beta_0 = 0$ and $\beta_1 = 1 - \gamma(1+\alpha)$.

When the change in the spot exchange rate is regressed on the lagged one-period forward premium, the slope coefficient, β_1, can be decomposed into two terms. The first term in β_1 is unity, which reflects the conventional force that drives the usual UIP result. The second term reflects investor overconfidence. The more overconfident investors are, the more negative the relationship between the forward premium and the subsequent exchange rate depreciation. If $\gamma > 1/(1+\alpha)$, then $\beta_1 < 0$.

To further understand Proposition 1 it is helpful to consider the responses of the domestic interest rate and the exchange rate to the arrival, at date 0, of a unit signal about domestic money growth at date 1. We first consider the case where individuals have rational expectations ($\lambda^C = \lambda^R$ and $\gamma = 0$). This is a case where UIP holds.

A unit signal shock at date 0 is any combination of u_0 and ϵ_0 such that $\zeta_0 = 1$. Figure 2(a) illustrates the effects of the signal shock for two such combinations.[14] In the first scenario, the signal is false ($u_0 = 0$ and $\epsilon_0 = 1$), whereas in the second scenario, the signal is true ($u_0 = 1$ and $\epsilon_0 = 0$). Since individuals do not observe u_0 and ϵ_0, but know their distributions, expectations can be calculated as if the two scenarios have probabilities $1-\lambda^R$ and λ^R.[15] Thus, individuals expect domestic money growth at date 1 to rise by λ^R. The expectation of higher future money growth causes the demand for real money balances to drop by $-\alpha\,(1+\alpha)^{-1}\,\lambda^R$. Consequently, the domestic price level rises and, from (20), the exchange rate depreciates by $\alpha\,(1+\alpha)^{-1}\,\lambda^R$.

At date 1, suppose the signal is revealed to be false. In this case, the price level and exchange rate go back to their original levels by falling $-\alpha\,(1+\alpha)^{-1}\,\lambda^R$. If, however, the signal is revealed to be true, the price level and exchange rate go to their new long-run levels (1 unit higher than their initial values) by rising a further $1-\alpha\,(1+\alpha)^{-1}\,\lambda^R$. Under rational expectations, the expected change in the exchange rate from date 0 to date 1 is a weighted average of these two possibilities:

$$E_0^R \Delta s_1 = (1-\lambda^R)\left(-\frac{\alpha}{1+\alpha}\lambda^R\right) + \lambda^R\left(1-\frac{\alpha}{1+\alpha}\lambda^R\right) = \frac{\lambda^R}{1+\alpha}.$$

By equation (18), the interest rate at date 0 rises by the same amount $\hat{\imath}_0 = \lambda^R/(1+\alpha)$. The forward premium regression's slope coefficient is the ratio of these two expressions because

[14] For the purposes of Figure 2(a), we set $\alpha = 7$ and $\lambda^R = 0.35$.

[15] Since u_0 and ϵ_0 are normally distributed, the signal extraction problem faced by individuals implies that when they see $\zeta_0 = 1$, they expect domestic money growth in the future to be higher by $\lambda^R = \sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2)$. This is equivalent to individuals believing that money growth will increase by 1 with probability λ^R and by 0 with probability $1-\lambda^R$.

there are no expected deviations of the interest rate from its steady state value from date 1 forward, and no expected changes in the exchange rate from date 2 forward. The slope coefficient is, therefore, 1.

Now consider the case where individuals are overconfident, which is illustrated in Figure 2(b).[16] In this case, we simply replace λ^R with λ^C in the above discussion. Thus, when overconfident individuals see a unit signal at date 0, they expect the exchange rate to change at date 1 by

$$E_0^C \Delta s_1 = (1-\lambda^C)\left(-\frac{\alpha}{1+\alpha}\lambda^C\right) + \lambda^C\left(1-\frac{\alpha}{1+\alpha}\lambda^C\right) = \frac{\lambda^C}{1+\alpha}.$$

By equation (18), the interest rate at date 0 rises by the same amount, $\hat{\imath}_0 = \lambda^R/(1+\alpha)$. Given the true values of σ_u^2 and σ_ϵ^2, however, the mean change of the exchange rate at date 1 is equal to

$$E_0^R \Delta s_1 = (1-\lambda^R)\left(-\frac{\alpha}{1+\alpha}\lambda^C\right) + \lambda^R\left(1-\frac{\alpha}{1+\alpha}\lambda^C\right) = \lambda^C\left(\frac{1}{1+\alpha}-\gamma\right). \tag{23}$$

Thus, the forward premium regression's slope coefficient is $1-\gamma(1+\alpha)$. If individuals are sufficiently overconfident, as they are in Figure 2(b), with $\gamma > 1/(1+\alpha)$, then the slope coefficient is negative.

The key factor driving the result in the overconfidence case is that individuals' expectations put excessive weight on the quality of the signal. As Figures 2(a) and 2(b) illustrate, the effects of overconfidence are twofold. First, the exchange rate overreacts to a signal at date 0. Second, individuals put too little weight on the possibility that the exchange rate will correct at date 1. The combination of these two effects, when sufficiently strong, leads to a negative slope coefficient in the forward premium regression.

We illustrate the magnitude of the forward premium bias generated by the model using realistic values for α and the overconfidence parameter γ. The average α estimate for six countries (Austria, Germany, Greece, Hungary, Poland, and Russia) in Table 3 of Cagan (1956) is 5.36. Cagan's estimate is relevant for our analysis, because, to first order, the expression for money demand in our model, (18), is equivalent to Cagan's. For the same six countries, the average α estimate in Table 2 of Sargent (1977) and Table 1 of Goodfriend (1982) are 4.1 and 3.7 respectively. Bailey (1956) uses $\alpha = 7$ in his specification. Lucas (2000) finds that $\alpha = 7$ best fits the U.S. data for the 1900-1994 period. Phylaktis and

[16] For the purposes of Figure 2(b), we set $\alpha = 7$, $\lambda^R = 0.35$ and $\gamma = 0.3$.

Taylor (1993) find higher α for five Latin American countries with an average estimate of 12.0. Based on these studies, we use α in the range between 3 and 11.[17]

For the overconfidence parameter γ, we rely on Friesen and Weller (2006) who measure the magnitude of analysts' overconfidence using earnings forecast data. They define overconfidence in the same manner as we do, and assume that analysts perceive the precision of their private signals to be $1+a$ times the true signal precision. That is, $\sigma_C^2 = \sigma_\epsilon^2/(1+a)$ in our notation. It is straightforward to verify that our overconfidence parameter γ is related to the parameter a in Friesen and Weller (2006) by

$$\gamma = \frac{a}{1+a}\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_u^2}.$$

The estimated value for a in Table 3 of Friesen and Weller (2006) is 0.94 with a standard error of 0.027. Thus, a value of 0.5 for $a/(1+a)$ is reasonable. Since $0 < \sigma_\epsilon^2/(\sigma_\epsilon^2+\sigma_u^2) < 1$, we use γ in the range between 0.1 and 0.4.

Table 1 provides estimates of β_1 using monthly data over the period 1983–2008 for 11 major currencies. As can be seen in Table 1, the average estimate of β_1 is roughly -1, but the estimates of β_1 vary considerably around this value. Figure 3 illustrates the magnitude of the forward premium bias generated by our model. As Figure 3 shows, under realistic parameter values ($3 < \alpha < 11$ and $0.1 < \gamma < 0.4$) our model generates forward premium bias that closely matches the magnitudes observed in the data.

4.2 Further Empirical Implications in the Time Series Dimension

For simplicity, throughout this subsection, we impose the symmetry condition that $\bar{\mu} = 0$ and we assume that the processes u_t, u_t^*, ϵ_t, and ϵ_t^* are such that $\lambda^R = \lambda^{R*}$. Therefore we have $\alpha = \alpha^*$, $\lambda^C = \lambda^{C*}$ and $\kappa = \kappa^*$.

4.2.1 Prediction Errors and the Forward Premium

As in Froot and Frenkel (1989) we can decompose the change in the spot rate as

$$\Delta s_{t+1} = E_t^C \Delta s_{t+1} + s_{t+1} - E_t^C s_{t+1}.$$

[17] The interest rate semi-elasticity, α, relates the logarithm of real balances to the nominal interest rate elasticity, which is the parameter α' in the log-log money demand function of the form $m_t - p_t = c - \alpha' \ln i_t$. Barro (1970) theoretically argues and empirically supports the hypothesis that $\alpha' = 0.5$, while Ball (2001) estimates a money demand elasticity $\alpha' = 0.05$ using U.S. post-war data. Given a 5 percent steady state interest rate, values of α between 3 and 11 translate into values of α' between 0.15 and 0.55.

The first component, $E_t^C \Delta s_{t+1}$, is individuals' forecast of the change in the spot rate. The second component is individuals' forecast error. By construction, the slope coefficient obtained from the forward premium regression can therefore be interpreted as the sum of the slope coefficients obtained from regressing $E_t^C \Delta s_{t+1}$ and the forecast error separately on the forward premium. This observation leads to the following proposition.

Proposition 2 *When investors are overconfident, their prediction errors are negatively correlated with the forward premium and account for all of the bias in the forward premium regression.*

Proof: The change of the exchange rate at date $t+1$ is given by (21). It follows that overconfident investors' forecast of the change in the exchange rate is

$$E_t^C \Delta s_{t+1} = -\kappa \bar{\zeta}_t + E_t^C \bar{u}_t = (\kappa/\alpha)\bar{\zeta}_t,$$

where $\bar{\zeta}_t = \zeta_t - \zeta_t^*$. This is the same as the expression for the forward premium derived from (22), $f_t - s_t = (\kappa/\alpha)\bar{\zeta}_t$. So the slope coefficient in a regression of $E_t^C \Delta s_{t+1}$ on $f_t - s_t$ is unity. It follows that the slope coefficient in a regression of the forecast error $s_{t+1} - E_t^C s_{t+1}$ on $f_t - s_t$ is $-\gamma(1+\alpha)$.■

The implication that forecast errors are negatively correlated with the forward premium is supported by the empirical findings in Froot and Frankel (1989), who regress $s_{t+1}^e - s_{t+1}$ on the forward premium, where s_{t+1}^e is the average investor forecast (based on survey data) of the spot exchange rate at date $t+1$. Froot and Frankel find that the slope coefficient in their pooled regressions is significantly greater than zero; this finding is robust across surveys. Since the left-hand-side of Froot and Frankel's regression equation is the negative of forecasters' expectational error, this indicates that forecast errors are negatively correlated with the forward premium.

Froot and Frankel (1989) also regress $s_{t+1}^e - s_t$ on the forward premium. Interpreting s_{t+1}^e as $E_t^C s_{t+1}$, our model predicts that the slope coefficient in this regression should be unity. For seven out of the nine surveys reported by Froot and Frankel the slope coefficient in this regression is insignificantly different from one.[18]

[18] Frankel and Chinn (1993) find similar results using a broader set of individual currency pairs. For 24 out of 34 currency/time period pairs they report the slope coefficient to be insignificantly different from one.

4.2.2 Longer-Horizon Regressions

In Section 4.1, we show that there can be a negative relationship between the one-period forward premium and the subsequent one-period change in the spot exchange rate. We now examine the relation between the forward premium and the future spot rate change in a longer-horizon regression. Specifically, we consider a regression of the n-period change in the spot exchange rate $s_{t+n} - s_t$ on the n-period forward premium $f_{nt} - s_t$, where f_{nt} is the n-period forward exchange rate. We examine whether the slope coefficient will be more or less negative for $n > 1$ than for $n = 1$.

Proposition 3 *In a regression of the n-period change in the spot exchange rate, $s_{t+n} - s_t$, on the n-period forward premium, $f_{nt} - s_t$, under our benchmark assumptions, the constant and slope coefficient are*

$$\beta_{n0} = 0 \text{ and } \beta_{n1} = 1 - \gamma\,(1 + \alpha)\,.$$

Proof: In the model

$$s_{t+n} - s_t = \sum_{j=1}^{n} \left(\kappa \Delta \bar{\zeta}_{t+j} + \bar{\eta}_{t+j} + \bar{u}_{t+j-1}\right)$$

and $f_{nt} - s_t = E_t^C\,(s_{t+n} - s_t)$. The latter condition follows from the fact that UIP holds for agents' expectations. Given our assumptions about ζ_t, η_t and u_t, $E_t^C\,(s_{t+n} - s_{t+1}) = E_t^R\,(s_{t+n} - s_{t+1}) = 0$. Hence $f_{nt} - s_t = E_t^C\,(s_{t+1} - s_t) = f_t - s_t$ and $s_{t+n} - s_t$ is equal to $s_{t+1} - s_t$ plus a mean zero error term that is orthogonal to time-t information. Therefore, the coefficients in a regression of $s_{t+n} - s_t$ on f_{nt} are the same as in a regression of $s_{t+1} - s_t$ on $f_t - s_t$. ■

Proposition 3 implies that when investors are overconfident, the n-period forward premium is still a biased predictor of the subsequent n-period exchange rate depreciation and the degree of bias does not vary with the forward horizon.

This implication of Proposition 3 is consistent with empirical results obtained with $n = 1$, 3, 6 and 12 months. These are shown in Table 1. Although there is some variation in the degree of bias, the average slope coefficient in the regressions does not vary systematically with the forward horizon. In fact, the typical slope coefficient remains very close to -1 at all forward horizons.

At much longer horizons of five and ten years, Chinn and Meredith (2004) find less negative forward premium bias. Our model can easily accommodate Chinn and Meredith's findings if we generalize our assumptions about $\bar{\eta}_t$, which is the difference, across the two countries, between the mean-zero unpredictable shocks to money growth. In particular, if we assume that $\bar{\eta}_t$ is persistent, and has considerably less variability than $\bar{\zeta}_t$, we can easily rationalize the stylized facts at both short and long horizons.

Proposition 4 *If the shock to the money growth differential, $\bar{\eta}_t$, is a persistent AR(1) process, with AR parameter ρ, then the slope coefficient in the n-period forward premium regression is increasing in n and is given by*

$$\beta_{n1} = 1 - \gamma (1 + \alpha) \varpi_n,$$

where

$$\varpi_n = \frac{(1+\alpha)^{-2} \left(\lambda^C\right)^2}{(1+\alpha)^{-2} \left(\lambda^C\right)^2 + \left[\frac{\rho}{1-\rho} (1 - \rho^n)\right]^2 \operatorname{var}(\bar{\eta}_t) / \operatorname{var}(\bar{\zeta}_t)}.$$

Proof: See the appendix. ■

We draw three main conclusions from Proposition 4. First, since $\varpi_n > 0$, as long as agents are overconfident ($\gamma > 0$) it follows that the slope coefficient $\beta_{n1} < 1$ for all n. Second, if $\gamma > (1+\alpha)^{-1}$, then, for any particular value of n, there is always a sufficiently small value of $\operatorname{var}(\bar{\eta}_t)$, such that $\beta_{n1} < 0$. Third, when $\rho > 0$, ϖ_n is decreasing in n, implying, as asserted, that β_{n1} is increasing in n. Therefore, the amount of forward premium bias diminishes as the investment horizon increases. In order for the value of β_{n1} to be negative and relatively insensitive to n at short horizons, and yet be close to 1 at horizons of five to ten years, this version of the model requires a value of ρ close to 1, and a very small value of $\operatorname{var}(\bar{\eta}_t)$ relative to $\operatorname{var}(\bar{\zeta}_t)$.

The reason that forward premium bias is less negative at longer horizons is that the persistence in $\bar{\eta}_t$ produces a foreseeable component of money growth that strengthens over time. This induces unbiased predictability in $s_{t+n} - s_t$ that eventually dominates the short-term (and biased) predictability that derives from the innovation $\bar{\zeta}_t$.

4.2.3 McCallum Regressions

McCallum (1994) runs a set of regressions

$$s_{t+1} - s_{t-n} = \bar{\beta}_{n0} + \bar{\beta}_{n1} (f_t - s_{t-n}) + \bar{\xi}_{t+1}$$

for $n \geq 1$. McCallum finds that the slope coefficient in the regression is typically very close to one and the R^2 is typically close to $n/(n+1)$. In Table 2 we show similar estimates for a set of 11 developed country currencies over the period 1983–2008. As McCallum points out these results are easily understood if we consider the fact that $s_{t+1} - s_{t-n} = \Delta s_{t+1} + s_t - s_{t-n}$ and $f_t - s_{t-n} = f_t - s_t + s_t - s_{t-n}$. That is, the regressand and the regressor in McCallum's regression are the regressand and the regressor in the forward premium regression plus a common term, $s_t - s_{t-n}$. Given that the variance of Δs_t is roughly 100 times that of $f_t - s_t$ for the currencies in our sample, it is not surprising that the coefficient in the regression approaches unity as n increases. For similar reasons, the R^2 ends up being close to $n/(n+1)$.

In our model, the results for these regressions depend not only on the degree of overconfidence, γ, and the interest elasticity parameter, α, but also on $\text{var}(\bar{\eta}_t)$. To derive model predictions we set $\alpha = 7$ and $\gamma = 0.25$, so that the model-implied value of the forward premium regression slope coefficient is $\beta_1 = -1$. We assume the shocks in the two countries have equal variances and calibrate σ_u, σ_ϵ and σ_η so that $\lambda^R = 0.35$ and the model-predicted values of $\text{var}(s_{t+1} - s_t)$ and $\text{var}(f_t - s_t)$ correspond to their average values in our 11-country data set. In this case we find that the slope coefficients in the McCallum regressions for $n = 1$, 2, 3 and 4 are, respectively, $\bar{\beta}_{11} = 0.89$, $\bar{\beta}_{21} = 0.94$, $\bar{\beta}_{31} = 0.96$ and $\bar{\beta}_{41} = 0.97$ (see details in the Appendix). Thus, for $n \geq 1$, in both the model and the data the slope coefficient is close to unity. As n increases, in both the model and the data, the slope coefficient becomes closer to one, although in the model it approaches from below one while in the data it approaches from above one for most currencies.

4.2.4 The Superiority of the Forward Premium Regression as a Predictor of the Future Spot Rate

In our model, the forward premium predicts exchange rate changes both for the traditional reason that it contains information about the future money growth differential, and for the reason that it contains information about spot rate mispricing. Therefore, the forward premium regression provides one method of forecasting the future value of the spot exchange rate. Similar reasoning implies that regressions based on the realized inflation differential, or the latest changes in the spot rate or forward rate, might also be useful for predicting future exchange rates. We also consider whether the current forward rate, or the current spot rate, is the best predictor of the future spot rate. We show that the forward premium regression provides a better predictor of the subsequent spot rate, in terms of mean squared error, than any of the alternatives.

Proposition 5 *In the model, the forward premium regression provides a minimum mean squared error predictor of the subsequent exchange rate. It dominates alternative predictors suggested by our model (regressions based on the inflation differential, or the latest changes in the forward rate or spot rate, or the simple levels of the current forward or spot rates).*

Proof: The proof of the first part of the proposition is straightforward. Given our results, we can write $\Delta s_{t+1} = \kappa\Delta\bar{\zeta}_{t+1} + \bar{u}_t + \bar{\eta}_{t+1}$. Since $\bar{u}_t$ is not observed at time t, and $E_t^R\bar{u}_t = \lambda^R\bar{\zeta}_t$, we can write

$$\Delta s_{t+1} = (\lambda^R - \kappa)\bar{\zeta}_t + w_{t+1} \tag{24}$$

where $w_{t+1} = \kappa\bar{\zeta}_{t+1} + \bar{\eta}_{t+1} + \bar{u}_t - \lambda^R\bar{\zeta}_t$ is orthogonal to time-t information. Suppose there is a variable x_t, such that $x_t = a_x\bar{\zeta}_t$ for some scalar $a_x \neq 0$. A regression of Δs_{t+1} on x_t has slope coefficient $(\lambda^R - \kappa)/a_x$ and the forecast of Δs_{t+1} generated by x_t is $(\lambda^R - \kappa)\bar{\zeta}_t$. Therefore, the forecast error associated with this forecasting rule is w_{t+1}. No other forecasting rule can improve on this result given that (24) holds, and w_{t+1} is unpredictable. Since $f_t - s_t = (\kappa/\alpha)\bar{\zeta}_t$, the forecasts generated by the forward premium regression are minimum mean squared error forecasts. The proof of the second part of the proposition is provided in the appendix.■

Intuitively, the only information that is useful in forecasting the exchange rate is $\bar{\zeta}_t$, and the forward premium is linked one-to-one with $\bar{\zeta}_t$. The reason for the second part of the proposition is that the regression-based alternative forecasts contain noise from the viewpoint of predicting exchange rate changes. These forecasts are linear combinations of $\bar{\zeta}_t$ and extraneous noise, such as η_t. Using the current forward (spot) rate as the forecast of the future exchange rate is suboptimal because it is equivalent to using the forward premium regression to forecast, while imposing the false restrictions that $\beta_0 = 0$ and $\beta_1 = 1$ ($\beta_1 = 0$).

The model's predictions correspond reasonably closely to empirical findings based on our 11-country sample. The calibrated version of the model described above predicts that there are only modest differences in the accuracy of the different forecasting methods, because variability in Δs_{t+1} is dominated by unpredictable components. In particular, the calibration of the model given in Section 4.2.3 implies that the forward premium regression has a root mean squared error (RMSE) of 2.89 percent, while the regressions based on the change in the forward and spot rates have RMSEs of 2.96 and 2.95 percent, respectively. Using the current forward and spot rates as the forecasts of s_{t+1} leads to RMSEs of 2.93 and 2.90 percent, respectively. These values are all close to their empirical counterparts in Table 3, which shows the RMSE associated with each of the forecasting methods. Consistent with the model, the forward premium regression typically provides the most accurate forecast.

4.3 Cross-Sectional Regressions

Suppose that Δs_{t+1} and $f_t - s_t$ are averaged over many time periods. When the steady state money growth rates are different in countries i and j but the shock processes are identical and independent of one another, it follows from (21) and (22), that in very large samples $\overline{\Delta s}_{ij} = \overline{f - s}_{ij} = \bar{\mu}_{ij}$, where $\bar{\mu}_{ij}$ is the steady state money growth rate differential between the two economies. Hence, in a cross sectional regression where the individual observations are pairs of $\overline{\Delta s}_{ij}$ and $\overline{f - s}_{ij}$ for individual pairs of countries, i and j, the slope coefficient obtained will be one and the constant will be zero.

Thus, the model predicts that in contrast with time series regressions, in cross-sectional regressions with a sufficiently long sample period there is no forward premium puzzle. Intuitively, over short time horizons forward and spot exchange rates are influenced by misperceptions about future money growth rates, but over long time horizons the average rates of exchange rate depreciation are determined by the average money growth differentials between pairs of countries. In general, such average effects tend to be picked up in cross-sectional regressions, while the short-term mispricing effects tend to be muted. If the sample period is sufficiently long, the effects of investor overconfidence will be eliminated in cross-sectional regressions.

4.4 The Carry Trade: An Extension

The model implies that a rational investor could, on average, earn excess returns by holding the short term debt of countries whose short term interest rates are temporarily higher relative to the interest rates of other countries. In other words, there are profits to be earned through carry trade strategies. *The Economist* (2007) quotes the estimate of one market analyst that as much as $1 trillion may be invested in the yen carry trade.

Since we have assumed that investors are identically overconfident, in equilibrium they do not engage in the carry trade to exploit the forward premium bias. However, all results of the model carry through identically if we introduce a set of risk averse rational investors with measure zero. In such an extended model, the risk-bearing capacity of rational investors is too small to allow them to affect prices non-negligibly. However, they would engage in carry trade activity to profit from the forward premium bias.[19]

It is worth emphasizing that the model predicts that profits are available by trading based on transitory interest rate differentials, not by holding the treasury bills of a country that persistently has a higher average nominal interest rate. This is consistent with the empirical findings that countries with steadily higher interest rates and inflation (than in the U.S.) have had steady currency depreciations (against the U.S. dollar), as predicted by UIP (see, for example, Cochrane, 1999).

5 A Model with Deviations from PPP

So far we have assumed that PPP holds for the aggregate consumption deflators in the two economies. In this section we extend the model to allow for violations of PPP for aggregate price deflators by introducing nontraded goods. We model nontraded goods producers as monopolistic competitors and we introduce price stickiness. This lets us study the role of overconfidence in a more realistic setting in which prices adjust slowly to inflationary shocks. Although introducing these new features in our model affects the magnitude of the implied forward premium bias, we find that the main implications of the model do not dependent upon the PPP assumption we made in Section 3. For a wide range of plausible parameter values, overconfidence continues to imply substantial bias in the forward premium regression.

In the expanded model there are two goods, traded and nontraded. Traded goods are modeled, as before, as an endowment. Each country produces its own nontraded good with labor. We separately consider households, who make decisions about consumption, labor effort, and money and asset holdings, and monopolistically competitive firms, who make decisions about production, labor input and the pricing of goods.

5.1 The Household's Problem

We now assume that representative household in the domestic economy has the instantaneous utility function

$$U(c_t, m_t, n_t) = \ln(c_t) + \varrho \frac{m_t^{1-\nu}}{1-\nu} - \vartheta \frac{n_t^{1+\phi}}{1+\phi}, \tag{25}$$

[19] If, instead, we were to introduce a set of rational investors with positive measure, we conjecture that this would reduce but not eliminate the forward premium bias. In many behavioral models, prices reflect the beliefs of both rational and irrational traders, with weights that depend on the risk bearing capacities of the different groups (see, e.g., Daniel, Hirshleifer and Subrahmanyam, 2001).

where c_t is aggregate consumption, m_t is the date-t stock of real money balances, n_t is labor, and $\varrho > 0$, $\nu > 0$, $\vartheta > 0$ and $\phi > 0$ are constants.

Aggregate consumption is given by

$$c_t = c_{Tt}^{\omega} c_{Nt}^{1-\omega}, \tag{26}$$

where c_{Tt} is the consumption of traded goods, c_{Nt} is the consumption of nontraded goods, and $0 < \omega < 1$. There is a single traded good. The output of this good, in each country, is a constant endowment. There are many differentiated nontraded goods within each country. In the home country, the consumption of nontraded goods is defined as

$$c_{Nt} = \left(\int_0^1 c_{Nit}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ and c_{Nit} is the date-t consumption of variety i of the nontraded goods. Households take the prices of all goods as given. Consequently, it is straightforward to show that the price index for nontraded goods is

$$P_{Nt} = \left(\int_0^1 P_{Nit}^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}, \tag{27}$$

where P_{Nit} is the price of variety i, and that the aggregate price index is

$$P_t = \frac{1}{\omega^{\omega}(1-\omega)^{1-\omega}} P_{Tt}^{\omega} P_{Nt}^{1-\omega}, \tag{28}$$

where P_{Tt} is the price of traded goods.

Given the sequence of budget constraints,

$$c_t + \int_{z_{t+1}} q_t(z_{t+1}) a_t(z_{t+1}) dz_{t+1} + m_t = y + w_t n_t + v_t + \tau_t + a_{t-1} + m_{t-1}\Pi_t^{-1}, \tag{29}$$

for $t \geq 0$, the household chooses c_0, n_0, m_0, a function $a_0(\cdot)$, and contingency plans for the future values of these variables in order to maximize

$$E_0^C \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, n_t).$$

Here $q_t(\cdot)$, $a_t(\cdot)$, y, τ_t and Π_t are defined as in Section 3, w_t is the real wage rate, and v_t represents the real value of any profits distributed by firms to households.

The household optimally sets its consumption of traded and nontraded goods according to

$$c_{Tt} = \omega c_t P_t / P_{Tt} \tag{30}$$

$$c_{Nt} = (1-\omega) c_t P_t / P_{Nt} \tag{31}$$

$$c_{Nit} = (P_{Nit}/P_{Nt})^{-\varepsilon} c_{Nt}. \tag{32}$$

The first order conditions for n_t, m_t and $a_t(z_{t+1})$ imply that

$$\vartheta n_t^{\phi} c_t = w_t, \tag{33}$$

$$c_t^{-1} = \varrho m_t^{-\nu} + \beta E_t^C c_{t+1}^{-1} \Pi_{t+1}^{-1}, \tag{34}$$

$$Q_t = \beta E_t^C (c_{t+1}/c_t)^{-1} \Pi_{t+1}^{-1}, \tag{35}$$

where Q_t is the price of a domestic nominal bond purchased at date t that pays one unit of domestic money in all states at date $t+1$.

5.2 The Firm's Problem

The nontraded good is produced by monopolistic competitors. For simplicity we assume that the output of variety i of the nontraded good is simply

$$x_{Nit} = n_{it}, \tag{36}$$

where n_{it} is the amount of labor hired by the producer of variety i.

We assume that prices are sticky as in Calvo (1983) and Yun (1996), and follow Galí (2008) in our description of the representative firm's problem. At each date firms have a probability $1-\theta$, of being able to change their prices, with drawings independent over time. As in the Calvo-Yun framework, we assume that firms choose their prices taking the demand curve for their product as given. Once they set price they satisfy demand by hiring enough labor to produce a sufficient quantity of their product. Suppose that a firm last set its price at time t, the price it set was $\bar{P}_{Nt}$, and it satisfies demand at this price at all future dates until able to change its price again.[20] Then if it has not changed its price by date $t+k$, its profit at date $t+k$ is

$$\left(\bar{P}_{Nt} - W_{t+k}\right) \left(\frac{\bar{P}_{Nt}}{P_{Nt+k}}\right)^{-\varepsilon} c_{Nt+k},$$

where W_t is the nominal wage rate. A firm that is able to choose its price at time t chooses $\bar{P}_{Nt}$ to maximize

$$E_t^C \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(\bar{P}_{Nt} - W_{t+k}\right) \left(\frac{\bar{P}_{Nt}}{P_{Nt+k}}\right)^{-\varepsilon} c_{Nt+k},$$

[20] We suppress the i subscript from $\bar{P}_{Nt}$ because all firms who are able to change price choose the same price.

where $M_{t,t+k} = \beta^k (c_t/c_{t+k})(P_t/P_{t+k})$, W_{t+k}, P_{Nt+k}, and c_{Nt+k} are taken as given for $k \geq 0$.[21] The θ^k term reflects the probability of the firm still not having changed its price by date $t+k$, while $M_{t,t+k}$ is the appropriate discount factor given that the firm is owned by the representative household.

Because the firm's problem is forward looking, and there is no capital in the model, the optimal decision does not depend on anything specific to the firm. Any firm that is able to change its price will choose the same price. The first order condition for the firm's problem is

$$E_t^C \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(\frac{\bar{P}_{Nt}}{P_{Nt+k}} \right)^{-\varepsilon} c_{Nt+k} \left(1 - \varepsilon + \varepsilon \frac{W_{t+k}}{\bar{P}_{Nt}} \right) = 0. \tag{37}$$

5.3 Market Clearing Conditions

The world endowment of traded goods is constant and given by $y + y^*$, so the market clearing condition for traded goods is

$$c_{Tt} + c_{Tt}^* = y + y^*. \tag{38}$$

In each country there are market clearing conditions for labor and nontraded goods. In the domestic economy these are

$$n_t = \int_0^1 n_{it} di \tag{39}$$

and

$$c_{Nit} = x_{Nit}, \forall i \in [0, 1]. \tag{40}$$

5.4 Characterizing the Equilibrium

We continue to assume that there are no trade frictions, so PPP holds for traded goods,

$$P_{Tt} = S_t P_{Tt}^*. \tag{41}$$

We continue to assume that the money growth rate is given by (13).

Given the laws of motion for domestic and foreign money growth, an equilibrium in our model is a set of sequences of prices and quantities such that the households in the domestic economy are maximizing utility subject to their budget constraints (i.e. equations (29)—(35) are satisfied), domestic firms make choices consistent with (36) and (37), foreign households

[21] For symmetry we assume that firms are also overconfident. Under the alternative assumption that firms have rational expectations, nontraded goods prices would presumably adjust more slowly to inflationary signals.

and firms make choices consistent with the foreign analogs of (29)—(35), (36) and (37), and markets clear (i.e. equations (38), (39), (40) and the foreign analogs of the latter two equations are satisfied).

Given the constant global endowment of traded goods, equilibrium consumption of traded goods in each country is constant across time and states of the world, $c_{Tt} = c_T$ and $c^*_{Tt} = c^*_T$ for all t. We linearize the equilibrium conditions to characterize an approximate solution to the model. Unfortunately, the model does not have a simple closed form solution as in the basic model of Section 3. In the appendix, however, we show that the equilibrium of the model can be characterized by the solution of a first-order difference equation in the deviations of the variables m_t, $p_{Nt} \equiv P_{Nt}/P_{Tt}$, $\Pi_{Tt} \equiv P_{Tt+1}/P_{Tt}$, and $\Pi_{Nt} \equiv P_{Nt+1}/P_{Nt}$ from their nonstochastic steady state values. For simplicity, we follow Galí (2008), by solving the model in the neighborhood of a zero inflation steady state (in our case, in both countries). So we set $\mu = \mu^* = 0$.

In the basic model the forward premium regression coefficient depended on the semi-elasticity of money demand, $\alpha = 1/(\nu i)$, and the degree of overconfidence, γ. In the expanded model it depends on these parameters as well as the "Calvo parameter," θ, the discount factor, β, the share of traded goods in utility, ω, and the parameter that determines the labor supply elasticity, ϕ. To illustrate the response of the interest differential and the exchange rate to shocks, we set $\alpha = 7$ and $\gamma = 0.3$. To calibrate the rest of the parameters we think of the natural length of a time interval as one quarter. Consistent with the basic sticky price models illustrated in Galí (2008) we therefore set $\theta = 2/3$ and $\phi = 1$. We set $\beta = 0.995$, consistent with a two percent real interest rate. Finally, we set $\omega = 0.35$.[22]

Figure 4 illustrates the responses of the domestic interest rate and the exchange rate to the date-0 arrival of a unit signal about domestic money growth at date 1. As Figure 4 indicates, the behavior of the interest rate and the exchange rate in response to a signal shock is very similar to that in the basic model. The exchange rate depreciates on impact and overshoots. Overconfident agents expect the exchange rate to keep depreciating, and this initially drives the domestic interest rate up.

Three differences between the expanded model and the basic model are worthy of note. First, domestic prices do not move one-for-one with the exchange rate. Because nontraded

[22] This value is approximately consistent with the share of nontraded goods in GDP used by Burnside, Eichenbaum and Rebelo (2006), and is similar to the CPI weight of tradables, 0.4, used by Burstein, Eichenbaum and Rebelo (2007).

goods prices are sticky, they initially rise by much less (0.21 percent) than the exchange rate depreciates (0.47 percent). As time passes, more firms adjust prices so that in the long run nontraded goods prices on average rise by as much as the exchange rate depreciates. Second, because of the slow adjustment of nontraded goods prices, the overall price level does not overshoot its long-run level. Third, on average there is a very small and temporary decline in the interest rate (relative to the initial steady state) after date 0. This decline in the interest rate reflects the fact that agents foresee a very slight appreciation of the domestic currency after date 1. This expected appreciation is driven by the real effects of the shock. Sticky prices cause an increase in the demand for nontraded goods, and after the impact period this has the net effect that the demand for real money balances rises, causing a slight appreciation of the exchange rate.

In our calibrated example, the forward premium regression coefficient is $\beta_1 = -1.05$, somewhat higher than the value of -1.4 implied by the basic model when $\alpha = 7$ and $\gamma = 0.3$. Figure 5 illustrates the sensitivity of β_1 to the Calvo price stickiness parameter, θ, and the overconfidence parameter γ. The basic model of Section 3 is equivalent to the expanded model when $\theta = 0$.

As before, we find that the amount of bias in β_1 is sensitive to the degree of agents' overconfidence. The amount of bias in β_1, on the other hand, is not very sensitive to the value of θ for $0 \leq \theta \leq 0.5$. Most calibrations of the Calvo parameter assume $\theta \leq 2/3$. Assuming θ lies in this range and $0.1 < \gamma < 0.4$, we see that the expanded model's quantitative predictions for the forward premium regression are similar to those of the basic model. Overall, we conclude that the conclusions we drew from the basic model are not dependent on the assumption that PPP holds at the level of aggregate consumption.

6 Monetary Policy and Interest Rate Rules

It is arguable that in reality the quantity of money is endogenous, and that monetary policy determines the nominal interest rate. We therefore modify the extended model of the previous section by characterizing monetary policy in each country as an interest rate rule.[23] Since the money growth rate is endogenous in this setting, we abandon the exogenous law of motion for money growth given by (13). Having done so, we need to modify the model to

[23] See, for example, Benigno and Benigno (2003), Clarida, Galí and Gertler (2001), Gali and Monacelli (2005), and McCallum and Nelson (2000).

incorporate an exogenous source of inflationary shocks that agents receive signals about.

There are many possibilities, but to keep our model as simple as possible we assume that the production technology in the nontraded goods sector, previously given by (36), is instead given by

$$x_{Nit} = e^{\chi_t} n_{it}. \tag{42}$$

Here, χ_t is a shock to the production technology. In standard sticky price models, negative shocks to technology lead to increases in inflation (see Galí, 2008). We assume that the law of motion for χ_t is given by

$$\chi_t = \eta_t + u_{t-1}, \tag{43}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$ and $u_t \sim N(0, \sigma_u^2)$ are i.i.d. processes. As we assumed before, at date t individuals observe a noisy signal about u_t, $\zeta_t = u_t + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is an i.i.d. process.

To close the model we assume that monetary policy in the domestic economy is given by

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\phi_\pi E_t^C \hat{\pi}_{t+1} + (1-\rho_i)\phi_y \hat{c}_{Nt}, \tag{44}$$

where, as before, $\hat{\imath}_t$, $\hat{\pi}_t$, and $\hat{c}_{Nt}$ represent deviations of the interest rate, the inflation rate, and the consumption of nontraded goods from their steady state values. Our interest rate rule has two important features. First, policy makers smooth the path of the interest rate if $\rho_i > 0$. Second, policy makers are forward looking and set the current interest rate by responding to agents' expectations of future inflation and output.[24] Our assumption that the interest rate rule is forward-looking with respect to inflation is consistent with the evidence provided by Clarida, Galí and Gertler (1998).

Apart from these changes the model remains the same as the model in the previous section. We work through the details of the model's solution in the appendix. Importantly, as with the models studied earlier, without overconfidence ($\gamma = 0$) there is no forward premium bias; $\beta_1 = 1$ regardless of other parameters. In other words, in the model, overconfidence is required to explain the forward premium bias.

To illustrate that the model continues to imply forward premium bias for reasonable parameterizations, we maintain the calibration of Section 5. We must also specify the parameters of the interest rate rule, ρ_i, ϕ_π and ϕ_y. Here we present impulse response functions

[24] Here, because the endowment of traded goods is assumed to be constant, deviations of output from the steady state are proportional to deviations of nontraded consumption from the steady state.

for two calibrations. In the first calibration we set $\rho_i = 0.95$, $\phi_\pi = 2$, and $\phi_y = 0.5$, values similar to those estimated by Clarida, Galí and Gertler (1998) for the Federal Reserve in the post-1982 period. In the second calibration we set $\rho_i = 0.95$, $\phi_\pi = 7.5$, and $\phi_y = 0$. Here, the monetary authority responds much more aggressively to inflation and has no concern with the output gap.

We now illustrate what happens to the economy when agents receive a negative signal about the level of technology at date 0. Impulse response functions for the first calibration are illustrated in Figure 6. In terms of dynamics, this case is quite similar to the one studied in the previous section (the expanded model with a money growth rule), but with two exceptions: the interest rate rises persistently in response to the shock, reflecting interest rate smoothing, and the price level slightly overshoots its long-run level. The model-implied value of the slope coefficient in the forward premium regression for this case is $\beta_1 = -3.1$.

Impulse response functions for the second calibration are illustrated in Figure 7. When the central bank responds more aggressively to inflationary expectations, as it does in this case, the exchange rate actually appreciates upon the arrival of the signal. On average the exchange rate continues to appreciate for one more period, overshooting its long-run level. Additionally, consumer prices rise immediately in response to the signal. This implies that in some parameterizations our model is consistent with evidence that in some cases unanticipated but temporary increases in inflation are associated with short-term exchange rate appreciation (see, for example, Andersen et. al., 2003 and Clarida and Waldman, 2008). Intuitively, investors foresee that the central bank will respond to news of high inflation with aggressive tightening. The model-implied value of the slope coefficient in the forward premium regression in this case is $\beta_1 = -0.6$.

Given the results of the two calibrations, it is clear that the value of the slope coefficient in the forward premium regression is highly sensitive to the parameterization of the interest rate rule. To investigate this further we computed β_1 for a wide range of values of ϕ_π and ϕ_y. Our results are shown in Figure 8. When the interest rate rule is less aggressive (ϕ_π is smaller), there is more forward premium bias (β_1 is more negative). This empirical implication of the theory merits further empirical testing. The degree of forward premium bias is relatively insensitive to ϕ_y in the range we consider.

We conclude that as long as investors are sufficiently overconfident, quantitatively plausible forward premium bias emerges from our model even when monetary policy takes the form of an interest rate rule. The model can, at the same time, capture the stylized fact that unanticipated increases in inflation are associated with exchange rate appreciation in the short run. In the model this occurs if monetary policy responds aggressively enough to changes in expected inflation.

7 Concluding Remarks

Motivated by evidence from the psychology of individual judgment, we offer an explanation for the forward premium puzzle in foreign exchange markets based upon investor overconfidence. In the model, investors overreact to information about future inflation, which causes greater overshooting in the forward rate than in the spot rate. Thus, the forward premium reflects the overreaction in the spot rate and predicts its subsequent correction. The forward premium bias results when this overreaction-correction effect dominates the conventional effect implied by UIP. The model can explain the magnitude of the forward premium bias, its greater strength at short time horizons than at long time horizons and in the time series than in the cross section, and other stylized facts about forward and spot exchange rates and professional forecasts. Our approach is also consistent with the availability of profitable carry trade strategies.

Our approach suggests some further directions for theoretical and empirical exploration. First, the model implies that the forward premium bias will be more pronounced in periods in which investors are more overconfident. Previous behavioral finance research suggests that overconfidence-induced overreaction is associated with greater trading volume, high return volatility, large cross-firm valuation dispersion, and strong return momentum and reversals (at different time horizons). This suggests testing whether the forward premium bias is more pronounced in periods where such variables are high.

Second is to consider whether overconfidence explains the ability of the term structure of domestic interest rates to predict bond returns. The bond pricing literature has identified that yield differentials between long and short-term bonds predict bond returns (e.g., Fama and Bliss 1987, Campbell and Shiller 1991, Cochrane and Piazzesi 2005). Our finding that overconfidence can explain the forward premium puzzle, which also involves return predictability based upon bond yield differentials, suggests that overconfidence may offer an integrated explanation of return predictability anomalies in domestic bond markets as well.

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TABLE 1: Forward Premium Regressions
Monthly Data, 1983–2008

	Forward Horizon, n				Sample
	1 Month	3 Months	6 Months	1 Year	
Australia	-0.726	-0.523	-0.529	-0.893	84:12–08:12
	(0.686)	(0.647)	(0.659)	(0.602)	
Canada	-1.047	-0.704	-0.305	-0.233	84:12–08:12
	(0.663)	(0.610)	(0.662)	(0.805)	
Switzerland	-0.634	-1.215	-1.256	-0.880	83:10–08:12
	(0.989)	(0.986)	(1.021)	(0.945)	
Denmark	-0.580	-0.213	-0.448	-0.571	84:12–08:12
	(0.677)	(0.780)	(1.013)	(1.039)	
UK	-2.077	-1.325	-0.750	-0.279	83:10–08:12
	(1.088)	(1.362)	(1.337)	(1.134)	
Japan	-0.511	-2.572	-2.976	-2.906	83:10–08:12
	(0.894)	(0.956)	(0.897)	(0.792)	
Norway	0.943	0.070	-0.258	-0.348	84:12–08:12
	(0.701)	(0.743)	(0.690)	(0.701)	
New Zealand	-1.146	-0.920	-0.981	-0.918	84:12–08:12
	(0.512)	(0.431)	(0.517)	(0.499)	
Sweden	-0.429	-0.055	-0.073	0.034	84:12–08:12
	(0.761)	(1.178)	(1.262)	(1.257)	
Singapore	-0.864	-0.997	-0.539	-0.625	84:12–08:12
	(0.667)	(0.562)	(0.566)	(0.567)	
Euro	-3.317	-1.782	-2.683	-4.512	99:1–08:12
	(2.231)	(2.365)	(2.194)	(1.574)	
Average	-0.944	-0.930	-0.982	-1.103	
Ave. Std. Err.	(0.897)	(0.965)	(0.983)	(0.901)	

Notes: The table reports estimates of β_{n1} from the regression $s_{t+n} - s_t = \beta_{n0} + \beta_{n1}(f_{nt} - s_t) + \epsilon_{t+n}$, where s_t is the logarithm of the spot exchange rate, and f_{nt} is the logarithm of the n-month forward exchange rate, measured in USD per FCU. Heteroskedasticity consistent standard errors are provided in parentheses. Data source: Datastream BBI end-of-month exchange rates.

TABLE 2: McCallum Regressions
Monthly Data, 1983–2008

	$n=1$		$n=2$		$n=3$		$n=4$	
	$\bar{\beta}_n$	R^2	$\bar{\beta}_n$	R^2	$\bar{\beta}_n$	R^2	$\bar{\beta}_n$	R^2
Australia	1.086	0.535	1.043	0.710	1.029	0.782	0.995	0.818
	(0.077)		(0.063)		(0.049)		(0.039)	
Canada	1.041	0.516	1.023	0.688	1.013	0.756	1.031	0.807
	(0.073)		(0.057)		(0.046)		(0.043)	
Switzerland	1.055	0.511	1.042	0.687	1.030	0.771	1.017	0.818
	(0.063)		(0.042)		(0.036)		(0.032)	
Denmark	1.068	0.522	1.061	0.703	1.040	0.788	1.021	0.830
	(0.062)		(0.043)		(0.036)		(0.031)	
UK	1.089	0.535	1.050	0.698	1.044	0.775	1.046	0.825
	(0.075)		(0.048)		(0.041)		(0.035)	
Japan	1.047	0.512	1.077	0.701	1.059	0.784	1.042	0.829
	(0.066)		(0.047)		(0.034)		(0.028)	
Norway	1.092	0.547	1.077	0.720	1.049	0.792	1.027	0.826
	(0.071)		(0.047)		(0.038)		(0.032)	
New Zealand	1.035	0.499	1.009	0.666	1.075	0.772	1.033	0.814
	(0.087)		(0.057)		(0.044)		(0.039)	
Sweden	1.168	0.578	1.084	0.734	1.066	0.808	1.050	0.847
	(0.076)		(0.049)		(0.035)		(0.030)	
Singapore	1.046	0.512	1.030	0.687	1.011	0.770	0.980	0.810
	(0.075)		(0.065)		(0.053)		(0.049)	
Euro	1.201	0.573	1.079	0.719	1.031	0.788	1.001	0.823
	(0.088)		(0.084)		(0.066)		(0.060)	
Average	1.084	0.531	1.052	0.701	1.041	0.781	1.022	0.822
Ave. Std. Err.	(0.074)		(0.055)		(0.043)		(0.038)	

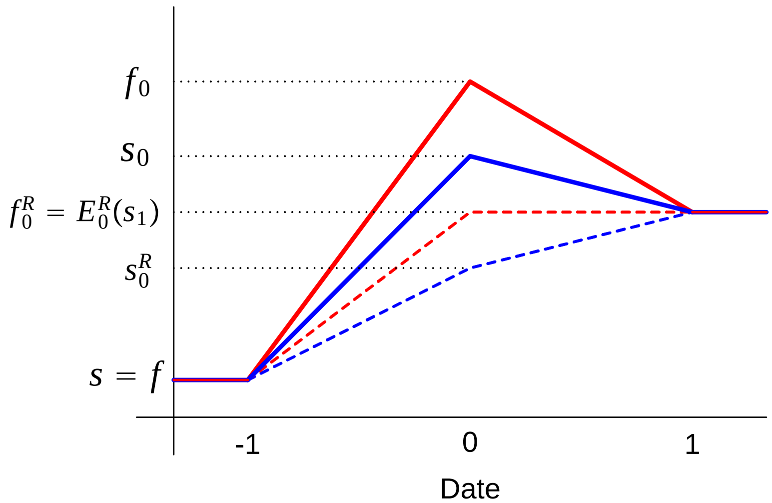
Notes: The table reports estimates of $\bar{\beta}_{n1}$ from the regression $s_{t+1} - s_{t-n} = \bar{\beta}_{n0} + \bar{\beta}_{n1}(f_t - s_{t-n}) + \epsilon_{t+1}$, where s_t is the logarithm of the spot exchange rate, and f_t is the logarithm of the one-month forward exchange rate, measured in USD per FCU. Heteroskedasticity consistent standard errors are provided in parentheses. Data source: Datastream BBI end-of-month exchange rates. The sample period for each currency is provided in Table 1.

TABLE 3: ROOT MEAN SQUARED ERROR OF EXCHANGE RATE FORECASTS (percent) Monthly Data, 1983–2008

	Simple Forecasting Rules		Regression-based Forecasts		
			$s^e_{t+1} = s_t + \hat{\alpha} + \hat{\beta} x_t$		
	$s^e_{t+1} = f_t$	$s^e_{t+1} = s_t$	$x_t = f_t - s_t$	$x_t = \Delta f_t$	$x_t = \Delta s_t$
Australia	3.326	3.295	3.289	3.285	3.285
Canada	1.880	1.856	1.848	1.857	1.856
Switzerland	3.414	3.403	3.392	3.395	3.395
Denmark	3.159	3.119	3.103	3.105	3.104
UK	3.068	3.031	3.004	3.021	3.021
Japan	3.327	3.319	3.301	3.306	3.307
Norway	3.096	3.101	3.083	3.091	3.094
New Zealand	3.390	3.273	3.238	3.273	3.272
Sweden	3.206	3.174	3.171	3.133	3.130
Singapore	1.528	1.503	1.487	1.491	1.490
Euro	2.894	2.869	2.826	2.809	2.805
Average	2.935	2.904	2.886	2.888	2.887

Notes: The table reports the root mean squared error (RMSE), in log-percent, of five forecasting rules for the logarithm of the exchange rate. For the simple forecasting rules, the forecast s^e_{t+1} is set equal to either the log of the one-month forward rate, f_t, or the log of the current spot rate, s_t. For the regression-based methods we use in-sample forecasts, in the sense that the regression coefficients used to generate the forecast are estimates based on the full sample of data. In each case the regression is $s_{t+1} - s_t = \alpha + \beta x_t + \epsilon_{t+1}$, and the forecast is formed as $s^e_{t+1} = s_t + \hat{\alpha} + \hat{\beta} x_t$, where $\hat{\alpha}$ and $\hat{\beta}$ are the estimated coefficients and x_t is the indicated right-hand-side variable. Data source: Datastream BBI end-of-month exchange rates. The sample period for each currency is provided in Table 1.

FIGURE 1: Overreaction and Correction of Exchange Rates

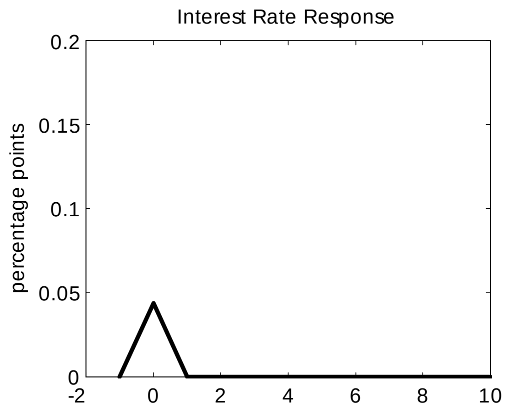


Notes: This graph illustrates the average path of movement for the spot and forward exchange rates from date -1 to date 1 conditional on a date-0 signal about the date-1 money growth (or inflation) differential. The figure depicts the case of a positive signal, that is a rise in domestic money growth relative to foreign money growth at date 1. Paths for the forward rate are indicated in red, for the spot rate in blue. Paths under rational expectations are indicated by dashed lines, under overconfidence by solid lines. In response to the signal, under rational expectations, at date 0 the spot and forward exchange rates rise from $s = f$ to s_0^R and f_0^R, respectively, with $f_0^R > s_0^R$. At date 1, the average realizations of the two rates are both $E_0^R(s_1)$. When agents are overconfident, at date 0 the spot and forward exchange rates rise from $s = f$ to $s_0 > s_0^R$ and $f_0 > f_0^R$, with $f_0 > s_0$. The graph illustrates the case where the overconfidence effect is strong enough that s_0 overshoots its long-run average level, $E_0^R(s_1)$. Thus, a rise in the forward premium predicts a downward correction in the spot exchange rate.

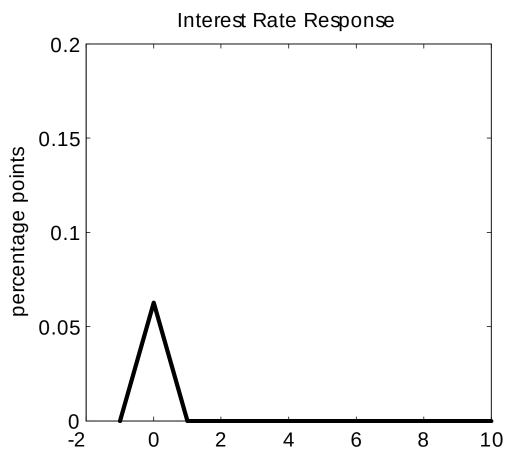
FIGURE 2

Dynamic Responses Functions to the Arrival of a Money Growth Signal

(a) Case 1: Investors Have Rational Expectations



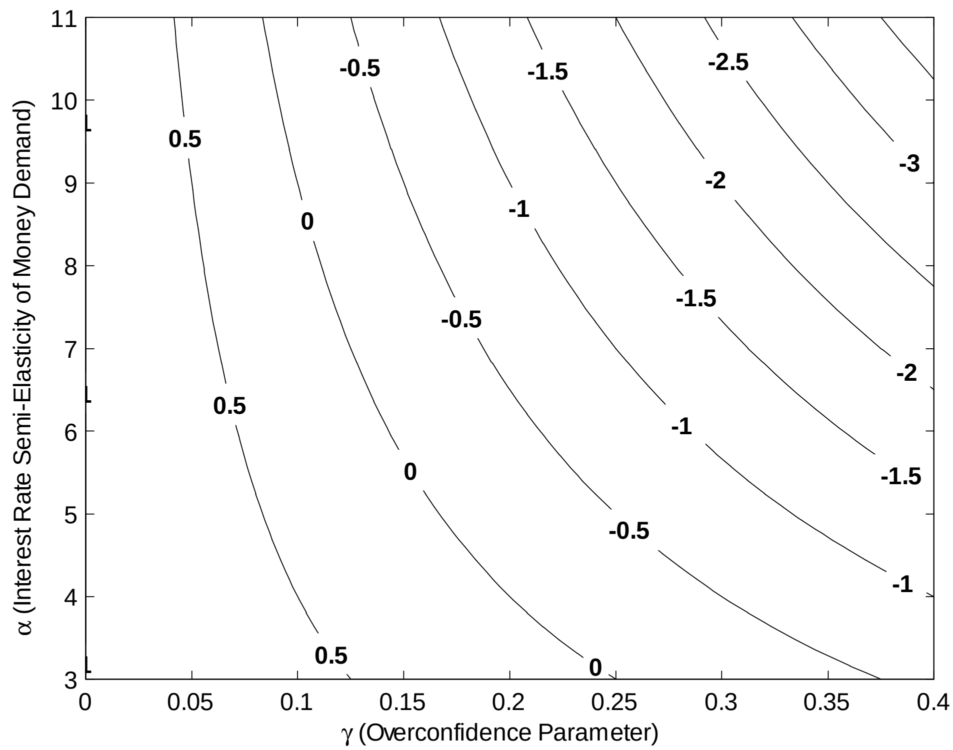
(b) Case 2: Investors are Overconfident



Notes: The graphs illustrate the paths of the interest rate and exchange rate in response to the arrival, at time 0, of a positive unit signal of an increase in the money growth rate differential. Panel (a) illustrates the case where investors have rational expectations. The paths are illustrated as deviations from the nonstochastic steady state. Panel (b) illustrates the case where investors are overconfident. In the exchange rate graphs we illustrate up to four paths for the exchange rate. The path labeled "true signal" is the path of the exchange rate if the unit signal, $\zeta = 1$, corresponds to a realized shock to the money growth rate, $u = 1$. The path labeled "false signal" is the path of the exchange rate if the unit signal, $\zeta = 1$, corresponds to a realized noise shock, $\epsilon = 1$. The path labeled "average signal" is the average path taken by the exchange rate given the distributions of u and ϵ. The path labeled "agents' expectations," which is only relevant in the case where investors are overconfident, is the path overconfident agents expect upon receiving the signal at time 0.

FIGURE 3

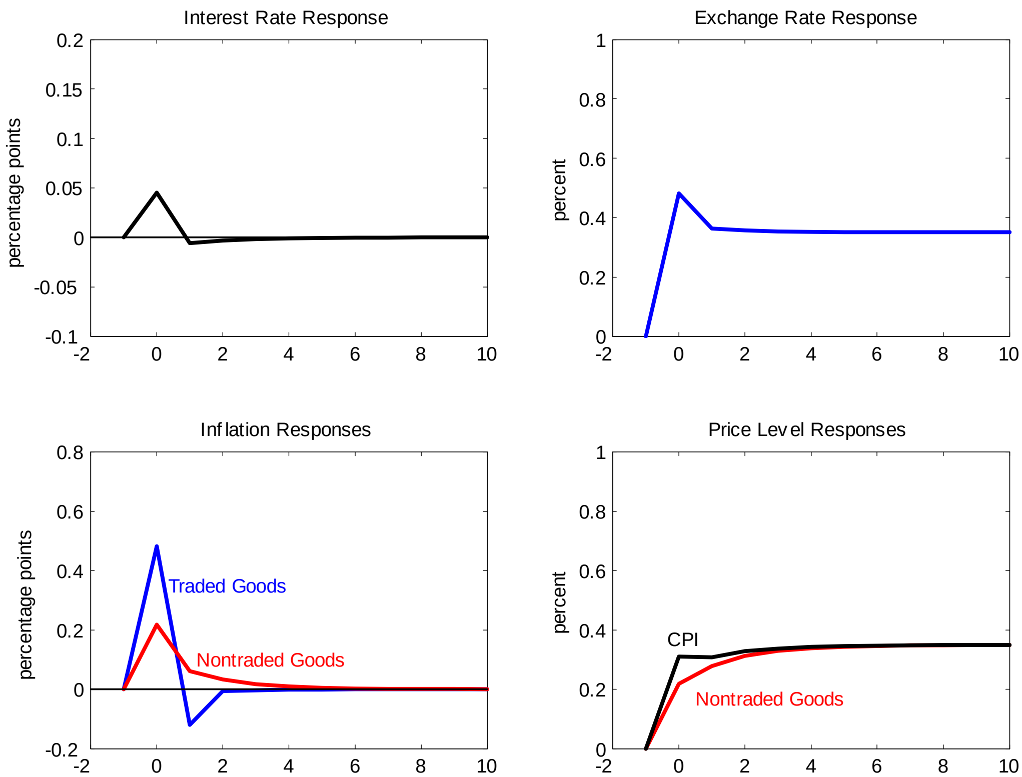
The Slope Coefficient in the Forward Premium Regression



Notes: The graph illustrates the model-implied value of the slope coefficient β_1 in the forward premium regression, $s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \epsilon_{t+1}$, where s_t is the logarithm of the spot exchange rate, and f_t is the logarithm of the one-month forward exchange rate. The value of β_1 is $1 - (1 + \alpha)\gamma$, where α is the interest rate semi-elasticity of money demand in the neighborhood of the model's nonstochastic steady state, and γ is the parameter that determines the degree of investor confidence.

FIGURE 4

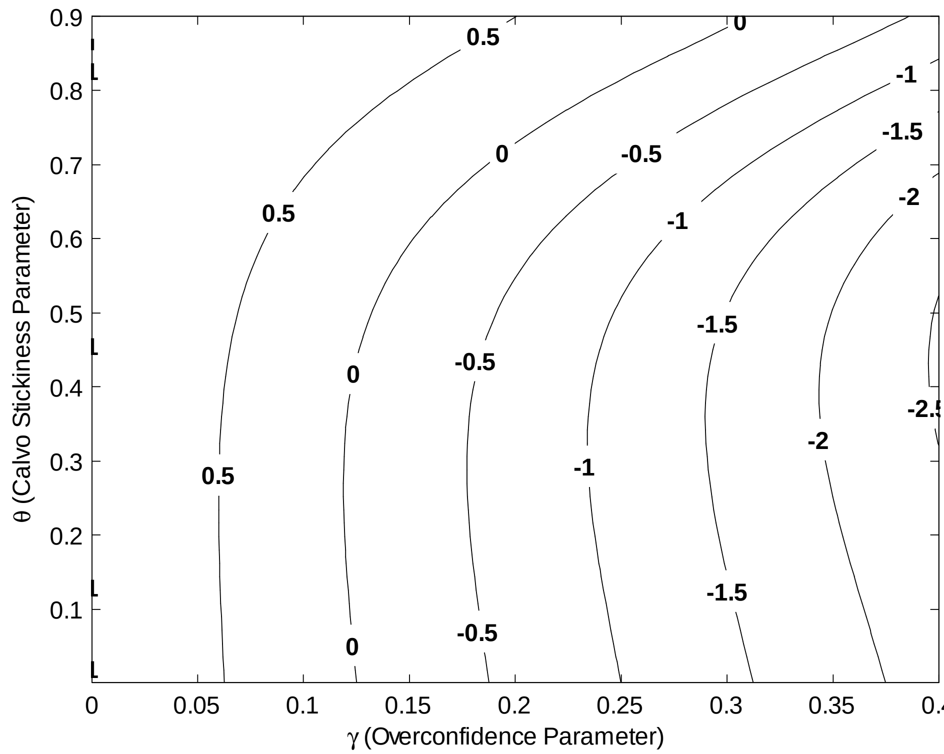
Dynamic Responses Functions to the Arrival of a Money Growth Signal in the Expanded Model



Notes: The graphs illustrate the paths implied by the expanded model with nontraded goods for the interest rate, the exchange rate, inflation rates and prices in response to the arrival, at time 0, of a positive unit signal of an increase in the money growth rate differential. The paths are illustrated as deviations from the nonstochastic steady state.

FIGURE 5

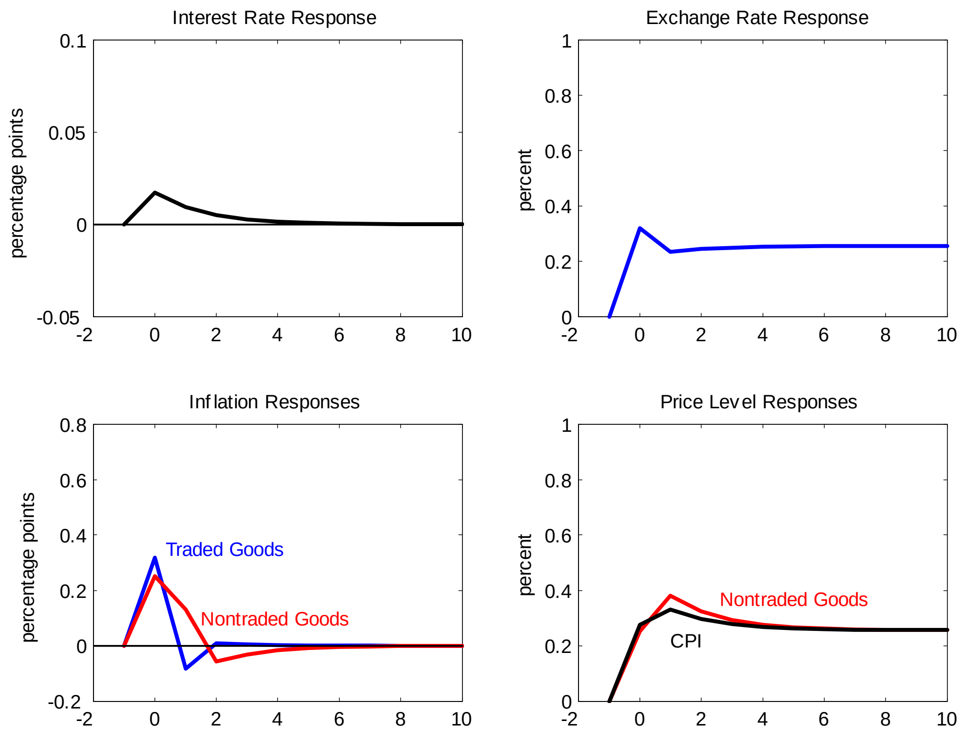
The Slope Coefficient in the Forward Premium Regression in the Expanded Model



Notes: The graph illustrates the expanded model-implied value of the slope coefficient β_1 in the forward premium regression, $s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \epsilon_{t+1}$, where s_t is the logarithm of the spot exchange rate, and f_t is the logarithm of the one-month forward exchange rate. The value of β_1 cannot be characterized in closed form in the expanded model with nontraded goods. It depends on θ and γ as well as α, the interest rate semi-elasticity of money demand, ϕ, the labor supply elasticity parameter, ω, the share of traded goods in utility, and β, the discount factor. In Figure 5 we set $\alpha = 7$, $\phi = 1$, $\omega = 0.35$, and $\beta = 0.995$.

FIGURE 6

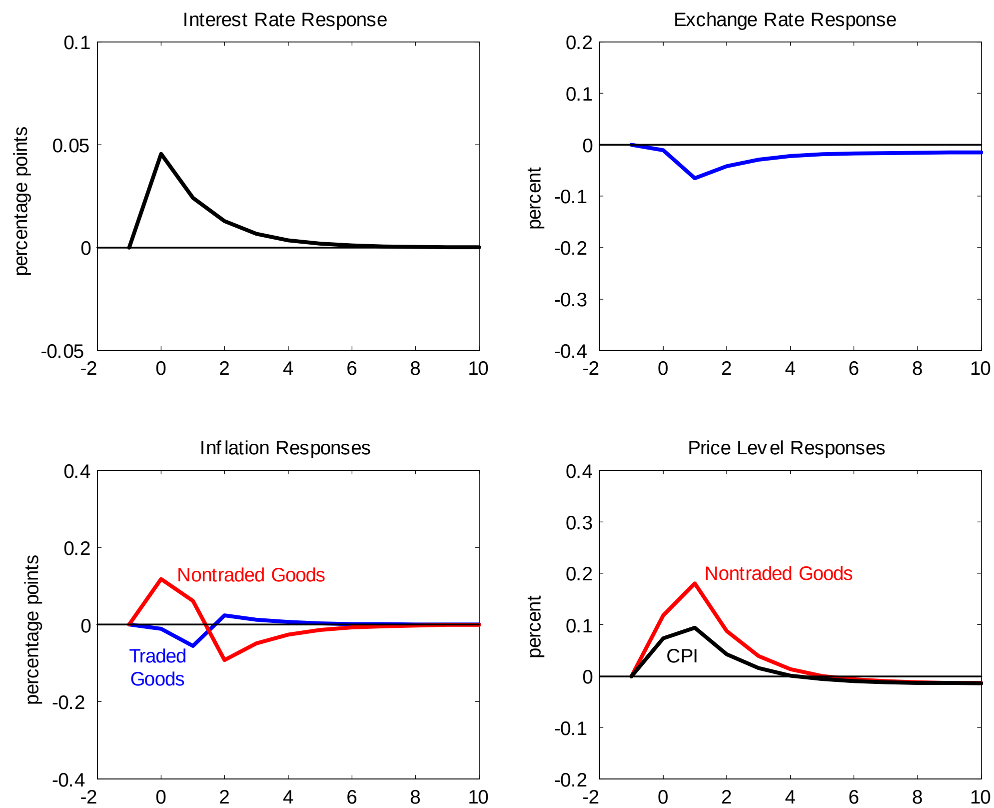
Dynamic Responses Functions to the Arrival of an Inflationary Signal in the Expanded Model with an Interest Rate Rule: Benchmark Calibration



Notes: The graphs illustrate the paths implied by the expanded model with an interest-rate monetary-policy rule for the interest rate, the exchange rate, inflation rates and prices in response to the arrival, at time 0, of a positive unit signal of a decrease in the future domestic level of technology. The interest rate rule uses the parameters $\rho_i = 0.95$, $\phi_\pi = 2$, and $\phi_y = 0.5$. The paths are illustrated as deviations from the nonstochastic steady state.

FIGURE 7

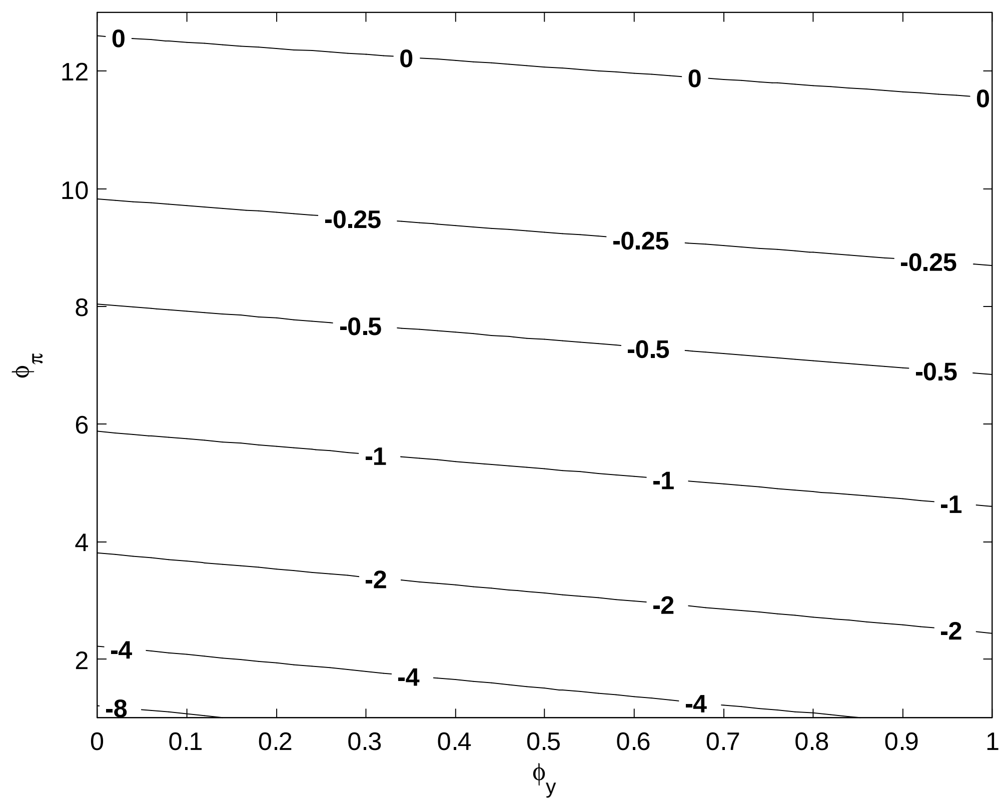
DYNAMIC RESPONSES FUNCTIONS TO THE ARRIVAL OF AN INFLATIONARY SIGNAL IN THE EXPANDED MODEL WITH AN INTEREST RATE RULE: ALTERNATE CALIBRATION



Notes: The graphs illustrate the paths implied by the expanded model with an interest-rate monetary-policy rule for the interest rate, the exchange rate, inflation rates and prices in response to the arrival, at time 0, of a positive unit signal of a decrease in the future domestic level of technology. The interest rate rule uses the parameters $\rho_i = 0.95$, $\phi_\pi = 7.5$, and $\phi_y = 0$. The paths are illustrated as deviations from the nonstochastic steady state.

FIGURE 8

The Slope Coefficient in the Forward Premium Regression in the Expanded Model with an Interest Rate Rule



Notes: The graph illustrates the expanded model with interest-rate rule-implied value of the slope coefficient β_1 in the forward premium regression, $s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \epsilon_{t+1}$ where s_t is the logarithm of the spot exchange rate, and f_t is the logarithm of the one-month forward exchange rate. The value of β_1 cannot be characterized in closed form in the expanded model. It depends on θ and γ as well as α, the interest rate semi-elasticity of money demand, ϕ, the labor supply elasticity parameter, ω, the share of traded goods in utility, β, the discount factor, and the parameters of the interest rate rule, ρ_i, ϕ_π and ϕ_y. In Figure 8 we set $\theta = 2/3$, $\gamma = 0.3$, $\alpha = 7$, $\phi = 1$, $\omega = 0.35$, $\beta = 0.995$, and $\rho_i = 0.95$.

A Appendix

A.1 Linearization of the Model

We begin by finding the nonstochastic steady state value of m. Setting $m_t = m_{t+1} = m$ and $\mu_{t+1} = \mu$ in (10) we have

$$m = \left[\varrho c / \left(1 - \beta e^{-\mu}\right)\right]^{1/\nu}. \tag{A1}$$

Taking a first order Taylor series approximation to (10) in the neighborhood of the nonstochastic steady state we have

$$0 = -\nu \varrho m^{-\nu} c \cdot \hat{m}_t - \beta e^{-\mu} \cdot E_t^C \hat{\mu}_{t+1} + \beta e^{-\mu} \cdot E_t^C \hat{m}_{t+1} - \beta e^{-\mu} \cdot \hat{m}_t \tag{A2}$$

where $\hat{m}_t \equiv (m_t - m)/m$ and $\hat{\mu}_t \equiv \mu_t - \mu$. Collecting terms and dividing by $\beta e^{-\mu}$ we obtain

$$E_t^C \hat{m}_{t+1} - \left(\frac{\nu \varrho m^{-\nu} c}{\beta e^{-\mu}} + 1\right) \hat{m}_t = E_t^C \hat{\mu}_{t+1} \tag{A3}$$

Using (A1) we see that $\varrho m^{-\nu} c/(\beta e^{-\mu}) = 1/(\beta e^{-\mu}) - 1 = i$. Therefore, using $\alpha = 1/(\nu i)$, we can rewrite (A3) as

$$E_t^C \hat{m}_{t+1} - \left(\frac{1+\alpha}{\alpha}\right) \hat{m}_t = E_t^C \hat{\mu}_{t+1}. \tag{A4}$$

We can rewrite (11) in terms of the interest rate:

$$(1 + i_t)^{-1} = 1 - \varrho m_t^{-\nu} c. \tag{A5}$$

Taking a first order Taylor series approximation to (A5) in the neighborhood of the nonstochastic steady state we have

$$-(1+i)^{-1} \cdot \hat{\imath}_t = \nu \varrho m^{-\nu} c \cdot \hat{m}_t$$

where $\hat{\imath}_t \equiv di_t/(1+i)$. Multiplying through by $-(1+i)$ and using the fact that $\varrho m^{-\nu} c = i/(1+i)$ we obtain

$$\hat{\imath}_t = -\nu i \hat{m}_t = -\hat{m}_t/\alpha. \tag{A6}$$

By (12) we have $s_t = \ln S_t = \ln(m_t^*/m_t) + \ln(M_t/M_t^*)$. Thus, to first order,

$$s_t = \ln(m^*/m) + \hat{m}_t^* - \hat{m}_t + \ln(M_t/M_t^*).$$

Using the solutions for $\hat{m}_t$ and $\hat{m}_t^*$ given by (16) and (17) gives equation (20). By covered interest rate parity we have $F_t/S_t = (1+i_t)/(1+i_t^*)$. The log of the forward premium is

$$f_t - s_t = \ln(1+i_t) - \ln(1+i_t^*).$$

Given (A5), a first order Taylor series approximation to this equation is

$$f_t - s_t = \mu - \mu^* + \hat{\imath}_t - \hat{\imath}_t^*.$$

A.2 Proof of Proposition 1

Given the results in Section 3, to first order,

$$\Delta s_{t+1} = \bar{\mu} + \kappa \Delta \zeta_{t+1} - \kappa^* \Delta \zeta_{t+1}^* + \bar{\eta}_{t+1} + \bar{u}_t,$$

and

$$f_t - s_t = \bar{\mu} + (\kappa/\alpha)\zeta_t - (\kappa^*/\alpha^*)\zeta_t^*.$$

The covariance of these quantities is

$$\begin{aligned} \operatorname{cov}(\Delta s_{t+1}, f_t - s_t) &= \operatorname{cov}\left[u_t - u_t^* - \kappa\zeta_t + \kappa^*\zeta_t^*, (\kappa/\alpha)\zeta_t - (\kappa^*/\alpha^*)\zeta_t^*\right] \\ &= \left(\frac{\kappa}{\alpha}\right)^2 [1 - \gamma(1+\alpha)] \operatorname{var}(\zeta_t) + \left(\frac{\kappa^*}{\alpha^*}\right)^2 [1 - \gamma(1+\alpha^*)] \operatorname{var}(\zeta_t^*) \end{aligned}$$

and the variance of the $f_t - s_t$ is

$$\operatorname{var}(f_t - s_t) = \left(\frac{\kappa}{\alpha}\right)^2 \operatorname{var}(\zeta_t) + \left(\frac{\kappa^*}{\alpha^*}\right)^2 \operatorname{var}(\zeta_t^*).$$

Since

$$\beta_1 = \frac{\operatorname{cov}(\Delta s_{t+1}, f_t - s_t)}{\operatorname{var}(f_t - s_t)}$$

we then have

$$\beta_1 = 1 - \gamma \frac{(1+\alpha)\left(\frac{\kappa}{\alpha}\right)^2 \operatorname{var}(\zeta_t) + (1+\alpha^*)\left(\frac{\kappa^*}{\alpha^*}\right)^2 \operatorname{var}(\zeta_t^*)}{\left(\frac{\kappa}{\alpha}\right)^2 \operatorname{var}(\zeta_t) + \left(\frac{\kappa^*}{\alpha^*}\right)^2 \operatorname{var}(\zeta_t^*)}.$$

Also

$$\beta_0 = E\Delta s_{t+1} - \beta_1 E(f_t - s_t) = (1 - \beta_1)\bar{\mu}.$$

When there is complete symmetry across the two countries $\beta_0 = 0$ and $\beta_1 = 1 - \gamma(1+\alpha)$.

A.3 Longer Horizon Regressions

We have $s_{t+n} - s_t = s_{t+1} - s_t + \sum_{j=2}^{n}\left(\kappa\Delta\bar{\zeta}_{t+j} + \bar{\eta}_{t+j} + \bar{u}_{t+j-1}\right)$. Given the baseline assumptions in the main text, $E_t^C\bar{\zeta}_{t+j}$, $E_t^C\bar{\eta}_{t+j}$ and $E_t^C\bar{u}_{t+j-1}$ are all zero for $j \geq 2$. Hence $f_{nt} - s_t = E_t^C(s_{t+n} - s_t) = E_t^C(s_{t+1} - s_t) = f_t - s_t$ for all $n \geq 2$ and $s_{t+n} - s_t$ is $s_{t+1} - s_t$ plus an error orthogonal to time t information. This implies that the coefficient in the forward premium regression is invariant to the investment horizon.

Here we consider an alternative case in which $\bar{\eta}_t$ is a persistent process, with $\bar{\eta}_t = \rho\bar{\eta}_{t-1} + \bar{\epsilon}_t$, for some $0 < \rho < 1$. We also assume that agents have rational expectations with respect to $\bar{\eta}_t$. We maintain the same assumptions, as before, for $\bar{u}_t$ and $\bar{\zeta}_t$. With this set of assumptions,

$$f_{nt} - s_t = (\kappa/\alpha)\,\bar{\zeta}_t + \frac{\rho}{1-\rho}\,(1-\rho^n)\,\bar{\eta}_t \text{ for } n \geq 1.$$

Also, $s_{t+n} - s_t$ is equal to $\left(\lambda^R - \kappa\right)\bar{\zeta}_t + [\rho/(1-\rho)]\,(1-\rho^n)\,\bar{\eta}_t$ plus an error that is orthogonal to time t information. Hence,

$$\text{var}\,(f_{nt} - s_t) = \left(\frac{\kappa}{\alpha}\right)^2 \text{var}(\bar{\zeta}_t) + \left[\frac{\rho}{1-\rho}\,(1-\rho^n)\right]^2 \text{var}(\bar{\eta}_t)$$

$$\text{cov}(s_{t+n} - s_t, f_{nt} - s_t) = (\kappa/\alpha)\left(\lambda^R - \kappa\right)\text{var}(\bar{\zeta}_t) + \left[\frac{\rho}{1-\rho}\,(1-\rho^n)\right]^2 \text{var}(\bar{\eta}_t).$$

So

$$\beta_{n1} = \frac{(\kappa/\alpha)\left(\lambda^R - \kappa\right) + \left[\frac{\rho}{1-\rho}\,(1-\rho^n)\right]^2 \frac{\text{var}(\bar{\eta}_t)}{\text{var}(\bar{\zeta}_t)}}{\left(\frac{\kappa}{\alpha}\right)^2 + \left[\frac{\rho}{1-\rho}\,(1-\rho^n)\right]^2 \frac{\text{var}(\bar{\eta}_t)}{\text{var}(\bar{\zeta}_t)}} = 1 - \gamma\,(1+\alpha)\,\varpi_n$$

where

$$\varpi_n = \frac{(1+\alpha)^{-2}\left(\lambda^C\right)^2}{(1+\alpha)^{-2}\left(\lambda^C\right)^2 + \left[\frac{\rho}{1-\rho}\,(1-\rho^n)\right]^2 \frac{\text{var}(\bar{\eta}_t)}{\text{var}(\bar{\zeta}_t)}}.$$

A.4 McCallum Regressions

In the model

$$\begin{aligned}
s_{t+1} - s_{t-n} &= \Delta s_{t+1} + s_t - s_{t-n} \\
f_t - s_{t-n} &= f_t - s_t + s_t - s_{t-n} \\
\Delta s_{t+1} &= \kappa\Delta\bar{\zeta}_{t+1} + \bar{\eta}_{t+1} + \bar{u}_t \\
f_t - s_t &= (\kappa/\alpha)\bar{\zeta}_t
\end{aligned}$$

and

$$s_t - s_{t-n} = \kappa\bar{\zeta}_t - \kappa\bar{\zeta}_{t-n} + \sum_{j=1}^{n}\left(\bar{\eta}_{t+1-j} + \bar{u}_{t-j}\right).$$

We denote the slope coefficient in a regression of $s_{t+1} - s_{t-n}$ on $f_t - s_{t-n}$ as $\bar{\beta}_{n1}$. The numerator and denominator of $\bar{\beta}_{n1}$ are

$$\begin{aligned}
\bar{\beta}_{n1}^{\text{num}} &= \text{cov}(\Delta s_{t+1}, f_t - s_t) + \text{cov}(\Delta s_{t+1}, s_t - s_{t-n}) + \text{cov}(s_t - s_{t-n}, f_t - s_t) + \text{var}(s_t - s_{t-n}) \\
\bar{\beta}_{n1}^{\text{den}} &= \text{var}(f_t - s_t) + 2\,\text{cov}(s_t - s_{t-n}, f_t - s_t) + \text{var}(s_t - s_{t-n}).
\end{aligned}$$

With some algebra, we have

$$
\begin{aligned}
\bar{\beta}_{n1}^{\text{num}} &= (\kappa/\alpha - \kappa + n)\operatorname{var}(\bar{u}) + \kappa^2 \operatorname{var}(\bar{\zeta}) + n \operatorname{var}(\bar{\eta}) \\
\bar{\beta}_{n1}^{\text{den}} &= (n - 2\kappa)\operatorname{var}(\bar{u}) + \left[\left(\lambda^C\right)^2 + \kappa^2\right]\operatorname{var}(\bar{\zeta}) + n \operatorname{var}(\bar{\eta}),
\end{aligned}
$$

and we can write

$$\bar{\beta}_{n1}^{\text{num}} = \bar{\beta}_{n1}^{\text{den}} + \lambda^C \operatorname{var}(\bar{u}) - \left(\lambda^C\right)^2 \operatorname{var}(\bar{\zeta}).$$

Hence

$$\bar{\beta}_{n1} = 1 - \gamma\left(\lambda^C\right)^2 \operatorname{var}(\bar{\zeta})/\bar{\beta}_{n1}^{\text{den}}.$$

To get quantitative predictions from the model we proceed as follows. We assume that the two countries have identical, independent money growth processes. We let $\gamma = 0.25$, $\alpha = 7$, and set $\lambda^R = 0.375$. These parameter values imply that $\lambda^C = 0.5$ and $\kappa = 0.4375$. The model implies that $\operatorname{var}(f_t - s_t) = \kappa^2 \operatorname{var}(\bar{\zeta})/\alpha^2$. Therefore, we set $\operatorname{var}(\bar{\zeta})$ equal to (α^2/κ^2) times the mean of the sample variances of $f_t - s_t$ for our sample of 11 countries. We then set $\operatorname{var}(\bar{u}) = \lambda^R \operatorname{var}(\bar{\zeta})$. Then we note that

$$\operatorname{var}(\Delta s_{t+1}) = 2\kappa^2 \operatorname{var}(\bar{\zeta}) + \operatorname{var}(\bar{\eta}) + (1 - 2\kappa)\operatorname{var}(\bar{u})$$

which implies that

$$\operatorname{var}(\bar{\eta}) = \operatorname{var}(\Delta s_{t+1}) - 2\kappa^2 \operatorname{var}(\bar{\zeta}) + (2\kappa - 1)\operatorname{var}(\bar{u}).$$

We set $\operatorname{var}(\Delta s_{t+1})$ equal to the mean of the sample variances of Δs_{t+1} for our sample of 11 countries in the above formula to obtain $\operatorname{var}(\bar{\eta})$. This allows us to calculate $\bar{\beta}_{n1}$.

A.5 Mean Squared Error of Forecasting Rules

For each forecasting rule, we evaluate the mean squared error (MSE).

(i) MSE of the forward premium Regression If the forward premium regression is used to predict the exchange rate, the forecast error is

$$
\begin{aligned}
e_{1t+1} &= \Delta s_{t+1} - [1 - \gamma(1 + \alpha)](f_t - s_t) \\
&= \kappa\bar{\zeta}_{t+1} + \bar{\eta}_{t+1} + \left(1 - \lambda^R\right)\bar{u}_t - \lambda^R \bar{\epsilon}_t,
\end{aligned}
$$

which is orthogonal to time t information. The MSE of this forecasting method is

$$MSE_1 = \kappa^2 \operatorname{var}(\bar{\zeta}) + \operatorname{var}(\bar{\eta}) + \left(1 - \lambda^R\right)^2 \operatorname{var}(\bar{u}) + \left(\lambda^R\right)^2 \operatorname{var}(\bar{\epsilon}).$$

(ii) MSE of a Regression on the Realized Inflation Differential Since PPP holds in the basic model, the realized inflation differential is equal to the rate of change of the exchange rate. That is $\bar{\pi}_t = \Delta s_t$. The results are therefore the same as for case (iv), below.

(iii) MSE of a Regression on the Change in the Forward Rate The regression is of $\Delta s_{t+1} = \kappa \Delta \bar{\zeta}_{t+1} + \bar{u}_t + \bar{\eta}_{t+1}$ on

$$\begin{aligned} \Delta f_t &= \Delta s_t + (1+\alpha)^{-1} \lambda^C \Delta \bar{\zeta}_t \\ &= \lambda^C \bar{\zeta}_t + \bar{u}_{t-1} - \lambda^C \bar{\zeta}_{t-1} + \bar{\eta}_t \end{aligned}$$

The regression coefficient is

$$b = \frac{\operatorname{cov}\left(-\kappa \bar{\zeta}_t + \bar{u}_t, \lambda^C \bar{\zeta}_t\right)}{\operatorname{var}(\Delta f_t)} = \frac{-\kappa \lambda^C \operatorname{var}(\bar{\zeta}) + \lambda^C \operatorname{var}(\bar{u})}{\operatorname{var}(\Delta f_t)}$$

The prediction error is

$$e_{3t+1} = \Delta s_{t+1} - \Delta f_t b = e_{1t+1} + \beta_1 (f_t - s_t) - \Delta f_t b.$$

As long as $b \neq 0$ or $\beta_1 \neq 0$, the MSE of e_{3t+1} exceeds that of e_{1t+1} because e_{1t+1} is orthogonal to lagged information. We can further work out that

$$e_{3t+1} = e_{1t+1} + \left[\beta_1 (\kappa/\alpha) - \lambda^C b\right] \bar{\zeta}_t + \left(\lambda^C - 1\right) b \bar{u}_{t-1} + \lambda^C b \bar{\epsilon}_{t-1} - b \bar{\eta}_t.$$

Therefore,

$$\begin{aligned} MSE_3 &= MSE_1 + \left[\beta_1 (\kappa/\alpha) - \lambda^C b\right]^2 \operatorname{var}(\bar{\zeta}) + \\ &\quad \left[\left(\lambda^C - 1\right) b\right]^2 \operatorname{var}(\bar{u}) + \left(\lambda^C b\right)^2 \operatorname{var}(\bar{\epsilon}) + b^2 \operatorname{var}(\bar{\eta}). \end{aligned}$$

(iv) MSE of a Regression on the Change in the Spot Rate The regression is of Δs_{t+1} on $\Delta s_t = \kappa \Delta \bar{\zeta}_t + \bar{u}_{t-1} + \bar{\eta}_t$. The regression coefficient is

$$b = \frac{\operatorname{cov}\left(-\kappa \bar{\zeta}_t + \bar{u}_t, \kappa \bar{\zeta}_t\right)}{\operatorname{var}(\Delta s_t)} = \frac{-\kappa^2 \operatorname{var}(\bar{\zeta}) + \kappa \operatorname{var}(\bar{u})}{\operatorname{var}(\Delta s_t)}$$

The prediction error is

$$e_{4t+1} = \Delta s_{t+1} - \Delta s_t b = e_{1t+1} + \beta_1 (f_t - s_t) - \Delta s_t b.$$

As long as $b \neq 0$ or $\beta_1 \neq 0$, the MSE of e_{4t+1} exceeds that of e_{1t+1} because e_{1t+1} is orthogonal to lagged information. We can further work out that

$$e_{4t+1} = e_{1t+1} + \left[\beta_1 (\kappa/\alpha) - \kappa b\right] \bar{\zeta}_t + (\kappa - 1) b \bar{u}_{t-1} + \kappa b \bar{\epsilon}_{t-1} - b \bar{\eta}_t.$$

Therefore,

$$\begin{aligned} MSE_4 &= MSE_1 + [\beta_1 (\kappa/\alpha) - \kappa b]^2 \operatorname{var}(\bar{\zeta}) + \\ & \quad [(\kappa - 1) b]^2 \operatorname{var}(\bar{u}) + (\kappa b)^2 \operatorname{var}(\bar{\epsilon}) + b^2 \operatorname{var}(\bar{\eta}). \end{aligned}$$

(v) MSE of the Forward Rate In this case the forecast error is

$$e_{5t+1} = \Delta s_{t+1} - (f_t - s_t) = e_{1t+1} - \gamma(1+\alpha)(f_t - s_t).$$

The MSE is

$$\begin{aligned} MSE_5 &= MSE_1 + [\gamma(1+\alpha)]^2 \operatorname{var}(f_t - s_t) \\ &= MSE_1 + (1 - \beta_1)^2 \operatorname{var}(f_t - s_t) \end{aligned}$$

since $f_t - s_t$ is orthogonal to e_{1t+1}. As long as $\beta_1 \neq 1$ the forward premium regression dominates the forward rate as a predictor.

(vi) MSE of the Spot Rate In this case the forecast error is

$$e_{6t+1} = \Delta s_{t+1} = e_{1t+1} + \beta_1(f_t - s_t)$$

The MSE is

$$MSE_6 = MSE_1 + \beta_1^2 \operatorname{var}(f_t - s_t),$$

since $f_t - s_t$ is orthogonal to e_{1t+1}. As long as $\beta_1 \neq 0$ the forward premium regression dominates the spot rate as a predictor.

A.6 Solving the Expanded Model

To solve the model we define the following variables $p_{Xt} \equiv P_t/P_{Tt}$, $w_{Xt} \equiv W_t/P_{Tt}$, $p_{Nt} \equiv P_{Nt}/P_{Tt}$ and $\Pi_{Tt} \equiv P_{Tt}/P_{Tt-1}$. Using the fact that $c_{Tt} = c_T$ for all t, and these newly defined variables, we can rewrite (28) as

$$p_{Xt} = \frac{1}{\omega^{\omega}(1-\omega)^{1-\omega}} p_{Nt}^{1-\omega}. \tag{A7}$$

We use (A7) to eliminate p_{Xt} from the other optimality conditions. We can rewrite (31) as

$$c_{Nt} = (\frac{1-\omega}{\omega}) c_T p_{Nt}^{-1}. \tag{A8}$$

We use (A8) to eliminate c_{Nt} from the other optimality conditions.

The demand function for each variety, (32), combined with the market clearing condition for each variety, (40), the production technology, (36), and the market clearing condition for labor imply that

$$n_t = c_{Nt} \int_0^1 (P_{Nit}/P_{Nt})^{-\varepsilon} di. \tag{A9}$$

Since we solve the model in the neighborhood of a zero inflation steady state, the integral in equation (A9) is equal to 1 up to a first order approximation. To see this, consider equation (27), which implies that

$$1 = \int_0^1 \exp\left[(1-\varepsilon) z_{it}\right] di, \tag{A10}$$

where $z_{it} \equiv \ln(P_{Nit}/P_{Nt})$. In the neighborhood of a zero inflation steady state in which $z_i = 0$ for all t, up to a first order approximation equation (A10) implies

$$1 \approx 1 + (1-\varepsilon) \int_0^1 z_{it} di$$

so that $\int_0^1 z_{it} di \approx 0$. The integral in equation (A9) is equal to $\int_0^1 \exp\left(-\varepsilon z_{it}\right) di$, which to first order is equal to $1 - \varepsilon \int_0^1 z_{it} di \approx 1$.[25] Therefore, up to first order we rewrite (A9), using (A8), as

$$n_t = c_{Nt} = (\frac{1-\omega}{\omega}) c_T p_{Nt}^{-1}. \tag{A11}$$

We use (A8) to eliminate n_t from the other optimality conditions.

Using the definition of aggregate consumption, (26), (A8), and (A11) the optimality condition for labor, (33), can be rewritten as

$$w_t = \vartheta c_T^{1+\phi} (\frac{1-\omega}{\omega})^{\phi+1-\omega} p_{Nt}^{\omega-\phi-1}. \tag{A12}$$

We use (A8) to eliminate w_t from the other optimality conditions.

Substituting out c_t using (26), and c_{Nt} using (A8) the household's first order condition for m_t, (34), can be rewritten as

$$1 = \varrho (\frac{1-\omega}{\omega})^{1-\omega} c_T m_t^{-\nu} p_{Nt}^{\omega-1} + \beta E_t^C \Pi_{Tt+1}^{-1}. \tag{A13}$$

To a first order approximation (A13) is

$$\nu i \hat{m}_t = i(\omega - 1)\hat{p}_{Nt} - E_t^C \hat{\pi}_{Tt+1} \tag{A14}$$

[25] An equivalent derivation is provided by Galí (2008).

where $\hat{m}_t \equiv (m_t - m)/m$, $\hat{p}_{Nt} \equiv (p_{Nt} - p_N)/p_N$, $\hat{\pi}_{Tt} \equiv \Pi_{Tt} - 1$ and $i \equiv \beta^{-1} - 1$.

Since $M_t/M_{t-1} = \exp(\mu_t)$ it follows that $m_t p_{Xt} \Pi_{Tt} / (m_{t-1} p_{Xt-1}) = \exp(\mu_t)$. Substituting out p_{Xt} using (A7) we have

$$m_t p_{Nt}^{1-\omega} \Pi_{Tt} p_{Nt-1}^{\omega-1} / m_{t-1} = \exp(\mu_t). \tag{A15}$$

To a first order approximation (A15) is

$$\hat{m}_t = (\omega - 1)\hat{p}_{Nt} - \hat{\pi}_{Tt} - (\omega - 1)\hat{p}_{Nt-1} + \hat{m}_{t-1} + \mu_t. \tag{A16}$$

By definition

$$\Pi_{Nt} = \frac{P_{Nt}}{P_{Nt-1}} = \frac{P_{Nt}}{P_{Tt}} \frac{P_{Tt}}{P_{Tt-1}} \frac{P_{Tt-1}}{P_{Nt-1}} = p_{Nt} \Pi_{Tt} / p_{Nt-1}. \tag{A17}$$

To a first order approximation (A17) is

$$\hat{\pi}_{Nt} = \hat{p}_{Nt} + \hat{\pi}_{Tt} - \hat{p}_{Nt-1}. \tag{A18}$$

Now consider the nontraded producing firm's first order condition, (37). To simplify this condition we follow Galí (2008). Multiplying (37) through by $\left(\bar{P}_{Nt}/P_{Tt-1}\right)(1-\varepsilon)$ we get

$$E_t^C \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(\frac{\bar{P}_{Nt}}{P_{Nt+k}}\right)^{-\varepsilon} c_{Nt+k} \left(\frac{\bar{P}_{Nt}}{P_{Nt-1}} \frac{P_{Nt-1}}{P_{Tt-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{W_{t+k}}{P_{Tt+k}} \frac{P_{Tt+k}}{P_{Tt-1}}\right) = 0.$$

Using the definitions $\bar{\Pi}_{Nt} \equiv \bar{P}_{Nt}/P_{Nt-1}$, $w_{Xt} \equiv W_t/P_{Tt}$, and $\Pi_{t-1,k}^T \equiv P_{Tt+k}/P_{Tt-1}$ this can be rewritten as

$$E_t^C \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(\frac{\bar{P}_{Nt}}{P_{Nt+k}}\right)^{-\varepsilon} c_{Nt+k} \left(\bar{\Pi}_{Nt} p_{Nt-1} - \frac{\varepsilon}{\varepsilon - 1} w_{Xt+k} \Pi_{t-1,k}^T\right) = 0.$$

Since the terms in parentheses are zero in the zero inflation steady state, this equation linearized is

$$E_t^C \sum_{k=0}^{\infty} (\beta\theta)^k \left(\tilde{\pi}_{Nt} + \hat{p}_{Nt-1} - \hat{w}_{Xt+k} - \hat{\Pi}_{t-1,k}^T\right) = 0 \tag{A19}$$

where $\tilde{\pi}_{Nt} = \bar{\Pi}_{Nt} - 1$, $\hat{p}_{Nt} = (p_{Nt} - p_N)/p_N$, $\hat{w}_{Xt} = (w_{Xt} - w_X)/w_X$ and $\hat{\Pi}_{t-1,k}^T = \Pi_{t-1,k}^T - 1$. For any variable x_t that does not depend on k, $E_t^C \sum_{k=0}^{\infty} (\beta\theta)^k x_t = x_t/(1-\beta\theta)$. Also $\sum_{k=0}^{\infty} (\beta\theta)^k \hat{\Pi}_{t-1,k}^T = \sum_{k=0}^{\infty} (\beta\theta)^k \hat{\pi}_{Tt+k}/(1-\beta\theta)$ where $\hat{\pi}_{Tt} = \pi_{Tt} - 1$. Therefore, equation (A19) may be written as

$$\tilde{\pi}_{Nt} + \hat{p}_{Nt-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t^C \hat{w}_{Xt+k} + \sum_{k=0}^{\infty} (\beta\theta)^k E_t^C \hat{\pi}_{Tt+k} \tag{A20}$$

Written as a difference equation, (A20) is equivalent to

$$\tilde{\pi}_{Nt} + \hat{p}_{Nt-1} = \beta\theta E_t^C \left(\tilde{\pi}_{Nt+1} + \hat{p}_{Nt}\right) + (1-\beta\theta)\hat{w}_{Xt} + \hat{\pi}_{Tt}. \tag{A21}$$

To proceed further we now derive an expression for $\tilde{\pi}_{Nt}$ using (27). Any firm that is setting its price has price $\bar{P}_{Nt}$ because all firms setting price at time t have the same first order condition. Denoting the set of firms who are not changing price as $\mathcal{S}_t$, we can write $P_{Nt} = \left[(1-\theta)\bar{P}_{Nt}^{1-\varepsilon} + \int_{\mathcal{S}_t} P_{Nit-1}^{1-\varepsilon} di\right]^{1/(1-\varepsilon)}$. Now $\int_{\mathcal{S}_t} P_{Nit-1}^{1-\varepsilon} di = \theta \int_0^1 P_{Nit-1}^{1-\varepsilon} di = \theta P_{Nt-1}^{1-\varepsilon}$ so that $P_{Nt} = \left[(1-\theta)\bar{P}_{Nt}^{1-\varepsilon} + \theta P_{Nt-1}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$. So then we have $\Pi_{Nt} = \left[(1-\theta)\bar{\Pi}_{Nt}^{1-\varepsilon} + \theta\right]^{1/(1-\varepsilon)}$ or where $\Pi_{Nt} = P_{Nt}/P_{Nt-1}$. Linearized this is $\hat{\pi}_{Nt} = (1-\theta)\tilde{\pi}_{Nt}$, where $\hat{\pi}_{Nt} = \pi_{Nt} - 1$. So we rewrite (A21) as

$$\hat{\pi}_{Nt} + (1-\theta)\hat{p}_{Nt-1} = \beta\theta E_t^C \hat{\pi}_{Nt+1} + (1-\theta)\beta\theta\hat{p}_{Nt} + (1-\theta)(1-\beta\theta)\hat{w}_{Xt} + (1-\theta)\hat{\pi}_{Tt}. \tag{A22}$$

Since $w_{Xt} = (W_t/P_{Tt})$ it follows that $w_{Xt} = (W_t/P_t)(P_t/P_{Tt}) = w_t p_{Xt}$ and $\hat{w}_{Xt} = \hat{w}_t + \hat{p}_{Xt}$. Hence, using (A7) and (A12) we have $\hat{w}_{Xt} = \hat{w}_t + \hat{p}_{Xt} = -\phi\hat{p}_{Nt}$. This allows us to write (A22) as

$$\hat{\pi}_{Nt} + (1-\theta)\hat{p}_{Nt-1} = \beta\theta E_t^C \hat{\pi}_{Nt+1} + \kappa_N \hat{p}_{Nt} + (1-\theta)\hat{\pi}_{Tt}, \tag{A23}$$

where $\kappa_N = (1-\theta)\left[\beta\theta - (1-\beta\theta)\phi\right]$.

The four equations (A14), (A16), (A18), and (A23), represent a first order linear difference equation in the variables $\hat{\pi}_{Nt}$, $\hat{p}_{Nt}$, $\hat{\pi}_{Tt}$ and $\hat{m}_t$, with μ_t being exogenous. From here, we solve the model using the method of King, Plosser and Rebelo (2002). The solution takes the form

$$\begin{pmatrix} \hat{m}_t \\ \hat{p}_{Nt} \\ \hat{\pi}_{Tt} \\ \hat{\pi}_{Nt} \end{pmatrix} = \Xi_1 \begin{pmatrix} \hat{m}_{t-1} \\ \hat{p}_{Nt} \end{pmatrix} + \Xi_2 \mu_t + \Xi_3 \zeta_t. \tag{A24}$$

where Ξ_1 is a 4×2 matrix of coefficients, while Ξ_2 and Ξ_3 are 4×1 vectors. Further details of the solution procedure are available from the authors.

The price of a domestic bond is given by equation (35). Combining this equation with (34) we obtain $Q_t = 1 - \varrho m_t^{-\nu} c_t$. Substituting out c_t using (26), and c_{Nt} using (A8) this becomes

$$Q_t = 1 - \varrho c_T \left(\frac{1-\omega}{\omega}\right)^{1-\omega} m_t^{-\nu} p_{Nt}^{\omega-1}. \tag{A25}$$

To solve for the interest rate in the domestic economy we linearize (A25):

$$\hat{\imath}_t = -\hat{Q}_t = -\nu i \hat{m}_t + i(\omega-1)\hat{p}_{Nt}. \tag{A26}$$

We can solve for the interest rate in the foreign economy in the same way. >From the PPP condition for traded goods, the rate of depreciation of the domestic currency is $\hat{\delta}_{t+1} = \hat{\pi}_{Tt+1} - \hat{\pi}^*_{Tt+1}$. Given the linear solution for $\hat{m}_t$, $\hat{p}_{Nt}$, $\hat{\pi}_{Tt}$ and $\hat{\pi}_{Nt}$ provided by equation (A24), it is straightforward to compute the probability limits of the regression coefficients discussed in Section 4, either analytically or by simulation.

A.7 Solving the Model with an Interest Rate Rule

The equilibrium conditions of the model, as in the previous case, reduce to four equations. Equations (A14) and (A18) are unchanged. Equation (A16) is dropped because the money growth rate is endogenous. It is replaced by (44). Finally, equation (A23) is modified to take into account the fact that the firm's marginal cost now depends on the stochastic level of technology:

$$\hat{\pi}_{Nt} + (1-\theta)\hat{p}_{Nt-1} = \beta\theta E_t^C \hat{\pi}_{Nt+1} + \kappa_N \hat{p}_{Nt} - \kappa_\chi \chi_t + (1-\theta)\hat{\pi}_{Tt}, \tag{A27}$$

where $\kappa_\chi = (1-\theta)(1-\beta\theta)(1+\phi)$.

Since (A8) and (A26) still hold, the interest rate rule can be rewritten as

$$\begin{aligned} -\nu i \hat{m}_t + \left[i(\omega-1) + (1-\rho_i)\phi_y \right] \hat{p}_{Nt} \quad = \quad & \rho_i \left[-\nu i \hat{m}_{t-1} + i(\omega-1)\hat{p}_{Nt-1} \right] + \\ & (1-\rho_i)\phi_\pi E_t^C \left[\omega \hat{\pi}_{Tt+1} + (1-\omega)\hat{\pi}_{Nt+1} \right]. \end{aligned} \tag{A28}$$

Equations (A14), (A18), (A27) and (A28), represent a first order linear difference equation in the variables $\hat{\pi}_{Nt}$, $\hat{p}_{Nt}$, $\hat{\pi}_{Tt}$ and $\hat{m}_t$, with χ_t being exogenous. As in the previous section, it is then straightforward to solve the model using the method of King, Plosser and Rebelo (2002). Once again, the solution takes the form (A24) with the matrices Ξ_i $i = 1, 2, 3$, being different nonlinear functions of the deep parameters than in the previous model. Further details of the solution procedure are available from the authors.