Residual Risk, Trading Costs, and Commodity Futures Risk Premia

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Trading costs, in the form either of explicit charges or of the costs of becoming informed, limit the participation of some classes of traders in commodity futures markets. When speculators face a fixed cost of participating in a futures market that is used by commodity producers to hedge their stochastic revenues, the futures risk premium deviates from the perfect markets prediction. The deviation rises in absolute value with the square root of the trading cost and with the standard deviation of residual returns, and it is unrelated to the covariance of the futures price with producers' nonmarketable wealths. The residual-risk premium depends not on the total magnitude of the risk that producers hedge (i.e., aggregate revenue variance), but on the variability of their revenue relative to its mean (i.e., the coefficient of variation). Hence, even a commodity that constitutes a minor fraction of aggregate consumption may have a large premium for residual risk if the revenue derived from it has a large coefficient of variation.

The two major approaches to the analysis of commodity futures risk premia can be distinguished by their assumptions about the marketability of assets. What can be called the perfect markets approach leads under conventional assumptions to the traditional capital asset

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pricing model (CAPM), which predicts that risk premia will be proportional to the covariance of the futures return with the return on the market portfolio of all (tradable) assets. The alternative imperfect markets approach is based upon the premise that market imperfections, such as adverse selection or moral hazard, limit the issuance of equity shares by agricultural producers. If so, the risk premia on agricultural futures contracts will depend not only on the covariance with the market portfolio of all *traded* assets but also on their covariance with nonmarketed endowments [as in the modified CAPM of Mayers (1972)]. An assumption of nonmarketability is implicit in the normal backwardation theory of Keynes and Hicks as well as in more recent work on the effects of hedging pressure by producers on futures prices.[1]

Telser (1958), in a classic paper, maintained that the effect of hedging pressure on the risk premium will be near zero because of the large number of outside speculators available to bear risk. This would suggest that even when revenue distributions are imperfectly marketable, the futures premium will satisfy with good approximation a perfect markets model of capital market equilibrium, such as the CAPM.

The CAPM, with either full marketability or nonmarketable assets, in general implies that all individuals hold a small amount of all marketable assets for diversification reasons. In reality, few noncommercial investors take positions in commodities (futures, forward, or spot), which suggests that there exist barriers to participation, perhaps arising from the fixed setup costs required to learn about these markets, or alternatively because of costs of taking small positions (as with minimum contract sizes).[2] Such setup costs, by limiting the spreading of agricultural risk, will tend to increase the magnitude of risk premia induced by producer hedging. Robert Merton's (1987) presidential address to the American Finance Association models security pricing when investors hold only those securities which they know enough about. He discusses a number of reasons why informational barriers to investing in a desirable security may be both pervasive and of long duration.

It does not take a large monetary or informational cost to deter individuals contemplating small positions from trading the futures contract on a given commodity on personal account,[3] and intermediaries have held positions on behalf of private investors only to a very limited extent.[4] This low participation is despite the attractiveness of commodity futures as investments. Bodie and Rosansky (1980) and Lintner (1983) drew the conclusion that substantial benefits were available to investors in the preceding three

[1] In these accounts, privately held producers reduce risk by selling futures short.

[2] The setup cost may be interpreted as the time investment required to avoid being at a severe informational disadvantage in the commodity market. It therefore includes the effort required to understand the mechanics of trading, principles of futures pricing, and the complex factors influencing supply-and-demand conditions.

[3] Blume and Friend (1975) describe a sample of investors who held few securities in their portfolios.

[4] Pension funds are limited by regulatory restrictions from trading commodities. Mutual funds do not combine commodity futures and stock investments; thus, to form a portfolio with commodities, an investor must resort to a specialized futures fund.

decades by combing portfolios of commodities and stocks, because of their negative correlation ($\rho = -.24$) arising from opposite inflation sensitivities. Although futures markets are highly liquid compared to commodity spot markets, the failure of investors to take advantage of commodity futures as investments indicates that there remain effective barriers (possibly informational) to trading in futures.

Constantinides (1986) examined the sensitivity of asset prices to transaction costs in a multiperiod setting. He demonstrated that even though transaction costs have a large effect on investors' portfolio decisions, the impact on prices is relatively small when the length of the interval between portfolio rebalancing is endogenous. On the other hand, he showed that the effect of transaction costs can be important when frequent trading (for example, every year) is exogenously imposed. This would apply to commodity futures, since most contracts have short expirations (a year or less) and since most of the trade in these contracts occurs in the months closer to expiration.[5]

This article analyzes the pricing of futures contracts in a context in which not only are agricultural revenues nonmarketable,[6] but participation in commodity futures markets is limited by the existence of fixed (setup) costs. This is in the spirit of Merton's (1987) general model of the effect of nonparticipation in securities markets. However, it differs in that the fixed setup cost of trading here presents each investor with an endogenous decision as to whether or not to hold the futures contract. In equilibrium, the commodity futures risk premium is determined by the marginal investor who is indifferent as to whether or not he participates in the futures market. Thus, the number of traders in the futures market, as well as the risk premium, are jointly determined, and we obtain a simple relation between the deviation of the futures price from the simple CAPM prediction, the residual risk of the futures contract, and the level of the fixed setup costs.

Empirical studies of futures pricing, beginning with Dusak (1973), have provided only mixed support for either the traditional or the consumption-based CAPM. The current model is consistent with evidence that producers are important participants in commodity futures market, as measured both by their share of open interest and by their impact on prices.[7] Chang (1985)

[5] Furthermore, Constantinides' numerical estimates were for stock returns, which are less volatile than futures percentage price changes.

[6] A nonmarketable source of wealth is defined here as one on which equity shares are not traded; note, however, that any gamble is partly "marketable" by trading a security that covaries with it.

[7] For example, from 1967 to 1987 the ratios of average monthly *commercial* futures positions to average *total* (commercial plus speculative) positions for three grains were:

Share of commercial futures positions, 7/67 to 3/87

	Wheat		Corn		Soybeans	
	Long	Short	Long	Short	Long	Short
Excluding spreads	.79	.87	.90	.93	.84	.90
Including spreads	.60	.68	.80	.82	.56	.62

Data source: *Commitments of Traders,* Commodity Futures Trading Commission; not reported January–November 1982.

found, consistent with the model, that commodity futures price changes are correlated with hedging positions taken previously by producers; see also Carter, Rausser, and Schmitz (1983) and Marcus (1984).

The remainder of this article is organized as follows. The economic setting of the model is described in Section 1. Section 2 relates the futures risk premium to residual risk, both with an exogenous number of speculators (Section 2.1) and with an endogenous number (Section 2.2). Section 3 relates the premium to the magnitude of revenue risks faced by producers in the spot market. Section 4 concludes the article.

1. The Economic Setting

A two-date mean-variance model is employed here.[8] There are two groups of participants in a competitive futures market and stock market, G producers (growers or handlers of the commodity) and N outside investors. They select positions at date 0 that influence their stochastic consumptions taking place at date 1.[9] For simplicity, market participants have the same risk aversion, although they differ in their initial endowments. Beliefs concerning the distributions of all variables are homogeneous. Producers and outside investors maximize the same mean-variance objective function[10]

$$U = E(C) - \left(\frac{\alpha}{2}\right)\mathrm{var}(C), \tag{1}$$

where C is consumption at date 1, and α is the coefficient of absolute-risk aversion.

Each producer $g = 1, \ldots, G$ receives a risky revenue from sales of the commodity, $\tilde{r}^g$. Equity shares in producers' businesses cannot be sold, creating an incentive to take a futures position with inversely correlated payoff. Revenue is random because of uncertainty about the size of the crop (for a grower) and about the price at which it can be sold. For an outside investor n, $r^n \equiv 0$; outside investors could also be given nonmarketable sources of wealth without affecting the results, so long as their wealths were independent of commodity output shocks.

There are two rounds of trading. The only securities tradable in the prior round are a futures contract and a risky asset suggestively termed the *stock market portfolio,* which represents those endowed risks which may be

[8] With only two dates, the model abstracts from the daily resettlement feature of futures contracts, which can cause small divergences between futures and forward prices when interest rates are stochastic; see, for example, Black (1976), Jarrow and Oldfield (1981), Richard and Sundaresan (1981), and Cox, Ingersoll, and Ross (1981).

[9] Similar results would apply in a model with two consumption dates and risk-free lending.

[10] This applies under the assumptions of normality and exponential (constant absolute risk aversion) preferences. Similar results could be derived without normality under quadratic utility.

divided into equity shares and traded costlessly.[11] A fixed transaction cost of t for trading in futures is assumed to deter some outsiders but no producers from the futures market,[12] so that the number of outside investors who actually trade futures ("speculators") is $\hat{N} \leq N$.

Some further definitions follow:

f = the futures price of wheat set at date 0.

P = the spot price of wheat set at date 1. The futures contract is assumed to be written in real rather than nominal terms, which abstracts from inflationary effects.

ξ^g, ξ^n = the number of contracts held by producer g (grower or handler) or speculator n.

S^g = gross dollar investment by a producer in the noncommodity risky asset (the stock market portfolio). Let S^n refer to the stock position of a speculator who trades in futures, and S^r to the position of one who refrains from trading futures. Let S^{eg}, S^{en}, S^{er} be the endowed positions in the stock market portfolio.

W^g, W^n, W^r = initial wealths (units of the date 1 consumption good) of members of the different classes of individuals.

R_m = net return on the stock market portfolio; that is, for every dollar the investor chooses to put at risk in the market, he receives the random net payoff of R_m. Since consumption takes place at a single date, there is no trade-off between current and future consumption in selecting the number of futures contracts and shares in the stock market; the decision is based on the impact of these positions on expected value and variance of consumption.

The unscripted variables r and W will refer more generally to the revenue and wealth endowments either of a producer or of a speculator (for whom $r \equiv 0$), and similarly the unscripted choice variables ξ and S refer to either a producer or a speculator.

[11] It is assumed that agricultural equity is not traded, so that the revenue distribution of a grower or handler is a nonmarketable risk. In fact, the vast bulk of planted acreage in the United States is owned by farms that are privately held [Lin, Johnson, and Calvin (1981)], and even some major processing firms are closely held (such as Cargill, Continental Grain, and Dreyfus). Moral hazard and adverse selection problems with issuing equity may be severe because of the high random variability of output in agricultural production. Furthermore, even in widely held firms, optimal contracts that impose risks on managers may provide an incentive to hedge the firm's risk by using futures [Diamond and Verrechia (1982)].

[12] This is intended to reflect the fact that when there are many outsiders relative to hedgers, each outside "speculator" in equilibrium contemplates only a small futures position. So it takes only a small fixed setup cost to drive many speculators from the market. Hedgers, in contrast, have a nontrivial risk-reducing incentive to trade futures and are less likely to be deterred by a small cost from taking positions of nonnegligible size. Margin requirements that favor commercial hedgers over speculators may also contribute to differences in participation. The nature of the results that follow would be similar if this condition were not imposed.

Futures trading opportunities are described by[13]

$$C = \begin{cases} W - t + r + (P - f)\xi + SR_m & \text{if trade futures} \\ W + r + SR_m & \text{otherwise.} \end{cases} \tag{2}$$

P is determined in the date 1 spot market for the commodity. For a futures-trading individual, terminal consumption is endowed wealth less the transaction cost, plus his random revenue endowment, plus the gain on the futures position taken and the profit or loss on the stock market position.

2. Residual Risk and the Risk Premium

As a preliminary, Section 2.1 derives from basic relations with futures and stock trading, holding constant the number of individuals trading on the futures market. Section 2.2 uses these relations to determine the risk premium and the number of traders simultaneously. All proofs will be found in the Appendix.

2.1 Equilibrium with the number of traders exogenous

This section takes as given the number of trading speculators $\hat{N}$.

The futures hedging problem. The problem solved by a futures trader is to maximize expected utility U with respect to the futures position ξ and the stock position S, subject to Equation (2). Let the risk premium $\Pi = -\text{bias} \equiv E(P - f)$, and $\tilde{\Pi} \equiv P - f$. A lowercase $\tilde{\pi}$ will indicate the percentage futures price appreciation $(P - f)/f$, and $\pi \equiv E(\tilde{\pi})$ *is the percentage premium. Concavity of the objective with respect to the choice variables ensures that the optimal stock and futures positions for an individual who trades in the futures market satisfy the first-order conditions*

$$\Pi = \alpha \operatorname{cov}(\tilde{\Pi}, C), \tag{3}$$

$$\bar{R}_m = \alpha \operatorname{cov}(R_m, C). \tag{4}$$

The linearity of the covariance operator implies that Equations (3) and (4) hold when C is interpreted to be the average consumption across all futures trading agents. As in the consumption-based CAPM,[14] the risk premium is proportional to the covariance of an average of consumption with the date 1 spot price. However, the first-order condition (3) will not hold for indi-

[13] While participation in the futures market is taken to be costly, for analytic simplicity trading in the stock market is assumed to be costless. This is to focus on the effect of excluding traders from the futures market. This assumption also may not be unrealistic, in that (1) far more investors trade in the stock market (including mutual and pension funds) than in commodity futures, and (2) for stocks, the setup costs (if viewed as an informational investment) may largely be avoided by investing in a passively managed mutual fund. Futures mutual funds, on the other hand, are very actively managed, owing partly to the short expirations of futures. Therefore, for futures, the difficult problem of monitoring and assessing fund managers remains.

[14] Breeden (1980) applies the intertemporal consumption CAPM to commodity futures; Richard and Sundaresan (1981) and Grauer and Litzenberger (1979) also relate the futures risk premium to its desirability as a consumption hedge.

viduals who do not trade futures. Since C as the average of consumption in Equation (3) includes the consumption only of those individuals who trade futures, the exclusion of some individuals by transaction costs leads to predictions (below) that differ from the CAPM. By Equations (3) and (4), substituting from (2), we have Proposition 1.

Proposition 1 (optimal futures and stock positions). *The optimal futures position ξ for producers or trading speculators is*

$$\xi = \frac{\Pi/\alpha - cov(\tilde{\Pi}, r + SR_m)}{var(\tilde{\Pi})}. \tag{5}$$

The optimal stock position S for an agent is

$$S = \frac{\bar{R}_m/\alpha - cov(R_m, r + \xi\tilde{\Pi})}{var(R_m)}, \tag{6}$$

where Equation (6) still applies with $\xi = 0$ for an individual who does not trade futures.

The first term in the numerator of Equation (5) is the component of the futures position taken to exploit bias. Its absolute size rises with the premium Π and with risk tolerance, $1/\alpha$. Note that t does not enter directly into the optimal-position sizes, because the assumption of exponential preferences eliminates wealth effects on risk aversion. Of course, since t affects the number of speculators in the futures market, it will also affect position sizes in equilibrium through the premium Π. For a given premium and expected return on the market, varying t leaves the futures position unchanged, unless t is high enough to deter trading altogether ($\xi = 0$). Since the futures payoff in general may be correlated with the market return, by Equation (6) participating on the futures market changes the optimal stock market position.

Let us consider now the special case of the optimal futures position taken by speculators, for whom $r \equiv 0$. Then

$$\xi^n = \frac{\Pi/\alpha - S^n \mathrm{cov}(R_m, \tilde{\Pi})}{\mathrm{var}(\tilde{\Pi})}. \tag{7}$$

When the spot price is uncorrelated with the market return, the second term in the numerator vanishes, in which case speculators trade only if there is a bias. With covariance between R_m and $\tilde{\Pi}$, even in the absence of a bias, if $S^n > 0$, positive (negative) correlation of the spot price with the market return leads to a short (long) futures position in order to diversify with respect to moves in the other risky asset, the stock market.

Futures market equilibrium. Let the endowed revenue distribution of a representative producer be defined as $r^p \equiv [\Sigma_{g=1}^G r^g]/G$, and let the stock position for such a producer be $S^p \equiv (\Sigma_{g=1}^G S^g)/G$. Let us define $b \equiv G/(G + \hat{N})$, the fraction of futures traders who are hedgers rather than

speculators. Also, let $\bar{S}$ be the average stock position among futures traders, defined by $\bar{S} \equiv (1 - b)S^n + bS^p$.

The remainder of this article will omit p superscripts so that r will be the revenue of the representative producer. Let a j superscript indicate the agent (either hedger or speculator). In equilibrium the futures market must clear,

$$\sum_{j=1}^{\hat{N}+G} \xi^j = 0, \tag{8}$$

which determines the futures price for given $\hat{N}$.[15] A basic futures pricing relation follows by Equations (5) and (8):

$$\pi = \alpha\bar{S}\,\mathrm{cov}(R_m, \tilde{\pi}) + \alpha b\,\mathrm{cov}(r, \tilde{\pi}). \tag{9}$$

Equation (9) characterizes the futures risk premium, *given* the number of speculators that trade in futures (as reflected in b, the fraction of futures traders that are hedgers). The risk premium has two components. The first is a stock market risk component, which shows that the risk premium increases with the covariance between the spot price and the market portfolio of tradable assets. The second term reflects the incentive of producers with nonmarketable assets to hedge. It is proportional to the covariance of the futures payoff with nonmarketable risks.[16]

Residual risk and the premium. By replacing $\tilde{\pi}$ in Equation (9) with its systematic and residual components, we can express the futures risk premium in terms of the contract's beta and its residual risk. Let us assume the market model for $\tilde{\pi}$ and for $\tilde{r}$:

$$\tilde{\pi} = \gamma_{\pi m} + \beta_{\pi m}\tilde{R}_m + \tilde{\epsilon}_{\pi m} \qquad \tilde{r} = \gamma_{rm} + \beta_{rm}\tilde{R}_m + \tilde{\epsilon}_{rm}. \tag{10}$$

Then, by Equation (9), we have Proposition 2.

Proposition 2 (residual risk with given participation). *With a given number of speculators $\hat{N}$, the futures premium is*

$$\pi = \alpha(\bar{S} + b\beta_{rm})\,var(R_m)\beta_{\pi m} + \alpha b\sigma(\tilde{\epsilon}_{rm})\sigma(\tilde{\epsilon}_{rm})\,corr(\tilde{\epsilon}_{\pi m}, \tilde{\epsilon}_{rm}). \tag{11}$$

Proposition 2 shows that the effects on the risk premium of the systematic and residual components of the futures return separate additively. This suggests some interesting points about tests of different versions of the CAPM. Consider the special case of full participation, $\hat{N} = N$ (i.e., the Mayers CAPM). In a cross section of securities with different return distri-

[15] Similarly, the expected return on the stock market is determined by the equilibrium condition that $\Sigma_{j=1}^{N+G} S^j = \Sigma_{j=1}^{N+G} S^{ej}$, where S^{ej} is the endowment position for individual j. It follows by Equation (6), letting $\underline{S}$ be the average stock market endowment and b' be the fraction of growers $b' = G/(N + G)$, that $\bar{R}_m = \alpha\,\underline{S}\,\mathrm{var}(R_m) + \alpha\,b'\,\mathrm{cov}(R_m, r)$.

[16] The division of risk premia into a marketable risk component and a nonmarketable risk component applies generally in the Mayers CAPM and has been described in a commodities context by Stoll (1979).

butions ($\tilde{\pi}$), the first term in Equation (11) indicates that the risk premium is linear with $\beta_{\pi m}$, just as in the traditional CAPM. The second term shows that the premium depends also on residual standard deviation $\sigma(\tilde{\epsilon}_{\pi m})$. However, this will not be a one-to-one linear relation, since this quantity is multiplied by another security-specific variable $\text{corr}(\tilde{\epsilon}_{\pi m}, \tilde{\epsilon}_{rm})$, the correlation of the security's residual with the residual component of nonmarketable wealth. Therefore, neither a nonlinear beta term nor residual variance are called for as regressors in a test between the Mayers and the traditional CAPM. It is residual standard deviation, not variance, that is relevant for pricing; and, in addition, some proxy is needed for the security's residual correlation with nonmarketable wealth. Furthermore, if the number of traders is *endogenous,* it would be premature to try to test either Equation (11) or (9) cross-sectionally by regressing mean futures price changes on right-hand-side variables. Such a regression will be misspecified if in equilibrium the unobserved b is systematically related to the other right-hand-side variables. The next section makes the number of speculators endogenous, which provides an estimatable pricing relation by eliminating not only b but also the unobserved correlation with nonmarketable wealth.

2.2 Endogenous number of traders

With these preliminary results, we are ready to examine the codetermination of the risk premium and the number of speculators who trade on the futures market. Speculators decide whether to trade in futures based on whether paying t and trading optimally at date 0 generates higher or lower expected utility than refraining ($\xi^r = 0$). Speculators enter the futures market until the equilibrium bias is just sufficient to compensate the marginal speculator for the cost he incurs in order to trade futures.

To find the implications of the indifference condition, we may calculate

$$\begin{aligned} U(W - t \text{ trade}) &= W - t + \Pi\, \xi^n + S^n \bar{R}_m - \frac{\alpha}{2} \text{var}\,(\xi^n \tilde{\Pi} + S^n R_m), \\ U(W; \text{refrain}) &= W + S^r \bar{R}_m - \frac{\alpha}{2} (S^r)^2 \sigma_m^2. \end{aligned} \tag{12}$$

The equilibrium condition that speculators be indifferent as to entry,

$$U(W - t; \text{trade}) = U(W; \text{refrain}), \tag{13}$$

implies that the setup cost t will determine the number of individuals who trade. In the result that follows, let us assume that $\text{cov}(r, P) \neq 0$; otherwise, in equilibrium no speculator would trade, because futures trading between speculators and producers would be jointly risk-increasing in comparison with producers trading only among themselves. Letting $\beta_{\Pi m}$, $\beta_{m\Pi}$ be regression coefficients and letting ρ be the correlation of R_m and Π, we have the following formula for the risk premium:[17]

[17] If different investors had different t's but were otherwise identical, then this formula would apply for the t of the marginal investor; with diverse α's, the formula applies for the marginal investor's risk aversion.

Proposition 3 (residual risk with endogenous participation). *With a fixed transaction cost that deters some speculators from the futures market, and if cov(r, P) ≠ 0, then the futures risk premia* Π *and* π *satisfy*

$$\begin{aligned} \Pi &= \beta_{\Pi m}\bar{R}_m \pm \sigma_\Pi\sqrt{2\alpha t(1-\rho^2)} \\ \pi &= \beta_{\pi m}\bar{R}_m \pm \sigma_\pi\sqrt{2\alpha t(1-\rho^2)}, \end{aligned} \tag{14}$$

where the +/− applies as hedging is short/long.

A number of implications should be noted. As before, the premium contains two terms, a stock market risk term and one due solely to hedging. The market risk component is proportional to $\beta_{\Pi m}$. In contrast, Tobin and Brainard (1977), Levy (1978), and Mayshar (1979, 1981) have contended that fixed transaction costs make a security's total variance a key determinant of its risk premium. In the market model, the total variance of a given security i may be decomposed into a nonlinear beta term $\beta_{im}^2\sigma_m^2$ and residual variance term $\sigma^2(\epsilon_i)$. So where the total variance implies a nonlinear beta effect, in Proposition 3 the effect of beta is linear.

The reason for this is quite simple. The risk premium on the futures contract is set to compensate the marginal speculator for his setup cost and for the incremental risk associated with his futures position. To the extent that the futures contract is correlated with the stock market, the speculator can offset the risk of his futures position by shifting his position in the stock market by an amount negatively proportional to the contract's beta. Since he can remove the systematic component of his futures payoff distribution by sacrificing the premium on quantity $\beta_{\pi m}$ of the stock market portfolio, the premium for this component must be $\beta_{\pi m}\bar{R}_m$. The total premium divides into this systematic risk term and a remaining component due solely to residual risk.

The residual-risk term has several interesting features. First is the broad prediction that the absolute hedging component of the premium rises with the costs of trading and/or information, approaching zero for costless trading. It is likely that the setup cost is higher in a thin market than in a broad one;[18] furthermore, thin markets are in general less familiar to investors. Gray (1960) contended that thin futures markets are characteristically biased, and he found that as some initially thin futures markets grew to successful maturity, their biases tended to diminish. Gordon (1984) also reports significant bias for some thin futures contracts. Consistent with the interpretation here, Gray viewed large bias in the futures price as indicative of an absence of speculative (noncommercial) interest in the market.

In particular, the premium rises with the *square root* of the setup cost.

[18] Since scale economies are important for organized trading on exchanges, it can be expected that thinly traded markets should be more costly to use than broad markets. In addition, contracts that are poorly designed, or illiquid for whatever reason, will tend to be both thinly traded and costly for speculators to learn to use effectively.

The positive infinite first derivative and negative infinite second derivative of the square-root function at the origin indicate that the marginal impact of the trading cost on the premium is highest when t is low, so that even relatively low costs may have a nonnegligible effect on the premium. This is because for low t there are many small speculators (ξ^n is low), so a given increase in the transaction cost must be compensated for by a larger increase in the price premium Π in order to increase sufficiently the expected dollar compensation $\xi^n \Pi$.

Second, unlike the Mayers CAPM and Proposition 2, the non-beta component of the premium is independent of the contract's covariance with nonmarketable risks (as long as they do covary)! At first glance, this seems to clash with the intuition that the incentive to hedge is driving the premium.[19] Producers will have a stronger inclination to hedge using a security that correlates highly with their risks, so it is tempting to believe that surely such a security will earn a larger premium than will a poorer hedging instrument.

Yet from another standpoint, the elimination of nonmarketable risk from the premium is obvious. The premium is determined by marginal speculators, who are concerned only with their own payoffs, not those of hedgers. If the premium for residual risk ever rose above the level appropriate to the dispersion of the residual return on the futures contract, then it would pay for more speculators to enter. Since the covariance with nonmarketable risks is irrelevant for their decisions, it is also for the premium.[20] In the spirit of Telser, the supply of speculative services is infinitely elastic (until all N of the outsiders participate), but at a *positive* instead of a zero absolute premium.

It is worth stressing that the "market" portfolio here refers only to *tradable* endowments. Because nonmarketable wealth does not influence the futures price here, the pricing relation provided here, in contrast with the CAPM, can be tested by estimating distributions of returns that are at least in principle observable.[21] Of course, any such test would face the problems of deciding what kinds of assets should be treated as "nonmarketable" and of obtaining price data on all "marketable" assets.

Third, the hedging component of the premium rises with the *standard deviation* of residual risk, $\sigma_\pi\sqrt{1-\rho^2}$. This is in contrast with the general presumption (à la Arrow and Pratt) that premia will be proportional to

[19] In fact, superficially this might seem inconsistent with Equations (11) and (9). However, as the hedging covariance in Equation (9) varies, what I have shown is that this leads to offsetting variations in the endogenously determined b.

[20] As an empirical matter, this point should not be carried to the absurd extreme of searching for hedging premia in contracts that have close to zero correlation with the producers' risks. If setup costs are faced by *producers* as well as speculators, then a virtually useless hedging instrument will be traded by neither group. Hirshleifer (1988) examines the pricing effects of differing levels of participation by suppliers at different production stages (growers versus processors).

[21] Roll (1977) described the problems with trying to test the standard CAPM with an unobservable market portfolio.

variance, and with Grinblatt and Titman's (1983) result that the premium for bearing idiosyncratic risk in a linear factor model is proportional to residual variance. Several empirical studies of capital asset pricing have found that residual variance has incremental explanatory power beyond beta for asset risk premia, but others have not been able to detect a significant residual-risk effect.[22] Proposition 3 suggests that for zero net supply securities whose returns are affected by incentives to hedge, standard deviation may be a more accurate measure of residual risk.

This comes about because speculators, having entered the futures market, protect themselves from higher residual risk by reducing the size of their positions. As a simple illustration, let us assume independence of stock and commodity shocks (so that all risk is residual). By Equations (7) and (14), the position of each speculator is

$$\xi^n = \pm\left(\frac{2t}{\alpha}\right)^{1/2}\frac{1}{\sigma_\Pi}, \tag{15}$$

which is inversely proportional to the standard deviation. So as the variance quadruples, the position size of each riskbearer halves. The variance of cash flows from the futures position $(\xi^n)^2\sigma_\Pi^2$ therefore remains the same. The Markowitz total dollar premium on the position, $\Pi\xi^n$, should be the same, since dollar variance is the same. For the dollar premium to be equally large when position ξ^n is halved, the price premium Π must be twice as large. So Π and π rise with the standard deviation, not with the variance. It is also of interest to note that the equilibrium futures position ξ^n rises in absolute value with the square root of the transaction cost, because [by Equation (14)] the premium is increasing as the square root of t.

Varying the dispersion of the futures return, holding ρ, the correlation with the market, constant brings out the similarity in scaling of Equation (14) with the CAPM. In the current model, as with the CAPM, increasing k in the multiplicative spread $\tilde{\pi}' - E(\tilde{\pi}') = k[\tilde{\pi} - E(\tilde{\pi})]$ causes the premium to vary linearly with the standard deviation. If the residual variance were priced, the relation would of course be nonlinear.[23]

Three qualifications to Proposition 3 should be mentioned. First, the assumption of identical speculators is extreme; more generally, setup costs, risk aversion, and beliefs will differ. With heterogeneous setup costs, as the hedging covariance in Equation (9) rises, larger premia will be required in order to draw higher-cost investors into the futures market. A similar effect will occur with heterogeneous risk aversion. However, Proposition 3 illustrates the general principle that potential entry or exit from the

[22] See Lehmann (1986) and references therein.

[23] Samuelson (1965) maintained that the variance of futures price changes should rise as expiration approaches, a prediction that has received some support for commodities. The comparative statics is suggestive of how daily premia may vary through the life of the contract or seasonally as volatility changes; however, an explicit multiperiod analysis is clearly called for.

market of large numbers of speculators can greatly reduce the sensitivity of a security's premium to the intensity of hedging pressure. With heterogeneous beliefs, Proposition 3 would apply with respect to the beliefs of the marginal speculator. An appeal to rational expectations, by limiting the range of disagreement between speculators, could justify its use as a prediction of the actual behavior of prices.

Second, although nonmarketable revenues of producers did not enter into the premium, the hedging side of the model is crucial in order for Proposition 3 to make economic sense. Since beliefs are homogeneous, without hedging pressure there would be no gain for speculators to trading futures; they would have to suffer a transaction cost for the privilege of betting against each other. The risk premium in this case is simply proportional to the stock market β.

Third, extensions of the model beyond the futures context to the problem of limited participation in many assets is likely to lead to results considerably more complex than Proposition 3. In the current model, hedging provides the primary reason for the zero net supply futures contract to be traded. In the context of positive net supply assets, such as stocks of small firms, the incentive to trade will typically arise from other sources, such as unevenness in the initial distribution of endowments of the security, the arrival of private information, and/or liquidity shocks. Therefore, use of the indifference condition as a method for pricing positive net supply assets, or financial securities not used as hedging vehicles, requires specific analysis of how the above-mentioned factors bear on the decision problem of the marginal participant. Even with homogeneous beliefs, this will involve a complex codetermination of the identities of marginal investors in each asset (i.e., what bundles of assets they are endowed with) and of the premium as based on their endowments and planned trades.[24]

3. Size of Risk and the Risk Premium

Telser's (1958) argument suggests that in a large capital market, where there are many potential riskbearers relative to the variance of aggregate revenue to be hedged, the hedging component of the premium will approach zero. I have shown that the hedging premium remains bounded from zero when there is a cost of trading futures. Section 2 related the premium to the standard deviation of the spot price, or, more strictly, to the residual after extracting its market component. To evaluate the likely importance of hedging for the futures risk premium, let us reexpress the premium in terms of the revenue risk to be hedged.

[24] Mayshar (1979) has derived a result [his Equation (45)] related to Proposition 3. His model is similar in spirit to the current article in that the choice of whether or not to trade securities is endogenous. Because the current model analyzes a zero net supply security for which hedging pressure provides the motive for trade, it avoids an artificial feature of Mayshar's model, that no investors are initially endowed with any of the risky assets traded on the capital market.

3.1 Absolute and relative risk

Since the futures price bias is a premium offered to speculators for bearing risk, the form of the well-known Arrow-Pratt risk-premium formula might lead one to expect that the hedging-induced component of the premium will rise with the variance of the risk to be hedged (i.e., variance of aggregate revenue). It has further been argued, in the spirit of Telser, that the magnitude of agricultural revenue risk is very small relative to the number of potential outside riskbearers, so that the hedging component of the premium must be very small. We will see, however, that both of these conjectures are invalid if the number of riskbearers is determined endogenously in a market with costly participation.

With respect to the latter, it is clear that with many outsiders, it is the transaction cost, not the number of outside riskbearers, that determines the bias. The marginal riskbearer will just satisfy Equation (13), and there will be a surplus of potential riskbearers who are deterred by the transaction cost from bearing any of the agricultural risk. In other words, it is not N, but $\hat{N}$, that determines the risk premium.

What is perhaps more surprising is that the total magnitude of the revenue risk is also not relevant for the percentage hedging premium. For a given revenue variability per acre, a small crop will have smaller aggregate revenue variance than a large one. We will see that it is possible for a small crop, with consequent low aggregate revenue variance, to have a large hedging-induced premium. Specifically, a proportional halving of the revenue distribution of a commodity, which reduces the variance by a factor of 4, leaves the risk premium approximately unchanged.

3.2 Spot market equilibrium

To relate the premium to the magnitude of revenue risk and the premium, let us add structure to the spot market side of the model. Consumers may be represented through the inverse demand function for the spot commodity,

$$P = g(Q^d) \qquad g' < 0, \tag{16}$$

where P is the spot price at date 1 and Q^d is the aggregate quantity demanded. The spot market clearing condition $Q = Q^d$, together with Equation (16), determines the spot price as a function of the exogenous output shock.

Some definitions prepare for the result that follows.

Definitions. Let $\bar{Q} = E(Q)$; R = aggregate revenue = PQ; σ_R, $\bar{R}$ = the standard deviation and expected value of R; $\mathrm{CV}(R)$ = coefficient of variation of revenue = $\sigma_R/\bar{R}$; $\mathrm{CV}^f(R)$ = quasi-coefficient of variation of revenue = $\sigma_R/f\bar{Q}$; and demand elasticity $\eta = g(\bar{Q})/\bar{Q}g'(\bar{Q})$.

$\mathrm{CV}^f(R)$ differs in a minor way from the standard coefficient of variation. The denominator, instead of being expected revenue, is the value of the expected output evaluated at the futures price.

The next result relates the percentage premium to beta and to the (quasi-) coefficient of variation of the crop's revenue. Proposition 4 shows that it is not size of the crop, but rather the variability in revenue generated per acre, that is important for the risk premium.

Proposition 4 (relative risk). *Assume that the number of speculators N is large, that revenue variability σ_R is low, and that demand elasticity is nonunitary, $\eta \neq -1$. Then*

$$\pi = \beta_{\pi m}\bar{R}_m \pm \frac{\sqrt{2\alpha t(1-\rho^2)}}{|1+\eta|} CV^f(R) + o(\sigma_R). \tag{17}$$

If $\beta_{\pi m} = 0$, then

$$|\pi| = \frac{\sqrt{2\alpha t}}{|1+\eta|} CV(R) + o(\sigma_R). \tag{18}$$

Example. Suppose that $\beta_{\pi m} = 0$, demand elasticity $\eta = -.2$, relative-risk aversion RRA = 8,[25] annual income is $W = \$40{,}000$, the coefficient of variation of revenue is CV$(R) = .4$,[26] and the cost of trading futures is $t = \$100$. Then $\alpha \equiv \text{RRA}/W^0 = 1/5000$, so $\pi = +.10$ (i.e., a percentage risk premium of 10 percent). Given the wide range of estimates of risk aversion in the literature, and the difficulty of assessing the time or cognitive cost of participating in the futures market, a much smaller or larger value could easily be estimated. ■

The first point to note about Proposition 4 is that demand elasticity affects the premium. Proposition 3 showed that (residual) price dispersion drives the risk premium. Proposition 4 translates price dispersion into revenue variability by way of demand elasticity for the spot commodity. For a given dispersion of price, demand elasticity indicates how great will be the dispersion of revenues. As $\eta \to -1$, there is a more perfect offset between variations in price and output.[27] Thus, greater price dispersion is needed in order to achieve a given revenue dispersion. This leads to a greater absolute hedging premium.

More importantly, the proposition indicates that the percentage bias depends on the relative, not the absolute, variability of revenue. A small crop with low revenue variance could have a large hedging-induced premium if the revenue is highly variable *relative to its mean.* So it does not follow from the large size of the capital market that hedging-induced

[25] See Blume and Friend (1975), Merton (1980), Wheatley (1985), and Jagannathan (1985) for estimates of risk aversion ranging from 1 to 16.

[26] Newbery and Stiglitz (1981, pp. 108–109) summarize empirical estimates of CV(R).

[27] In the extreme case of (locally) unitary elastic demand, price and output are inversely proportional, so that revenue is to a first-order approximation nonstochastic; in this case the expression in Equation (17) is indeterminate.

premia need be small.[28] Suppose that commodities A and B each have zero β's and have revenues $R^A = 10 \times R^B$ in each state of nature. Then commodity A has a revenue variance that is 100 times as great as that of B, but its coefficient of variation is the same, and so it will have approximately the same premium. It is not the magnitude of risk, but the variability relative to the mean, that counts. So the fact that wheat is a more important crop than rye does not give any reason to expect that hedging effects on the wheat risk premium should be greater than for the rye premium.

The reason that relative rather than absolute risk matters may be linked heuristically to Proposition 3, in which the hedging component of the premium was proportional to σ_π by relating σ_π, the dispersion of futures price changes, to revenue dispersion. Since

$$\tilde{\pi} = \frac{P - f}{f} = \frac{Q(P - f)}{Qf}$$

if we treated Q as virtually constant (which would still be consistent with variable price with highly inelastic demand), the random term in the numerator would essentially be aggregate revenue shifted by a constant. The denominator, Qf, is "similar" to expected revenue, although the futures price f is not exactly equal to the expected spot price. In rough terms this suggests that it is the standard deviation of revenue variability relative to its mean, rather than total revenue variability, that matters for the premium.[29]

For a given coefficient of variation, it may be argued that it is actually the *smaller* crop that will have the higher premium (despite its lower variance of aggregate revenue), for an unimportant commodity is less likely to be traded on a highly liquid futures market. If the smaller crop is less familiar to investors or harder to learn to trade, then it will command the higher premium. So it may not be surprising to find that the rice and sunflower futures markets have large risk premia [Gordon (1984)].

4. Conclusion

When individuals are endowed with nonmarketable assets, the distribution of a security's market-model error will influence risk premia. For commodity futures, the imperfectly marketable revenue risks of producers lead to a futures price bias that deviates from proportionality to stock market beta. Setup costs or informational barriers that reduce participation by speculators on the futures market, by limiting the number of riskbearers, increase the impact of hedging on the risk premium (Proposition 3).

When some but not all potential outside speculators trade, the premium

[28] This observation is equally valid when risk arises from demand rather than supply shocks. For a demand curve of the form $P = \tilde{\delta} g(Q)$ where only $\tilde{\delta}$ is stochastic, the premium may be expressed exactly as $\pi = \beta_{\pi m}\bar{R}_m \pm \sqrt{2\alpha t(1 - \rho^2)}\mathrm{CV}^f(R)$.

[29] With quantity risk (which tends to offset price variability in determining revenue), demand elasticity will also come into play in relating price variability to revenue variability.

must compensate the marginal speculator both for the incremental systematic and residual risk that holding a futures position imposes. As in the traditional CAPM, the systematic component of the return on the futures contract receives a premium linear in its beta (Proposition 3), for the speculator can trade away the systematic component of the futures return by varying his holdings in the stock market. This contrasts with the predictions of previous writers that transaction costs will cause total security variance to be priced.

Furthermore, the premium for bearing residual risk is proportional not to its variance, but to its standard deviation. This is because when residual variance is large, speculators are able to protect themselves by holding smaller futures positions, so the premium they demand grows less than in proportion to residual variance (Proposition 3). In addition, the risk premium rises with (the square root of) the setup cost of trading, which is consistent with previous empirical findings of bias on thin markets.

Surprisingly, even though the model is rooted in the Mayers CAPM, the hedging component of the risk premium is entirely determined by the dispersion of residual returns; given the dispersion, the premium is unrelated to the covariance of return with nonmarketable risks. This is because residual dispersion of return measures the cost to speculators of holding the futures contract. In consequence, to test the model, it is not necessary to measure the return on the market portfolio of all invested wealth, but only the returns on traded securities.

The two main branches of the futures pricing literature differ in the degree to which they stress the influence of hedging by producers on the futures risk premium, as compared with stock market risk. An argument of Telser suggests that because the size of the agricultural risk to be hedged is small relative to the number of potential outside riskbearers, the component of bias induced by producer hedging should be small.

The model demonstrates (Proposition 4) that the percentage risk premium depends not on the size of the risk to be hedged (i.e., aggregate revenue variance), but on the variability of revenue relative to its mean (i.e., the coefficient of variation). When it is costly for outsiders to trade futures, a crop which is small compared to the economy as a whole and which is uncorrelated with the stock market may still have a large premium if its coefficient of variation is high.

Several variables suggested here may be used for empirical tests of nonbeta influences on the risk premium: residual standard deviation, fixed setup costs (proxied by thinness of the market), the square root of aboluterisk aversion, demand elasticity, and the coefficient of variation of producers' revenues. One of the more interesting questions raised by this article is whether the influence of residual risk on the premium may be expected to operate in financial futures or options markets. For example, what, if any, premium results from hedging on foreign exchange markets or from hedging in CPI, municipal bond index, and over-the-counter index futures? If these instruments are correlated with some otherwise poorly

marketable risks, then their prices should reflect the need for hedgers to induce marginal speculators to participate. These and other possible extensions remain for future theoretical and empirical work.

Appendix

Proof of Proposition 3

First, we rule out the case $\mathrm{cov}(r, P) = 0$, in which no speculator would trade the futures contract. Solving jointly the optimality conditions (5) and (6) for ξ^n and S^n gives

$$\xi^n = \frac{\Pi - \beta_{\Pi m}\bar{R}_m}{\alpha\sigma_\Pi^2(1-\rho^2)},$$
$$S^n = \frac{\bar{R}_m - \beta_{m\Pi}\Pi}{\alpha\sigma_m^2(1-\rho^2)}, \tag{19}$$

where $\beta_{\Pi m}$, $\beta_{m\Pi}$ are regression coefficients, and ρ is the correlation of R_m and Π. The first-order conditions for the simpler problem of the nontrading speculator, for whom we constrain $\xi^r = 0$, give

$$S^r = \frac{\bar{R}_m}{\alpha\sigma_m^2}. \tag{20}$$

Condition (13), by (12), implies

$$0 = -t + \Pi\,\xi^n + (S^n - S^r)\bar{R}_m - \frac{\alpha(\xi^n)^2\sigma_\Pi^2}{2} - \alpha\xi^n S^n\sigma_{\Pi m} - \frac{\alpha[(S^n)^2 - (S^r)^2]\sigma_m^2}{2}. \tag{21}$$

To substitute optimal values into condition (21), it is convenient to note that

$$S^n - S^r = \frac{\rho^2\bar{R}_m - \beta_{m\Pi}\Pi}{\alpha\sigma_m^2(1-\rho^2)},$$
$$(S^n)^2 - (S^r)^2 = \frac{(2\rho^2 - \rho^4)\bar{R}_m^2 - 2\beta_{m\Pi}\bar{R}_m\Pi + \beta_{m\Pi}^2\Pi^2}{\alpha^2\sigma_m^4(1-\rho^2)^2}. \tag{22}$$

Substituting into (21) gives

$$0 = -\alpha t + \frac{\Pi^2 - \beta_{\Pi m}\bar{R}_m\Pi}{\sigma_\Pi^2(1-\rho^2)} + \frac{\rho^2\bar{R}_m^2 - \beta_{m\Pi}\Pi\bar{R}_m}{\sigma_m^2(1-\rho^2)} - \frac{\Pi^2 - 2\beta_{\Pi m}\bar{R}_m\Pi + \beta_{\Pi m}^2\bar{R}_m^2}{2\sigma_\Pi^2(1-\rho^2)^2} - \sigma_{\Pi M}\left[\frac{\Pi\bar{R}_m - \beta_{\Pi m}\bar{R}_m^2 - \beta_{m\Pi}\Pi^2 + \rho^2\bar{R}_m\Pi}{\sigma_\Pi^2\sigma_m^2(1-\rho^2)^2}\right]$$

$$- \frac{(2\rho^2 - \rho^4)\bar{R}_m^2 - 2\beta_{m\Pi}\bar{R}_m\Pi + \beta_{m\Pi}^2\Pi^2}{2\sigma_m^2(1 - \rho^2)^2}. \tag{23}$$

With this step the worst is over. Equation (23) is a quadratic equation in the risk premium Π of the form

$$0 = a\Pi^2 + b\Pi + c. \tag{24}$$

With some algebraic manipulation, it can be shown that

$$a = \frac{1}{2\sigma_\Pi^2(1 - \rho^2)},$$

$$b = -\frac{\sigma_{\Pi m}\bar{R}_m}{\sigma_\Pi^2\sigma_m^2(1 - \rho^2)},$$

$$c = -\alpha t + \frac{\sigma_{\Pi m}^2\bar{R}_m^2}{2\sigma_\Pi^2\sigma_m^4(1 - \rho^2)}. \tag{25}$$

The proof is completed by applying the quadratic formula, and noting that the result stated in absolute terms is equivalent to the result expressed in percentage terms. ■

Proof of Proposition 4

By Equation (16),

$$\mathrm{var}(R) = \mathrm{var}(PQ) = \mathrm{var}[Qg(Q)]. \tag{26}$$

A Taylor expansion of $Qg(Q)$ about $\bar{Q}$ shows that

$$\mathrm{var}(R) = [\bar{Q}g'(\bar{Q}) + g(\bar{Q})]^2\sigma_Q^2 + o(\sigma_Q^2). \tag{27}$$

Similarly, a Taylor expansion of $g(Q)$ about $\bar{Q}$ shows that

$$\mathrm{var}(P) = [g'(\bar{Q})]^2\sigma_Q^2 + o(\sigma_Q^2). \tag{28}$$

Substituting for σ_Q^2 in Equation (27) from (28) gives

$$\mathrm{var}(R) = \left[\frac{\bar{Q}g'(\bar{Q}) + g(\bar{Q})}{g'(\bar{Q})}\right]^2 \mathrm{var}(P) + o(\sigma_Q^2), \tag{29}$$

so

$$\sigma_R = \bar{Q}|1 + \eta|\sigma_\Pi + o(\sigma_Q). \tag{30}$$

Dividing by f in Equation (30) and substituting for σ_π in Equation (14) gives

$$\pi = \beta_{\pi m}\bar{R}_m \pm \frac{\sqrt{2\alpha t(1 - \rho^2)}}{|1 + \eta|}\frac{\sigma_R}{f\bar{Q}} + o(\sigma_Q). \tag{31}$$

By definition of $\mathrm{CV}^f(R)$, and noting by Equations (28) and (30) that σ_R is of the same order as σ_Q, the first part of the proposition is proved.

For the case $\beta_{\pi m} = 0$, it remains to be seen whether $f\bar{Q}$ can be replaced with $\bar{R}$ in Equation (31). By definition of Π, $f = E(P) - \Pi$. So

$$f\bar{Q} = \bar{Q}E(P) - \bar{Q}\Pi. \tag{32}$$

This, combined with Equation (31), gives

$$|\pi| = \frac{K\sigma_R}{\bar{Q}E(P) - \bar{Q}\Pi} + o(\sigma_R), \tag{33}$$

where $K = \sqrt{2\alpha t}/|1 + \eta|$. By Equation (31), $\Pi = O(\sigma_R)$. Therefore, Equation (33) takes the form

$$|\pi| = K\frac{X}{Y - f(X)} + o(X), \tag{34}$$

where $X = \sigma_R$, $Y = \bar{Q}E(P)$, and $f(X) = \bar{Q}\Pi = O(X)$. A Taylor expansion in X about $X = 0$ gives

$$|\pi| = K\frac{X}{Y} + o(X) = K\frac{X}{\bar{Q}E(P)} + o(X). \tag{35}$$

Again, Taylor expansions about $\bar{Q}$ show that

$$\begin{aligned} \bar{R} = E(PQ) &= \bar{Q}g(\bar{Q}) + o(\sigma_Q), \\ E(P) &= g(\bar{Q}) + o(\sigma_Q), \end{aligned} \tag{36}$$

so that $\bar{Q}E(P) = \bar{R} + o(\sigma_R)$. Substituting into Equation (35) completes the proof. ■

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