Limited Investor Attention and Stock Market Misreactions to Accounting Information

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Abstract

We provide a model in which a single psychological constraint, limited investor attention, explains both under- and over-reaction to different earnings components. Investor neglect of information in current-period earnings about future earnings induces post-earnings announcement drift, the strength of which is increasing with the persistence of earnings. Neglect of earnings components causes accruals and cash flows to predict abnormal returns. We derive new untested empirical implications relating the strength of the drift, accruals, and cash flow anomalies to the quality of earnings, to the number of distracting events, and to the volatilities of and correlation between accruals and cash flows.
1 Introduction

Market reactions to earnings and earnings components present a striking puzzle. Stock prices on average under-react to earnings surprises (post-earnings announcement drift), but over-react to the operating accruals component of earnings.¹ In this paper we argue that a single psychological constraint, limited investor attention, offers a parsimonious explanation for both under- and over-reactions to earnings and earnings components. The model of limited attention that we provide is consistent with post-earnings announcement drift, the accruals anomaly, and the cash flow anomaly. In addition to providing an integrated explanation for effects that have previously been considered separately, this paper offers new empirical implications about the determinants of the strength of different effects.

Earnings- and accruals-related patterns of return predictability are often referred to as ‘anomalies,’ ‘under-’ and ‘over-reactions,’ or as reflecting investor ‘optimism,’ ‘pessimism,’ or ‘naiveté’. Such labels offer little guidance as to the sources of these effects. Furthermore, a procedure of conjecturing a separate psychological bias for each misreaction pattern creates a problem of model overfitting; explanatory power is bought at the expense of predictive power. We offer here an alternative approach based upon a well-established psychological constraint, limited attention. We explore the implications of limited investor attention for the ability of both earnings and earnings components to forecast abnormal returns. Specifically, we derive new untested implications for how return predictability is related to the volatilities, correlations, and earnings forecasting power of these components.

The psychological underpinning of our approach is limited attention. Attention requires effort, and since the amount of information available is vast, must be selective (see, e.g., Kahneman (1973)). There is evidence from the experimental laboratory that limited attention affects how both naive and sophisticated individual investors, as well as financial professionals, interpret accounting data (this evidence is reviewed by Libby, Bloomfield, and Nelson (2002)).

¹Evidence of post-earnings announcement drift is provided by Ball and Brown (1968) and Bernard and Thomas (1989); on the accrual and cash flow anomalies, see Sloan (1996) and Collins and Hribar (2000). Most empirical tests that control for possible risk factors have not supported a rational risk premium explanation for these return patterns (see the abovementioned studies, and Bernard, Thomas, and Wahlen (1997)).
Empirical research provides several indications that limited investor attention contributes to post-earnings announcement drift. Market reactions to earnings surprises are muted when the news is released to the media during low attention periods. After-hours earnings announcements are impounded into price gradually in the days after the disclosure (Francis, Pagach, and Stephan (1992)). Recent evidence also indicates that the volume-reaction and two-day stock price reaction to news that is released to the media on Fridays are much weaker than when news is released on other days of the week, and that post-earnings announcement drift is 70 percent larger for news that is released on Fridays than on other weekdays (DellaVigna and Pollet (2004)).

There are also indications that limited investor attention may play a role in the accruals anomaly. Institutional investors, as professionals, may be more attentive to earnings components than individuals. The accruals anomaly is stronger among stocks with lower ownership by active institutional investors (Collins, Gong, and Hribar (2003)). Furthermore, managers seem to use their accounting discretion to exploit investors’ neglect of accruals information. Analysts, whether for agency or psychological reasons, tend to neglect accruals information in forming their forecasts (Teoh and Wong (2002)). Finally, the accruals anomaly is not present among the subset of firms that disclose the level of accruals at the date of the original earnings announcement. This suggests that the accruals anomaly among non-disclosing firms is caused by a subset of investors who attend to the earnings announcement but not to the later financial reporting of accruals.

The business media focus on earnings much more than on cash flow and accruals numbers, contributing to a focus on earnings per share by both individual investors and financial professionals. As discussed by the famous stock analyst Abby Joseph Cohen, “Many participants in the investment business still rely on EPS [earnings per share],

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2Furthermore, firms seem to take limited investor attention into account in timing the release of earnings news. There is evidence that firms defer the release of bad earnings news to Fridays rather than other weekdays (Bagnoli, Clement, and Watts (2005), DellaVigna and Pollet (2004)), to late in the trading day (Patell and Wolfson (1982)) and to after-hours rather than trading hours (Bagnoli, Clement, and Watts (2005)).

3This is confirmed in an extensive literature on earnings management prior to new issues of equity (Teoh, Welch and Wong (1998a, 1998b)), in order to meet benchmarks (DeGeorge, Patel, and Zeckhauser (1999)), and for other purposes; and evidence in general samples that investor misvaluation of the abnormal component of accruals (a proxy for the use of discretion by firms) is especially strong (Teoh, Welch and Wong (1998a, 1998b) and Xie (2001)).

4Among those firms that do not disclose, part of the subsequent abnormal returns associated with abnormal accruals occurs at the date of the later SEC filing when earnings and accruals are reported, and part occurs in the year subsequent to this filing (Louis, Robinson, and Sbaraglia (2005)).
to the exclusion of important measures of firm performance, such as revenues and cash flow..." (Cohen (2005)). This emphasis is useful for investors with limited attention and processing power. If an investor must select between earnings and cash flow to focus upon, the best choice is the signal that is most informative about firm value. Since stock returns are more strongly related to news about contemporaneous earnings than about cash flow (Dechow (1994)), earnings is a better choice. Nevertheless, since cash flow is incrementally informative relative to earnings, the neglect of how earnings is divided between cash flow and accruals causes systematic bias.

Consistent with some past literature on limited attention, in our model some investors condition only on subsets of publicly available information signals in valuing a stock. Risk averse investors who are fully attentive to the relevant information item are willing to bear only a limited amount of risk in order to exploit mispricing. In consequence, equilibrium stock prices reflect a weighted average of the beliefs of investors who attend to different signals, with weights that depend on the relative numbers in each investor group and their risk tolerances; see also the discussion in footnote 12. This approach is not new; it serves as the building block for our new findings about the effects of limited attention toward earnings and earnings components.

In the model, some investors attend to the information in current-period earnings about future prospects, and a subset of these investors also attend separately to the accrual and cash flow components of earnings. (Further motivation of the assumption that some investors neglect earnings information is provided in Subsection 3.1 and Section 4.) Using the information signals they attend to, investors forecast future cash flows and earnings and form valuations of the firm. In equilibrium, prices underreact to earnings surprises because some investors form expectations that do not reflect the information about future earnings contained in earnings news.

Investors who do attend to earnings but do not distinguish between earnings components misvalue firms with abnormal levels of accruals. Empirically, the level of accruals is a less favorable forecaster than cash flow of firm profitability (Sloan (1996)). Regardless of whether this difference in forecasting power is a consequence of earnings management or of the general nature of the accounting system, a rational investor should take this fact into account in valuing firms. In our model, an investor who does not impound the information in earnings components into his valuation overvalues high-accruals firms and undervalues low-accruals firms. Since misvaluation is eventually corrected, high accruals predict low subsequent abnormal returns, and high cash flows predict high subsequent abnormal returns. Thus, the analysis reconciles underreaction to earnings with
overreaction to accruals.

The model also provides untested empirical implications about the strength of the forecasting power of earnings surprises, accruals, and cash flow for future returns. The intuition starts from the fact that, conditional upon high accruals, contemporaneous earnings also tend to be high, which causes investors to forecast higher future earnings. In this situation, an investor who neglects accruals is overoptimistic because he forecasts high future earnings based upon high current-period earnings. Similarly, when cash flow is high, an individual who neglects this fact is overpessimistic.

These effects imply that the strength of accruals or cash flows as return predictors depends on how favorably these variables predict future earnings relative to the forecasting power of current-period earnings as a predictor of future earnings. The sensitivities of expected future earnings to these variables are influenced by the relative variability of cash flows and accruals, the correlation of these earnings components, and the quality of accruals. In consequence, the more variable are accruals relative to cash flows, the stronger is the cash flow anomaly relative to the accruals anomaly; higher correlation between cash flows and accruals tends to weaken the accruals anomaly; and lower quality of accruals (a weaker incremental ability of accruals to forecast future earnings) tends to strengthen the accruals anomaly.

When some investors neglect earnings, and others attend to earnings but neglect accruals, stock prices underreact to earnings, and overreact to accruals relative to cash flow. If enough attention is paid to earnings, the relative overreaction to accruals outweighs the general underreaction to earnings, so that overall price overreacts to accruals. Furthermore, there tends to be stronger underreaction to cash flow than to earnings; the ratio of the cash flow effect on returns to the accruals effect on returns is greater in absolute value than the ratio of the variance of accruals to the variance of cash flow.

This paper is part of a recent theoretical literature on how constraints on information processing affect investor behavior. The approach followed here is similar in spirit to that of Hirshleifer and Teoh (2003), who study the effects on market prices of investors neglecting relevant accounting information or strategic aspects of the disclosure and reporting environment. A key difference here is that we examine the implications of limited attention for market misvaluation in relation to earnings surprises, accruals, and cash flows. Other recent papers model the allocation of attentional resources (Gabaix and Laibson (2004), Peng (2005), and Hirshleifer, Lim, and Teoh (2004)), how limited learning capacity affects asset price comovement (Peng and Xiong (2004)) and the speed of price adjustment to fundamental shocks (Peng (2005) and Peng and Xiong (2004)).
how delayed processing of new information affects the dynamics of asset price volatility (Peng and Xiong (2002)), how neglect of demographic information affects asset prices (DellaVigna and Pollet (2003)) and how informed parties make disclosure decisions when observers have limited attention (Hirshleifer, Lim, and Teoh (2004)). A somewhat different behavioral approach to modelling post-earnings announcement drift is provided by Fischer (2004), who does not, however, analyze the accruals anomaly.

2 The Economic Setting

As in Hirshleifer and Teoh (2003), we assume that each of a continuum of investors has a probability between zero and one of attending to a given information signal. They form their beliefs using only a subset of all publicly available information. We assume that inattentive investors, apart from the specific signals that they ignore, update beliefs as rational Bayesians.

There are 2 dates. At date 1, cash flow $c_1$, accruals $a_1$, and earnings $e_1 = c_1 + a_1$ are realized. Investors update their prior beliefs based upon whatever public signals they observe. At date 2 terminal earnings, cash flows and accruals, $e_2$, $c_2$, and $a_2$, are realized, where $e_2 = c_2 + a_2$, and the firm is liquidated. Since under so-called ‘clean surplus accounting’ accruals must reverse out, $a_2 = -a_1$.\(^5\)

Prices are set by trading in a securities market with no private information. Since no investor has private information, a fully rational individual has nothing to learn from market price. An inattentive individual who is unaware of his signal neglect will also think he has nothing to learn from market price. We therefore assume that inattentive investors do not update their beliefs based upon market price.\(^6\,7\)

Nevertheless, if an investor understood his attentional limits, he could in principle adjust for them by deferring to the belief implicit in market price. Indeed, a discrepancy

\(^5\)Date 1 cash flows, accruals and earnings can be interpreted as the latest flow of these variables for a firm. Alternatively, we can interpret date 1 as being a relatively long time period, such as 5 to 10 years. In that case, we interpret $c_1$, $a_1$ and $e_1$ as cumulative cash flows, accruals and earnings over a longer time interval.

\(^6\)Observing the ‘wrong’ price is an event which, as perceived by the investor, is not supposed to occur in equilibrium. In the Perfect Bayesian Equilibrium concept of game theory, setting the individual’s posterior beliefs in such a situation equal to the prior belief can be consistent with equilibrium. More generally, if liquidity shocks were introduced to the model (as in many models of information and securities markets), an investor with limited attention would attribute price fluctuations to noise, and therefore would have no reason to change his beliefs.

\(^7\)Similar results would hold so long as some disagreement remains between the attentive and inattentive investors, i.e., inattentive investors do not always abandon their beliefs in favor of the information implicit in the market price.
between an investor's valuation and the market price could alert the investor to his information neglect. In general, however, the same constraints on processing power and memory that make it hard to attend to an aspect of the environment also make it hard to compensate optimally for the failure to attend to an item. As with any public signal, they may simply fail to make use of the information it contains. So long as some fraction of inattentive investors have imperfect self-awareness, results similar to those derived here will obtain. Furthermore, even if individuals always attend to market price and are fully aware of their information neglect, similar results to those we derive here could be obtained in a setting so long as there is ‘noise’ in market price arising from liquidity trading.

We assume that individuals are identical except for differences in how much of the public information set they process. There is trade in equilibrium owing to imperfect rationality. Let \( \phi^i \) denote a date 1 information set attended to by investor group \( i \). For the fully attentive investor group, \( \phi^i \) is equal to the set of all date 1 publicly available information \( \phi \). For investors with limited attention, \( \phi^i \subset \phi \).

Investors have mean-variance preferences, so the date 1 optimization problem for investors in group \( i \) is

\[
E[C_2|\phi^i] - \left( \frac{A}{2} \right) \text{var}(C_2|\phi^i),
\]

where \( C_2 \) is terminal consumption, and \( A \) is the coefficient of absolute risk aversion. Such preferences are consistent with the combination of normality of returns and Constant Absolute Risk Aversion (CARA) utility.

We assume an initial wealth endowment (i.e., claims to terminal consumption) of \( W \) and the per capita endowment of the single risky security is \( x_0 \). At date 1, the individual can buy or sell the security in exchange for ‘cash’ (claims to terminal consumption) at price \( S_1 \). The position in the security he attains is denoted \( x^i \). Let \( S_2 \) be the true value of the stock, which is conclusively revealed to all at date 2. Then the consumption of

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8 In reality there are many relevant signals. Individuals can leverage their attention by focusing on more important information items, but it is hard to know which are more important before processing them. This makes it hard to determine how to optimally compensate for information neglect. Section 4 discusses evidence suggesting that individuals fail to compensate fully for the consequences of limited attention in making decisions.

9 An investor may just not think about the source of the discrepancy between the market price and his own valuation.

10 In such a setting, an individual who attends to a given public signal in effect has a sort of ‘private’ information, so different individuals who attend to different public signals will trade and profit at the expense of liquidity traders, in the spirit of the models of Grossman and Stiglitz (1976) and Diamond and Verrecchia (1981). This approach is discussed more fully in Section 4.
an individual in attention group $i$ is
\[ C_i^t = W - (x^i - x_0)S_1 + x^iS_2. \] (2)

Thus, an individual who attends to information set $\phi^i$ solves
\[ \max_{x^i} x^i(E[S_2|\phi^i] - S_1) - \left( \frac{A}{2} \right) \text{var}(x^iS_2|\phi^i). \] (3)

As a preliminary building block, we verify in our setting a standard finding about stock prices as a weighted average of investor expectations of terminal cash flows. Differentiating the objective with respect to $x^i$, equating to zero and solving yields
\[ x^i = \frac{E[S_2|\phi^i] - S_1}{A\text{var}(S_2|\phi^i)}. \] (4)

In several behavioral models, there are both rational and imperfectly rational investors who are risk averse and who maximize their expected utilities with respect to their correct or incorrect beliefs. In this literature, prices in equilibrium reflect a weighted average of the valuations of the rational and irrational traders. Even if there are no market imperfections, both groups influence prices significantly owing to the finite risk-bearing capacity of each group. As a preliminary step in the analysis, we verify that the market valuation of the firm in our setting is a weighted average of the beliefs of investors who attend to different parts of the public information set.

Letting $f_i$ denote the fraction of investors who attend to information set $\phi_i$, market price is determined by the security market clearing condition
\[ \sum_i f^i x^i = x_0. \] (5)

Substituting for $x^i$ from (4), and solving for $S_1$ gives
\[ S_1 = \sum_i \lambda^i E[S_2|\phi^i] - \frac{Ax_0}{\sum_i \alpha^i}, \] (6)

where
\[ \alpha^i \equiv \frac{f^i}{\text{var}(S_2|\phi^i)}, \quad \lambda^i \equiv \frac{\alpha^i}{\sum_i \alpha^i}. \] (7)

By normality, the $\lambda^i$’s are constants independent of the signal realizations used by investors to condition beliefs.
The final term in (6) is the risk premium that the security earns by virtue of being in positive net supply ($x_0 > 0$). Nothing in our analysis requires risk premia, so without loss of generality we eliminate this nuisance term by setting $x_0 = 0$ to obtain

$$S_1 = \sum \lambda E[S_2|\phi^i].$$

(Setting $x_0 = 0$ is solely to reduce clutter; all the model predictions carry through more generally.) This confirms that in equilibrium prices are a weighted average of the beliefs about terminal cash flows of different investors, with weight $\lambda^i$ on each information set. By (7), *ceteris paribus* $\alpha^i$ and $\lambda^i$ are increasing in $f^i$. Thus, the greater the likelihood of each investor being inattentive, the greater the weight that inattentive investors play in determining prices.

In this setting rational investors exploit a trading strategy that earns predictable abnormal returns relative to fully rational asset pricing benchmark. Nevertheless, even though markets are perfect and there are no restrictions on either long positions or short-selling, fully attentive investors do not completely arbitrage away the mispricing generated by inattentive investors. The reason they do not is because doing so is risky.

The intuition behind the traditional notion that rational investors dominate price is that rational investors trade to arbitrage away mispricing. However, if prices were set solely by the rational investors, imperfectly rational investors would perceive a profit opportunity to trading against what they regard as mispricing. If all investors are risk averse, the equilibrium outcome, as in equation (8), reflects a weighted average among these beliefs.\footnote{11}{It is often argued that even if there are irrational investors, the ‘marginal investor’ is rational, so that prices must be set rationally. However, under perfect markets, the behavior of all investor groups in equilibrium affect prices (see, e.g., Ball and Kothari (1991)), so all investors are marginal, as the $\lambda^i$ weights above indicate. As discussed in Hirshleifer and Teoh (2003), market price is determined by the aggregate of all investors’ security demand curves, not just by the demands of some ‘marginal’ investor group.}

\footnote{12}{Weighted average forms are found in previous models; see, e.g., Kandel and Pearson (1995), who provides evidence supportive of prices reflecting average trader perceptions; and in the context of limited attention, Hirshleifer and Teoh (2003). Equation (8) differs somewhat from the pricing equations in the heuristic trader models of Fischer and Verrecchia (1999) and Verrecchia (2001), which allow for non-competitive price effects and liquidity trading.}
3 Reconciling the Accruals Anomaly with Post-Earnings Announcement Drift

In the next subsection we describe the basic structure of cash flow, earnings and accruals over time. Subsection 3.2 establishes that there is post-earnings announcement drift in the model. Subsection 3.3 shows that the same setting also generates overreaction to the accruals component of earnings, and that cash flows also predict subsequent abnormal returns. Subsection 3.4 derives new empirical predictions about the circumstances in which the accrual and cash flow anomalies will be stronger or weaker.

3.1 The Relation of Earnings, Accruals and Cash Flow Over Time

We apply equation (8) letting \( i = 1 \) refer to the category of investors who use only the basic public information \( \phi_1 \) that is used by all investors, \( i = 2 \) to the investors who in addition attend to earnings information, and \( i = 3 \) to investors who additionally attend separately to each of accruals and cash flow, and thereby attends to all publicly available information.

Bernard and Thomas (1989) have proposed that post-earnings announcement drift is a result of investors mistakenly forming expectations of future earnings based upon a seasonal random walk model. In other words, they propose that investors form their expectation of future earnings based upon the level of earnings one year in the past while neglecting more recent quarterly earnings news. The seasonal random walk misperception explanation is therefore also based upon a form of limited attention. Our model has only 2 dates and therefore does not capture the full dynamics of seasonal random walk expectations. However, a key feature of the seasonal random walk misperception perspective which our model does capture is that at a given point in time some investors (our Category 1 investors) neglect the information contained in the latest earnings announcement.

The opportunity cost \( \gamma^i \) of achieving a probability \( \lambda^i \) that an investor processes a given subset of the public information pool can be modeled as a convex increasing function of the probability of attending to the given information set, and increasing in the size of the subset. Specifically, suppose that the cost of attending only to \( \phi_1 \) is zero, and that \( \gamma(\lambda^2) \), the cost of achieving probability \( \lambda^2 \) of attending to \( \phi^2 \) (but not to the additional information contained in \( \phi^3 \)) is increasing and convex. Suppose further that
the cost of increasing the probability of processing the full information set \( \phi^3 \) is \( K\gamma(\lambda^3) \), where \( K > 1 \). After the individual selects \( \lambda^2 \) and \( \lambda^3 \) subject to the constraint that \( \lambda^2 + \lambda^3 \leq 1 \), \( \lambda^1 \) is determined as the residual \( 1 - \lambda^2 - \lambda^3 \).

With sufficient convexity of the cost function \( \gamma \) there will be an interior solution for the allocation of attention. We do not analyze the attentional optimization decision (a topic analyzed in Hirshleifer, Lim, and Teoh (2004), Gabaix and Laibson (2004) and Peng (2005)). We merely note that since attending to more information is costly, those who devote more cognitive resources to an information signal (such as a given firm’s earnings, accruals or cash flow) need not do better overall. Attending to a given signal allows individuals to value the firm more accurately, at the cost of withdrawing attentional resources from some other activity. For example, attention demands time, which has a monetary opportunity cost. Since there is both a benefit and a cost to attending to a given public signal, in a long-run equilibrium in which wealths of investors shift over time, it does not follow that fully attentive investors dominate.\(^{13}\) Indeed, on a priori grounds such an outcome is impossible since human information processing capacity is finite.

Suppose that at date 1 fraction \( \lambda \) of investors ignore current-period earnings \( e_1 \) and remain at their prior belief,\(^{14}\) that fraction \( \lambda^e \) of investors attend to earnings \( e_1 \) but not to accruals and cash flow separately, and that the remaining fraction \( 1 - \lambda - \lambda^e \) attend to both accruals and cash flow (and therefore fully take into account the information in earnings), since

\[
e_1 = c_1 + a_1. \tag{9}\]

Then by equation (8), the date 1 stock price is

\[
S_1 = \lambda E[S_2] + \lambda^e E[S_2|e_1] + (1 - \lambda - \lambda^e)E[S_2|a_1, c_1]. \tag{10}\]

For tractability, we assume multivariate normality of the stochastic variables. As a result, date 2 earnings can be expressed as a linear function of the date 1 variables \( a_1 \)

\(^{13}\)Fischer and Verrecchia (1999) and Verrecchia (2001) have emphasized that investors who are modeled as influencing price should be able to earn enough profits to survive as important players in a capital market. Owing to the cost functions \( \gamma^1(\lambda^1) \) and \( \gamma^1(\lambda^1) \), our model satisfies this criterion. Investors who allocate greater or lesser attention to earnings or accruals can coexist in the long run since more attentive investor incur higher opportunity costs of attention. In addition, the literature on long-run survival suggests that imperfectly rational investors can under some circumstances remain influential in the long run.

\(^{14}\)It is not essential for the main conclusions that these investors completely ignore earnings. They could attend to the direct effects but ignore the implications of immediate earnings for future earnings, which would slightly modify the algebra but would not substantively affect the nature of the paper’s predictions.
and $c_1$ and a noise term $\delta_2$ as

$$e_2 = \beta_0 + \beta_1 c_1 + \beta_2 a_1 + \delta_2,$$  \hfill (11)

where $\text{cov}(\delta_2, c_1) = \text{cov}(\delta_2, a_1) = 0$. Consistent with past empirical evidence (Dechow (1994)), we assume that cash flow is a more positive predictor of future earnings than accruals, $\beta_1 > \beta_2$.\(^{15}\) Since earnings $e_1 = a_1 + c_1$, equation (11) directly implies that in a regression of future earnings $e_2$ on current period earnings and accruals, there is a negative coefficient on $a_1$. Similarly, in a regression of future earnings on current period earnings and cash flow, there is a positive coefficient on cash flow.

Letting bars denote unconditional expectations, by (11) we have

$$\bar{e}_2 = \beta_0 + \beta_1 \bar{c}_1 + \beta_2 \bar{a}_1,$$  \hfill (12)

so the deviation of earnings from its unconditional mean is

$$e_2 - \bar{e}_2 = \beta_1 (c_1 - \bar{c}_1) + \beta_2 (a_1 - \bar{a}_1) + \delta_2.$$  \hfill (13)

The terminal realized value of the stock is the sum of the cash flows at the two periods. Under clean surplus accounting, this is also equal to the sum of the earnings, so

$$S_2 = c_1 + c_2 = e_1 + e_2$$

$$E[S_2] = \bar{e}_1 + \bar{e}_2 = \bar{c}_1 + \bar{c}_2.$$  \hfill (14)

High earnings at date 1 is linearly associated with high earnings at date 2. The strength of this relation is given by the regression coefficient, which by (9) and (11) is

$$\beta_{e_2e_1} = \frac{\text{cov}(\beta_1 c_1 + \beta_2 a_1 + \delta_2, c_1 + a_1)}{V_{c_1}}$$

$$= \left( \frac{V_c + C}{V_c + V_a + 2C} \right) \beta_1 + \left( \frac{V_a + C}{V_c + V_a + 2C} \right) \beta_2,$$  \hfill (15)

\(^{15}\)An extra unit of either cash flow or accruals at date 1 increases date 1 earnings by one dollar. Thus, precisely because $\beta_1 > \beta_2$, expected firm value $E[S_2|a_1, c_1] = E[e_1 + e_2|a_1, c_1]$ increases more when cash flow increases by one dollar than when accruals does.

An equivalent way of viewing this is that ceteris paribus an extra dollar of date 1 cash flow by definition increases $c_1$ by a dollar, whereas an extra dollar of accruals does not. If $\beta_1 = \beta_2$, then an extra dollar of cash flow or of accrual predicts the same additional quantity of date 2 earnings. But if there is an extra date 1 accrual, then to achieve the same additional quantity of date 2 earnings despite the reversal of the date 1 accrual, the date 2 cash flow would have to be one dollar higher. Thus, if $\beta_1 = \beta_2$, $E[S_2|a_1, c_1] = E[e_1 + e_2|a_1, c_1]$ would be the same in both cases. Thus, the condition for expected firm value to be higher with an additional dollar of cash flow rather than accrual is precisely $\beta_1 > \beta_2$.\(^{11}\)
where $C \equiv \text{cov}(a_1, c_1)$, and where $V$ denotes the variance of a variable, with the abbreviations $V_a$ for $V_{a_1}$ and $V_c$ for $V_{c_1}$.

Thus, $\beta_{e_2e_1}$ is a weighted average of $\beta_1$ and $\beta_2$. Since $\beta_2 > \beta_1$, it follows that $\beta_1 > \beta_{e_2e_1} > \beta_2$ so long as both weights are positive. This indicates that high current-period cash flow is a more favorable forecaster of future earnings than is high current-period earnings, which in turn is a more favorable forecaster than high current-period accruals. It follows that the relation between date 2 and date 1 earnings is stronger the more variable are cash flows relative to accruals.

### 3.2 Post-Earnings Announcement Drift

We next examine the relation between the date 1 earnings surprise and the true expectation of future abnormal stock returns. We define the earnings surprise as $e_1 - \bar{e}_1$, where $\bar{e}_1$ is the prior expectation of date-1 earnings before they are realized. Since $\bar{e}_1$ is a constant, conditioning on the earnings surprise is equivalent to conditioning on the date 1 earnings $e_1$. We begin by calculating, conditional on earnings $e_1$, the expected future value of the stock, $E[S_2|e_1]$. For tractability, we examine price changes rather than percentage returns, as is standard in much of the literature on information in securities markets (see, e.g., Fischer and Verrecchia (1999), Verrecchia (2001), and Peng (2005)). Using standard properties of conditional expected values with multivariate normal distributions,

$$E[S_2|e_1] = e_1 + E[e_2|e_1] = e_1 + \bar{e}_2 + (e_1 - \bar{e}_1)\beta_{e_2e_1}.$$  

To calculate the conditional expected return given $e_1$, we also need $E[S_1|e_1]$. Drift in our model means that the expected change in the stock price conditional on the level of earnings increases with the level of earnings, with zero effect when realized earnings is equal to its prior expectation. By (10), we calculate drift as

$$E[S_2 - S_1|e_1] = e_1 + \bar{e}_2 + (e_1 - \bar{e}_1)\beta_{e_2e_1} - \lambda E[S_2] - \lambda' E[S_2|e_1] - (1 - \lambda - \lambda')E[E[S_2|a_1, c_1]|e_1]. \quad (16)$$

To evaluate this expression, we derive a slight generalization of the law of iterated expectations.

---

16The required regularity condition for the claim that $\beta_1 > \beta_{e_2e_1} > \beta_2$ is that each of higher accruals and higher cash flow is associated with higher earnings, i.e., $\beta_{e_1c_1}, \beta_{e_1e_1} > 0$. The proof is in the appendix.
Theorem 1  If $z$ is an integrable random variable, and random variable $y$ is a sufficient statistic for random variable $x$ (all on the same probability space), then
\[
E[E[z|y]|x] = E[z|x].
\] (17)

The proof is in the appendix.

Since $e_1 = a_1 + c_1$, the vector random variable $y ≡ (a_1, c_1)$ is a sufficient statistic for $e_1$. It follows by Theorem 1 that
\[
E[E[S_2|a_1, c_1]|e_1] = E[S_2|e_1].
\] (18)

$E[S_2|e_1]$ is given in equation (16). It follows by (16) that
\[
E[S_2 - S_1|e_1] = e_1 + \bar{e}_2 + (e_1 - \bar{e}_1)\beta_{e_{2e1}} - \lambda(\bar{e}_1 + \bar{e}_2) - \lambda^e[e_1 + \bar{e}_2 + (e_1 - \bar{e}_1)\beta_{e_{2e1}}]
\]
\[
- (1 - \lambda - \lambda^e)[e_1 + \bar{e}_2 + (e_1 - \bar{e}_1)\beta_{e_{2e1}}]
\]
\[
= \lambda(1 + \beta_{e_{2e1}})(e_1 - \bar{e}_1).
\] (19)

This proves:

**Proposition 1**

1. If fraction $\lambda > 0$ of investors neglect the information in current-period earnings (as well as cash flow and accruals), then there is post-earnings announcement drift.

2. Drift is proportional to:
   (i) the earnings surprise $e_1 - \bar{e}_1$;
   (ii) the fraction $\lambda$ of investors who neglect date 1 earnings; and
   (iii) the persistence of earnings, $\beta_{e_{2e1}}$.

As a special case, suppose that earnings follows a random walk, and consider a previous date 0 at which time the firm earns $e_0 = \bar{e}_1$. Then the surprise is the change in earnings $e_1 - e_0$.

Part 1 of Proposition 1 implies that drift is stronger when a greater fraction $\lambda$ of investors are inattentive. Consistent with this, firms whose shares are held heavily by individuals instead of institutions have stronger drift (Bartov, Krinsky, and Radhakrishnan (2000)). However, Bartov et al recommend caution in interpreting institutional shareholdings as a proxy for investor sophistication.

The implication of Part 2(ii) that greater neglect of earnings intensifies drift is consistent with the evidence discussed in the introduction that drift is stronger when earnings
is announced after hours rather than during trading hours, or on Friday rather than other weekdays. Holding constant the size of the earnings surprise and other model parameters, greater neglect of earnings in our model causes a weaker immediate reaction to earnings news, and a greater long term reaction (drift).

An untested empirical prediction of Part 2(ii) is that when there is a greater number of distracting news events such as earnings announcements by other firms, there should be greater neglect of earnings (higher $\lambda$). In consequence, the immediate stock price reaction to earnings announcement is predicted to be weaker, and post-earnings announcement drift is predicted to be stronger, when there is a greater number of distracting news events.

A recent body of research provides evidence which is generally supportive of the persistence prediction of Part 2(iii). Livnat (2003b) provides evidence that higher earnings persistence is associated with stronger drift, consistent with Part 2. Furthermore, since revenues have greater persistence than expenses (Jegadeesh and Livnat (2004a)), drift should be especially strong if the earnings surprise comes largely from a surprise in revenues. Livnat (2003b) finds that PEAD is stronger when the revenue surprise is in the same direction as the earnings surprise. His study controls for a variety of possible interfering effects. Jegadeesh and Livnat (2004b) and Gu, Jain, and Ramnath (2005) find that analyst forecasts also do not fully reflect differences in persistence between revenues and expenses. Livnat (2003a) relates the differential persistence of positive versus negative earnings surprises in the fourth quarter versus the first through third quarters to subsequent drift. He finds that the strength of drift is aligned with the degree of persistence of the earnings surprises. Sun (2005), extending the work of Koch and Sun (2004), uses dividend levels to identify when earnings is likely to be most persistent, and finds, consistent with the prediction in Part 2, that post-earnings announcement drift is stronger when earnings is more persistent. Chen (2004) also finds that drift is strongest among high persistence firms, but reports a reverse drift relation for firms with very low persistence, a phenomenon our model does not explain.
3.3 The Accruals and Cash Flow Anomalies

We now examine whether accruals or cash flows are forecasters of future returns. Specifically, we examine whether, conditional on the level of accruals (or cash flows), the stock is mispriced, thereby implying subsequent abnormal returns. To do so, we calculate the true expectation of the date 2 value of the stock conditional on accruals, $E[S_2|a_1]$, and of the date 1 stock price conditional on accruals, $E[S_1|a_1]$. The difference between these two values is the expected price change conditional on the level of accruals.

We first calculate the expected fundamental value of the firm of an individual who rationally conditions on both $a_1$ and $c_1$. By (13),

$$S_2 = c_1 + c_2 = e_1 + e_2$$
$$= a_1 + c_1 + \bar{e}_2 + \beta_1(c_1 - \bar{c}_1) + \beta_2(a_1 - \bar{a}_1) + \delta_2,$$  \hspace{1cm} (20)

$$E[S_2|a_1, c_1] = E[e_1 + e_2|a_1, c_1]$$
$$= e_1 + \bar{e}_2 + \beta_1(c_1 - \bar{c}_1) + \beta_2(a_1 - \bar{a}_1).$$  \hspace{1cm} (21)

By the rules for conditional expectations of multivariate normal random variables,

$$E[e_1|a_1] = \bar{e}_1 + (a_1 - \bar{a}_1)\beta_{e_1a_1}$$
$$E[c_1|a_1] = \bar{c}_1 + (a_1 - \bar{a}_1)\beta_{c_1a_1},$$  \hspace{1cm} (22)

So the expected price change conditional on accruals $a_1$ is

$$E[S_2 - S_1|a_1]$$
$$= E[e_1|a_1] + E[e_2|a_1] - \lambda E[S_2] - \lambda E[E[S_2|e_1]|a_1] - (1 - \lambda - \lambda^e)E[E[S_2|a_1, e_1]|a_1]$$
$$= \bar{e}_1 + (a_1 - \bar{a}_1)\beta_{e_1a_1} + E[e_2|a_1] - \lambda(\bar{e}_1 + \bar{e}_2) - \lambda E[E[S_2|e_1]|a_1]$$
$$- (1 - \lambda - \lambda^e)E[E[S_2|a_1, e_1]|a_1].$$  \hspace{1cm} (23)

We next calculate the various expectations above. By (13) and (22),

$$E[e_2|a_1] = \bar{e}_2 + \beta_1(E[c_1|a_1] - \bar{c}_1) + \beta_2(a_1 - \bar{a}_1)$$
$$= \bar{e}_2 + \beta_1(\bar{c}_1 + \beta_{c_1a_1}(a_1 - \bar{a}_1) - \bar{c}_1) + \beta_2(a_1 - \bar{a}_1)$$
$$= \bar{e}_2 + (\beta_1\beta_{c_1a_1} + \beta_2)(a_1 - \bar{a}_1).$$  \hspace{1cm} (24)

---

17Since earnings is the sum of cash flows and accruals, a bivariate regression of future returns on earnings and on accruals is essentially equivalent to a bivariate regression of future returns on earnings and on cash flows. However, the univariate regressions of returns on accruals or on cash flows are not equivalent.

18$S_1$ is stochastic from the perspective of an econometrician who performs a test that conditions only on the information $a_1$. 

15
By (16), (22) and multivariate normality,
\[
E[E[S_2|e_1]|a_1] = E[e_1|a_1] + \bar{e}_2 + (E[e_1|a_1] - \bar{e}_1)\beta_{e_2e_1} \\
= \bar{e}_1 + (a_1 - \bar{a}_1)\beta_{e_1a_1} + \bar{e}_2 + (a_1 - \bar{a}_1)\beta_{e_1a_1}\beta_{e_2e_1} \\
= \bar{e}_1 + (a_1 - \bar{a}_1)(1 + \beta_{e_2e_1})\beta_{e_1a_1} + \bar{e}_2.
\] (25)

Since the vector random variable \((a_1, c_1)\) is a sufficient statistic for \(a_1\), by Theorem 1,
\[
E[E[S_2|a_1, c_1]|a_1] = E[S_2|a_1].
\] (26)

So by (22), (24) and multivariate normality,
\[
E[S_2|a_1] = E[e_1|a_1] + E[e_2|a_1] \\
= \bar{e}_1 + (a_1 - \bar{a}_1)\beta_{e_1a_1} + \bar{e}_2 + (a_1 - \bar{a}_1)\beta_1\beta_{c_1a_1} + \beta_2 \\
= \bar{e}_1 + \bar{e}_2 + (a_1 - \bar{a}_1)(\beta_{e_1a_1} + \beta_1\beta_{c_1a_1} + \beta_2). 
\] (27)

It follows that
\[
E[S_2 - S_1|a_1] = \bar{e}_1 + (a_1 - \bar{a}_1)\beta_{e_1a_1} + \bar{e}_2 + (a_1 - \bar{a}_1)(\beta_1\beta_{c_1a_1} + \beta_2) - \lambda(\bar{e}_1 + \bar{e}_2) \\
- \lambda\bar{e}_1 + (a_1 - \bar{a}_1)(1 + \beta_{e_2e_1})\beta_{e_1a_1} + \bar{e}_2 \\
- (1 - \lambda - \lambda\bar{e}_1 + \bar{e}_2 + (a_1 - \bar{a}_1)(\beta_{e_1a_1} + \beta_1\beta_{c_1a_1} + \beta_2)] \\
= (a_1 - \bar{a}_1)[\lambda(\beta_{e_1a_1} + \beta_1\beta_{c_1a_1} + \beta_2) \\
+ \lambda\bar{e}_1 + \beta_1\beta_{c_1a_1} + \beta_2 - \beta_{e_2e_1}\beta_{e_1a_1})]. 
\] (28)

On the right hand side of the last equality, the first term within the brackets reflects the misvaluations of the fraction \(\lambda\) of investors who ignore earnings entirely. This leads to direct underreaction to the higher date 1 earnings associated with higher date 1 accruals \(a_1\) (the \(\beta_{e_1a_1}\) piece), and underreaction to the implications of a higher accrual for future earnings (the \(\beta_1\beta_{c_1a_1} + \beta_2\) piece). (On the \(\beta_{c_1a_1}\) piece, see also footnote 14.) Since these investors ignore the implications of higher accruals and resulting earnings, they underreact. So if \(\lambda^e = 0\), the future return is increasing with the deviation of accruals from their mean.

The other term reflects the fraction \(\lambda^e\) of investors who attend to earnings but not how earnings is divided between accruals and cash flows. The expected error made by these investors given current-period accruals is the difference between the correct forecast given current-period accruals, and the forecast of future earnings based on the effect of current-period accruals on current-period earnings, \(\beta_{e_2e_1}\beta_{e_1a_1}\). The correct forecast takes
into account the direct implications of accruals for future earnings, $\beta_2$, and the indirect effect of accruals on future earnings operating through the implications of accruals for current-period cash flow, $\beta_1 \beta_1 \beta_1 \beta_1$. We will show that the difference $\beta_1 \beta_1 \beta_1 + \beta_2 - \beta_2 \beta_2 \beta_2 \beta_2$ is negative, so that the $\lambda^e$ term contributes to overreaction to accruals.

It is not hard to confirm that under mild conditions the $\lambda^e$ term in (28) is negative, which is Part 1 of the following lemma:

**Lemma 1** If $\text{corr}(a_1,c_1) < 1$ and $\beta_1 > \beta_2$, then

\[
\beta_1 \beta_1 \beta_1 + \beta_2 - \beta_2 \beta_2 \beta_2 \beta_2 < 0 \quad (29)
\]

\[
\beta_2 \beta_2 \beta_2 \beta_2 + \beta_1 > \beta_2 \beta_2 \beta_2 \beta_2 \beta_2 > 0. \quad (30)
\]

The proof is in the appendix.

The model is symmetric with respect to $c_1$ and $a_1$, except for the assumption that $\beta_1 > \beta_2$ in (11). Thus, replacing $a_1$ with $c_1$, $c_1$ with $a_1$, $\beta_2$ with $\beta_1$, and $\beta_1$ with $\beta_2$ in (28), we obtain the expected return conditional on the date 1 cash flow,

\[
E[S_2 - S_1 | c_1] = (c_1 - \bar{c}_1)[\lambda(\beta_1 \beta_1 \beta_1 + \beta_2 \beta_2 \beta_2 \beta_2 + \beta_1) + \lambda^e(\beta_2 \beta_2 \beta_2 \beta_2 + \beta_1 - \beta_2 \beta_2 \beta_2 \beta_2)] \quad (31)
\]

It follows from Lemma 1 that the $\lambda^e$ term in (31) contributes to underreaction to cash flow. Since the $\lambda$ term does so as well, overall there is underreaction to cash flow.

The above analysis is summarized in the following proposition.

**Proposition 2** Suppose that at date 1 fraction $\lambda$ of investors do not attend to any of earnings $e_1$, cash flow $c_1$ and accruals $a_1$; that fraction $\lambda^e$ attend to earnings but not to its decomposition between cash flow and accruals; and that the remaining fraction $1 - \lambda - \lambda^e$ attend to all publicly available information. Then:

1. A firm with date 1 cash flows that are above (below) their unconditional mean is undervalued (overvalued), and subsequently on average earns positive (negative) abnormal returns.

2. Greater investor neglect of a firm intensifies the cash flow anomaly.

3. If $\lambda$ is sufficiently small relative to $\lambda^e$, then overvaluation is positive and abnormal returns are on average negative subsequent to date 1 accruals that are above their unconditional mean, and overvaluation is negative and abnormal returns on average positive subsequent to date 1 accruals that are below their unconditional mean.
Part 1 indicates that under our assumption that cash flow is a more favorable incremental predictor of future earnings than is accruals ($\beta_1 > \beta_2$ in equation (11)), then there is under-reaction to cash flow. As discussed after footnote 15, this assumption is equivalent to the assumption that after controlling for current-period earnings, accruals are a negative incremental predictor of future earnings).

Part 2 reflects the fact that neglect of earnings and neglect of earning components both reinforce the cash flow anomaly.

Part 3 indicates that, again if cash flow is a more favorable incremental predictor of future earnings than is accruals, if enough attention is paid to earnings, there is overreaction to the accruals component of earnings.

The only asymmetry in the model between accruals and cash flows comes from our assumption $\beta_1 > \beta_2$ as in (11). It follows that if the assumptions were reversed (i.e., accruals were a positive incremental predictor of future earnings and cash flows were a negative incremental predictor), then it would be accruals that positively predict returns, and it would be cash flow which could predict either positively or negatively. If the two were equally positive predictors of future earnings, then there would be underreaction to both accruals and cash flow, just as there is underreaction to earnings in Proposition 1.

In the case of $\lambda = 0$, Part 3 applies. The ability of accruals to forecast returns in this case reflects the fact, as discussed at the end of Subsection 3.1, that $\beta_1 > \beta_{ce1} > \beta_2$. In other words, high current-period cash flow is a more favorable forecaster of future earnings than is high current-period earnings, which is a more favorable forecaster than high current-period accruals. Those investors who focus on earnings without attention to its components do not take into account that for a given level of earnings, the true expectation of future earnings is higher when cash flow is high than when it is low. In consequence, such investors undervalue firms with high cash flow and overvalue firms with low cash flow. Such misvaluation is subsequently corrected, causing abnormal returns.

Similarly, those investors who focus on earnings without attention to its components do not take into account that for a given level of earnings, the true expectation of future earnings is higher when accruals are low than when they are high. In consequence, such investors overvalue firms with high accruals and undervalue firms with low accruals, leading to subsequent abnormal returns.

\footnote{Proposition 2 does not require the regularity condition described in footnote 16, but the case in which this condition is violated is unrealistic.}
When $\lambda > 0$, so that some investors neglect earnings, there is general underreaction to earnings. Since cash flow is a more favorable predictor of earnings than accruals, $\lambda^c > 0$ further implies underreaction to cash flow relative to accruals. This reinforces the underreaction to cash flows, which provides the intuition for the cash flow anomaly asserted in Part 1.

With regard to Part 2, a shift in investor probability mass from attending to both earnings and its components to either neglecting earnings components (increasing $\lambda^c$) or to neglecting earnings as well as its components (increasing $\lambda$) intensifies the cash flow anomaly. A shift in investor probability mass from attending only to earnings to neglecting earnings (decreasing $\lambda^e$, increasing $\lambda$ by the same amount) has two effects on drift of opposing sign. However, the effect of the increase in $\lambda$ is stronger, because this shift means that when cash flow is high (for example), a set of investors is now neglecting completely the fact that cash flow is high, instead of just neglecting the fact that it is cash flow that is high rather than just earnings that is high. An untested empirical implication of Part 2 is that the cash flow anomaly will be stronger when the number of distracting news events, such as earnings announcements by other firms, is greater.

Even though investors overreact to accruals relative to cash flow, if $\lambda$ is sufficiently large relative to $\lambda^e$, there is underreaction even to the accruals component of earnings. This explains why the Part 3 conclusion that high accruals are associated with low returns requires that the fraction of investors who attend only to earnings (and not its components) be sufficiently large relative to the fraction of investors who neglect earnings entirely.

Thus, the analysis is consistent with the accruals anomaly, but the condition under which it applies is slightly different from that proposed by Sloan (1996), that accruals be a less favorable predictor than cash flows of future earnings ($\beta_2 < \beta_1$ here). Here $\beta_2 < \beta_1$ is a necessary but not sufficient condition for the accruals anomaly to apply. Thus, an empirical implication of Proposition 2 is that the positive relation of cash flows to subsequent returns should be more robust across different firms, trading venues and time periods than the negative relation of accruals to subsequent returns. Specifically, the accruals anomaly could reverse for firms that have greater neglect of earnings (higher $\lambda$). Proposition 1 indicates that a proxy for $\lambda$ that can be used for empirical testing is provided by the strength of the firm’s post-earnings-announcement drift. Alternatively, other proxies for the sensitivity of a firm’s misvaluation to its earnings surprises could be used based upon contemporaneous misvaluation proxies. Either proxy for $\lambda$ can also
be used to test Part 3 of Proposition 2 as well.

### 3.4 The Relative Strength of Accruals- and Cash Flow-Based Return Predictability

Our analysis provides insight regarding the strength of the cash flow and accruals anomalies. Let \( b_a \) be the slope coefficient in the relation between accruals surprises \( a_1 - \bar{a}_1 \) and subsequent abnormal stock returns, and let \( b_c \) be the slope coefficient in the relation between accruals surprises \( a_1 - \bar{a}_1 \) and subsequent abnormal stock returns. Then by equations (28) and (31),

\[
\begin{align*}
    b_a &= \lambda (\beta_{e1}a_1 + \beta_1\beta_{c1}a_1 + \beta_2) + \lambda^e (\beta_1\beta_{c1}a_1 + \beta_2 - \beta_2\beta_{c1}\beta_{e1}a_1) \\
    b_c &= \lambda (\beta_{e1}c_1 + \beta_2\beta_{a1}c_1 + \beta_1) + \lambda^e (\beta_2\beta_{a1}c_1 + \beta_1 - \beta_2\beta_{a1}\beta_{e1}c_1). \\
\end{align*}
\]

(32)

#### 3.4.1 Effects of Variations in Attention on the Accruals- and Cash Flow-Anomalies

Since the \( \lambda \) terms are positive, in the cash flow equation the \( \lambda^e \) terms is also positive, and in the accruals equation the \( \lambda^e \) term is negative, we have:

**Proposition 3**

1. The strength of the relation between cash flow and subsequent abnormal returns, \( b_c \), is increasing in \( \lambda \) and \( \lambda^e \).

2. The strength of the relation between accruals and subsequent abnormal returns, \( |b_a| \), is decreasing in \( \lambda \) and increasing in \( \lambda^e \).

Intuitively, if the fraction of investors that attends to earnings but neglects accruals and cash flow, \( \lambda^e \), increases (at the expense of fully attentive investors), then this neglect favors underreaction to cash flows (which are more persistent than earnings), and overreaction to accruals (which are less persistent than earnings). If the fraction of investors that neglects earnings, \( \lambda \), increases, the underreaction to cash flows is reinforced, whereas the overreaction to accruals is weakened. As mentioned in the discussion following Proposition 2, an empirical proxy for \( \lambda \) for a stock is the strength of its post-earnings announcement drift anomaly.

A somewhat more complex proxy is the share ownership of institutional versus individual investors. If institutional investors are attentive to accruals, cash flows, and earnings, then high institutional ownership and low individual ownership should be associated with low \( \lambda \) (neglect of both earnings and accruals) and \( \lambda^e \) (neglect of earnings).
Low \( \lambda \) and low \( \lambda^e \) both weaken the cash flow anomaly. Furthermore, if institutional ownership is high enough to drive both \( \lambda \) and \( \lambda^e \) toward zero (so that few investors neglect the split of earnings between cash flow and accruals), the accruals anomaly should become arbitrarily weak. Empirically, Collins, Gong, and Hribar (2003) find that the accruals anomaly is stronger in stocks with lower ownership by active institutional investors.

3.4.2 Effects of Earnings Quality on Accrual and Cash Flow Anomalies

It has often been alleged that earnings management reduces the quality of earnings, in the sense that earnings is a less accurate indicator of long-run firm performance. In our setting such a reduction in earnings quality associated with accruals would be reflected in a low \( \beta_2 \) in equation (11), so that current-period level of accruals becomes a much less favorable predictor of future earnings than current-period cash flow.

We now consider the implications of lower earnings quality in this sense for the strengths of the accrual and cash flow anomalies. Substituting from (15) for \( \beta_{eze1} \) in the first equation of (32) and differentiating with respect to \( \beta_2 \), by (11) yields

\[
\frac{\partial b_a}{\partial \beta_2} = \lambda + \lambda^e - \lambda^e \left( \frac{V_a + C}{V_a + V_c + 2C} \right) \beta_{e1a1}
\]

\[
= \lambda + \lambda^e - \lambda^e \left[ \frac{(V_a + C)^2}{(V_a + V_c + 2C)V_a} \right]
\]

\[
= \lambda + \lambda^e \left( \frac{V_cV_a - C^2}{V_a + V_c + 2C} \right)
\]

\[
> 0,
\]

where the final inequality holds because \( \text{corr}(a_1, c_1) < 1 \). This derivative indicates that if there is an accruals anomaly (\( b_a < 0 \)), then when earnings quality increases (accruals becomes a more favorable incremental predictor of future earnings), the accruals anomaly becomes weaker (the negative coefficient increases, moving it closer to zero). If there is no accruals anomaly or a reverse accruals effect (\( b_a \geq 0 \)), then the effect becomes even more reversed.
Differentiating the second equation in (32) with respect to $\beta_2$ yields

$$
\frac{\partial b_c}{\partial \beta_2} = \lambda \beta_{a1c1} + \lambda e \left[ \beta_{a1c1} - \left( \frac{\partial \beta_{e2c1}}{\partial \beta_2} \right) \beta_{e1c1} \right] \\
= \lambda \beta_{a1c1} + \lambda e \left[ \frac{C}{V_c} - \left( \frac{V_a + C}{V_a + V_e + 2C} \right) \left( \frac{V_e + C}{V_e} \right) \right] \\
= \lambda \beta_{a1c1} + \lambda e \left[ \frac{C^2 - V_a V_e}{V_e (V_a + V_e + 2C)} \right].
$$

(34)

The $\lambda$ term is positive, but the numerator of the $\lambda e$ term is negative since $\text{corr}(a_1, c_1) < 1$. This indicates that the cash flow anomaly could be either strengthened or weakened by an increase in earnings quality.

**Proposition 4** Consider an increase in accruals quality that increases $\beta_2$, i.e., makes the incremental forecasting power of current-period accruals for future earnings closer to that of current-period cash flows. Then an increase in earnings quality:

1. Weakens the accruals anomaly; and
2. Can either weaken or strengthen the cash flow anomaly.

The $\beta_2$ parameter can be estimated by running the regression in (11). Therefore the prediction about the relation of accruals quality to the accruals anomaly is empirically testable.

Intuitively, the accruals anomaly is driven by the low quality of the accruals component of earnings, so if this component becomes a more favorable forecaster of earnings, investor neglect of accruals and cash flows leads to less overreaction. This is reinforced by a subtler effect. When earnings quality increases owing to an increase in $\beta_2, \beta_{e2c1}$, the persistence of earnings increases (see equation (15). Investor neglect of the implications of current-period earnings for future earnings (as reflected in $\lambda$) therefore causes greater underreaction to earnings. This greater general underreaction further weakens the overreaction to accruals.

The intuition for the effect of a change in accruals quality on the cash flow anomaly reflects the same two effects. On the one hand, higher accruals quality eases the misperceptions associated with the neglect of accruals and cash flow information. This is reflect in the negative $\lambda e$ term in (34), indicating a weakening of the cash flow anomaly.

On the other hand, more subtly, higher accruals quality makes high current period earnings a more favorable forecaster of future earnings. Therefore, the neglect of current
period earnings, as reflected in the $\lambda^e$ term in (34), tends to increase underreaction. This reinforces the underreaction to cash flows, strengthening the cash flow anomaly. Thus, the overall effect of improving accruals quality on the cash flow anomaly is ambiguous.

Testing of Proposition 4 requires controlling for other exogenous parameters such as the variances of accruals and cash flows. This is important, for example, in evaluating differences in earnings quality that derive from differing earnings management practices, since a firm that manages earnings heavily may have greater variability in accruals. We next examine the effects on anomalies of varying volatilities and correlations as well.

3.4.3 Effects of Accruals and Cash Flow Volatilities and Correlations on Accruals- and Cash Flow- Anomalies

The $\lambda^e$ term, which reflects investor inattention to the division of earnings between cash flow and accruals, is the sole source of mispricing when $\lambda = 0$. We define $b^0_a$ and $b^0_c$ as the values of the slope coefficients that come from neglect of earnings components, not from neglect of earnings itself (i.e., the $\lambda^e$ term),

$$b^0_a = \lambda^e (\beta_1 \beta_{c1a1} + \beta_2 - \beta_{e2e1} \beta_{e1a1})$$

$$b^0_c = \lambda^e (\beta_2 \beta_{a1c1} + \beta_1 - \beta_{e2e1} \beta_{e1c1}).$$

(35)

We will see that the relative size of the slope coefficients in the accruals regression and in the cash flow regression is determined by the ratio of the variances of accruals and of cash flows. To verify this algebraically, we start with the fact that $e_1 = a_1 + c_1$, so that

$$\beta_{e1a1} = 1 + \frac{C}{V_a}$$

$$\beta_{e1c1} = 1 + \frac{C}{V_c}.$$

By (35),

$$\frac{b^0_a}{\lambda^e} = \beta_1 \left( \frac{C}{V_a} \right) + \beta_2 - \beta_{e2e1} \left( 1 + \frac{C}{V_a} \right)$$

$$\frac{b^0_c}{\lambda^e} = \beta_2 \left( \frac{C}{V_c} \right) + \beta_1 - \beta_{e2e1} \left( 1 + \frac{C}{V_c} \right).$$

(36)

Multiplying $b^0_a/\lambda^e$ in the first equation by $V_a$, and $b^0_c/\lambda^e$ in the second equation by $V_c$, we obtain

$$V_a b^0_a/\lambda^e = \beta_1 C + \beta_2 V_a - \beta_{e2e1} V_a - \beta_{e2e1} C$$

$$V_c b^0_c/\lambda^e = \beta_2 C + \beta_1 V_c - \beta_{e2e1} V_c - \beta_{e2e1} C.$$

(37)
As shown in Lemma 1, there is overreaction to accruals, so that the first quantity is negative; and there is underreaction to cash flow, so that the second quantity is positive. Adding these two quantities, multiplying by $\lambda^c$, and by (15) we find that

$$V_a b_a^0 + V_c b_c^0 = 0,$$

or

$$\frac{b_a^0}{b_c^0} = -\frac{V_c}{V_a},$$

consistent with the earlier intuitive discussion. For example, if $\lambda = 0$ and $V_c = V_a$, then $b_a^0 = -b_c^0$, so that the accruals and cash flow anomalies are equally strong.

The assumption so far in this discussion, that $\lambda = 0$, eliminates post-earnings announcement drift, as shown in Proposition 1. More generally, when $\lambda > 0$ in (32), the slope coefficients $b_a$ and $b_c$ are larger than the corresponding $b_a^0$ and $b_c^0$. Since $b_a^0 < 0$ and $b_c^0 > 0$, it follows that if $\lambda > 0$,

$$\frac{b_a}{b_c} > -\frac{V_c}{V_a},$$

i.e., the relative strength of the cash flow effect is greater than the negative variance ratio. For example, if cash flow and accruals are equally variable, then (38) indicates an equally strong cash flow and accrual effect. But if some investors ignore earnings, so that $\lambda > 0$, by (39) the cash flow effect is stronger than the accruals effect. Thus, although the empirical literature has focused more on accruals than cash flow, our analysis predicts that the cash flow effect can be stronger.

This analysis is summarized in the following proposition.

**Proposition 5**

1. If investors always attend to earnings, so that $\lambda = 0$, then the ratio of the slopes in the univariate regressions of subsequent returns on accruals and on cash flows, $b_a^0/b_c^0$, is negatively proportional to the ratio of the variance of cash flow to the variance of accruals, $-V_c/V_a$.

2. If some investors neglect earnings, so that $\lambda > 0$, then the ratio of the accruals misvaluation/abnormal returns coefficient to the cash flow misvaluation/abnormal returns coefficient, $b_a/b_c$, exceeds the negative of the ratio of the variance of cash flow to the variance of accruals, $-V_c/V_a$.

Intuitively, consider an investor who focuses on current-period earnings in forecasting future earnings. For simplicity, suppose that the covariance of current-period accruals and current-period cash flow is zero. High accruals increase current-period earnings,
making the investor more optimistic about future earnings. The investor on average becomes too optimistic, because he forecasts high future cash flow based on the unconditional relation between current-period earnings and future earnings. Thus, the investor’s overoptimism will be greater to the extent that an extra unit of earnings is on average associated with higher future earnings more strongly than is an extra unit of accruals.

When accruals are highly variable relative to cash flow, most variation in earnings comes from variations in accruals. In this case, the forecasts of investors with limited attention who condition on earnings are almost equivalent to fully attentive forecasts based on accruals. High accruals bring about little over-optimism, so the slope \( b_0 \) approaches zero. In contrast, when the variance of accruals is low relative to the variance of cash flow, the forecasts of investors with limited attention who condition on the a given deviation of earnings from mean earnings are similar to the forecasts that would be made based on a comparable deviation of cash flow from its mean. If in fact it is accruals, not cash flow, that is high, then this misattribution brings about a great deal of overoptimism.

A higher value of \( b_a \) means that it is less negative, a weaker accruals effect. So Part 2 indicates that the cash flow effect can be stronger than the accruals effect even if the relative variability is equal.

One reason why a firm’s accruals might be highly variable is if it engages in extensive earnings management. If so, then accruals may be a less favorable predictor of future earnings (lower \( \beta_2 \)). It is therefore striking that the prediction of Proposition 5 is independent of the forecasting powers of cash flow or accruals, \( \beta_1 \) or \( \beta_2 \), for future earnings.\(^{20}\) The independence of the prediction of Proposition 5 from \( \beta_1 \) and \( \beta_2 \) simplifies empirical testing.

Analytical models predict that market price should be a proxy both for misvaluation and for rational risk premia. In consequence, accruals-to-price and cash-flow-to-price ratios should predict returns even if the asset-scaled variables do not. Thus, examining price-scaled versions of these variables does not sharply distinguish between a limited attention explanation for these effects versus a rational risk premium explanation or other behavioral hypotheses. This paper focuses on the distinctive aspects of the accruals anomaly, that accruals predict returns when the scaling is by assets rather than price.

\(^{20}\)Intuitively, the prediction in (38) is about the relative sizes of these anomalies. If \( \beta_2 << \beta_1 \), so that the neglect of accruals makes a big difference for value, the accruals anomaly will be stronger (as indicated by equation (33) in the derivation of Proposition 4). However, in such a case the cash flow anomaly also tends to be stronger (as indicated by the \( \lambda^c \) term in equation (34)).
Similarly, we draw predictions for cash flow not scaled by price.

A price-scaled variable such as cash flow/price or earnings/price can subsume part of the accruals effect (for example) even if the source of predictability is limited attention about accruals. Since market price reflects mispricing, price-related ‘controls’ such as size, book-to-market and/or factor models derived from these characteristics can remove part of the limited attention effects we seek to identify. Thus, to test the predictions of this paper a non-price scaling such as assets is appropriate. Nevertheless, to the extent that these price-containing variables are imperfect proxies for mispricing, we would not expect them to entirely eliminate the limited attention effects we would seek to measure to test the model. For example, empirically the accruals anomaly remains strong after controlling for book-to-market and other price-related controls (for a confirmation in a recent sample, see Hirshleifer, Hou, Teoh, and Zhang (2004)).

By Proposition 5, a higher variance of accruals relative to cash flow implies a weaker accruals effect. With regard to Part 1, if the accruals or cash flow effects are empirically evaluated using hedge profits based upon fractile sorts, the spread in the independent variables is affected by their variances. Under normality, a higher variance of accruals implies a higher mean high-low fractile spread in accruals. Since the relative weakness in the slope is proportional to the relative variances, whereas the fractile spread increases only as the square root of the variances, there is partial cancellation. Overall, Proposition 5 indicates that for fractile spreads, the ratio of the hedge profits based upon accruals versus those based upon cash flow will be negatively inversely proportional to the ratio of the standard deviations (instead of variances) of accruals and of cash flows.

We next evaluate the effects of variances on each of the slope coefficients separately.

**Proposition 6** If \( \beta_1 > \beta_{e1} > \beta_2 \), investors are fully attentive to earnings \( (\lambda = 0) \), but some do not attend to accruals and cash flow separately \( (\lambda^* > 0) \), and if the correlation between cash flows and accruals, \( \rho \), exceeds a critical value \( \rho^* \), where

\[
\rho^* = -\frac{R}{2} + \sqrt{\frac{R^2}{4} - 1} < 0, \tag{40}
\]

and

\[
R \equiv \frac{\sigma_a}{\sigma_c} + \frac{\sigma_c}{\sigma_a}, \tag{41}
\]

then the slope coefficient \( b_a^* \) is increasing, and the slope coefficient \( b_c^* \) is decreasing, with \( \rho \). In other words, as \( \rho \) increases, both the accruals and cash flow anomalies become weaker.
The proof is in the Appendix. Empirically, the parameter $\rho$ can be estimated using accruals and cash flow data.

The intuition is that both accruals and cash flow effects derive from investors neglecting the differing implications for long-run earnings of current-period cash flow versus accruals. When accruals and cash flow are highly correlated with each other, each is highly correlated with earnings, so that ignoring the distinction between different earnings components is on average of little consequence.\footnote{In Part 3, the critical value $\rho^*$ can be quite low, so that the implication can apply even when there is a strong negatively correlation between cash flows and accruals. For example, if $\sigma_c = \sigma_a$, then by (41) $\rho^* = -1$.}

If $\lambda > 0$ so that some investors neglect earnings, not just its components, and if $\rho \geq \rho^*$ as given in (40), higher $\rho$ further weakens the accruals effect (increases the negative slope $b_a^0$):

$$b_a = \lambda \left[ (1 + \beta_2) + (1 + \beta_1) \frac{2\rho \sigma_c}{\sigma_a} \right] + b_a^0, \quad (42)$$

so

$$\frac{\partial b_a}{\partial \rho} = 2\lambda (1 + \beta_1) \frac{\sigma_c}{\sigma_a} + \frac{\partial b_a^0}{\partial \rho}. \quad (43)$$

Since $2(1 + \beta_1)\sigma_c/\sigma_a > 0$, it follows that $\partial b_a/\partial \rho > 0$. The following proposition summarizes this finding:

**Proposition 7** If $\beta_1 > \beta_{e_2e_1} > \beta_2$ and if $\rho > \rho^*$ as defined in equation (40), then the slope coefficient of future returns on accruals $b_a$ is increasing with the correlation $\rho$ between cash flows and accruals.

Intuitively, the first term reflects that fact that when the correlation between accruals and cash flow is higher, high accruals tend to be more positively (or less negatively) associated with high cash flow. Since the market underreacts to earnings ($\lambda > 0$), there is underreaction to the higher cash flow, which tends to oppose the pure accruals effect (make $b_a$ less negative).

### 4 Do Investors Fully Compensate for Limited Attention?

A key assumption of our model is that individuals with limited attention trade based upon their beliefs. As a result, limited attention affects the equilibrium price. However, an individual who understands that he is neglecting a relevant public information item
in forming his beliefs is free to retreat into autarky, thereby deferring completely to the market price. Such withdrawal into passivity would be based upon the belief that the individual’s belief about market value is less precise than the market price. The model could easily be generalized to allow for the possibility that individuals are sometimes sophisticatedly passive in this fashion. So long as individuals with limited attention are not always passive, similar results will hold.

Casual observation suggests that investors often do make trades based on beliefs that do not fully reflect publicly available information. For both psychological and economic reasons, we do not expect that individuals with limited attention would or even should always withdraw into passivity. Evidence from both psychology and market behavior indicates that individuals often fail to adjust appropriately for the fact that they have left some information unprocessed.

Such double neglect makes sense when processing power is limited. Attention and mental processing are needed to adjust for the fact that information items that are not being attended to in detail may affect the reliability of the judgments being formed. An extra cognitive step is required, after forming a valuation of the firm, to recognize that the market may be using useful information that was not a part of the information used by the individual. Indeed, the cognitive cost of calculating how to adjust for the fact that a relevant information item has been neglected can be as great or greater than the cognitive cost of simply processing the information item. So in many cases, ignoring the information item and failing to adjust for the fact that the item has been neglected go hand in hand.

The neglect of both a signal and of the fact that it is neglected is reflected in the availability heuristic of Tversky and Kahneman (1973). In this heuristic, individuals assess the frequency or likelihood of some characteristic in a population according to their ability to retrieve confirmatory examples from memory. This tends to overweight examples with vivid, salient characteristics. If individuals were on the whole highly sophisticated they would largely debias the availability heuristic by downgrading their frequency estimates for items that are easy to recall because of vivid, salient characteristics.

A second type of evidence of this double neglect is provided by studies that show that the form of presentation of information affects individuals judgments and decisions (see, e.g., Slovic (1972), Payne, Bettman, and Johnson (1993), and the review of Libby, Bloomfield, and Nelson (2002)). Experimental studies have found that different presentations of equivalent information about a firm affects the valuations and trades of
investors and experienced financial analysts. In principle, if an investor understood that owing to limited attention certain formats were hard to process, he could debias himself by, for example, mentally rearranging the format of presentation. However, such rearrangement itself requires mental processing.

Furthermore, experimental research has found that the presentation of one-sided arguments and evidence to subjects (call ‘jurors’) asked to judge a legal dispute were biased in favor of the side they heard (Brenner, Koehler, and Tversky (1996)). According to the authors, “The results indicate that people do not compensate sufficiently for missing information even when it is painfully obvious that the information available to them is incomplete. Another indication of failure to adjust for unprocessed information is that individuals tend to underweight the probabilities of event contingencies that are not explicitly available for consideration; e.g., in a list of possible causes of an event, the probability of ‘other causes’ is underestimated (Fischoff, Slovic, and Lichtenstein (1978)).

Why are individuals with limited cognitive resources subject to availability bias? And more generally, why don’t individuals fully adjust beliefs about their own precisions to take into account the set of cues they are neglecting? Because thinking about how the neglect of a cue creates bias or reduces precision requires an extra layer of cognitive processing. Just as cognitive resource constraints cause a cue to be neglected, they often also cause the individual to fail to take the extra step of assessing the consequences of that neglect. Even if on average the individual is correct, in those cases in which the cues he neglects are especially important, he will tend to overestimate the precision of his beliefs; and when the cues he neglects are minor, he will underestimate his precision. Such miscalibration encourages individuals who have neglected important cues to trade and influence price.

Our model shares with informal behavioral explanations for anomalies the assumption that investors who neglect public signals such as current-period earnings are important for price setting. For example, the seasonal-random-walk explanation for post-earnings-announcement drift of Bernard and Thomas (1989) assumes that investors who

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22Presentation effects have been found in the context of recognition versus disclosure of pension liabilities (Harper, Mister, and Strawser (1987)), classification of the same hybrid financial instrument as debt, equity or mezzanine financing in the balance sheet (Hopkins (1996)), the previewing of negative earnings news with an adverse qualitative preannouncement (Libby and Tan (1999)), the use of the purchase method of accounting for business combinations with the premium ratably amortized versus the use of pooling-of-interest (Hopkins, Houston, and Peters (2000)) and the inclusion of other comprehensive income items in the income statement rather than in the statement of changes in shareholders’ equity (Hirst and Hopkins (1998)), as well as in market settings (Dietrich et al (2001)).
focus on year-ago earnings neglect the latest earnings surprise. There are a number of reasons why an individual who neglects important information such as the implications of current-period earnings would still trade actively. An investor who has a perspective about the firm’s long-term business strategy and competitive environment may trade based upon his valuation target without updating his forecasts of future earnings in response to every earnings report.\textsuperscript{23}

There is evidence that some individual investors are net providers of liquidity to the market, absorbing the demand by institutions for trading immediacy.\textsuperscript{24} If institutions (perhaps for agency reasons) sometimes make uninformed trades, a simple contrarian trading strategy will in general be profitable. Indeed, the evidence indicates that contrarian trading allows U.S. individual investors to earn positive excess returns in the month after their trades (Kaniel, Saar, and Titman (2004)).

A naive contrarian strategy of trading in opposition to market price movements without regard to their source will on average induce trading against earnings announcements.\textsuperscript{25} If institutions are willing to pay for trading immediacy, a naive contrarian strategy can be profitable on average. However, such a strategy will on average be unprofitable after an earnings announcement, because the price move is being triggered by actual news rather than by another investor’s demand for trading immediacy. Thus, limited attention on the part of investors traders who are following a generally-reasonable contrarian strategy can induce unprofitable contrarian trading in response to earnings announcements.

A further reason why an investor with limited attention would actively trade as a

\textsuperscript{23}The tendency to hold a fixed valuation in mind may be reinforced by the practice of analysts of announcing target prices below which the stock is recommended as a good buy. An investor who follows such a recommendation without adjusting the target to reflect further news about earnings fits the analysis. Plausible-sounding media soundbites can reinforce the tendency of investors with limited attention to trade as contrarians. There is much editorializing in the business media against ‘obsession with the short-term, and in favor of ‘buying on the dips. An investor can excuse a failure to update valuations frequently as “focusing on the long-term” rather than “myopically” focusing on quarterly earnings. A naive application of soundbites with high availability in the mass media is itself a possible consequence of limited attention.

\textsuperscript{24}For example, in the U.S, there is evidence that individuals as a group trade as contrarians to the previous week’s stock return (Kaniel, Saar, and Titman (2004)). A similar finding has been documented among Finnish and Korean individual traders (Grinblatt and Keloharju (2000), Choe, Kho, and Stulz (1999)).

\textsuperscript{25}For example, a limit order is one means by which an investor can implement a naïve contrarian strategy. Since limit orders are triggered by moves in market price, places such an order allows the investor to trade against market price moves, providing liquidity to the market, without continuously monitoring prices. Linmainia (2003) finds that individual day traders in Finland provide liquidity to the market through limit orders and on average profit during the day by doing so.
contrarian to earnings news is that analysts on average do not fully adjust their earnings forecasts in response to earnings announcements.\textsuperscript{26} Owing to limited attention, some investors may rely on analyst earnings forecasts without performing the additional cognitive processing needed to adjust for this state-contingent bias in analyst forecasts. The beliefs of such investors will therefore underreact to earnings surprises, just as analyst forecasts do.\textsuperscript{27} Consistent with this interpretation, there is evidence that firms that are followed by analysts whose forecasts are more responsive to earnings announcements have less post-earnings-announcement drift (Zhang (2005)).

There are economic reasons why even an investor who is aware of his own limited attention and rationally adjusts for it should trade based upon his expectations rather than withdrawing into autarky. Such an individual is not wholly at a disadvantage relative to other traders. Time and physical constraints imply that all individuals are subject to limited attention. An individual who neglects a public information signal trades against other individuals who neglect other public signals. In equilibrium the market price aggregates the beliefs of other investors with limited attention. Any individual is informationally disadvantaged with respect to signals he neglects, but is advantaged with respect to the signals to which he attends. For example, an investor who does not attend to quarterly earnings announcements but analyzes the implications of strategic information such as demographics has both an advantage and a disadvantage relative to another investor with a different attentional focus.

Furthermore, standard models of information and securities markets such as Grossman and Stiglitz (1976) provide the insight that, owing to liquidity or noise trading, prices aggregate information imperfectly, and as a result, that even an informationally disadvantaged individuals should trade based upon his beliefs rather than deferring completely to market price.\textsuperscript{28} In a setting with limited attention, an isomorphic situation arises in which a set of investors has a kind of pseudo-private information. Attending

\textsuperscript{26}This phenomenon is well-documented (e.g., Abarbanell and Bernard (1992) and Zhang (2005)), and has theoretical motivation. Models of analyst reputation predict this phenomenon because an analyst who has a prior forecast tends to hold too strongly to that forecast in response to a public signal in order to maintain a reputation for having highly accurate private information. A general model of analyst neglect of relevant information is provided by Trueman (1994).

\textsuperscript{27}A similar argument applies to investors who neglect accruals. Such investors may form beliefs based upon analysts’ earnings forecasts, which underutilize the information contained in accruals (Teoh and Wong (2002)).

\textsuperscript{28}In forming his beliefs, an investor combines any private signals he may possess (none, for an uninformed trader), together with the information implicit in market price. The weight on his own signals and prior is positive because market price is noisy. The noise in market price derives both from liquidity shocks and, if there are multiple private signals, from confounding between signals.
to a public signal that some other investors neglect is analogous to observing a private signal. Neglecting a signal that other individuals attend to is akin to being uninformed about that signal. However, in standard models of information and securities markets, individuals with informational advantages and disadvantages all trade based upon their beliefs, and in so doing profit at the expense of liquidity traders.\textsuperscript{29}

Indeed, securities market models that are ostensibly about private information are often interpreted in ways that are more consistent with different individuals processing publicly available information differently. Applied discussions based on these models do not usually interpret private information as being limited to inside information (e.g., information obtained by rummaging through a firm’s trash, eavesdropping on cell phone conversations, or persuading employees to leak information). Instead, investors become privately informed through some unspecified process—presumably involving analysis of publicly available signals such as financial statements and media reports. This broader interpretation of Grossman and Stiglitz (1976) implicitly involves trade based upon processing versus neglect of publicly available information.

In order to develop our implications in the simplest and most parsimonious possible way, we dispense with liquidity trading and instead directly assume that investors with limited attention do not incorporate market price into their beliefs. In reality investors with limited attention probably do sometimes fail to process the information about future earnings implicit in market price. However, qualitatively identical results to those in our model could be derived in a setting in which investors with limited attention draw rational inferences from market price, and in which liquidity trading makes prices noisy. In such a setting liquidity trading causes investors rationally to place less weight upon market price in updating beliefs. Thus, such a setting would endogenize the conclusion that individuals with limited attention trade actively based upon their beliefs instead of deferring completely to market price. Such an analysis would have more algebraic details, but conceptually would be essentially equivalent to our model. The main difference is that in such a setting prices would depend upon the liquidity shock as well as on the beliefs of different investors.

There is other empirical evidence that limited attention affects capital markets; indeed, Daniel, Hirshleifer, and Teoh (2002) argue that limited attention may underly a

\textsuperscript{29}In analogy to the fact that in standard securities market models uninformed traders profitably trade by bearing the risk created by liquidity trading, in a limited attention setting even an investor who is attentionally inferior (does not attend to any signal that someone else attends to) still profitably trades based upon the deviation between his valuation and the market price, since such trading causes him to absorb order flow, leading to profit at the expense of liquidity traders.
wide range of anomalous patterns in securities market trading and prices. Many short-horizon event studies confirm that stock markets react immediately to relevant news. Long-horizon event studies provide evidence suggesting that there is underreaction to various kinds of public news events (see, e.g., the review of Hirshleifer (2001)). However, there has been a great deal of debate as to the appropriate methodology for testing market efficiency using long-run abnormal returns. There is also evidence suggesting that investors’ and analysts’ assessments are influenced by the format and salience with which public signals are presented (see footnote 3 and Schrand and Walther (2000)).

Although this paper uses a pure limited attention approach, psychological studies also indicate that individuals tend to be overconfident about the precision of their beliefs. Overconfidence can reinforce the tendency for individuals who fail to process an information item to also fail to adjust for this neglect. An individual who overconfidently thinks that he has already taken into account the most important information would underestimate the urgency of working hard to adjust for his neglect of other relevant signals (Peng and Xiong (2004) analyze the how the combination of limited attention and overconfidence generates asset price comovement).

It can be argued that arbitrageurs such as hedge funds will eliminate almost all of the mispricing created by limited attention. However, a literature in behavioral finance and accounting has argued that arbitrage is limited by risk-bearing capacity and market frictions; see, e.g., Shleifer and Vishny (1997), Hirshleifer (2001), and Lee (2001). We would argue that an important constraint on arbitrage is the aggregate attentional capacity in the market (see, e.g., Peng (2005)). Attention can be leveraged by such means as computers and financial intermediation, but even large institutions face tradeoffs in allocating their attentional resources.

30 In an experimental setting, Gillette et al (1999) document investor misreactions to public information arrival. Perhaps the most striking indication of limited attention in public markets is that stock prices react to news that is already public information (Huberman and Regev (2001), and Ho and Michaely (1988)), and even to confusions in ticker symbols between stocks (Rashes (2001)). More broadly, Hong, Torous, and Valkanov (2004) report evidence that industry stock returns lead aggregate market returns, potentially consistent with gradual diffusion of information about fundamentals across markets. Hou and Moskowitz (2005) provide a measure of investor neglect of a stock, the lag in the relation between the return on the overall market and the stock’s return. They find that stocks with long delay (which can be viewed as low-attention stocks) have stronger post-earnings announcement drift. DellaVigna and Pollet (2003) provide evidence that the stock market neglects demographic information; DellaVigna and Pollet (2004) find that the daily timing of earnings announcements is related to post-earnings announcement drift.

31 More generally, as emphasized by Kothari (2001), some findings of apparent return predictability in the literature may come from methodological problems. On the other hand, some patterns of return predictability have proven hard to reconcile with an efficient markets perspective.
5 Conclusion

This paper offers a model of stock market misreactions to earnings-related information based upon limited investor attention. It is motivated by evidence of stock return predictability based upon earnings, accruals, and cash flows, including the striking fact that there are both under-reactions and over-reactions to different kinds of earnings-related information. To understand the sources of these return anomalies, it is important to be parsimonious in assumptions about psychological biases, to avoid overfitting the data. We therefore explore the consequences of a single psychological constraint, which we use to derive a rich set of untested empirical implications.

We assume that some investors neglect information contained in the latest earnings surprise, and that some investors further neglect the information contained in accruals and cash flow. In equilibrium, there is underreaction to earnings surprises, because some investors do not attend to the newly-arriving earnings news. Cash flows positively and accruals negatively predict future stock returns, because some investors do not impound the information contained in the division of earnings between cash flow and accruals. Since cash flow is more favorable than accruals as a forecaster of future earnings, high accruals are associated with overvaluation, and high cash flow with undervaluation. The model does not in general predict that either of the cash flow or the accruals effects on future returns will completely subsume the other.

The model provides a rich set of new empirical implications. When some investors neglect earnings, and others attend to earnings but neglect accruals, price underreacts to earnings, but overreacts to accruals relative to cash flow. In consequence, there is stronger underreaction to cash flow than to earnings. Furthermore, if enough attention is paid to earnings, there is overreaction to accruals. A lower quality of accruals as an earnings forecaster (induced, for example, by earnings management) strengthens the accruals anomaly, but has an ambiguous effect on the cash flow anomaly. The more variable are accruals relative to cash flows, the stronger is the cash flow anomaly relative to the accrual anomaly. The ratio of the cash flow effect to the accruals effect is stronger than the ratio of the variances (variance of accruals over variance of cash flow). Higher correlation between cash flows and accruals tends to weaken the accruals anomaly.

Another appealing set of empirical implications is provided by events that shift attention to or away from the firm. For example, a greater number of distracting events such as earnings announcements by other firms is predicted to intensify both post-earnings announcement drift and the cash flow anomaly. The implications of shifts of attention
toward or away from a firm for the accruals anomaly is less clearcut. Neglect of earn-
ings components (cash flow versus accruals) strengthens the accruals anomaly, whereas
neglect of the earnings innovation itself weakens the accruals anomaly.

The empirical implications of the model associated with under- and over-valuation
can be expressed in terms of the relation of earnings or its components either to sub-
sequent abnormal stock returns, or to contemporaneous misvaluation measures. For
example, the residual income model of Ohlson (1995) provides a contemporaneous mea-
sure of fundamental firm value. As a result, the ratio of market price to the residual
income model valuation provides a measure of misvaluation (as applied, for example, by
Frankel and Lee (1998), Lee, Myers, and Swaminathan (1999), and Dong, Hirshleifer,
Richardson, and Teoh (2005)).

Our reconciliation of the drift and accruals effects is potentially consistent with some
of the informal intuitions about investor ‘naiveté’ offered in the empirical literature. It
is of course reassuring that such insights can be captured within an equilibrium model.
However, the main contribution of the paper is to develop a unified framework based
upon a well-established psychological constraint that offers a rich set of unttested empir-
ical predictions about earnings, cash flow, accruals, and stock market prices.

Our model is, of course, very stylized. A natural further direction of extension is
to consider finer components of accruals and cash flow. An analysis similar to that
developed here would show that overvaluation will be increasing with the level of an
earnings component if that component is a relatively strong incremental forecaster of
future earnings, and will be decreasing with that component if it is a weak incremen-
tal forecaster of future earnings. Thus, our approach is consistent with the evidence
on earnings components and returns of Richardson et al (2004) and Dechow and Ge
(2005). Furthermore, it would be simple to apply such a generalized analysis to normal
versus abnormal levels of accruals (where ‘normal’ is evaluated relative to industry or
another benchmark), in order to develop predictions about how managerial discretion
affects misvaluation (as studied empirically in Teoh, Welch and Wong (1998a, 1998b),
Xie (2001) and others. Also, by adding another time period, the relation between the
forecasting power of variables for short-term versus long-term earnings with misvaluation
could be considered. The framework can be applied to investment-related anomalies, as
in Hirshleifer and Teoh (2005). These and other possible extensions suggest that the
modeling approach offered here potentially has a wide range of applicability to circum-
stances in which market misvaluation derives from investor neglect of publicly available
information.
Appendix

Proof of Theorem 1: The law of iterated expectations implies that for any random variables $y'$ and $z$ satisfying the relevant regularity conditions,

$$E[E[z|y']] = E[z]. \quad (44)$$

Let $g(z|y')$ on the left hand side be the density for the inner expectation, $h(y')$ be the density for the outer expectation, and let

$$f(z) = \int g(z|y')h(y')dy' \quad (45)$$

be the density for the expectation on the right hand side. Since (44) obtains for any densities $g$ and $h$, for any given values of random variables $x$ and $y$, we can set

$$g(z|y' = y) = g^*(z|y, x), \text{ and } h(y) = h^*(y|x), \quad (46)$$

where $g^*(z|y, x)$ is the conditional density of $z$ given $y$ and $x$, and $h^*(y|x)$ is the conditional density of $y$ given $x$. It follows by (45) that

$$f(z) = \int g^*(z|y, x)h^*(y|x)dy,$$

$$= f^*(z|x),$$

where $f^*(z|x)$ is the conditional density of $z$ given $x$. Thus, by (44),

$$E[E[z|y, x]|x] = E[z|x].$$

Since $y$ is a sufficient statistic for $x$, the $x$ in the inner expectation can be removed, completing the proof. ||

Proof of Lemma 1: We begin with (29). Since $corr(a_1, c_1) < 1$ and $\beta_1 > \beta_2$,

$$\beta_1(V_aV_c - C^2) > \beta_2(V_aV_c - C^2). \quad (47)$$

Adding the quantity

$$\beta_1V_aC + \beta_1V_cC + \beta_1C^2 + 2\beta_2V_aC + \beta_2(V_a)^2$$

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to both sides of the equation, factoring, and dividing both sides by \( V_a(V_a + V_C + 2C) \) yields
\[
\frac{(V_a + C)[\beta_1 V_c + \beta_2 V_a + (\beta_1 + \beta_2)C]}{V_a(V_a + V_c + 2C)} > \frac{\beta_1 C + \beta_2 V_a}{V_a}. \tag{48}
\]
It follows that
\[
\frac{\text{cov}(a_1 + c_1, a_1) \text{cov}(\beta_0 + \beta_1 c_1 + \beta_2 a_1 + \delta, a + c)}{V_a} > \beta_1 \left( \frac{C}{V_a} \right) + \beta_2, \tag{49}
\]
so by (13),
\[
\beta_{e_{2e1}} \beta_{e_{1a1}} > \beta_1 \beta_{e_{1a1}} + \beta_2.
\]

The proof of (30) follows the steps for Part 1 in a precisely analogous fashion based upon the symmetry between \( c_1 \) and \( a_1 \) discussed in the main text, and is for brevity omitted (available upon request). \( \parallel \)

**Proof that** \( \beta_1 > \beta_{e_{2e1}} > \beta_2 \): To show that \( \beta_1 > \beta_{e_{2e1}} \), note first that since \( \beta_{e_{1a1}} > 0 \), \( C > -V_a \). Since \( \beta_1 > \beta_2 \), it follows that
\[
(\beta_1 - \beta_2)(V_a + C) > 0. \tag{50}
\]
In consequence,
\[
\beta_2 V_a + (\beta_1 + \beta_2)C < \beta_1 V_a + 2\beta_1 C, \tag{51}
\]
and adding \( \beta_1 V_c \) to both sides,
\[
\beta_1 V_c + \beta_2 V_a + (\beta_1 + \beta_2)C < \beta_1 (V_c + V_a + 2C). \tag{52}
\]
Dividing both sides by \( V_c + V_a + 2C \), and by (15), it follows that \( \beta_1 > \beta_{e_{2e1}} \).

To show that \( \beta_{e_{2e1}} > \beta_2 \), note first that since \( \beta_{e_{1c1}} > 0 \), \( C > -V_c \). Since \( \beta_1 > \beta_2 \), it follows that
\[
(\beta_1 - \beta_2)(V_c + C) > 0. \tag{53}
\]
In consequence,
\[
\beta_1 V_c + (\beta_1 + \beta_2)C < \beta_2 V_c + 2\beta_2 C, \tag{54}
\]
and adding \( \beta_2 V_a \) to both sides,
\[
\beta_2 V_a + \beta_1 V_c + (\beta_1 + \beta_2)C < \beta_2 (V_c + V_a + 2C). \tag{55}
\]
Dividing both sides by \( V_c + V_a + 2C \), and by (15), it follows that \( \beta_2 < \beta_{e_{2e1}} \). \( \parallel \)

**Proof of Proposition 6:**
By (35) and (15),

\[
\frac{b_o^\nu}{\lambda_e} = \rho \left( \frac{\sigma_c}{\sigma_a} \right) (\beta_1 - \beta_{e_1}) + \frac{\beta_2 - \beta_{e_1}}{\sigma_c^2 + \sigma_a^2 + 2\rho\sigma_a\sigma_c}
\]

\[
= \frac{(\rho^2 - 1)\sigma_c^2(\beta_1 - \beta_2)}{\sigma_c^2 + \sigma_a^2 + 2\rho\sigma_a\sigma_c}
\]

\[
= \frac{(\rho^2 - 1)(\beta_1 - \beta_2)}{(\frac{\sigma_a}{\sigma_c} + \rho)^2 + 1 - \rho^2}.
\]

(56)

Similarly, by (35) that

\[
\frac{b_o^o}{\lambda_e} = \rho \left( \frac{\sigma_a}{\sigma_c} \right) (\beta_2 - \beta_{e_1}) + \beta_1 - \beta_{e_1}
\]

\[
= \frac{(1 - \rho^2)\sigma_a^2(\beta_1 - \beta_2)}{\sigma_c^2 + \sigma_a^2 + 2\rho\sigma_a\sigma_c}
\]

\[
= \frac{(1 - \rho^2)(\beta_1 - \beta_2)}{(\frac{\sigma_a}{\sigma_c} + \rho)^2 + 1 - \rho^2}.
\]

(57)

Differentiating \(b_o^o\) in (56) and \(b_o^o\) in (57) with respect to \(\rho\):

\[
\frac{\partial(b_o^o/\lambda_e)}{\partial \rho} = \frac{\sigma_a^2(\beta_1 - \beta_2)[2\sigma_a \sigma_c (1 + \rho^2) + 2\rho(\sigma_c^2 + \sigma_a^2)]}{(\sigma_c^2 + \sigma_a^2 + 2\rho\sigma_a\sigma_c)^2}. \quad (58)
\]

Earnings \(e_1 = c_1 + a_1\) has positive variance, so that the denominator in the above expression is positive (i.e., is is not the case that both \(\rho = -1\) and \(\sigma_a = \sigma_c\)). Since \(\beta_1 < \beta_2\), It follows that \(\partial(b_o^o/\lambda_e)/\partial \rho\) is positive if and only if the term in brackets in the numerator is positive, i.e.,

\[
1 + \rho^2 + \rho \left( \frac{\sigma_a}{\sigma_c} + \frac{\sigma_c}{\sigma_a} \right) = \left( \rho + \frac{\sigma_a}{\sigma_c} \right) \left( \rho + \frac{\sigma_c}{\sigma_a} \right) > 0. \quad (59)
\]

It follows that both roots of the quadratic equation corresponding to the quadratic expression in \(\rho\) of (59) are negative. By inspection (59) holds for all \(\rho \geq 0\), so the quadratic function (59) is increasing in \(\rho\) at the larger root \(\rho^*\). It follows that (59) holds for all \(\rho > \rho^*\), and therefore that for all \(\rho > \rho^*\), \(\partial(b_o^o/\lambda_e)/\partial \rho > 0\).

Differentiating \(b_o^o\) in (57) with respect to \(\rho\), similar reasoning shows that

\[
\frac{\partial(b_o^o/\lambda_e)}{\partial \rho} = \frac{-(\beta_1 - \beta_2)[2\sigma_a \sigma_c (1 + \rho^2) + 2\rho(\sigma_c^2 + \sigma_a^2)]}{(\sigma_c^2 + \sigma_a^2 + 2\rho\sigma_a\sigma_c)^2} < 0 \quad (60)
\]

for all \(\rho > \rho^*\). ||
References


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