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Risk, Futures Pricing, and the Organization of Production in Commodity Markets

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This paper examines equilibrium in a spot and futures market with both primary producers (growers) and intermediate producers (processors). For a commodity that is subject to output shocks, processors tend to hedge long, in contrast with Hicks's theory of futures hedging. Nevertheless, if transaction costs are low, the two-stage production process brings about a downward futures price bias, consistent with Hicks's pricing prediction. But if costs of trading futures are high, growers tend to be differentially driven from the futures market, reversing the direction of the bias. Futures trading may also affect the organization of industry; when demand is inelastic, futures trading can serve as a substitute for vertical integration as a means of diversifying risk because the risk positions of growers are complementary with those of processors.

I. Introduction

The classical literature on futures hedging and price determination, from Keynes (1927) and Hicks (1939) through Telser (1958) and Cootner (1960), focused on the futures trading decisions of processors or storage firms, to the exclusion of growers. The source of risk was viewed as variability either in the cost of purchasing the raw commodity input or else in the value of the output produced. Furthermore, the production level was taken as fixed so that revenue (or input cost) was assumed to be perfectly correlated with the price of the output (or input).

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Several more recent writers have included growers in their analyses (see McKinnon 1967; J. Hirshleifer 1977; Newbery and Stiglitz 1981; Britto 1984). This has permitted the analysis of futures hedging and equilibrium in which quantity risk and price risk jointly determine the overall revenue variability of producers. However, most of this literature, while properly assigning a role for growers, does not consider intermediate processors of the commodity.

The few exceptions, papers that cover the decisions of processors as well as growers, are all incomplete in important respects. O'Hara (1985) examines Hicks's theory but does not explicitly set out the hedging decision problem of producers and is not specific about the nature of the contracts examined.[1] Anderson and Danthine (1983) assume that the raw commodity is costlessly transformed into the finished product.[2] Baesel and Grant (1982) consider processors under three scenarios each of which is special in crucially limiting respects.[3] The current paper closes the model by making futures trading and spot purchases endogenous choices for outside speculators, for primary suppliers (growers), and for intermediate handlers with increasing marginal costs of processing the commodity.

The analysis here leads to a set of predictions that differs from those in both the classical literature and more recent papers on hedging and futures prices. In contrast with Hicks's theory of hedging, I show that processors tend to take long positions. However, as long as transaction costs are low, the presence of a second production stage promotes downward price bias, which is consistent with Hicks's pricing prediction. On the other hand, when costs of trading futures are high, growers tend to be differentially driven from the futures market, reversing the direction of futures price bias.

The classical writers, who focused their analyses on intermediate producers, were aware that most hedging on organized futures exchanges was done by intermediate handlers of the commodity (processors and storage firms), not growers (Paul, Heifner, and Helmuth 1976). However, one reason for low participation in futures markets by growers is that they engage in forward trading (individualized contracting between producers), which acts as a substitute for hedging on the exchange. Growers, storers, and processors of grain are

[1] O'Hara's pricing results do not apply to futures contracts, which are based on delivery of a fixed quantity of the commodity. Possibly the assertions of the paper refer to crop share contracts.

[2] They allow for possible spoilage of the commodity, but not for any other costly inputs.

[3] Under the first scenario, there is no futures market. In the second, processors are assumed to have nonrandom profit margins, so that there is no risk to be hedged. In the third, processors sell forward their product to unidentified third parties outside the model.

connected by a web of extensive forward trading.[4] To understand commodity market equilibrium, one must examine not just the futures market in isolation, but what is really a single joint futures/forward market.[5]

A second reason why many growers do not hedge on the organized exchange is that farms are small businesses, so that costs of learning how to use futures contracts may be a significant deterrent to trading futures. And of course, the direct transaction costs are a heavier burden on small rather than large traders, in both forward and futures contracting.

Rather than assuming either complete participation or complete absence of growers from the futures/forward market, this paper allows for the likelihood that for the reasons just stated some growers of at least some commodities may effectively remain unhedged. I demonstrate that the degree of participation in the futures market by different groups (speculators, growers, and processors) is an important determinant of how futures contracts are priced.[6]

A relatively unexplored topic is the extent to which financial risk-sharing markets affect the industrial organization of product markets. I show that the risks faced by processors versus growers may be complementary, which leads to a risk-reducing benefit to forward contracting or to vertical integration. This analysis casts light on whether futures trading can effectively serve as a substitute for vertical integration as a means of diversifying risk. The remainder of the paper is structured as follows. The economic setting is laid out in

[4] Paul et al. (1976) and Helmuth (1977) describe how storers and processors who contract forward with farmers typically hedge their commitments, by either futures trading or a forward contract with a buyer at the next level.

[5] However, futures and forward contracts are not *perfect* substitutes; among their differences is the daily resettlement ("marking to market") feature of futures contracts. A number of authors have shown that if daily interest rates are nonstochastic, then futures and forward prices must be identical (Cox, Ingersoll, and Ross 1981; Jarrow and Oldfield 1981). More generally, futures and forward prices should be very close since their differences are due to shifts in the timing of cash flows over periods of only a few months. Cornell and Reinganum (1981) and French (1983) found empirically that the differences between futures and forward prices for metals and foreign exchange were small and were not explained by models of the daily vs. terminal settlement features.

[6] Two significant criticisms of the traditional hedging pressure approach to futures pricing that I will not address here are that (1) it excludes risky assets other than a futures contract, ruling out the diversification effects underlying the capital asset pricing model, and (2) with many outside risk bearers, the impact of hedging on prices should be small (Telser 1958). In a hedging pressure/capital asset pricing hybrid model (D. Hirshleifer 1988), I have argued that even in a large capital market, hedging by undiversified producers can influence futures prices, an influence that is strengthened by nonparticipation by outsiders in the futures market. In such a setting, the futures price bias depends both on hedging pressure effects and on the "stock market risk" of the futures contract (see also Stoll 1979).

Section II. Market equilibrium is determined in Section III, which contains all the paper's results. Section IV concludes the paper.

II. The Economic Setting

There are four types of individuals: (1) pure consumers (who are involved neither in supplying the good nor in trading futures), (2) outside speculators (who are not suppliers, but who trade in futures), (3) growers (primary suppliers of the raw commodity Y), and (4) processors (intermediate suppliers of the final good Z). All markets are assumed to be competitive, and beliefs about distributions are all agreed and match the corresponding actual distributions. A tilde will be used to stress that a variable is random as viewed from date 0.

The pure consumers enter only the terminal (date 1) spot market for the finished commodity Z. Since there is no need to model their optimizing decisions directly, they are represented only by the market demand curve

$$\tilde{Q} = \delta(\tilde{P}^Z)^\eta, \tag{1}$$

where $\tilde{Q}$ is aggregate demand for the finished good, $\tilde{P}^Z$ is its spot price at date 1, and η is demand price elasticity.

The optimizing decision for the "traders" (a term reserved here for the three classes of individuals other than pure consumers) involves maximizing a mean-variance utility function[7]

$$U \equiv E(\tilde{c}) - \frac{\alpha}{2}\operatorname{var}(\tilde{c}), \tag{2}$$

where $\tilde{c}$ is terminal (date 1) consumption, and α is absolute risk aversion, the same for all traders.

Trading occurs at two dates in the model. The sequence of events is as follows. The primary production (planting) decisions have been made before the analysis proper begins, but the actual output of Y is a stochastic variable whose realization will not be known until date 1. At date 0 the futures market[8] opens, generating a futures price P^0 for the raw commodity Y as a result of the trading decisions of the growers, processors, and speculators. At the final date 1 the following events occur: (i) the realization of the stochastic output of Y becomes universally known; (ii) deliveries and financial settlements are made

[7] This applies under the assumptions of normality and constant absolute risk aversion preferences. Similar results would apply without constant absolute risk aversion by use of Stein's lemma (Stein 1973) or without normality assuming quadratic utility.

[8] The term "futures" is to be interpreted as covering *forward* contracts or markets as well; in a two-date setting, the distinction between daily resettlement (futures contracts) and expiration date settlement (forward contracts) vanishes.

on any outstanding futures contracts; (iii) a spot market for the raw good Y opens, in which the intermediate processors purchase the entire realized output of Y at an equilibrium date 1 price $\tilde{P}^Y$; (iv) intermediate processing converts Y into finished good Z;[9] (v) the processed output is sold to consumers at the equilibrium final price $\tilde{P}^Z$.

The superscripts G, A, and S will identify growers, processors, and speculators, respectively. There are n_G, n_A, and n_S of each type of trader. For typical individuals in each category the initial wealth endowments, in terms of a numeraire commodity (say dollars) apart from whatever value attaches to their initial holdings or entitlements of Y or Z, are w^G, w^A, and w^S; wealths could differ within each group as well without affecting any results. In addition, the growers (and only they) are endowed with a risky distribution $\tilde{q}^G$ of the raw commodity Y. By convention, one unit of the raw good transforms to one unit of finished good. The typical processor therefore purchases the raw commodity and sells the finished commodity in necessarily equal amounts q^A. Finally, let net revenue from spot purchase or sale of Y or Z, as the case may be, be symbolized by $\tilde{R}^i$, where

$$\begin{aligned} \tilde{R}^S &\equiv 0, \\ \tilde{R}^G &= \tilde{P}^Y \tilde{q}^G, \\ \tilde{R}^A &= (\tilde{P}^Z - \tilde{P}^Y)\tilde{q}^A - f(\tilde{q}^A), \end{aligned} \tag{3}$$

and where $f(q^A)$ is the processing cost function, with $f', f'' > 0$.

For each group, let ξ^i ($i = G, A, S$) represent the size of the date 0 futures position, and let t^i be the fixed cost of trading.[10] Then the consumption constraints for the optimizing decisions of a grower, processor, or speculator all take the form[11]

[9] In a more realistic model, processing would take time, which would require a third date after the purchase of the raw good. The current simplified model captures some essential points that would be unaffected by the introduction of storage in the intermediate process as long as the process does not carry over into a second stochastic harvest. There has been considerable modeling of carryover across harvests (Newbery and Stiglitz 1982; Scheinkman and Schechtman 1983; Turnovsky 1983), a topic not addressed here. Because of serial interactions of risk between harvests, the problem of carryover calls for a multiperiod consumption framework.

[10] The fixed cost represents such one-time setup costs as learning about contracting procedures and establishing trading contacts, which are likely to be more important as deterrents to trading than explicit brokerage fees. A different interpretation is that the fixed cost represents the minimum investment in learning needed to avoid trading at an informational disadvantage in the futures market.

[11] The consumption constraint rules out sale of equity shares in the producer's business; in fact most U.S. farms and agricultural firms are closely held. The imperfect marketability of revenue risk leaves producers with an incentive to hedge using futures. Scale economies in going public, as well as moral hazard and adverse selection problems, may explain the limited equity issuance by small producers. Even in widely held firms, optimal contracts that impose risks on managers may provide an incentive to hedge the firm's risk using futures (Diamond and Verrecchia 1982).

$$\tilde{c} = \begin{cases} w - t + \tilde{R} + (\tilde{P}^Y - \tilde{P}^0)\xi & \text{if trade futures} \\ w + \tilde{R} & \text{otherwise.} \end{cases} \quad (4)$$

Growers are assumed to sell all their output at the raw good price $\tilde{P}^Y$ and the processors all theirs at the finished good price $\tilde{P}^Z$, with settlement of all futures contracts taking place by financial balancing rather than actual delivery and acceptance of the good. It will also be assumed that all the traders in aggregate are only a negligible factor on the demand side of the final-product market. Thus the aggregate demand for the finished good Z is assumed to be unaffected by the various parties' gains and losses in futures trading, as well as the realization of the output of the raw good Y.

III. Industry Structure, Market Participation, and Futures Pricing

A. *Spot Market Equilibrium*

Let us begin with equilibrium in the spot market for the raw and finished commodity at date 1 and then move backward to the futures hedging decisions and equilibrium at date 0. After output uncertainty has been resolved, consumption variance is zero, and the processor's production decision is to select $q^A(P^Y, P^Z)$ to maximize net revenue. The processing cost function in (3) is assumed to be quadratic, $f(q^A) = \gamma_0 q^A + (\gamma_1/2)(q^A)^2$, where $\gamma_0, \gamma_1 > 0$.[12]

The optimality condition price = marginal cost yields an optimal output of

$$q^A = \frac{P^Z - P^Y - \gamma_0}{\gamma_1}. \quad (5)$$

I assume that all the raw output is processed, so that aggregate demand by processors for the input is equal to the total quantity supplied by growers, $n_A q^A = Q$. So in equilibrium the spread between final and raw product price by (5) is

$$P^Z - P^Y = \gamma_0 + \gamma_1 \frac{Q}{n_A}. \quad (6)$$

Note that higher Q raises the spread between the raw and finished product price. Intuitively, a greater demand for processing services increases the wedge between input and output prices and, as will be shown below, the rents to processors.

[12] The quadratic cost function is especially tractable because it leads to linear first-order conditions, but similar results would hold more generally.

By (1), the equilibrium spot price for the finished good is

$$P^Z = \left(\frac{Q}{\delta}\right)^{1/\eta}, \tag{7}$$

which with (6) determines the raw product price

$$P^Y = \left(\frac{Q}{\delta}\right)^{1/\eta} - \gamma_0 - \frac{\gamma_1 Q}{n_A}. \tag{8}$$

Having described the spot market equilibrium at date 1, I will proceed backward to determine futures hedging choices at date 0 in the next subsection. Then, in Section III*C*, I will examine how futures prices are determined at date 0.

B. The Futures Hedging Decision

Let us now turn to the date 0 futures hedging problem for growers, speculators, and processors. The futures position is found by maximizing expected utility in (2) with respect to ξ subject to the upper equality in (4) for a grower, processor, or speculator. Expected utility becomes

$$\begin{aligned} U = w - t + E(\tilde{R}) + \xi E(\tilde{P}^Y - P^0) \\ - \frac{\alpha}{2}[\mathrm{var}(\tilde{R}) + \xi^2 \mathrm{var}(\tilde{P}^Y) + 2\xi \mathrm{cov}(\tilde{R}, \tilde{P}^Y)]. \end{aligned} \tag{9}$$

Let the futures price bias be defined as $B = P^0 - E(\tilde{P}^Y)$. A downward-biased futures price ($B < 0$) is often called "backwardation" and an upward bias "contango." Differentiation yields an optimal futures position for any of these types of traders if he takes a futures position at all, which takes the form

$$\xi = -\frac{\mathrm{cov}(\tilde{R}, \tilde{P}^Y - P^0) + (B/\alpha)}{\mathrm{var}(\tilde{P}^Y)}. \tag{10}$$

The decision whether or not to trade futures is made by comparing the expected utility that arises from either alternative (see [14] below).

For a speculator the covariance term vanishes, so that ξ^S is positive or negative according to the sign of the futures price bias B. In the absence of covariation between profits and the futures payoff, the futures contract cannot be used to hedge, so the optimal position will be long or short depending on whether a long position in the commodity generates an expected profit or loss.

Rolfo (1980) and Newbery and Stiglitz (1981) have pointed out that in mean-variance hedging problems, the optimal futures position contains two components, one for risk reduction and one to exploit the expected profit that can be achieved when bias is nonzero (the first and second terms in the numerator of [10]). If B were zero (unbiased futures prices), there would be no expected profit to either a long or short position, so only the risk reduction component would remain. In this case, the direction in which processors hedge is given by the sign of the covariance term, which is found by seeing how R^A and P^Y covary across aggregate output states. By (5) and (6)

$$\frac{dR^A}{dQ} = \frac{\gamma_1 Q}{(n_A)^2} > 0. \tag{11}$$

Thus net revenue increases with aggregate output, so that processors do best when spot prices are low. By (8), P^Y declines with output. Therefore, R^A and $P^Y - P^0$ are inversely ordered in the sense of Hardy, Littlewood, and Pólya (1952) (as one goes up, the other goes down), so $\text{cov}(\tilde{R}^A, \tilde{P}^Y - P^0) < 0$. It follows by (10) that as long as bias is nonpositive, processors hedge long, in contrast with Hicks's theory and O'Hara's further development of it.

PROPOSITION 1. When output is stochastic and there is a positive marginal cost of processing the commodity for resale, if bias is not upward ($B \leq 0$), processors take long futures positions.

Processors tend to hedge long because their net revenues (and also their revenues gross of processing costs, $[\tilde{P}^Z - \tilde{P}^Y]\tilde{q}^A$) are highest when output is high and price low. The specialized resources of processors are most valuable when the crop is plentiful because this is when the demand for their services is highest. So their profits are inversely related to the futures payoff, and a long hedge reduces risk.[13]

If demand elasticity were unitary, then by (1) gross revenue received by producers as a group, $\tilde{P}^Z\tilde{Q} = \delta$, would be a constant. Or if demand were inelastic ($\eta > -1$), by (7), revenue $\tilde{P}^Z\tilde{Q} = \delta^{-(1/\eta)}\tilde{Q}^{1+(1/\eta)}$ would decrease with $\tilde{Q}$. (Intuitively, when demand is inelastic, price fluctuations are large compared to output fluctuations in percentage terms, so that total revenues are high when output is low.) But by (6), multiplying on the left and right by $n_A\tilde{q}^A = \tilde{Q}$, we see that gross revenues to processors are *increasing* in $\tilde{Q}$. It follows in either case that gross revenue to growers is lowest when output is high, a complementary risk position. Recalling by (8) that $\tilde{P}^Y$ decreases with $\tilde{Q}$, we see that the covariance in (10) is positive, so that without a bias, growers hedge short.

[13] The hedging demand for futures by processors is akin to the "convenience yield" on storage described by Newbery and Stiglitz (1981, p. 196). They observe that a producer whose revenue covaries inversely with the spot price has an incentive to reduce risk by storing, i.e., a long spot position.

COROLLARY. If demand is inelastic or unitary elastic, the risk reduction component of each grower's optimal hedge is negative.

In models without processors, the risk reduction component of a typical grower's hedge is positive or negative according to demand elasticity and is zero for unitary elastic demand. Here, even with *unitary* elasticity, growers hedge short, so the presence of a second production stage promotes short hedging by growers. Basically, processors absorb a profit wedge that covaries negatively with the spot price, which would have accrued to growers if the good sprang from the earth in finished form.

A relatively unexplored topic is the relation of futures trading to industry structure (see, however, Carlton 1983). We have just seen that under inelastic or unitary elastic demand, growers and processors have complementary risk positions. In the absence of a futures market, there is an incentive to vertically integrate the negatively correlated payoffs of a grower and a processor. This could be done by combining the farm and processing assets under a single ownership or by forward contracting between the grower and the processor (for either fixed quantities or variable crop shares).[14] Opening a futures market introduces an alternative means by which these groups can transfer risk. It thereby encourages a greater degree of productive specialization. Thus organized futures trading can act as a substitute for vertical integration or share contracting as a means of diversifying risk.[15]

With sufficiently elastic demand, on the other hand, $P^Z Q$ will increase with output, so that growers also will do best when output is high. In this case risk positions are no longer complementary; futures trading, instead of substituting for vertical integration, promotes it. The ability of integrated producers to trade futures then mitigates the adverse effects of combining positively correlated risks.

C. *Futures Market Equilibrium*

Let us next examine equilibrium futures price bias as a predictor of the later spot price (B). The clearest baseline case is one in which demand elasticity is unitary. In a mean-variance model in which the commodity can be immediately transferred from primary producer

[14] Historically, farming cooperatives for wheat have owned a significant share of the U.S. wheat elevator business. Paul et al. (1976) describe the agreements of vegetable shippers and canners with farmers deducting the packing and processing costs from the proceeds of sales and then giving the residual return to the grower.

[15] It is interesting that the rise of the oil futures markets occurred in the 1970s, when the rise of OPEC segregated the production stages of oil extraction from refining in what had been a highly vertically integrated industry.

to consumer without processing, bias is upward or downward according to demand elasticity, so that unitary elasticity leads to zero bias.[16] The intuition is very simple. With unitary demand elasticity, price is inversely proportional to aggregate output, so for a typical grower revenue is nonrandom. This eliminates any hedging pressure effect on the futures price. Let us turn to a commodity that must be processed for final consumption. Superficially it might appear that the bias should now be upward since, as shown in the preceding subsection, processors have an incentive to hedge long.

We now find the equilibrium price bias, taking as given the number of traders of each type in the futures market. It is assumed initially that t^A and t^G are zero so that all producers trade futures. But speculators may be deterred from participating fully by transaction cost $t^S > 0$ so that $\hat{n}_S \leq n_S$ actually trade.[17] Later I allow for the possibility that producers are also deterred by the trading cost. The bias in the futures price may be found by employing the market-clearing condition that the individual futures positions sum to zero:

$$0 = n_A \xi^A + n_G \xi^G + \hat{n}_S \xi^S. \tag{12}$$

Substituting the optimal futures positions ξ^i for the different traders from (10) using (3) and noting that, with none of the commodity wasted, $n_A q^A = n_G q^G$ gives[18]

$$B = -\frac{\alpha}{\hat{n}_S + n_A + n_G} \operatorname{cov}\left[\tilde{P}^Z \tilde{Q} - n_A f\left(\frac{\tilde{Q}}{n_A}\right), \tilde{P}^Y - P^0\right]. \tag{13}$$

The direction of the bias is determined by the sign of the covariance. In the case of unitary demand elasticity for the finished good, the product $P^Z Q$ is nonstochastic. Since $f(Q/n_A)$ is increasing in Q and P^Y is decreasing, by similar ordering the covariance is positive. It follows that $B < 0$ (downward bias) and, by proposition 1, processors hedge long. In other words, two-stage production leads to *backwardation*, not contango!

If demand is inelastic, the futures price is still downward biased because if $\eta > -1$, by (7) $P^Z Q$ decreases with Q, so that the two arguments of the covariance are still similarly ordered. Note also that

[16] Single-stage models relating bias to demand elasticity include Britto (1984) and D. Hirshleifer (1988).

[17] This asymmetric assumption is meant to reflect the fact that while futures or forward trading is common among producers, only a small minority of outside investors trade commodity futures, either directly or through financial intermediaries such as futures mutual funds.

[18] If different traders have different risk aversions (α), then the same equation obtains with α replaced by the harmonic mean of the α^i's of the traders on the futures market.

since $B < 0$, speculators are long by (10), so by futures market clearing (12), growers are short. This shows the following proposition.

PROPOSITION 2. When demand for the finished good is inelastic or unitary elastic and there is a positive marginal cost of processing the commodity for resale, then processors hedge long, growers hedge short, and the futures price is downward biased.

It is remarkable that even though processors are long in futures, the reverse of Hicks's predicted hedge, the necessity of processing the commodity brings about a downward bias, confirming his conclusion about price. This seeming anomaly arises because the activities of processors affect the risks and hedging choices of growers. The second production stage leads growers to go short by an amount that more than offsets the long positions of processors.

This may be seen more clearly by considering the profits of a vertically integrated producer who both grows and processes the commodity. The gross revenue he receives from consumers with unitary demand elasticity is nonrandom because price and quantity move in exact inverse proportion. But the total processing cost incurred rises with output, so net profit decreases with output. So a vertically integrated producer's profit covaries positively with the spot price, which implies that a short hedge is risk reducing. To induce outsiders to take on risky long positions, a downward bias (backwardation) is called for.

Combining propositions 1 and 2, we have shown that processors hedge long because their net revenues covary negatively with the spot price. Yet as argued above, a vertically integrated producer's revenues would covary positively with the spot price. It follows that growers' revenues covary positively with the spot price. (The covariance would be zero if the commodity could be consumed without a second stage of production.) So for growers short hedging is optimal. The growers' incentive to hedge short outweighs the impact of the long-hedging activity of processors, and hence, the futures price is downward biased.

This contrasts with Anderson and Danthine's (1983) prediction, for which the second production stage was essentially irrelevant. They found upward or downward bias according to demand elasticity,[19] a result that, as mentioned at the start of this subsection, obtains equally in a framework with a single production stage. The discrepancy be-

[19] This refers to their "input flexibility case." In their inflexible input case, futures positions are taken after the production level of processors has been committed and output is nonrandom. They find that bias is signed according to the expected value of an exogenous shock to the demand for the raw commodity by intermediate firms. These trades are determined outside the model, so their analysis does not indicate in which direction the bias would ordinarily go.

tween their result and that of this paper arises from their assumption that the marginal dollar cost of processing the commodity is zero, whereas here it is positive. The effect of demand elasticity is still reflected here in the revenue ($P^Z Q$) term of (13), but here there is also a subtracted term (f) reflecting the cost of production.

Nonparticipation by Growers

Let us now consider how $\hat{n}_S$, $\hat{n}_A$, and $\hat{n}_G$ are determined in equilibrium to see if differences in participation by different kinds of producers are likely to affect the pricing of futures contracts. A speculator or producer's decision whether to trade in the futures market is based on whether his expected utility is higher from trading or refraining. Let $V(w$; refrain) be the expected utility of an individual with wealth w who behaves optimally in his production decisions (if he is a grower or processor) and does not trade in the futures market. Let $V(w - t$; participate) similarly denote the expected utility attained by trading optimally in the futures market. If all speculators face the same transaction cost t^S, then the number of speculators $\hat{n}_S$ is determined by an indifference condition that

$$V^S(w - t^S; \text{participate}) = V^S(w; \text{refrain}), \tag{14}$$

where the S superscripts denote speculators.[20] When $t^G, t^A > 0$, similar indifference conditions determine $\hat{n}_A$ and $\hat{n}_G$.

Suppose for simplicity now that the transaction cost is the same for all three groups, $t^A = t^G = t^S > 0$. Without detailed formal analysis, we can make an intuitively reasonable statement about which types of producers will tend to participate more or less. A hedger who by trading futures attains little risk reduction is less willing to pay a given fixed cost of trading. So (with the correlation of the hedger's revenue with the futures payoff held constant) a producer with a large revenue variance to be hedged is less likely to be deterred by a given fixed transaction cost than one with only a small risk.

Many processors are large-scale enterprises, unlike most growers. (Some economic reasons are mentioned by Newbery and Stiglitz [1981, p. 197].) For example, 52 percent of U.S. milling capacity is accounted for by the top four flour-milling firms (Goldberg 1983). This suggests that just as more speculators than growers are driven from the futures market by transaction costs,[21] so more growers than

[20] If individuals have different transaction costs, then this condition will obtain only for the marginal speculator.

[21] In a perfect market, very many small speculators would hold a fraction of the futures contract to profit off of any bias, however small. A small transaction cost will suffice to deter most of them.

processors will be deterred from trading futures. Suppose that only $\hat{n}_G < n_G$ growers and $\hat{n}_A < n_A$ of the processors trade futures. If $\hat{n}_G$ is near zero and $\hat{n}_A$ is near n_A, then while many short-hedging growers are driven out, the long-hedging processors for the most part remain in the market.

The next proposition follows immediately.

PROPOSITION 3. With unitary elastic demand and a sufficiently large transaction cost that deters growers rather than processors from the futures market, the futures price is upward biased.

Proof. Let $g \equiv \hat{n}_G/(\hat{n}_G + \hat{n}_A + \hat{n}_S)$ and $a \equiv (\hat{n}_A/\hat{n}_G)g$. Then following steps analogous to those leading to (13) gives

$$B = -\alpha[a \operatorname{cov}(\tilde{R}^A, \tilde{P}^Y - P^0) + g \operatorname{cov}(\tilde{R}^G, \tilde{P}^Y - P^0)]. \qquad (15)$$

We have already seen by (11) that the first covariance is negative. By (8)

$$\frac{dR^G}{dQ} = \frac{1}{n_G}\frac{dP^YQ}{dQ} = -\frac{1}{n_G}\left(\gamma_0 + \frac{2\gamma_1 Q}{n_A}\right) < 0, \qquad (16)$$

so the second covariance is positive (indicating the tendency for growers to hedge short). If transactions costs lead $\hat{n}_G$ to be sufficiently small relative to $\hat{n}_A$, then the right-hand side of (15) will be positive, implying a positive instead of a negative bias. Q.E.D.

More generally, with inelastic demand a lack of participation by growers will tend to algebraically increase the bias, although it need not be upward. The intuition of proposition 3 is that the transaction cost affects the relative importance of hedging by growers versus processors, who have negatively correlated risks. We saw above that processors have a long hedging incentive (promoting upward bias) whereas growers have a short hedging incentive (promoting downward bias). So a sufficiently large transaction cost will lead to an upward-biased futures price.

The assumption that transaction costs drive growers from the futures market is in the spirit of the classical theorists. Yet in this case, the result here of upward bias (contango) is the opposite of the downward bias (normal backwardation) their discussions predicted. This highlights the importance of modeling two-sided uncertainty with both price and quantity risk to fully describe equilibrium price determination.

It is worth stressing that the current analysis has focused on output shocks while assuming demand to be constant. For most agricultural commodities, output shocks are probably the dominant source of price variability. But when demand rather than supply is subject to shocks, hedging pressure will typically promote downward bias because high demand will be good news for both growers and processors. This leads to a short hedging incentive, tending to reduce the futures price. So a possible interpretation of Hicks's backwardation theory is that it was intended for markets with stable output and stochastic demand. More generally, the tendency toward upward bias suggested here will tend to be muted if demand as well as output is stochastic.

IV. Conclusion

This paper has examined how futures contracts are priced for commodities whose production is associated with quantity risk and that must be processed for final resale. The model allows for uncertainty in both the cost of purchasing the raw commodity and the price at which it can be sold. Furthermore, the ability of processors to vary their level of production in response to the prices they face affects their overall risk. The paper also includes costs of processing the commodity, which were shown to be crucial if the inclusion of a second stage of production is to make a substantive difference in the model.

In the absence of transaction costs, intermediate producers hedge long, in contrast with Keynes's and Hicks's theories, but the bias in the futures price as a predictor of the later spot price is downward, confirming their pricing prediction. A lack of participation in the futures market by growers can reverse the traditional prediction, leading to upward bias, or contango.

When demand is inelastic or only mildly elastic, processors and growers have complementary risk positions. This suggests that for closely held producers, there is a risk diversification benefit to vertical integration or to forward contracting. However, futures trading can act as a substitute for vertical integration as a means of reducing risk. Thus not only does the nature of the production process affect futures pricing, but the presence or absence of a futures market can affect how production is optimally organized.

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