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Price Discrimination through Offers to Match Price*

In this paper, a firm discriminates between two classes of customer who have different cost of information by coupling a list price with an offer to match the price of any other shop. If the list price elsewhere is lower, the firm will be successful in discrimination. The list price of each firm is increasing in the number of sellers, and the total sales are decreasing in the number of sellers. Furthermore, if sellers coordinate, they discriminate more efficaciously and increase their profits by increasing their total sales.

I. Introduction

Offers by one seller to match the prices of others are a feature of both consumer and industrial marketing. In industrial marketing, meet-or-release provisions are common in long-term contracts of sale: these provide the buyer with assurance that, should he be offered a lower price, the original seller will either match that price or release the buyer from the contract. Such provisions assure the buyer that he will benefit from subsequent drops in the price of the item to be purchased. They protect the buyer from subsequent ex post opportunism on the part of the seller (Klein and Kenney 1985). Such provisions, however, have the effect of deterring price competition, as argued by Salop (1986) and Holt and Scheffman (1985).

In retail markets, sellers frequently offer to match any other advertised price—we have noticed such offers by major department stores, a renter of furniture, and several electronics goods stores. It is not obvious why such practices should arise outside a long-term sale contact. If a retailer wishes to lower his price, he could do so directly by cutting his price. A

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distinguishing feature of retail markets is that the cost of information about price may be large relative to the value of the item to be purchased. Moreover, consumers may well differ in their cost of information. In these circumstances, an offer to match any other advertised price has the effect of screening customers by their cost of information.

There have been several recent papers on the subject of price discrimination in retail markets. Shilony (1977), Rosenthal (1980), and Varian (1980) present models in which identical competing sellers of a homogeneous good select strategies in price to discriminate between two classes of customer with different elasticities of demand. In their analyses, the unique symmetric equilibrium was a mixed-strategy equilibrium. This was interpreted as a model of sales or discounts.

Sobel (1984) introduced time explicitly into the model to study the timing of the discounts. Jeuland and Narasimhan (1985) gave a monopoly analysis of a similar problem. Narasimhan (1984) presented a theoretical and empirical analysis of the use of cents-off coupons as a method of price discrimination.[1] Lazear (1986) solved for the profit-maximizing price path where a monopolistic seller learns about customer demand through the price that he sets.

In this paper, we consider a larger strategy space in which firms may not only choose price but also choose to offer to match the lowest price listed by a competitor. It is shown that, in symmetric equilibrium with more than one firm, each firm will attempt to discriminate by randomization of its list price and, in addition, by offering to match the price of its competitors. Unlike other methods of discrimination such as periodic sales and coupons, price matching is a device that depends on the existence of competition: it cannot be implemented by a single firm.

In equilibrium, the list price of each firm is increasing in the number of firms, and the total sales are decreasing in the number of firms. Furthermore, for a given number of firms, if firms coordinate their pricing, they will be able to discriminate more efficaciously and increase their profit by selling more to the customers with low cost of information. Thus increased competition through the entry of more sellers reduces social welfare, while coordination increases social welfare.

The remainder of the paper is organized as follows. The setting is introduced in Section II, and the equilibrium in the case of two firms is presented in Section III. In Section IV, this is compared with the equilibrium with correlated strategies. In Section V, the model is extended to the case of more than two firms, and the number of firms is made endogenous. Some comparative statics are presented in Section VI. The paper concludes with some remarks on directions for future work.

1. For a general review of the literature on coupons, see Vilcassim and Wittink (1985).

II. Basic Model

Assume, for simplicity, that all firms produce under identical conditions of zero fixed cost and constant marginal cost, c. The strategy of firm i consists of a list price p_i and, possibly, a qualification in the form of an offer to match the competitor's list price. The equilibrium concept is Nash in the firms' strategies. The firms announce their strategies simultaneously. Once the strategies are announced, price may alter only through the effect of offers to match. For instance, if one firm announces a higher list price but has offered to match, in equilibrium the customers of that firm who take advantage of its offer to match will be charged a price equal to the lowest list price announced.

There are two classes of customer. One class, tourists, has an infinite cost of search and an inelastic demand for $2Q_t$ units of the good at any price at or below a reservation level, r. The tourists choose between the firms at random: each firm receives an equal fraction of the tourist demand. The other class, locals, can collect information about the firms' pricing strategies at zero cost and has an elastic demand, $2Q_\ell(p)$, for the good. If all firms offer the same price (either by the same list or through matching), the demand of the locals is divided equally between the firms.

Let

$$\Pi_\ell(p) \stackrel{\text{def}}{=} Q_\ell(p) \cdot (p - c) \tag{1}$$

and

$$\Pi_t(p) \stackrel{\text{def}}{=} \begin{cases} Q_t \cdot (p - c) \text{ for } p \leq r, \\ 0 \quad \text{otherwise.} \end{cases} \tag{2}$$

Assume that the tourist demand and the local demand are such that the following two assumptions hold.

ASSUMPTION 1. $\arg\max\,[\Pi_\ell(p)] < r$.

ASSUMPTION 2. $\Pi_\ell(\cdot)$ is twice differentiable and $\Pi_\ell''(p) < 0$.

Assumption 2 is a sufficient condition for there to exist a unique profit-maximizing price for a monopolist that caters to a population of locals only.

For purposes of comparison, it will be helpful first to study the profit-maximizing strategy in the case in which there is only one seller in the market. In these circumstances, if the seller could discriminate, he would charge a price r to the tourists and $\arg\max[2Q_\ell(p) \cdot (p - c)] = \arg\max[\Pi_\ell(p)] \stackrel{\text{def}}{=} p_\ell$ to the locals. The snag is how to distinguish a tourist from a local.

Suppose that the monopolist is unable to discriminate. Then he will choose price to maximize

$$2Q_\ell(p) \cdot (p - c) + 2Q_t \cdot (p - c) = 2[\Pi_\ell(p) + \Pi_t(p)].$$

Let the profit-maximizing price for the (nondiscriminating) monopolist be denoted p_m, that is,

$$\Pi_\ell'(p_m) + \Pi_t'(p_m) = 0. \tag{3}$$

This will be a compromise between the two prices that a discriminating monopolist would charge. Since, by assumption 1, $p_\ell < r$, it follows that

$$p_\ell < p_m < r.$$

The total sales will be $2Q_\ell(p_m)$ to the local customers and, since $p_m < r$, a quantity of $2Q_t$ to the tourists.

III. Duopoly Equilibrium with Price Matching

Now let there be two firms in the industry.[2] A natural mechanism by which to screen the two classes of customer that arises when there are several sellers is the offer to match. The tourists do not know the list prices of the various firms; only the locals do. Hence only the locals can take advantage of offers to match prices. Let $\Pi(p_i : \Phi)$ denote the profit of firm i when the list price of firm i is p_i and the strategy of firm j is given by the distribution $\Phi(\cdot)$. In the particular case in which the strategy of firm j is a single price p_j, the function will be written as $\Pi(p_i : p_j)$.

In the following lemma, it will be shown that both firms will couple their list prices with offers to match the price of the other. The reason is very simple: a firm loses nothing by doing so, but in the event that its list price is higher than that of the other firm, an offer to match will prevent the defection of its local customers.[3]

LEMMA 1. For each firm, it is a weakly dominant strategy to couple its list price with an offer to match.

In the following lemma, it is proved that, in the model, all strategies that have support outside the interval $[p_m, r]$ are dominated.

LEMMA 2. Any strategy with support in $[0, p_m)$ or in $(r, +\infty)$ is dominated.

It remains to analyze the equilibria of the game. In the remainder of this section, it will be shown that there are two pure-strategy equilibria, both of which are asymmetric. Next, it will be shown that there is a unique symmetric differentiable mixed-strategy equilibrium.

A. *Pure-Strategy Equilibrium*

The character of the pure-strategy equilibria is quite simple. Each equilibrium consists of one firm listing price p_m and offering to match and the other firm listing price r and offering to match.

2. In a subsequent section, we consider the case of more than two firms.
3. The proofs of all results will be presented in the App.

PROPOSITION 1. There are two pure-strategy equilibria: (*a*) one in which firm 1 sets list price p_m and offers to match and firm 2 sets list price r and offers to match; (*b*) another with the pricing strategies interchanged between the firms.

The intuition of the equilibrium is simple. Suppose that one seller prices at p_m. Then the other surely will price at r, for by doing so he gains more profit from his tourists while his offer to match ensures that he loses no customers to the lower-list-price firm. On the other hand, if one firm prices at r, the other is in the position of being a price leader. His decision is exactly like that of the nondiscriminating monopolist; hence he will choose p_m.

The local customers utilize the offer to match of the higher-priced firm. In equilibrium, all locals pay price p_m, and each firm produces an identical output $Q_\ell(p_m) + Q_t$. The combined output of the two firms is $2[Q_\ell(p_m) + Q_t]$, which is exactly equal to the output of a nondiscriminating monopoly.

By taking advantage of the low-price firm, the high-price firm (which sets list price r) effects price discrimination and realizes profit of

$$Q_\ell(p_m) \cdot (p_m - c) + Q_t \cdot (r - c) = \Pi_\ell(p_m) + \Pi_t(r).$$

The low-price firm realizes a smaller profit of

$$Q_\ell(p_m) \cdot (p_m - c) + Q_t \cdot (p_m - c) = \Pi_\ell(p_m) + \Pi_t(p_m).$$

Because some discrimination occurs, the combined profit of the duopoly exceeds the profit of a nondiscriminating monopoly.

B. Mixed-Strategy Equilibrium

In the previous subsection, it was shown that there are only two pure-strategy equilibria and that both are asymmetric. In pure-strategy equilibrium, the firm that sets the higher list price will earn larger profit. Immediately the question arises, In a situation in which the two firms are identical, which will be the one to price higher?

Intuitively, a theory in which identical firms pursue identical strategies may seem more plausible. The next proposition presents the main result of the paper. There is a unique symmetric differentiable mixed-strategy equilibrium. Under the equilibrium strategy, each firm sets price on the range $[p_m, r)$ with continuous density $-Q_t\Pi_\ell''(p)/[\Pi_\ell'(p)]^2$ and at r with probability mass $-Q_t/\Pi_\ell'(r)$.

This strategy may be interpreted as one in which each firm has a standard list price, r, but may offer discounts from that according to the distribution given. Such discounts in themselves have a discriminatory effect.[4]

4. The discriminatory effect of such discounts is the focus of Shilony (1977), Rosenthal (1980), and Varian (1980).

PROPOSITION 2. The unique symmetric differentiable equilibrium strategy is given by the distribution function

$$F(p) = \begin{cases} 1 + Q_t/\Pi_\ell'(p) & \text{if } p \in [p_m, r), \\ 1 & \text{if } p = r. \end{cases} \tag{4}$$

It was shown earlier that there are no symmetric pure-strategy equilibria. Proposition 2 shows that there is a unique symmetric differentiable mixed-strategy equilibrium. The support of this equilibrium is the interval $[p_m, r]$. This has significant welfare implications. Under nondiscriminating monopoly, the price to locals is p_m always. Proposition 2 implies that, under duopoly, the (effective) price to locals will exceed p_m with positive probability.

This implies the striking result that, under duopoly, the expected combined sales will be less than under nondiscriminating monopoly. The presence of competition allows each firm an additional method of discrimination, namely, the offer to match price. This leads each firm to try to extract more from its tourists by raising its list price with the result that total expected sales are reduced. In contrast, an increase in the number of sellers alone would lead to larger expected sales.[5]

To summarize, in the mixed-strategy equilibrium, the total sales to tourists are $2Q_t$, while the total expected sales to locals are $2\int_{p_m}^{r} Q_\ell(p)dF(p) < 2Q_\ell(p_m)$. The expected profit of each firm is

$$\mathscr{E}\Pi = \int_{p_m}^{r} [Q_\ell(p) + Q_t](p - c)dF(p) = \Pi_\ell(p_m) + \Pi_t(p_m), \tag{5}$$

where $\mathscr{E}(\cdot)$ denotes the expectation operator.

IV. Correlated Strategies

The problem that confronts the two sellers is inherently one of coordination. Profits may be increased by discrimination between the two classes of customer. There is, however, only one means of discrimination—the offer to match price. But this will work for a seller only if the other seller sets a lower list price.

In the mixed-strategy equilibrium, both sellers randomize over the prices in the interval $[p_m, r]$. But the profits that result are dominated by the profits in the pure-strategy equilibrium, in which one seller sets price at p_m and the other at r. The essential reason is that, in the mixed-strategy equilibrium, each firm must be indifferent between setting list price at some $p > p_m$ and setting it at p_m. Therefore the expected profit must be $\Pi_\ell(p_m) + \Pi_t(p_m)$, which is exactly the profit of the low-price firm in pure-strategy equilibrium.

5. For a detailed analysis of the effect on output of the introduction of (third-degree) price discrimination, see Schmalensee (1981) and Varian (1985).

It is likely that the firms in such a situation will appreciate that there are gains from coordination. In a static model, they may exploit external devices to achieve a correlated equilibrium.[6] That is, they may adopt strategies that specify moves contingent on external signals. For instance, one strategy might be, Set list price at p_m if the closing Dow Jones the previous day is even and at r otherwise. The best response to such a strategy is, Set a list price at r if the closing Dow Jones the previous day is even and at p_m otherwise.[7]

There is a multiplicity of possible correlated equilibria. First, there are many possible external signals on which the equilibrium may be based. Second, for any given signal, there are multiple equilibria that differ in the division of profits between the two sellers. We shall focus attention on a symmetric equilibrium, in which the external signal is such that, with probability one-half, one firm sets list price p_m and the other list price r and, with probability one-half, the roles are reversed between the two firms. In this equilibrium, the expected profit of a firm will be

$$\begin{aligned} &\tfrac{1}{2}[\Pi_\ell(p_m) + \Pi_t(p_m)] + \tfrac{1}{2}[\Pi_\ell(p_m) + \Pi_t(r)] \\ &\quad = \Pi_\ell(p_m) + \tfrac{1}{2}\Pi_t(p_m) + \tfrac{1}{2}\Pi_t(r). \end{aligned} \tag{6}$$

This is larger than the profit under the mixed-strategy equilibrium.

A fundamental difficulty for a conspiracy to support prices in an oligopoly is that each seller has an incentive to shade his price to draw more business. In coordination to price discriminate through offers to match price, however, neither seller has an incentive to deviate from the (agreed) pattern of pricing. The effect of the offers to match is to make deviations unprofitable and hence support the coordinated price discrimination.

It is worth noting that, in the equilibrium of correlated strategies, the realized list prices always are r for one firm and p_m for the other. Hence, in all instances, the effective price to local customers is p_m. Therefore it is unambiguous that, relative to the mixed-strategy equilibrium, the industry output in correlated equilibrium is larger—it is equal to the output of a nondiscriminating monopoly. One possible measure of welfare is the expectation of combined supplier and buyer surplus. Since price is above marginal cost in both the independent

6. In a setting of repeated plays, a seller may play a strategy of the form, Price at p_m on odd dates and at r on even dates. The best response to this is the strategy, Price at r on odd dates and at p_m on even dates. By alternating their prices, the two firms can achieve an equal division of profit. A similar practice occurred in the Great Electrical Equipment Conspiracy of 1955–56: General Electric, Westinghouse, I-T-E, and Allis-Chalmers coordinated their bids to supply large turbine generators by the phases of the moon (see Sultan 1974, p. 39).

7. For the original reference on the subject of correlated strategies, see Aumann (1974).

mixed strategy and the correlated equilibria, the increase in output leads to an increase in expected total surplus. In this sense it may be argued that welfare is higher in the correlated equilibrium; however, there are distributive differences between the two equilibria as well.

V. Equilibrium Number of Firms

Up until this point, it has been assumed that the number of firms in the market is fixed at two. In this section, we allow the number of firms, n, to be an endogenous function of a fixed cost of production, K. Assume that this cost satisfies the following assumption.

ASSUMPTION 3. $K \leq \Pi_\ell(p_m) + \Pi_t(p_m)$.

To recall, the total demand of tourists is $2Q_t$ if the price is no greater than r, and the total demand of locals is $2Q_\ell(p)$. Each firm will sell a quantity $2Q_t/n$ to tourists if its price is no greater than r. If all firms charge the same effective price p to locals, the locals also divide themselves equally, so that each firm will sell quantity $2Q_\ell(p)/n$ to locals.

The unique symmetric differentiable equilibrium strategy is for each firm to set price on the range $[p_m, r)$ with continuous density

$$\frac{1}{n-1}\left[-\frac{Q_t}{\Pi_\ell'(p)}\right]^{1/(n-1)} \frac{\Pi_\ell''(p)}{\Pi_\ell'(p)}$$

and at r with mass probability $[-Q_t/\Pi_\ell'(r)]^{1/(n-1)}$.

PROPOSITION 3. There are n pure-strategy equilibria: one firm lists price p_m and the remaining firms list price r. The unique symmetric differentiable equilibrium strategy is given by the distribution function

$$F_n(p) = \begin{cases} 1 - [-Q_t/\Pi_\ell'(p)]^{1/(n-1)} & \text{if } p \in [p_m, r), \\ 1 \quad \text{if } p = r. \end{cases} \tag{7}$$

The effective price to the locals is the minimum price quoted by the n firms. The distribution of the minimum price is

$$\begin{aligned} \text{pr}[\min(p_1, \ldots, p_n) \leq p] &= 1 - \text{pr}[p_i > p, \quad \text{for all } i = 1, \ldots, n] \\ &= 1 - [1 - F_n(p)]^n. \end{aligned} \tag{8}$$

The total sales to the locals are $2Q_\ell[\min(p_i)]$.

Notice that the distribution F_n is decreasing in n. Hence the distribution of the minimum price is decreasing in n. That is, the effective price to locals is higher and the total sales to locals smaller in the sense of first-order stochastic dominance the larger is the number of firms.

The intuitive explanation of this result is that setting a low price is a public good to all the other firms; once one firm sets a low price, the others are free to set list price at r to maximize earnings from discrimi-

nation. The larger the number of firms, the smaller is the pressure on any firm to set a low price.[8]

In equilibrium, the profit for a firm if it sets list price $p \in [p_m, r]$ is given by

$$\Pi(p : F_n) = \begin{cases} \int_{[p_m,p)} Q_\ell(x)(x - c) \cdot dG_n(x) \\ \quad + [1 - G_n(p)] \cdot Q_\ell(p)(p - c) \\ \quad + \Pi_t(p) \quad \text{if } p \in [p_m, r), \\ \int_{[p_m,r)} Q_\ell(x)(x - c) \cdot dG_n(x) \\ \quad + [-Q_t/\Pi_\ell'(r)] \cdot Q_\ell(p)(p - c) \\ \quad + \Pi_t(r) \quad \text{if } p = r, \end{cases} \tag{9}$$

where $G_n(\cdot)$ is the distribution of the minimum of the list prices of the other $n - 1$ firms. In the proof of proposition 3, it is shown that the expected profit from the strategy $F_n(\cdot)$,

$$\mathscr{E}\Pi = \int_{p_m}^{r} \Pi(p : F_n) dF_n(p) = \frac{2}{n}[\Pi_\ell(p_m) + \Pi_t(p_m)]. \tag{10}$$

This is decreasing in n, the number of sellers.

In equilibrium, it must not be profitable for another firm to enter the industry, that is,

$$\frac{2}{n}[\Pi_\ell(p_m) + \Pi_t(p_m)] \geq K \geq \frac{2}{n+1}[\Pi_\ell(p_m) + \Pi_t(p_m)]. \tag{11}$$

Thus the equilibrium number of firms, n, in the industry is bounded by

$$\frac{2}{K}[\Pi_\ell(p_m) + \Pi_t(p_m)] - 1 \leq n \leq \frac{2}{K}[\Pi_\ell(p_m) + \Pi_t(p_m)].$$

It is interesting to compare these results with the equilibrium with correlated strategies. Consider a symmetric correlated equilibrium with N firms and with an external signal that takes values $i = 1, \ldots, N$ with equal probability: if the value of the signal is i, firm i sets list price p_m, and all the other firms set list price r. The expected profit of each firm will be

$$\begin{aligned} &\frac{1}{N}\left[\frac{2}{N}\Pi_\ell(p_m) + \frac{2}{N}\Pi_t(p_m)\right] + \frac{N-1}{N}\left[\frac{2}{N}\Pi_\ell(p_m) + \frac{2}{N}\Pi_t(r)\right] \\ &\quad = \frac{2}{N}\left[\Pi_\ell(p_m) + \frac{1}{N}\Pi_t(p_m) + \frac{N-1}{N}\Pi_t(r)\right]. \end{aligned} \tag{12}$$

8. Rosenthal (1980) presented a model in which an increase in the number of sellers led to a higher price in the sense of first-order stochastic dominance. In this model, the strategy space of a seller was simply price. It was assumed that each additional seller brought along with him a captive customer base ("tourists") but competed with the existing sellers for a fixed group of price-sensitive customers ("locals"). In our paper, each additional seller competes for a share of fixed tourist and local markets.

For a given number of firms, the expected profit is larger in correlated equilibrium than in the mixed-strategy equilibrium. An increase in the number of firms has two effects on the expected profit in correlated equilibrium. With more sellers, the fixed market must be divided into smaller shares; hence the sales of each firm will be smaller. On the other hand, with more sellers, the probability that any one seller must set the low price p_m is reduced. On balance, the net effect is that the expected profit is decreasing in the number of firms.

New firms will enter the industry until

$$\frac{2}{N}\left\{\Pi_\ell(p_m) + \Pi_t(r) + \frac{1}{N}[\Pi_t(p_m) - \Pi_t(r)]\right\}$$

$$\geq K \geq \frac{2}{N+1}\left\{\Pi_\ell(p_m) + \Pi_t(r) + \frac{1}{N+1}[\Pi_t(p_m) - \Pi_t(r)]\right\}.$$

This provides bounds on the equilibrium number of firms, N, in the industry.

VI. Comparative Statics

In this section, we study the effect of shifts in demand and cost on the character of the mixed-strategy equilibrium of the industry. To recall from (7), the equilibrium strategy is

$$F_n(p) = \begin{cases} 1 - [-Q_t/\Pi_\ell'(p)]^{1/(n-1)} & \text{if } p \in [p_m, r), \\ 1 \quad \text{if } p = r. \end{cases} \tag{7}$$

Suppose that the total local demand is $2\alpha Q_\ell(p)$, where α is a multiplicative factor. The distribution of the equilibrium strategy is $F_n(p) = 1 - [-Q_t/\alpha\Pi_\ell'(p)]^{1/(n-1)}$, which is increasing in α. Thus it is unambiguous that the larger is the total local demand, the lower will be the list price of each firm in the sense of first-order stochastic dominance. By (8), it follows that the effective price to locals will be lower and the total sales to locals larger.

From (10), the expected profit of a firm from the strategy $F_n(\cdot)$ is

$$\mathscr{E}\Pi = \frac{2}{n}[\alpha\Pi_\ell(p_m) + \Pi_t(p_m)].$$

Hence

$$\frac{d}{d\alpha}\mathscr{E}\Pi = \frac{2}{n}\left\{[\alpha\Pi_\ell'(p_m) + \Pi_t'(p_m)]\frac{dp_m}{d\alpha} + \Pi_\ell(p_m)\right\}.$$

Now by definition of p_m, $\alpha\Pi_\ell'(p_m) + \Pi_t'(p_m) = 0$; hence

$$\frac{d}{d\alpha}\mathscr{E}\Pi = \frac{2}{n}\Pi_\ell(p_m) > 0.$$

Therefore $\mathscr{E}\Pi$ is increasing in α. The larger is the local demand, the larger will be the expected profit and hence the larger the equilibrium number of firms in the industry.

Next we consider the effect of an increase in the total tourist demand, $2Q_t$. Now

$$\frac{dF_n(p)}{dQ_t} = -\frac{1}{n-1}\left[-\frac{Q_t}{\Pi_\ell'(p)}\right]^{\frac{2-n}{n-1}}\left[-\frac{1}{\Pi_\ell'(p)}\right] < 0.$$

Hence it is unambiguous that an increase in the total tourist demand leads firms to set higher list prices in the sense of first-order stochastic dominance. As a result, the effective price to locals will be higher and the total sales to locals will be smaller. It is straightforward to show that the expected profit of each firm is increasing also.

Finally, we consider the effect of an increase in marginal cost, c. Now

$$\frac{d}{dc}\Pi_\ell'(p) = \frac{d}{dc}[Q_\ell'(p)\cdot(p-c) + Q_\ell(p)] = -Q_\ell'(p) > 0.$$

Hence

$$\frac{d}{dc}F_n(p) = -\frac{1}{n-1}\left[-\frac{Q_t}{\Pi_\ell'(p)}\right]^{\frac{2-n}{n-1}}\left\{\frac{Q_t}{[\Pi_\ell'(p)]^2}\right\}\frac{d}{dc}\Pi_\ell'(p) < 0.$$

Therefore it is unambiguous that the list price of each firm will be higher in the sense of first-order stochastic dominance. Furthermore, the expected profit of each firm is decreasing in c. Therefore, as might be expected, when marginal cost is higher, list prices are higher, profits lower, and the number of firms smaller.

VII. Concluding Remarks

In this paper, competing sellers seek to discriminate between two classes of customer by means of offers to match price. Although the discussion was couched in terms of "locals" and "tourists," these labels were only a convenient shorthand. "Tourists" should be taken to mean customers with loyalty to a particular seller because of imperfect information, the distance between competing sellers, brand loyalty, or other reason. In retail marketing, the seller's offer to match price may also take the form of accepting discount coupons issued by competing firms, as occurred is the battle between American Airlines and United Airlines in 1979, and as in common among supermarkets in Los Angeles. In industrial marketing, the offer may take the form of a meet-or-release provision. The model may be generalized to allow nonconstant marginal cost, provided that assumptions 1–3 continue to be satisfied.

The analysis shows clearly not only that is it possible for price discrimination to exist in the presence of competition but indeed that there are situations in which the seller makes use of its competitors to effect discrimination. Discrimination is effected when one firm lists a low price: the competing sellers then list a higher price so as to extract more profit from the tourists while effectively charging the lower price to locals.

One of the major results is that, starting from a nondiscriminating monopoly, the introduction of competition that enables discrimination through offers to match price would lead to a reduction in total sales. Moreover, the entry of additional firms would lead all sellers to raise their list price and thus further reduce total sales. If sellers coordinate their pricing, they will increase their profit by increasing their sales to local customers. The essential reason for these outcomes is that each firm would like to discriminate by setting a high list price for tourists and taking advantage, through its offer to match, of a lower list price elsewhere to charge a lower list price to local customers. Without coordination, the larger the number of competing sellers, the less the pressure on any particular firm to be the low-price firm. With coordination, the firms can economize on setting the low list price—only one need do so at any instant for all firms to benefit.

From the standpoint of formulating marketing strategy, the analysis indicates that a firm with several outlets should not impose a uniform price on all its outlets. In this regard, our conclusion concurs with a result of Salop (1977). Salop analyzed a model in which consumers differed in their cost of information about prices at various outlets and showed that a firm with several outlets would increase expected profit by randomization of prices. The model of this paper shows, that the firm should not give its outlets independence in pricing: it will maximize its profit by arranging that, at any time, only a single firm chosen at random sets a low list price.

A possible extension would be to examine the implications of allowing price-beating as well as price-matching offers. It may be conjectured that this would lead to competitive pressures to have high list prices since under price beating it may be the high-list-price firm that captures the local patronage by undercutting the lower-priced competitor.

Another interesting line of further inquiry is to analyze the incentive for a low-price seller to advertise his price—for it is such information that enables his competitors to discriminate. (There is one "discount" electronics store in Los Angeles that maintains a strict policy of not quoting prices over the telephone.) Another avenue for future research is to study the seller's choice between alternative methods of price discrimination. This could build on the existing work on various methods of discrimination, which has treated each method in isolation.

Appendix

Proof of lemma 1. Suppose that the strategy of firm j is Φ. Consider the strategy of firm i.

First, consider list prices $p_i \leq r$. Suppose that firm j does not offer to match. Hence, if firm i also does not offer to match, its sales to locals will depend on the realized price of firm j. In particular, if $p_i < p_j$, firm i will capture all the local customers, while if $p_i > p_j$, it will receive no local customers. The sales of firm i to tourists will be Q_t regardless. Thus

$$\Pi(p_i : \Phi) = \begin{cases} 2Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i < p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i = p_j, \\ Q_t \cdot (p_i - c) & \text{if } p_i > p_j. \end{cases}$$

If firm i does offer to match, it will capture all the local customers if $p_i < p_j$, but if $p_i \geq p_j$, firm i will receive exactly half the local customers—because they will take advantage of the offer to match. Thus profit will be

$$\Pi(p_i : \Phi) = \begin{cases} 2Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i < p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i = p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i > p_j. \end{cases}$$

By offering to match, firm i can increase its profit in the event that $p_i > p_j$. Hence, if firm j does not offer to match, it is weakly dominant for firm i to offer to match.

The next case to consider is that in which firm j does offer to match. In this case, firm i will receive exactly half the local customers when $p_i < p_j$; it will not capture the whole local business. If firm i does not offer to match, its profit will be

$$\Pi(p_i : \Phi) = \begin{cases} Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i < p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i = p_j, \\ Q_t \cdot (p_i - c) & \text{if } p_i > p_j. \end{cases}$$

If firm i does offer to match, its profit will be

$$\Pi(p_i : \Phi) = \begin{cases} Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i < p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i = p_j, \\ Q_\ell(p_i) \cdot (p_i - c) + Q_t \cdot (p_i - c) & \text{if } p_i > p_j. \end{cases}$$

Again, it is weakly dominant for firm i to offer to match.

A similar argument provides the proof for the case that $p_i > r$. The only difference is that then firm i will sell nothing to tourists.

Proof of lemma 2. In lemma 1, it was proved that all firms will offer to match. Here, the style of proof will be to suppose that the strategy of one firm is given by some distribution function Φ, with $p \stackrel{\text{def}}{=} \min[p : \Phi(p) > 0]$, and then to consider the expected profit $\Pi(p : \Phi)$ to the remaining firm of alternative list prices p.

a) For any $p < p_m$,

$$\Pi(p : \Phi) = [1 - \Phi(p)] \cdot [\Pi_\ell(p) + \Pi_t(p)] + \int_{\underline{p}}^{p} [\Pi_\ell(x) + \Pi_t(p)] d\Phi(x),$$

while

$$\Pi(p_m : \Phi) = [1 - \Phi(p_m)] \cdot [\Pi_\ell(p_m) + \Pi_t(p_m)] + \int_{\underline{p}}^{p_m} [\Pi_\ell(x) + \Pi_t(p_m)] d\Phi(x),$$

Hence

$$\begin{aligned}\Pi(p_m : \Phi) - \Pi(p : \Phi) &= [1 - \Phi(p_m)] \cdot [\Pi_\ell(p_m) + \Pi_t(p_m)] \\ &\quad + \left\{\int_{\underline{p}}^{p} [\Pi_\ell(x) + \Pi_t(p_m)]d\Phi(x) \right. \\ &\quad \left. + \int_{p}^{p_m} [\Pi_\ell(x) + \Pi_t(p_m)]d\Phi(x)\right\} \\ &\quad - \left[1 - \Phi(p_m) + \int_{p}^{p_m} d\Phi(x)\right] \cdot [\Pi_\ell(p) + \Pi_t(p)] \\ &\quad - \int_{\underline{p}}^{p} [\Pi_\ell(x) + \Pi_t(p)]d\Phi(x) \\ &= [1 - \Phi(p_m)] \cdot [\Pi_\ell(p_m) + \Pi_t(p_m) - \Pi_\ell(p) \\ &\qquad - \Pi_t(p)] \\ &\quad + \int_{p}^{p_m} [\Pi_\ell(x) + \Pi_t(p_m) - \Pi_\ell(p) - \Pi_t(p)]d\Phi(x) \\ &\quad + \int_{\underline{p}}^{p} [\Pi_\ell(x) + \Pi_t(p_m) - \Pi_\ell(x) - \Pi_t(p)]d\Phi(x) \end{aligned} \tag{A1}$$

Now $p_m = \arg\max [\Pi_\ell(x) + \Pi_t(x)]$; hence the first term in (A1) is positive. Next, $\Pi_\ell(\cdot)$ is concave, and $\Pi_t(\cdot)$ is linear; therefore the function $[\Pi_\ell(\cdot) + \Pi_t(\cdot)]$ is concave. The function attains a maximum at $p = p_m$. Hence for $p \leq x \leq p_m$ we have $\Pi_\ell(x) + \Pi_t(x) \geq \Pi_\ell(p) + \Pi_t(p)$. But $\Pi_t(p_m) > \Pi_t(x)$ since $p_m > x$. Therefore $\Pi_\ell(x) + \Pi_t(p_m) > \Pi_\ell(p) + \Pi_t(p)$, and hence the second term in (A1) is positive.

The third term in (A1) is

$$\int_{\underline{p}}^{p} [\Pi_t(p_m) - \Pi_t(p)]d\Phi(x) = \int_{\underline{p}}^{p} Q_t \cdot (p_m - p)d\Phi(x),$$

which is positive. Thus $\Pi(p_m : \Phi) > \Pi(p : \Phi)$.

b) For any $p > r$, the demand from tourists is zero; hence $\Pi_t(p) = 0$,

$$\Pi(p : \Phi) = [1 - \Phi(p)] \cdot [\Pi_\ell(p)] + \int_{\underline{p}}^{p} \Pi_\ell(x)d\Phi(x),$$

while

$$\Pi(r : \Phi) = [1 - \Phi(r)] \cdot [\Pi_\ell(r) + \Pi_t(r)] + \int_{\underline{p}}^{p} [\Pi_\ell(x) + \Pi_t(r)]d\Phi(x).$$

Thus

$$\begin{aligned}\Pi(r : \Phi) - \Pi(p : \Phi) &= \left\{[1 - \Phi(p)] + \int_{r}^{p} d\Phi(x)^r\right\} \cdot [\Pi_\ell(r) + \Pi_t(r)] \\ &\quad + \int_{\underline{p}}^{r} [\Pi_\ell(x) + \Pi_t(r)]d\Phi(x) - [1 - \Phi(p)]\Pi_\ell(p) \\ &\quad - \left[\int_{\underline{p}}^{r} \Pi_\ell(x)d\Phi(x) + \int_{r}^{p} \Pi_\ell(x)d\Phi(x)\right]\end{aligned}$$

$$= [1 - \Phi(p)] \cdot [\Pi_\ell(r) + \Pi_t(r) - \Pi_\ell(p)] + \int_r^p [\Pi_\ell(r) + \Pi_t(r) - \Pi_\ell(x)]d\Phi(x) + \int_{\underline{p}}^r [\Pi_\ell(x) + \Pi_t(r) - \Pi_\ell(x)]d\Phi(x). \tag{A2}$$

The function $\Pi_\ell(\cdot)$ is concave and attains a maximum at $p_\ell < r < p$. Hence, for $x \in [r, p]$, $\Pi_\ell(r) \geq \Pi_\ell(x)$. Thus the first and second terms in (A2) are positive. The third term is $\int_{\underline{p}}^r \Pi_t(r)d\Phi(x) > 0$; therefore $\Pi(r : \Phi) > \Pi(p : \Phi)$.

Proof of proposition 1. In lemma 1, it was shown that each firm will couple its list price with an offer to match. To determine the equilibrium strategies, it remains to solve for the list prices.

a) We first prove that, in pure-strategy equilibrium, the list price of each firm will be either p_m or r. From lemma 2, we know that any strategy with support outside $[p_m, r]$ is weakly dominated and hence not an equilibrium strategy. Therefore, in pure-strategy equilibrium, $p_i \in [p_m, r]$, $i = 1, 2$.

Let firm 2 price at $p_2 \in [p_m, r]$, and study the pricing decision of firm 1. Consider those $p_1 > p_2$. Notice that, for such values of p_1, the locals will take advantage of firm 1's offer to match; hence its effective price to local customers will be p_2. Since both firms charge an identical price to locals, each will receive half the local demand, that is, $Q_\ell(p_2)$.

For all $p_1 \in [p_2, r]$, the profit of firm 1 is

$$\Pi_1(p_1 : p_2) = Q_t \cdot (p_1 - c) + Q_\ell(p_2) \cdot (p_2 - c),$$

which is less than $Q_t \cdot (r - c) + Q_\ell(p_2) \cdot (p_2 - c)$. Moreover, for all $p_1 > r$, the profit of firm 1 is

$$\Pi_1(p_1 : p_2) = 0 + Q_\ell(p_2) \cdot (p_2 - c),$$

which is less than $Q_t \cdot (r - c) + Q_\ell(p_2) \cdot (p_2 - c)$. Therefore the best response that satisfies the constraint $p_1 > p_2$ is $p_1 = r$.

Consider those $p_1 \leq p_2$. For such values of p_1, the effect of firm 2's offer to match will be that the price to locals will be p_1 at both firms. Hence the profit to firm 1 will be

$$\Pi_1(p_1 : p_2) = Q_t \cdot (p_1 - c) + Q_\ell(p_1) \cdot (p_1 - c).$$

The profit-maximizing value of p_1 is the solution to

$$\max_{p1} \Pi_1(p_1 : p_2) \quad \text{subject to } p_1 \leq p_2.$$

This is the same program as that for the nondiscriminating monopolist with the addition of the constraint $p_1 \leq p_2$. Now the unconstrained solution for the nondiscriminating monopolist is p_m. But $p_2 \in [p_m, r]$, and hence $\min(p_2, p_m) = p_m$. Thus the best response for firm 1 is to set list price $p_1 = p_m$.

By a similar argument beginning with some price p_1 for firm 1, it may be demonstrated that firm 2 will choose a list price of either p_m or r. This com-

pletes the proof that, in pure-strategy equilibrium, the list price of each firm will be either p_m or r.

b) Now suppose that firm 2 sets $p_2 = r$. By the definition of p_m,

$$Q_t \cdot (p_m - c) + Q_\ell(p_m) \cdot (p_m - c) \geq Q_t \cdot (p - c) + Q_\ell(p) \cdot (p - c),$$

for all p. This implies that $\Pi_1(p_m : r) \geq \Pi_1(r : r)$ and hence that firm 1 will set $p_1 = p_m$.

Suppose that seller 1 sets $p_1 = p_m$. Now, profit of seller 2

$$\Pi_2(p_m : p_m) = Q_t \cdot (p_m - c) + Q_\ell(p_m) \cdot (p_m - c),$$

and

$$\Pi_2(r : p_m) = Q_t \cdot (r - c) + Q_\ell(p_m) \cdot (p_m - c).$$

Since $p_m < r$, it follows that $\Pi_2(r : p_m) > \Pi_2(f_m : f_m)$. Therefore seller 2 will set $p_2 = r$. Thus strategies of a list price p_m coupled with an offer to match for firm 1 and a list price r coupled with an offer to match for firm 2 are an equilibrium. Similarly, it may be shown that the same strategies, reversed between the firms, also are an equilibrium.

Proof of proposition 2. Suppose that the distribution F is a symmetric differentiable equilibrium strategy. Let S be its support and f the density function.

a) The first step is to prove that $p_m \in S$ and $r \in S$. Suppose that

$$\min (p : p \in S) \stackrel{\text{def}}{=} \underline{p} > p_m.$$

Then

$$\Pi(\underline{p} : F) = \Pi_\ell(\underline{p}) + \Pi_t(\underline{p}).$$

Since $p_m < \underline{p}$,

$$\Pi(p_m : F) = \Pi_\ell(p_m) + \Pi_t(p_m) > \Pi_\ell(\underline{p}) + \Pi_t(\underline{p}),$$

by the definition of p_m. Thus $\Pi(p_m : F) > \Pi(\underline{p} : F)$, which is a contradiction.

Suppose that $\max (p : p \in S) \stackrel{\text{def}}{=} \bar{p} < r$. Then

$$\Pi(\bar{p} : F) = \int_{\underline{p}}^{\bar{p}} [\Pi_\ell(x) + \Pi_t(\bar{p})]dF(x),$$

and, since $\bar{p} < r$,

$$\begin{aligned}\Pi(r : F) &= \int_{\underline{p}}^{r} [\Pi_\ell(x) + \Pi_t(r)]dF(x) \\ &= \int_{\underline{p}}^{\bar{p}} [\Pi_\ell(x) + \Pi_t(r)]dF(x).\end{aligned}$$

But $\Pi_t(r) > \Pi_t(\bar{p})$; hence $\Pi(r : F) > \Pi(\bar{p} : F)$, which is a contradiction.

b) Since F is an equilibrium strategy, it must be that

$$\Pi(p : F) = \Pi(p_m : F) \tag{A3}$$

for all $p \in [p_m, r]$. Now for any $p \in [p_m, r]$,

$$\Pi(p : F) = \int_{p_m}^{p} \Pi_\ell(x) \cdot dF(x) + [1 - F(p)]\Pi_\ell(p) + \Pi_t(p).$$

Since the function $\Pi(p : F)$ is differentiable in p, (A3) implies that

$$\frac{d\Pi}{dp} = 0,$$

for all $p \in (p_m, r)$, or

$$\Pi_\ell(p)dF(p) + [1 - F(p)]\frac{d\Pi_\ell}{dp} - dF(p) \cdot \Pi_\ell(p) + Q_t = 0,$$

that is, $F(p) = 1 + Q_t/\Pi_\ell'(p)$.

It remains to show that the strategy

$$F(p) = \begin{cases} 1 + Q_t/\Pi_\ell'(p) & p \in [p_m, r), \\ 1 & p = r, \end{cases}$$

is a symmetric equilibrium. First, note that, by the definition of p_m, $\Pi_\ell'(p_m) + \Pi_t'(p_m) = 0$, and hence $\Pi_\ell'(p_m) = -\Pi_t'(p_m) = -Q_t$. Now, by assumption 2, $\Pi_\ell''(p) < 0$. Hence, for all $p \in [p_m, r]$, $\Pi_\ell'(p) \leq -Q_t$.

Since $F(\cdot)$ is differentiable on (p_m, r), the density is

$$f(p) = -\frac{Q_t\Pi_\ell''(p)}{[\Pi_\ell'(p)]^2}.$$

At any $p \in [p_m, r)$,

$$\begin{aligned}
\Pi(p : F) &= \int_{p_m}^{p} \Pi_\ell(x)f(x)dx + [1 - F(p)]\Pi_\ell(p) + \Pi_t(p) \\
&= \int_{p_m}^{p} \Pi_\ell(x) \cdot \left\{-\frac{Q_t \cdot \Pi_\ell''(x)}{[\Pi_\ell'(x)]^2}\right\}dx + \left[-\frac{Q_t}{\Pi_\ell'(p)}\right]\Pi_\ell(p) + \Pi_t(p) \\
&= Q_t\int_{p_m}^{p} \left\{\frac{[\Pi_\ell'(x)]^2 - \Pi_\ell(x)\Pi_\ell''(x)}{[\Pi_\ell'(x)]^2} - 1\right\}dx - \frac{Q_t\Pi_\ell(p)}{\Pi_\ell'(p)} + \Pi_t(p) \\
&= Q_t\left[\frac{\Pi_\ell(x)}{\Pi_\ell'(x)} - x\right]_{p_m}^{p} - \frac{Q_t\Pi(p)}{\Pi_\ell'(p)} + \Pi_t(p) \\
&= -\frac{Q_t\Pi_\ell(p_m)}{\Pi_\ell'(p_m)} - Q_t(p_m - c).
\end{aligned}$$

Now $\pi_\ell'(p_m) = -Q_t$; hence, for all $p \in [p_m, r)$,

$$\Pi(p : F) = \Pi_\ell(p_m) + \Pi_t(p_m) = \Pi(p_m : F).$$

Likewise,

$$\begin{aligned}
\Pi(r : F) &= \int_{p_m}^{r} \Pi_\ell(x)f(x)dx + \mathrm{pr}(r) \cdot \Pi_\ell(r) + \Pi_t(r) \\
&= Q_t\left[\frac{\Pi_\ell(x)}{\Pi_\ell'(x)} - x\right]_{p_m}^{r} - \frac{Q_t\Pi(r)}{\Pi_\ell'(r)} + \Pi_t(r) \\
&= -\frac{Q_t\Pi_\ell(p_m)}{\Pi_\ell'(p_m)} + Q_t(p_m - c) = \Pi(p_m : F).
\end{aligned}$$

Thus, for all $p \in [p_m, r]$, $\Pi(p : F) = \Pi(p_m : F)$.

In lemma 2 it was proved that any strategy with support outside $[p_m, r]$ is weakly dominated. Therefore $F(p)$ is the unique symmetric differentiable equilibrium strategy.

Proof of proposition 3. The proof of the pure-strategy equilibrium is a direct extension of the proof of proposition 1 and will be omitted. Likewise, the proof of the mixed-strategy equilibrium is an extension of the proof of proposition 2, and only a sketch will be given here. Consider the equilibrium strategy of some firm i. As in the proof of proposition 2, it may be shown that the support of a symmetric differentiable equilibrium strategy is the interval $[p_m, r]$.

Let G_n be the distribution of the minimum of the prices listed by the other $(n - 1)$ firms. Then the expected profit of firm i for list price $p \in [p_m, r]$ is

$$\Pi(p : G_n) = \frac{2}{n}\left\{\int_{[p_m,p)} \Pi_\ell(x)dG_n(x) + [1 - G_n(p)]\Pi_\ell(p) + \Pi_t(p)\right\}.$$

In mixed-strategy equilibrium, the firm must be indifferent among all the list prices in the interval $[p_m, r]$, which implies that

$$\frac{d}{dp}\Pi(p : G_n) = \frac{d}{dp}\left(\frac{2}{n}\left\{\int_{[p_m,p)} \Pi_\ell(x)dG_n(x) + [1 - G_n(p)]\Pi_\ell(p) + \Pi_t(p)\right\}\right) - 0.$$

Notice that this is identical to the equation for the symmetric equilibrium in the case of two firms. Hence the solution must be

$$G_n(p) = \begin{cases} 1 + Q_t/\Pi_\ell'(p) & \text{if } p \in [p_m, r), \\ 1 & \text{if } p = r, \end{cases}$$

and therefore $\Pi(p : G_n) = (2/n)[\Pi_\ell(p_m) + \Pi_t(p_m)]$.

Now $G_n(p)$ is the distribution of the minimum of the list prices of the other $(n - 1)$ firms. Thus

$$\begin{aligned} G_n(p) &= \text{pr[minimum of } (n - 1) \text{ prices is no greater than } p] \\ &= 1 - \text{pr[all } (n - 1) \text{ prices are greater than } p] \\ &= 1 - [1 - F_n(p)]^{n-1}. \end{aligned}$$

Hence $1 - F_n(p) = [1 - G_n(p)]^{1/(n-1)}$, and thus

$$\begin{aligned} F_n(p) &= 1 - [1 - G_n(p)]^{1/(n-1)} \\ &= 1 - \{1 - [1 + Q_t/\Pi_\ell'(p)]\}^{1/(n-1)} \\ &= 1 - [-Q_t/\Pi_\ell'(p)]^{1/(n-1)}. \end{aligned}$$

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