Risk, Managerial Effort, and Project Choice

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In our model risk-neutral shareholders need to motivate a manager to select among projects with different risks, and to work hard in implementing the chosen project. Curvature of the manager’s compensation contract as a function of profit affects his attitude toward project risk. The optimal curvature depends on the trade-off between controlling project risk and motivating effort. The analysis predicts greater option-based compensation when there are desirable risky growth opportunities (proxied by Tobin’s q or R&D expenditures) and less option compensation when there are effective monitoring institutions (such as outside directors and bank lenders). Journal of Economic Literature Classification Numbers: 522, 521. © 1992 Academic Press, Inc.

1. INTRODUCTION

Managers are important both for selecting projects and for implementing them. There has been little research on the relation between these different managerial roles. The principal objective of this paper is to examine the dual-agency problem of motivating managers first to select the appropriate project among a menu of alternatives with differing risks and then to work diligently and efficiently to implement the project.

With respect to project choice, a compensation scheme increasing in profits creates managerial incentives that are largely aligned with those of shareholders if projects have similar riskiness. However, it is harder to motivate a manager to make the right decisions when projects differ in risk. Amihud and Lev (1981) theorized that managers with undiversified

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human capital may be more conservative than shareholders would like
and provided evidence suggesting that conglomerate mergers may be un-
dertaken by managers to reduce risk. Greenwald and Stiglitz (1990) argue
that asymmetric information between suppliers of capital and managers
leads to risk-averse behavior by firms,¹ and that this explains numerous
stylized facts which are not consistent with the traditional neoclassical
model. Hirshleifer and Thakor (1992) show that managers will avoid
projects that are subject to early and conspicuous failure in order to
maintain their reputations as good judges of project quality.

Given these problems, an important issue is the extent to which com-
penation contracts can be designed to motivate effective project choice.²
An endogenously chosen compensation scheme can give managers per-
fomance-based compensation such as incentive stock options, stock ap-
preciation rights, performance share/unit plans, and phantom stock plans
to induce any desired levels of risk tolerance. In practice, incentive com-
pensation is widely used and large in magnitude.³

Several papers examine this topic. Ross (1974) was the first to discuss
the effect of contract curvature on the choice among risky projects, al-
though his main focus was on risk sharing in the absence of moral hazard.
Lambert (1986) examines the problem of motivating managers to devote
effort to investigation of alternative projects and, having investigated, to
selecting the right project when project choice is ex post observable to the
principal. Campbell et al. (1989) examine the adverse selection problem
that arises from managerial mobility when the manager has private infor-
mation about project quality and his own ability. Noe and Rebello (1991)
analyze managers' risk incentives in relation to the degree of managerial
entrenchment and whether the firm is in financial distress. Brander and
Poitevin (1991) analyze how compensation contracts that motivate the
choice of low-risk projects can help resolve debt/equity conflicts when
there is no problem of motivating effort.

In these papers, once the project is initiated, its payoff distribution is
given. The authors therefore do not examine the problem of motivating
the manager to exert costly effort to increase the return of the selected

¹ They also discuss the possibility that managerial contracts lead to risk aversion.
² Holmstrom and Ricart i Costa (1986) examine the managerial decision of whether to
undertake a project of given riskiness, in a setting where the contract is contingent on the
choice of whether to initiate, though not on the final outcome. Banker et al. (1989) examine
the decision of whether to initiate a project when the contract can depend on the outcome.
³ For example, in 1989, August Busch III of Anheuser Busch Co. received $7.4 million
from exercise of options, in comparison with $1.5 million in salary and bonus. The article
"Pay for Performance" (Wall Street Journal 4/15/90 R7-8) describes several other examples
of large stock and option compensation and the growing trend toward "megagrants," such
as Disney Chairman Michael Eisner's option to buy 2 million shares with value of $142
million.
project. In reality, managerial effort is crucial for maintaining efficiency and incentives in ongoing projects. Such effort can be extremely demanding, and managers face a serious trade-off between work and such other goods as leisure, health, and family life.

The problem of designing compensation schemes to motivate managerial effort has been examined by Holmstrom (1979), Harris and Raviv (1979), and Grossman and Hart (1983). These papers show that when the agent is risk averse, there is a trade-off between providing incentives and achieving a desirable distribution of risk bearing. To motivate the manager to work hard, it becomes necessary to force him to bear more risk than would be efficient if effort were observable.

In this paper, we examine the problem of simultaneously motivating managers to select the right project among alternatives with varying riskiness and to work hard in implementing the project. According to the standard principal-agent theory, an effort-motivating contract will impose risk on risk-averse managers. One might therefore expect managers to be too conservative in selecting projects from the point of view of risk-neutral owners. We show that the optimal contract can induce the manager to choose projects that are either too conservative or too risky from the owners' point of view. Even though compensation contracts can be written to regulate the project decision of the manager, since the compensation contract must deal with another moral hazard problem as well, it is not optimal to control the project choice perfectly. The insufficient or excessive risk chosen by the manager is a predictable though undesirable by-product of a payoff scheme that is needed to motivate the manager to work hard.

More generally, we analyze the induced risk preference that arises from the combination of the manager's risk aversion and the curvature of the compensation contract as a function of outcome. The importance of induced risk preference is highlighted by the evidence of DeFusco et al. (1990) that the introduction of executive stock option compensation plans is on average followed by an increase in implied option volatility. Furthermore, Agrawal and Mandelker (1987) found that managers of firms whose return variance rose after an acquisition had higher option (and stock) compensation than managers of firms whose return variance declined. Induced risk preference explains why the argument that managers will always be too conservative is invalid. When it is optimal to motivate effort through a convex compensation scheme, his risk-inducing compen-

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4 In more recent work, Kihlstrom and Matthews (1990) analyze the efficiency of competitive equilibrium under moral hazard.

5 Holmstrom (1982) mentioned that when there is a need to create work incentives, a manager's compensation scheme will depend on output in a way that will affect risk preferences. He did not, however, develop this theme.
sation scheme may overcome the manager's inherent risk aversion, leading to excessive risk-taking.

The risk preference of managers induced by managerial contracts causes the problems of motivating project choice and effective implementation of projects to interact in a number of interesting ways. First, a risk-neutral owner will, ceteris paribus, prefer a safe project because this reduces the noisiness of output as a signal of the manager's effort. If the owner can observe the project choice, he will force the manager to select a safer project, because this reduces the cost of motivating the manager to work hard in implementing the selected project.

Second, if the owner cannot observe the manager's choice from a menu of projects with equal expected return, then he may have to modify the contract to induce the manager to select a low-risk project. Let us refer to the functional relationship between profit and pay when project choice is observable as the OP (Observable Project) contract. It is possible that under the OP contract, compensation is a concave function of output, or only mildly convex (compared to the concavity of the manager's utility function). If so, then even if the project choice is unobservable, the same compensation scheme remains optimal, because under it the manager will prefer the safe project desired by the owner. However, if the OP contract is highly convex, the induced risk preference of the compensation scheme may more than offset the manager's inherent risk aversion, leading him to prefer a risky project. This can necessitate a contractual adjustment.

Third, even if the manager is inclined to favor high risk under the OP contract, the owner may not wish to modify the contract to control the project choice. If projects are of discretely different types, the owner may prefer to accept the contracting cost of a riskier project rather than to modify the contract to induce low risk. The potential benefit of the safe project is that the observed outcome becomes a more precise signal of effort. Nevertheless, inducing the manager to choose the safe project may render the contract more costly in motivating effort than a contract that permits a high-risk project to be chosen.

Fourth, if the OP contract is concave or is not sufficiently convex compared to the manager's risk aversion, then the manager may become overly cautious in his project choice. This problem will arise when there is a risk-return trade-off, so that a higher risk project has higher return than a lower-risk one. Under observable project choice, the owner may prefer

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6 A related idea is examined by Aron (1988), in which a conglomerate is formed in order to obtain multiple observations on the manager's effort choice.

7 In general, even if the OP contract is concave or not too convex, the interests of owner and manager are not necessarily fully aligned. If there is a positive risk-return trade-off in available projects, the manager may be either more or less averse to risk than the owner would prefer.
the higher-risk project despite the higher contracting costs associated with it. However, with an unobservable project, the OP contract could lead the manager to select the lower-risk project. Again, it may be optimal for the owner to accept the inferior project and establish the OP contract that would apply if the safe project alone were available, or it may be optimal to induce the higher-risk project, although the need to do so raises the contracting cost of motivating effort.

Fifth, when a desirable but risky growth opportunity is likely to be available to the firm, there is an incentive to offer the manager a convex contract to motivate him to exploit the opportunity when it appears. Thus, option-based compensation should be positively associated with such measures of growth opportunities as the price/earnings ratio, the price/book value ratio, or Tobin's $q$. Option-based compensation should also be associated with other proxies for growth opportunities, such as high R&D expenditures.

The remainder of the paper is as follows. Section 2 describes the basic economic setting. In Section 3, we examine the choice among projects with different risk but the same expected return. The cases of observable and unobservable projects are dealt with in Subsections 3.1 and 3.2, respectively. In Section 4, we allow for a risk–return trade-off. We examine cases in which there is no conflict of interest with regard to project choice (Subsection 4.1) and in which the owner prefers a different level of risk from that chosen by the manager (Subsection 4.2.). Section 5 examines the problem of motivating project choice when the owners are uncertain about the menu of projects available to the manager, in order to make predictions about the kind of firm that uses option-based compensation. Section 6 considers the effect of monitoring by large shareholders or banks on project choice. Section 7 concludes.

2. THE ECONOMIC SETTING

The firm has a single risk-neutral owner. The "owner" is interpreted as public shareholders, and his assumed risk neutrality reflects shareholders' ability to diversify idiosyncratic risk. Let $a$ be the manager's level of effort and $b$ be a second parameter chosen by the manager that affects the riskiness of the project (and also possibly its expected return) in a manner

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8 We do not analyze systematic risk in the sense of the Capital Asset Pricing Model or Arbitrage Pricing Theory. As shown by Diamond and Verrecchia (1982) and Ramakrishnan and Thakor (1984), optimal managerial contracts impose specific risk on managers in order to motivate them to work diligently. Diamond (1984) provides a model in which it is optimal to allow a financial intermediary to hedge away factor risks, leaving it only with idiosyncratic risk. The effect in our model of allowing systematic risk is discussed in the concluding remarks.
described further below, and let $\theta$ be the state of the world. Then output can be written as $X = X(a, b, \theta)$. We assume that the manager chooses $a$ and $b$ before observing $\theta$. The manager is both risk averse and effort averse. Thus, the manager's utility is assumed to be

$$H(S(X), a) = u(S(X)) - v(a),$$

where $S(X)$ is the value of the manager's compensation as a function of $X$. We assume that there is rising marginal disutility of effort, $u''(a) > 0$.

We examine cases in which project choice is either observable or unobservable. In the observable case, the owner can base compensation on the project that is chosen and therefore can force the manager's project choice by imposing a large penalty on the manager unless the desired project is chosen. In the unobservable case, it is impossible to write a contract based on the project choice. Even if the owner can perceive the nature of the selected project, if the properties of different projects are difficult to specify succinctly, it may be impossible to base a contract explicitly on the project choice.\footnote{Harris and Raviv (1979) analyzed the effects of action choice occurring after an agent observes private information.}

### 2.1. The Owner's Decision when Project Choice Is Observable

We first describe the decision problem of an owner who can observe project choice $b$ but not effort $a$. The owner's maximization problem is constrained by the condition that the manager obtains at least his reservation utility by agreeing to the contract and the condition that the effort $a$ be optimal for the manager given the compensation scheme. Given that the owner chooses to motivate a project choice of $b$, the optimal contract in the observable project case, which we call $OP(b)$ contract, solves the problem

$$\max_{a, S(X)} E[X - S(X)|a, b]$$

such that $E[u(S(X))|a, b] - v(a) \geq \bar{H}$,

$$a \in \arg\max_{a'} E[u(S(X))|a', b] - v(a'),$$

where $\bar{H}$ represents the minimum utility offered in the managerial labor market. Suppose that output $X$ is a continuous random variable, where

\footnote{For sufficiently important projects, the owner can insist that they be submitted for his approval. However, this defeats the purpose of delegating authority to a manager. Since any managerial decision can be regarded as a "project," in the limit requiring ratification by the owner of all projects is equivalent to having an owner-managed firm.}
\( f(X) \) denotes its probability density. Then if the first-order approach for the agent's action choice is valid, the second constraint becomes

\[
\int u(S(X)) f_a(X|a, b) dX = v'(a).
\]

In this case, the first-order condition for the optimal contract is

\[
\frac{1}{u'(S(X))} = \lambda + \mu \frac{f_a}{f}
\]

for each \( X \). Here \( \lambda > 0 \) and \( \mu > 0 \) are Lagrangian multipliers corresponding to the two constraints which do not depend on \( X \).

2.2. The Owner's Decision when Project Choice Is Unobservable

We next describe the decision problem of an owner who can observe neither the project choice \( b \) nor the effort \( a \). This adds a third constraint, that the manager's choice of \( b \) be optimal given the compensation scheme. Given that the owner wishes to motivate a project choice of \( b \), the optimal contract for \( b \) when project choice is unobservable, which we call the \( UP(b) \) contract, solves the problem

\[
\max_{a, b, S(X)} E[X - S(X)|a, b]
\]

such that

\[
E[u(S(X))|a, b] - v(a) \geq H
\]

\[
a \in \arg\max_{a'} E[u(S(X))|a', b] - v(a')
\]

\[
b \in \arg\max_{b'} E[u(S(X))|a, b'] - v(a).
\]

3. NO RISK-RETURN TRADE-OFF

To focus on the pure effect of project riskiness, we first examine the case in which all alternatives have the same expected return. In the next subsection, we show that when the project choice is observable, the risk-neutral owner will prefer the low-risk project in order to monitor most efficiently the manager's implementation of the project. Then, in Subsection 3.2, we show that the unobservability of project choice will force the owner either to modify the contract or to accept that the manager will select an excessively high level of risk.

Throughout this section, we assume that the set of available projects is such that \( b \in [b, \bar{b}] \). We also assume that output is

\[
X(a, b, \theta) = a + b\theta, \quad E[\theta] = 0,
\]
so that a higher value of risk parameter $b$ increases the dispersion of possible outcomes.

3.1. Observable Project Choice

We first show that when the project choice $b$ is observed but effort $a$ is unobserved, the owner wants to minimize the project risk $b$. We then have

**Proposition 1.** Suppose that output is $X = a + b\theta$, with $\theta \sim N(0, 1)$. Then if project choice is observable (the OP case), the owner will motivate the choice of the lowest risk project, $b = b$.

The proof of this proposition is a direct extension of Proposition 2 in Kim and Suh (1991). The optimal solution for $b$ is $b$; i.e., the owner will require the manager to choose the least risky project, because a lower-risk project makes it easier to disentangle the manager’s effort $a$ from the noise in the project’s outcome.\(^1\)

The outcome of risk minimization may be sensitive to the additive separability between effort and risk in the output function $X = a + b\theta$. This implies that the sensitivity of output to the manager’s effort is independent of project risk. If instead, high risk were associated with greater sensitivity of outcome to effort, the owner might prefer a riskier project in order to elicit more information about the manager’s effort.\(^2\) It should be noted, however, that while sometimes high risk can increase effort sensitivity (because the circumstances are volatile, but still controllable), sometimes increasing risk can reduce effort sensitivity (because the factors outside the manager’s control render his actions moot in many states of the world). The case of additive separability therefore provides a useful benchmark case.

Let us define the power, exponential, and logarithmic utility functions, respectively, as

\[
\begin{align*}
  u &= \frac{S^\alpha}{\alpha}, \quad \alpha < 1, \alpha \neq 0, \\
  u &= -e^{-rS}, \quad r > 0, \\
  u &= \ln(S).
\end{align*}
\]

\(^1\) If the compensation function is not restricted to a finite interval, the existence of an optimal solution to the principal-agent problem may not be guaranteed in the case of normally distributed output, as noted by Mirrlees (1974). Recently, Ziv (1990) showed that the solution to the principal-agent problem with compensation constrained to a finite interval converges to the unconstrained solution for a sufficiently high reservation utility of the agent.

\(^2\) This possibility can be brought out conjecturally with the following example. Suppose that $X = ab\theta^2 + e$, where $e$ is a random variable independent of $\theta$. Higher $b$ increases the sensitivity of $X$ to $a$ and increases risk. In so doing, higher $b$ should make it easier for the owner to attribute fluctuations in $X$ to high versus low level of effort ($a$) rather than $e$. 

The following proposition describes the degree of convexity or concavity of the optimal contract in terms of the manager's preferences. The curvature of the utility function is important because it determines the induced risk preference of the manager and thus how much risk-taking the OP contract will bring about when the project choice is unobservable.

**Proposition 2.** Suppose that the likelihood ratio \( f_o/f \) is linear in \( X \). Then

(i) if the manager has a power utility function, the optimal contract is strictly convex (concave) in \( X \) if \( \alpha > (\leq) 0 \), is strictly concave if the manager's utility function is exponential, and is linear in \( X \) if utility is logarithmic, and

(ii) if the manager has a power utility function, \( u(S(X)) \) is strictly convex in \( X \) if \( \alpha > \frac{1}{2} \), is linear in \( X \) if \( \alpha = \frac{1}{2} \), and is strictly concave if \( \alpha < \frac{1}{2} \). Furthermore, \( u(S(X)) \) is strictly concave for logarithmic and exponential utility functions.

**Proof.** See Appendix. ■

Both the likelihood ratio function and the preferences will affect the curvature of the optimal contract. In the above proposition, we have assumed that the likelihood ratio is linear in order to focus on the effects of different preferences. In some of the examples of the following sections we fix the preferences by examining the baseline case of square root utility while varying the production technology (i.e., the likelihood ratio function). In other examples we assume that the likelihood ratio function is linear and vary the utility function.

Intuitively, the effect of the likelihood ratio function \( f_o/f \) on the curvature of the compensation scheme is as follows. The likelihood ratio measures how strongly the owner can infer from output whether the manager has taken the assumed level of effort. If \( f_o/f \) is convex, the increase in the likelihood ratio is more pronounced as the outcome increases. Thus, it will be optimal for the owner to increase the manager's compensation more rapidly as the outcome increases. If \( f_o/f \) is concave, the argument is reversed, so the manager's compensation tends to increase less rapidly as the outcome increases.

Murphy (1986) has shown that the effect of the utility function on the curvature of the compensation scheme is related to the magnitude of absolute risk aversion and to whether absolute risk aversion is decreasing or increasing. His result is based on the assumption that the likelihood ratio \( f_o/f \) is linear in outcome \( X \). By the optimality condition (3), a linear

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13 Many well-known families of distributions such as the exponential, normal, gamma, and Poisson satisfy linearity of the likelihood ratio (see Banker and Datar, 1989).

14 In discrete examples, the analogous likelihood ratio is \( \frac{\Pr(X|a_H) - \Pr(X|a_L)}{\Pr(X|a_H)} \), where \( a_H \) is a higher and \( a_L \) is a lower effort, where \( a_H \) is the equilibrium effort.

15 Grossman and Hart (1983) provide a closely related result.
likelihood ratio implies that the optimal contract $S(X)$ will be convex in $X$ if and only if $1/u'$ is concave in $S$. The condition for convexity of the optimal contract therefore becomes

$$\frac{dr}{dS} + r^2 < 0,$$

where $r$ is the coefficient of absolute risk aversion. Thus, risk aversion per se promotes concavity, while decreasing risk-averse preferences promote convexity. To understand why, let us rewrite the optimality condition in (3) as

$$1 = u'(S(X)) \left[ \lambda + \mu \frac{f_a}{f} \right]$$

for each $X$. The left-hand side is the marginal disutility to the owner of raising management compensation by $1$ for each outcome $X$. The right-hand side is the marginal benefit to the owner of increasing management compensation by $1$, which arises from motivation of higher effort by the manager. This is the sum of two determinants of the manager's incentive to sacrifice effort. The first, $u'\lambda$, is the marginal (shadow) benefit to the owner that an increase in management compensation has in satisfying the manager's minimum utility constraint. The second, $u'\mu f_a/f$, is the marginal effect of an increase in effort on the owner's likelihood of attaining the outcome $X$, leading to wealth of $S(X)$. Since $f_a/f$ is assumed to increase linearly with $X$, an increase in $X$, which raises the marginal effect of effort on the likelihood ratio, must be offset at the optimum by a corresponding decrease in the marginal utility from extra wealth. Given risk aversion ($u''(S) < 0$), the manager's pay $S(X)$ must rise.

Now compare a small increase in $X$ starting from a low level versus a high level. To focus on the direct effect of risk aversion, we begin by assuming constant absolute risk aversion, denoted by $r$. Let us call the bracketed term above the "motivation coefficient" of the manager (which reflects motivating him both to work and to participate in the contract). Recall that constant absolute risk aversion means that utility curvature is constant; i.e., marginal utility declines by a constant percentage as wealth is increased by a given amount. It follows that if management compensation rose linearly with $X$, then marginal utility would fall at a constant percentage rate with $X$. But this would not equate marginal benefit to marginal cost because the motivation coefficient rises only linearly with

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16 This is a constant because of the assumed risk neutrality of the owner. If the owner is risk averse, then by arguments similar to that provided here, the intensity of the owner's risk aversion will also promote concavity, while a downward slope of the owner's risk aversion as a function of his wealth will promote convexity.
As $X$ increases, if $S$ were to grow linearly with $X$, eventually the cost to the owner of raising $S$ would exceed the benefit. Thus, $S$ must be concave. Furthermore, since marginal utility declines more rapidly with wealth when risk aversion is high, greater risk aversion intensifies the need to restrain management compensation increases at higher levels of output, strengthening the tendency toward a concave contract.

To understand the effect of decreasing risk aversion, recall the classical trade-off in agency models between motivating the manager to work and reaching a desirable allocation of risk between the owner and the manager. Decreasing risk aversion implies that when the shock $\theta$ is large, the manager is relatively wealthy, so that he should become less risk averse. In the region of high outcomes, it therefore becomes mutually profitable to motivate the manager by imposing greater risk on him. Thus, decreasing risk aversion tends to bring about a contract that becomes steeper as outcome increases. In other words, decreasing risk aversion leads to greater convexity.

3.2. **Unobservable Project Choice**

In the preceding subsection, we showed that the owner prefers lower risk ($b$) in order to improve monitoring of the manager's performance. When project choice is unobservable, he may have to adjust the OP contract in order to achieve low $b$. Since such a change in the contract will raise the cost of motivating effort, it is not obvious that the owner will find it optimal to motivate the manager to select $b$.

We show that, for some parameter values, the owner can motivate the choice of the low-risk project and a corresponding level of effort in implementing it with the same contract as that in the case of observable project choice. For other parameter values, the owner accepts a more costly motivation of effort by the manager than if the project choice had been observable. This higher cost may arise because a riskier project is chosen, so that the output upon which compensation is based is a less accurate measure of effort. Alternatively, the owner may modify the contract to motivate choice of the low-risk project. Since the OP contract was optimal given observability, such a modification forced by unobservability raises the cost of motivating any given level of effort and makes the owner worse off.

17 These two effects are consistent with part (i) of Proposition 2. A concave management compensation schedule arises from the exponential (constant absolute risk-averse) utility because of the direct risk-aversion effect. The linearity of pay under logarithmic utility arises from an exact offset of the risk-aversion effect and the effect of decreasing risk aversion. Under power utility, the decreasing risk-aversion effect is stronger than the direct risk-aversion effect, leading to convexity.
We first establish conditions under which unobservability of project choice does not lead to efficiency loss. Let the OP contract corresponding to choice of $b$, the lowest risk project, be denoted the OP($b$) contract. Then let the OP'($b$) contract be defined as the project-unconditional contract corresponding to OP($b$) contract; i.e.,

$$S^{UP}(X) = S^{OP'(b)} = S^{OP(b)}(X|b = b),$$

where $S^{UP}$ denotes the contract when project choice is not observable.

**PROPOSITION 3.** Suppose that (i) $X = a + b\theta$, $\theta \sim N(0, 1)$, and (ii) the manager has either exponential utility or logarithmic or power utility $u = [S(X)]^{a/\alpha}$, with exponent $\alpha \leq \frac{1}{2}$, $\alpha \neq 0$. Then the unobservability of project choice does not raise the cost of motivating effort; i.e., the manager chooses $b$ under the OP'($b$) contract.

*Proof.* See Appendix. $\blacksquare$

The next proposition describes a condition under which unobservability causes an efficiency loss, in the sense that the owner is worse off than under observability (while the manager is equally well off, at his reservation level of utility).

**PROPOSITION 4.** Suppose that (i) $X = a + b\theta$, $\theta \sim N(0, 1)$, and (ii) $u = [S(X)]^{a/\alpha}$, with exponent $\frac{1}{2} < \alpha < 1$. Then under the OP'($b$) contract the manager chooses $b$; thus the owner's welfare is reduced by the unobservability of project choice.

*Proof.* See Appendix. $\blacksquare$

Proposition 4 follows by noting that whatever values $a$ and $b$ the manager might contemplate when the project choice is unobservable, by the induced convexity of $u(S(X))$ (Proposition 2), his utility is increased by increasing $b$ to $b$. This follows by Jensen's inequality since raising $b$ increases the variance of $X$ with mean unchanged.\(^{18}\)

We have established that unobservability of project choice will sometimes lead to inefficiencies compared to the OP case associated with the manager's temptation to favor high risk. In the next section, allowing for a

\(^{18}\) Assuming that $f_x/f$ is linear, which is implied by normality, Jewitt's (1988) condition for the validity of the first-order approach is satisfied for exponential, logarithmic utilities and for power utility so long as $\alpha \leq \frac{1}{2}$. It is not clear whether the first-order approach is valid when $\frac{1}{2} < \alpha < 1$. However, by Jewitt (1988, p. 1179), we can deal with the case of $\alpha = \frac{1}{2}$. Jewitt's result implies that the chosen effort level will satisfy the original incentive compatibility conditions if, at the solution to (3) (the OP contract derived based on the first-order approach), the manager's expected utility is a concave function of his effort. This leads to the following observation, which is proved in the appendix.

**Observation.** For $b$ sufficiently close to zero, the first-order approach is valid for $\alpha = \frac{1}{2}$.\n
risk–return trade-off, we show that inefficiencies can arise from a managerial bias toward either low or high risk, and that even when the contract is set optimally, this bias can lead to distortion in the choice of projects.

4. RISK–RETURN TRADE-OFF

Since our goal is to illustrate some possible outcomes of the contracting process, we examine a series of examples and special cases in this section. In the absence of a risk–return trade-off, we found that under normality of the output distribution the owner always prefers the safest project (given observability), but that when the project choice is unobservable, the owner might decide to tolerate an additional contracting cost to motivate the safe project. In this section, we assume that the expected return increases with risk (at least among the set of efficient projects). If return rises mildly with risk, the owner may, given project observability, still prefer a low-risk project. This can lead to a conflict of interest and a distortion of the project choice if the manager favors high risk. Hirshleifer and Suh (1991) provide an example in which this distortion can occur.

Letting \( a_H \) refer to high effort and \( a_L \) to low effort, in the example, the likelihood ratio \( \frac{\Pr(X|a_H) - \Pr(X|a_L)}{\Pr(X|a_H)} \) is a convex function of output \( X \). As discussed in Subsection 3.1, the likelihood ratio describes the extent to which the owner can infer high managerial effort from each outcome, so that convexity of the likelihood ratio tends to lead to a convex compensation schedule (if project choice is observable). A conflict of interest arises because such a convex contract induces the manager to choose a high-risk rather than a low-risk project. In equilibrium, because of the value of convexity in motivating effort, the high-risk project is in fact selected.

However, if there is a strong risk–return trade-off, the owner may prefer a high-risk project, given project choice observability. This need not lead to a conflict of interest if the manager also prefers high risk, as shown in Subsection 4.1. However, if management prefers safety under the OP contract, then in the case of unobservable project choice, the manager will need to be motivated to be more aggressive. In Subsection 4.2, using a discrete example we show that sometimes this is too costly, so that the project will be distorted toward low risk. We also show that equilibrium effort can be distorted by the unobservability of project choice.

4.1. No Conflict over Project Riskiness

Unobservability of project choice need not lead to a reduction in contractual efficiency, because the optimal contract for motivating effort, taking project choice as given, may in fact induce the manager to choose
the right project. We describe an example in which both the owner and the manager prefer the risky project, so that the unobservability of the project is inconsequential.

We modify the assumption on the distribution of output of Section 3. Let output satisfy $X = a + b\theta + Kb$, $b = 0$ or $\hat{b}$, $E[\theta] = 0$ so that $E[X] = a + K\hat{b}$. $K$ is the reward for risk-bearing provided by the project, and assume $\theta \sim N(0, 1)$, $\text{var}(X) = b^2$. Suppose that $K$ is sufficiently large that the owner prefers $b = \hat{b}$ when the project choice is observed (the OP case). Then $S(X|\hat{b})$ will solve (2) with $b = \hat{b}$, where the first-order condition (3) applies. We now turn to the UP case. Since the project is unobservable, consider the contract that corresponds to the OP($\hat{b}$) contract by giving the manager the same payoff for any given level of output (regardless of project choice) as he would under the optimal OP contract had he chosen $\hat{b}$. In other words, OP($\hat{b}$) sets

$$S^{UP}(x) = S^{OP(\hat{b})}(X) = S^{OP(\hat{b})}(X|b = \hat{b}),$$

where the RHS is the solution to (2). The following corollary provides conditions under which this contract is optimal because the induced aggressiveness of the manager leads him to select the higher-return project desired by the owner.

**Corollary to Proposition 4.** Suppose $u = [S(X)]^\alpha/\alpha$, with exponent $\frac{1}{2} \leq \alpha < 1$. Then, under the OP($\hat{b}$) contract (6), the manager strictly prefers to choose the risky project (i.e., $b = \hat{b}$). Thus, the unobservability of project choice does not lead to a loss in contractual efficiency.

**Proof.** See Appendix.

### 4.2. The Owner Prefers a Higher Risk than the Manager

For brevity we summarize the assumptions and results of the example verbally. Details may be found in the Appendix. Let there be two action choices, $a \in \{a_H, a_L\}$, and two projects, safe and risky. Depending on both effort and the state of the world, the safe project generates $X_M$ or 0, and the risky project generates $X_L$, $X_M$, or $X_H$. Specifically, for the safe project, output is $X = X_M$ if the manager chooses $a_H$ and 0 if the manager chooses $a_L$; the payoffs for the risky project are described in Table 1.

In this subsection $p = q = \frac{1}{2}$; the more general case is relevant later. Let the manager’s utility function be $u = 2\sqrt{S}$, $v(a_H) = 1$, $v(a_L) = 0$, and his reservation level of utility $\bar{H} = 0$. We select parameters such that $X_H$ is sufficiently greater than $X_M$ so that the owner will prefer to motivate the risky project.

We derive in the Appendix conditions under which, given observable project choice, the owner prefers the risky project (because of its higher
TABLE I
RISKY PROJECT OUTCOMES

<table>
<thead>
<tr>
<th>$a_H$</th>
<th>$X_L$</th>
<th>$X_M$</th>
<th>$X_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_L$</td>
<td>$X_L$</td>
<td>$X_L$</td>
<td>$X_L$</td>
</tr>
<tr>
<td>Pr($\theta$)</td>
<td>$p$</td>
<td>$1 - p - q$</td>
<td>$q$</td>
</tr>
</tbody>
</table>

expected return). We then turn to the unobservable project case to examine whether the manager is still willing to choose the risky project if the owner offers him the same contract with payoff now independent of the project, the $OP'(Risky)$ contract. We show that in this example the manager prefers the safe project.

4.2.1. Conflict of Interest

The manager is strictly better off with the safe project, implying a conflict of interest with the owner. The intuition for this is as follows. Under the $OP'(Risky)$ contract, the manager will prefer to choose the safe project, owing to the concavity of the contract. The concavity of the contract arises from the probability structure assumed in this example, which causes the likelihood ratio $1 - \{\Pr(X|a_L)/\Pr(X|a_H)\}$ to be concave in $X$.

4.2.2. Distortion of Project Choice

Consider now the unobservable project case. Being aware of the conflict of interest in project choice, the owner has to choose between designing a UP contract that motivates selection of the risky project and one that motivates selection of the safe project.

Now, UP(Risky) must be more convex than OP(Risky) because when project choice is unobservable, the manager must be given an indirect outcome-based incentive to choose the risky project. The benefit in higher expected return from inducing selection of the risky project may or may not be worth the greater compensation cost of motivating any given level of effort. There exist parameter values in this example such that the equilibrium choice of the project will be distorted toward lower risk due to the unobservability of project choice.

4.2.3. Distortion of Managerial Effort

Intuitively, one expects effort as well as project choice to be distorted by the unobservability of project choice, because the attempt to motivate a better project may lead to a contract that is too costly for motivating the
same level of effort. We now extend the example illustrated in Table I to show that effort distortion can also occur. We provide a discrete example, but one would expect distortion to occur even more easily in a continuous setting.

We now assume that \( V(a_H) = 100 \) and that \( V(a_L) = 0 \). By direct calculation, it is not hard to show that \( OP(\text{Risky, } a_H) \) is optimal when the project choice is observed, but that \( UP(\text{Risky, } a_L) \) is optimal when the project choice is not observed. Details of this calculation are provided in the Appendix.\(^9\)

4.3. When Are Managers Too Conservative?

We have described several determinants of whether a manager will be too conservative or aggressive in his project choice: the coefficient of absolute risk aversion and its rate of decrease (see Section 3) and the curvature of the likelihood ratio \( (f_{ij}/f_{in}) \) in a continuous setting. It is difficult to predict which firms and industries will display greater convexity of the likelihood ratio and which managers will have more rapidly decreasing absolute risk aversion.

However, since managers of large firms tend to be wealthier than their counterparts in smaller firms, and since wealth is associated with lower absolute risk aversion, excessive managerial conservatism is likely to be a greater problem among small rather than large firms.\(^20\) It is also reasonable to expect self-selection of managers and firms so that more risk-tolerant managers work in riskier firms. Thus, we expect a greater bias toward conservatism in low-risk firms. Thus, the bias to conservatism occurs most strongly where it will do the least damage. More generally, the analysis implies that firms with desirable risky growth opportunities (e.g., small high-tech firms) should attempt to hire managers with preferences that promote risk-taking, e.g., low absolute risk aversion, and high rate of decrease of risk aversion with wealth, in order to reduce the conflict between optimal project and effort incentives.

5. COMPENSATION AND RISKY GROWTH OPPORTUNITIES

We have assumed thus far that while shareholders may be unable to observe the manager's project choice, they do have the same knowledge

\(^9\) Our finding that the equilibrium effort choice will be distorted owing to moral hazard in the project choice is consistent with Demski and Sappington (1987), who find that designing a contract to motivate an expert to become informed at the planning stage of a project can conflict with the goal of controlling effort in implementing the project.

\(^20\) There may, of course, be organizational considerations beyond the scope of our model operating in the other direction.
TABLE II

<table>
<thead>
<tr>
<th>Low-risk project</th>
<th>High-risk project</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$\theta_3$</td>
</tr>
</tbody>
</table>

as managers regarding the menu of available projects. However, investors in firms that operate in unstable environments or in industries with rapid innovation may have only imperfect knowledge of alternatives.

We show that when there is a high probability that a desirable but risky growth opportunity is available to the firm, there is an incentive to offer the manager a convex contract to motivate him to seize the opportunity when it appears. Suppose then that firms are one of two types, those with a high and those with a low ex ante probability of a risky growth opportunity. Managers of firms with a high probability should receive more option-based compensation. Firms that are more likely to discover and exploit risky growth opportunities will also tend to have higher stock prices relative to earnings, book value, or replacement value of assets. Thus, option-based compensation should be positively associated with such measures as the price/earnings ratio, the price/book value ratio, or Tobin’s $q$. Since uncertainty about growth opportunities is likely to be present in high-technology firms and industries, option-based compensation should also be associated with high R&D expenditures.

To develop this prediction more formally, consider a setting in which there are two possible projects with high or low risk and in which the high-risk project is superior owing to a higher expected payoff. Suppose now that there is a probability $p$ that the high-risk project will be available to the manager. The project outcomes and probabilities are summarized in Table II.

\[ \begin{array}{cccccc}
  a_L & X_L & X_L & X_L & X_L & X_L \\
  a_H & X_L & X_M & X_H & X_L & X_M & X_H \\
  \text{Pr}(\theta) & r & 1 - 2r & r & q & 1 - 2q & q \\
\end{array} \]

21 Given that option-based compensation is associated with the adoption of desirable risky projects, such compensation may be associated with high R&D if such projects are associated with high R&D expenditure. This linkage between R&D and option compensation differs from the example we develop here, in which the project choice is not directly observable ex post. However, the same implication could be derived in a setting with observable project choice, so long as the menu of projects is not observed by owners. The same intuition applies, that for a low ex ante probability that the high-risk project will be available, it is not worth the cost to modify the contract to motivate choice of the high-risk project, but with high probability this becomes profitable.
We assume that $0.5 \,(X_L + X_H) > X_M$. Thus, a shift in probability toward extreme outcomes not only increases risk, but also raises the expected payoff. We assume that $r < q < \frac{1}{2}$, so that the high-risk project is indeed more risky and has a higher expected payoff. Thus, the owner would like the manager to choose the high-risk project if it is available. It may be too costly for the owner to motivate this behavior, however. Any shift in the contract to motivate the high-risk project will distort the motivation of effort in states of the world in which only the low-risk project is available. We show that the owner is willing to modify the contract to motivate selection of the high-risk project only when the probability that the high-risk project becomes available is sufficiently high.22

Let $V_H = V(a_H)$ and $V_L = V(a_L)$. We first consider the $UP(HR)$ program that the owner solves if he chooses to motivate the manager to choose the high-risk project, when available:

$$\begin{align*}
\max (1 - p)[&r(X_L - S_L) + (1 - 2r)(X_M - S_M) + r(X_H - S_H)] \\
&+ p[q(X_L - S_L) + (1 - 2q)(X_M - S_M) + q(X_H - S_H)]
\end{align*}$$

such that

$$\begin{align*}
(1 - p)[&ru(S_L) + (1 - 2r)u(S_M) + ru(S_H)] \\
&+ p[qu(S_L) + (1 - 2q)u(S_M) + qu(S_H)] - V_H \geq \overline{H} \quad (8) \\
ru(S_L) + (1 - 2r)u(S_M) + ru(S_H) - V_H \geq u(S_L) - V_L \quad (9) \\
qu(S_L) + (1 - 2q)u(S_M) + qu(S_H) - V_H \geq u(S_L) - V_L \quad (10) \\
qu(S_L) + (1 - 2q)u(S_M) + qu(S_H) - V_H \\
\geq ru(S_L) + (1 - 2r)u(S_M) + ru(S_H) - V_H. \quad (11)
\end{align*}$$

If the owner does not choose to motivate the choice of the high-risk project, his $UP(LR)$ program becomes

$$\begin{align*}
\max r(X_L - S_L) + (1 - 2r)(X_M - S_M) + r(X_H - S_H)
\end{align*}$$

such that

$$\begin{align*}
ru(S_L) + (1 - 2r)u(S_M) + ru(S_H) - V_H \geq \overline{H} \quad (13) \\
ru(S_L) + (1 - 2r)u(S_M) + ru(S_H) - V_H \geq u(S_L) - V_L. \quad (14)
\end{align*}$$

22 Even as the probability is varied within a range where the high-risk project is motivated when available, the contract changes because of the changing trade-off between efficiently motivating effort with the low-risk and the high-risk project.
For this problem, when the owner motivates the choice of the low-risk project, the constraint that the manager choose low risk is unnecessary. This is because a concave likelihood ratio function leads to a concave OP contract, as indicated in Section 4.2.1. Given a concave unconditional OP contract, the manager will prefer the low-risk project even when the project choice is not observed. Thus, the optimal UP contract which motivates the choice of the low-risk project is just the unconditional optimal OP contract.

The optimal contracts that either motivate or does not motivate the choice of the high-risk project when available are solved for in the Appendix. For our purposes, the key point is that the contract that motivates high risk, $\text{UP}^*(\text{HR})$, is convex, whereas the contract that does not motivate high risk, $\text{UP}^*(\text{LR})$, is concave. This brings us to the main conclusion of this section.

**Proposition 5.** When $u = 2\sqrt{5}$, there exists a value $\hat{X}_H$ such that two possible cases apply depending on the size of $X_H$:

1. Unfavorable risk–return trade-off ($X_H \leq \hat{X}_H$). In this case, the owner always motivates the low-risk project.
2. Favorable risk–return trade-off ($X_H > \hat{X}_H$). In this case, there exists a value $0 < \hat{p} < 1$ such that the owner motivates the low-risk project if $p \leq \hat{p}$ and motivates the high-risk project if $p > \hat{p}$.

**Proof.** See Appendix.

In the first case, the risk–return trade-off is unfavorable, so the owner always motivates the low-risk project. In the second case, the risk–return trade-off is more favorable, so for low values of $p$ the low-risk project is still chosen, while for large values the owner induces a choice of the high-risk project. Intuitively, with a low probability of the high-risk project, it is too costly to motivate the high-risk project because this distorts effort incentives in states of the world in which only the low-risk project is available.

The case of a favorable risk–return trade-off implies that for large $p$ the optimal contract will be convex, while for small values of these variables the contract is concave. In other words, a greater likelihood that the riskier project will become available and a higher expected return on the risky project, both lead to a more convex contract being offered. Larcker

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23 We add asterisks to indicate a contract that optimally motivates choice of the high-risk or low-risk projects given an ex ante probability of $p$ that the high-risk project is available. These contracts can in general differ from contracts that motivate the choice of a project given the certainty that both projects are available. The uncertainty regarding the availability of HR affects the trade-off between motivating effort efficiently given that HR is undertaken and given that LR is undertaken.
(1983), Hagerty et al. (1990), and Kumar and Sopariwala (1992) provide empirical evidence consistent with this prediction. Larcker found that firms that adopt performance plans\(^{24}\) have high growth in capital expenditures (relative to nonadopting firms) and have a positive average abnormal stock price reaction to the announcement of adoption. Kumar and Sopariwala found a positive association between the adoption of long-term performance plans and subsequent changes in profitability, and also found positive average abnormal stock returns associated with the announcement of adoption.

Hagerty et al. found that the use of option-based compensation was associated with lower earnings/price ratios and with higher levels of R&D expenditures.\(^{25}\) This evidence is also consistent with their theory, which is based on the design of optimal contracts to motivate managers to sacrifice short-term in favor of long-term earnings.

6. MONITORING AND EQUILIBRIUM PROJECT CHOICE

One way to reduce the moral hazard problem in project choice is to have large shareholders (see Demsetz, 1983) or bank lenders monitor the manager's decision. We assume that the monitor acts loyally on behalf of shareholders. To illustrate the effect of monitoring, we focus on the problem that the manager may select a project with inappropriately low risk. Since the monitor in such a case attempts to verify that the manager is taking sufficient risk, in this example the monitor is best interpreted as a large shareholder rather than as a lender.\(^{26}\) The larger shareholder could be either a wealthy individual or an institutional investor such as a pension fund, or their surrogate.

We extend the example of Table I in Subsection 4.2 by allowing the owner to pay to monitor the project choice of the manager at a cost of \(C^M\).

\(^{24}\) According to Larcker, "... performance plans exhibit 'option-like' characteristics similar to stock options, SAR's [share appreciation rights], and phantom stock, in that the payoff is bounded by zero from below and increases as the performance measure exceeds some target."

\(^{25}\) Interpreted narrowly, our example implies that there will be two categories of firms, one with high \(E/P\) and little option-based compensation and the other with low \(E/P\) and a great deal of option compensation. However, in reality firms are heterogeneous in their \(E/P\) ratios and R&D levels for reasons other than the magnitude of risky growth opportunities, so that in any real sample this step function relationship will be smoothed into a finitely sloped increasing relationship.

\(^{26}\) It is also possible to construct examples in which the manager is tempted to choose excessively risky projects from the point of view of the owner. Despite the well-known conflict of interest between debt and equity with regard to risk, if the expected return on the riskier project is sufficiently low, both the lender and the owner may favor a lower-risk project. The problem for both is to prevent the manager from taking excessive risks. Thus, in such a setting a bank lender can act as a monitor on behalf of shareholders.
Suppose that monitoring is perfect, so that payment of $CM$ makes the project choice transparently observable. Then the value of monitoring is

$$E[X - S|OP] - E[X - S|UP] - CM. \quad (15)$$

Following the analysis of Subsection 4.2, OP(Safe) = UP(Safe), so $S = \frac{1}{4}$.

$$E[X - S|OP(Safe)] = E[X - S|UP(Safe)] = X_M - \frac{1}{4}. \quad (16)$$

By reasoning analogous to that of Subsection 4.2, in the OP(Risky) contract,

$$S_L = 0 \quad (17)$$

$$S_M = S_H = \left(\frac{1}{2(1 - p)}\right)^2 > \frac{1}{4}, \quad (18)$$

a concave contract. Thus,

$$E[X - S|OP(Risky)] = pX_L + (1 - p - q)X_M + qX_H - \frac{1}{4(1 - p)}. \quad (19)$$

Similarly, in the UP(Risky) contract,

$$S_L = 0 \quad (20)$$

$$S_M = \frac{1}{4} \quad (21)$$

$$S_H = \left(\frac{p + q}{2q}\right)^2 = \left(\frac{p}{2q} + \frac{1}{2}\right)^2 > \frac{1}{4}. \quad (22)$$

Thus,

$$E[X - S|UP(Risky)] = pX_L + (1 - p - q)X_M + qX_H - \frac{1}{4} (1 - p - q) - \frac{(p + q)^2}{4q}. \quad (23)$$

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27 Equivalent results can be derived from a noisy signal with a support that moves as a function of the project choice. If with positive probability the signal reveals the project choice conclusively, then a forcing contract can be written on project choice. Even if penalties are bounded, if the probability of revelation is sufficiently high, the manager can be forced to choose the correct project.
The UP(Risky) contract is less concave than the OP(Risky) contract.\footnote{Note that as the outcome rises from \(X_L\) to \(X_M\) to \(X_H\), the OP(Risky) contract rises to greater than \(\frac{a}{2}\) and then becomes flat, whereas the UP(Risky) contract rises at first only to \(\frac{a}{2}\) and then continues to increase.} This is because greater convexity is needed when the project is unobservable in order to motivate the manager to select the risky project.

Consider first parameter values under which, if there is no monitoring, the owner will want to induce choice of the risky project. Then calculating the difference \(E[X - S|\text{OP(Risky)}] - E[X - S|\text{UP(Risky)}] - CM\) and differentiating with respect to \(p\) and \(q\) show that the value of monitoring decreases with the likelihood of the high outcome and increases with the likelihood of the low outcome. It is less costly to induce the choice of the risky project contractually (instead of through monitoring) when the risky project generates greater outcomes (in the first-order stochastic dominance sense), because for a given (nondecreasing) contract, the manager becomes more willing to choose the risky project over the safe one.

Consider next parameter values under which, if there is no monitoring, the owner is unwilling to incur the contracting cost needed to induce the choice of the risky project. Then the value of monitoring is the difference \(E[X - S|\text{OP(Risky)}] - E[X - S|\text{UP(Safe)}] - CM\). Differentiating shows that the gain to monitoring is increasing in \(q\), but decreasing in \(p\). Intuitively, the value of monitoring increases when there is a greater benefit to undertaking the risky project. Higher \(q\) increases that benefit, while higher \(p\) reduces it.

We therefore obtain a single-peaked graph as in Fig. 1, in which monitoring is performed for intermediate risky project qualities, but not if the risky project is very good or very poor. (Moving to the right on this graph, project quality can be measured either by \(q\) or by \(-p\).) The graph may be divided into four regions depending on project quality. In Regions 1 and 2, the project is of sufficiently low quality that, without monitoring, project choice will be distorted to the safe project. In Regions 3 and 4, the project is better, so that the project will be undertaken even without monitoring. Monitoring can have value in these regions, because motivating the risky project may still be costly. In Region 1, the project is so poor that it is not worth the monitoring cost needed for it to be undertaken. In Regions 2 and 3, the project is better, and it will be undertaken and monitored. In Region 4, the project is so good that it can be motivated contractually without need for monitoring. Eventually, for sufficiently high quality, the cost of motivating the project reaches zero.

Monitoring occurs in the model for intermediate qualities of the investment opportunity. One interpretation of “monitoring” is the accumulation of a large shareholding in the firm that will provide the shareholder with a significant incentive to investigate the firm’s investment decisions. Thus, the analysis suggests that we should observe large accumulations of
shares held by active monitors in firms with medium investment opportunities, but that these shareholdings are less crucial when the investment opportunities are either very good or very poor.

Beatty and Zajac (1992) find a negative relationship between monitoring institutions (e.g., percentage of outside directors, separation of chairman of board and CEO position) and both (1) managers’ equity holdings and (2) option compensation of top managers. In our example with monitoring, the UP(Risky) contract is less concave than the OP(Risky) contract, because greater convexity is needed when the project is unobservable to motivate choice of the risky over the safe project. Thus, the example is consistent with this evidence. More generally, for power utility with \( \alpha < 0 \) (relative risk aversion greater than 1), the general features of this example will still hold: OP(Risky) will be concave, while UP(Risky) will need to be modified in the direction of convexity in order to encourage choice of risky project.29

7. CONCLUDING REMARKS

We have shown that contracts designed to motivate managers to work diligently lead to an induced risk preference over alternative projects. If the project riskiness is observable and can be contracted on directly, then

29 Several portfolio and capital market studies have estimated relative risk aversion to be much greater than 1, although a few studies offer lower ranges of values (see references in Kimball, 1988). Based on a numerical simulation and an intuitive notion of plausibility, Kimball argues that the relative risk aversion of most individuals exceeds \( \frac{1}{2} \).
the manager can be forced to choose the desired project. But if project choice is unobservable (or not directly contractible), then induced risk preference can cause managers to be either excessively conservative or excessively aggressive. Taking into account induced risk preferences, shareholders may do best by offering the manager a contract that implicitly accepts second-best project choices (either too aggressive or too conservative). Alternatively, they may do better with modified contracts in which managers choose projects correctly, but with a higher cost of motivating the manager to work diligently in implementing the chosen project.

We find that (under appropriate assumptions) a higher probability that a desirable risky investment is available causes shareholders to offer the manager greater option compensation, in order to decrease his induced risk aversion. Thus, the analysis is consistent with evidence that firms that adopt option or option-like compensation schemes (such as performance plans) have high growth in capital expenditures, low earnings/price ratios, and high R&D expenditures (see Larcker, 1983; Hagerty et al., 1990). The analysis is also consistent with the positive average abnormal stock price reaction to the announcement of performance plans (Larcker, 1983).

The analysis suggests a role for monitoring of the project choice by a bank lender or large shareholder. In the case where there is a possibility that the manager will be too conservative, we provide an example that illustrates that monitoring is a substitute for contract convexity, which weakens induced risk aversion. Either mechanism can be used to ensure selection of the risky project. Whether one or the other or neither is used is determined by the quality of the risky project. Thus, our analysis is consistent with the evidence of Beatty and Zajac (1992) of a negative relationship between monitoring institutions (such as a high percentage of outside directors) and option compensation of top executives.

The analysis suggests that distortions in investment choices are more likely to occur when the riskiness of alternative project choices is difficult to observe. Thus, we expect relatively small distortion arising from contracts written for a portfolio manager, since reasonable (though imperfect) measures of portfolio risk can be calculated based on the past history of security returns. In other words, mutual funds (and to some degree, pension funds) are usually "transparent" financial intermediaries in the sense of Ross (1989). At the opposite extreme, for a biotech firm it may be impossible to describe in advance the riskiness of alternative projects, and for proprietary reasons it may be undesirable to reveal ex post full details about the nature of the project that was undertaken. Thus, we expect larger potential distortions of investments in a high-tech industry than in an industry in which it is at least partly possible to define in advance the riskiness of alternative investment decisions.
In practice, contracts for top managers contain numerous features which, by inducing convexity, should shift managers’ preferences toward higher risk. First, having a compensation component that is independent of performance combined with a component such as a bonus that is contingent on achieving some minimum target performance leads to convexity. Second, option compensation leads to convex payoffs. Third, convexity is accentuated to the extent that managers are protected from termination by golden parachutes or pensions.

Recently controversy over compensation of top U.S. executives has prompted proposals to bound executive pay, including an annual limit on corporate deductibility of pay of $1 million proposed by candidate Bill Clinton in the closing weeks of the 1992 presidential campaign. Jensen and Murphy (1990) maintain that constraints that limit pay at the high end will reduce the incentive for managers to work. However, truncation at the top will tend to reduce the convexity as well as the slope of the compensation schedule. This suggests that the imposition of constraints on executive pay may lead to excessive caution by managers in their project choices.30

The general conclusions of our analysis would continue to hold in a setting with systematic risk. Systematic risk would be similar to modifying the expected payoffs as perceived by shareholders (the owner) and the manager by the appropriate risk premium. It would not be difficult to construct examples in which project choice is distorted toward either excessively low or excessively high idiosyncratic risk. Which case occurs will depend on the convexity or concavity of the compensation scheme that would be given to the manager under observable project choice. Such risk distortions will occur whether or not the manager is permitted to hedge the firm’s systematic risk (see also footnote 8).

Since refraining from any project may be viewed as a special case of choosing a safe project, our analysis is also relevant for motivating the manager to decide whether to undertake a project so long as this decision is unobservable to the owner, as will be the case for some, though not all, managerial decisions.31 Thus, our analysis has a bearing on the debate over whether managerial incentives lead to myopic underinvestment. Our

30 However, as pointed out by Harris and Holmstrom (1982) in a multiperiod learning model, and by Murphy (1986) in a multiperiod incentive model, the ability of managers to quit can put a lower bound on compensation, thus tending to make compensation schemes more convex. The incentive to quit can be reduced by offering the manager a bonus contingent on staying with the firm, i.e., “a golden handcuff.”

31 The ability to contract on the decision of whether to invest is assumed in Holmstrom and Ricart i Costa (1986) and in Banker et al. (1989). Holmstrom and Ricart i Costa also examine reputation building by managers. In their paper, compensation cannot be made contingent on profit outcomes.
analysis shows that with effort-aversion and project-choice moral hazards, managers may be biased in the direction of either under- or over-investment.32

We have assumed in this paper that the problem of agency is entirely between the manager and shareholders. More generally, there can be conflicts of interests about the level of project risk between other stakeholders, such as shareholders and debtholders (e.g., Jensen and Meckling, 1976). Brander and Poitevin (1990) have argued that concave managerial contracts can be used to mitigate the agency problem between debt and equity. However, such contracts would encourage shareholders to renegotiate with the manager immediately after issuing debt and before the manager acts. By giving the manager a more convex contract, the shareholders would induce the manager to choose a higher-risk project and so profit at the expense of the bondholders.

Our analysis suggests that sometimes the shareholders will give a manager a concave contract in order to motivate diligence in implementing the project. This can mitigate the agency problem between shareholders and bondholders even if shareholders are free to renegotiate with the executive after the firm has issued debt.33 Thus, it would seem to be possible that shareholders’ inability to observe the manager’s effort might actually make them better off ex ante.

Some empirical predictions derive from the trade-off emphasized here between motivating effort and motivating project choice. First, when it becomes more important to motivate the choice of risky projects, greater option-based compensation should be used. Thus, the analysis predicts greater option-based compensation when there are desirable risky growth opportunities (proxied by Tobin’s q or R&D expenditures). Second, when there are specialists monitoring project choice, it becomes less important to motivate the choice of risky projects choice through option compensation. Thus, the analysis predicts that less option compensation is used when institutions for monitoring are present that can act as substitutes (such as the inclusion of outsiders on the board of directors, or the existence of bank loans).

APPENDIX

Proof of Proposition 2. (i) When \( u = [S(X)]^{\alpha}/\alpha, \alpha < 1 \), the optimal contract has

32 Narayanan (1985) shows that a managerial concern for reputation for ability can lead to a preference for short-run cash flows. However, Hirshleifer and Chordia (1991) show that if project decisions affect the timing of news arrival as well as cash flows, that reputational concerns may bias managers toward late rather than early cash flows.

33 Dewatripont (1988) provides a general analysis of commitment through renegotiation-proof contracts with third parties.
\[ S(X) = \left[ \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right) \right]^{1/(1-\alpha)}. \]

If \( f_a(x)/f(x) \) is linear in \( X \), then \( S(X) \) is convex (concave) in \( X \) if \( \alpha > (\leq) 0 \).

When \( u = -e^{-rS} \),

\[ S(X) = \frac{1}{r} \ln \left( r \left[ \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right) \right] \right). \]

When \( u = \ln(s) \),

\[ S(X) = \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right). \]

(ii) When \( u = [S(X)]^{\alpha/\alpha} \),

\[ u(S(X)) = \frac{1}{\alpha} \left[ \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right) \right]^{-\alpha/(1-\alpha)}. \]

It follows that \( u(S(X)) \) is strictly convex (concave) in \( x \) iff \( \alpha > (\leq) \frac{1}{2} \) and linear in \( x \) iff \( \alpha = \frac{1}{2} \). For exponential utility,

\[ u(S(X)) = -\left( r \left[ \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right) \right]^{-1}, \]

which is strictly concave in \( x \). For logarithmic utility, we have

\[ u(S(X)) = \ln \left[ \lambda + \mu \left( \frac{f_a(x)}{f(x)} \right) \right], \]

which is strictly concave in \( X \).

**Proof of Proposition 3.** When \( \theta \sim N(0, 1) \), \( f_a/f \) is linear in \( x \). Then, by Proposition 2, \( u(S(X)) \) is strictly concave in \( x \) for exponential, logarithmic, or power utility with \( \alpha < \frac{1}{2} \). In these cases, since higher \( b \) implies higher variance with the same mean of \( X \), it follows by Jensen’s inequality that the manager strictly prefers \( b \). The manager is indifferent when \( \alpha = \frac{1}{2} \) for the power utility function.

**Proof of the observation in footnote 18.** It suffices to show that \( \int uf_{aa} \, dX - v''(a) < 0 \) at the solution to (2) when \( \alpha = \frac{3}{4} \). By normality of output,
so that

\[ f_{aa}(X|a, b) = f \cdot \frac{1}{b^2} [(X - a)^2 - 1], \quad (24) \]

Through a tedious calculation, for \( \alpha = \frac{3}{2} \) we obtain

\[ \int u_{faa} dX = \frac{3}{2b^2} \left( 3\mu^2 + b^2\lambda^2 - \frac{\mu^2}{b^2} - \lambda^2 \right). \]

Since \( v''(a) > 0 \), and as \( b \to 0 \), \( \int u_{faa} dX < 0 \) (since \( \mu^2/b^2 \to \infty \)), it follows that \( \int u_{faa} dX - v''(a) < 0 \). \( \blacksquare \)

**Proof of corollary to Proposition 4.** Let \( \hat{a} \) be defined as the manager’s optimal effort \( \text{given that } b = 0 \), \( a^* \) as the optimal level of effort \( \text{given that } b = \hat{b} \), and \( S(X) \) as the OP' contract corresponding to \( b = \hat{b} \). Since the power utility function has an exponent of \( \alpha > \frac{1}{2} \), by Proposition 2, \( H(S(X)) \) as defined in (1) is convex in \( X \). Then

\[ E[H(S(X), a^*, b = \hat{b})] > E[H(S(X), \hat{a}, b = \hat{b})] \]
\[ > E[H(S(X), \hat{a}, b = 0)], \]

where the last inequality follows from the induced convexity of \( H(S(X)) \), so that the dispersion associated with a positive \( b \) leads to second-order stochastic dominance, and by the additional first-order stochastic dominance arising from the positive mean effect of higher \( b \). When \( \alpha = \frac{1}{2} \), the last inequality follows from the positive mean effect of higher \( b \). \( \blacksquare \)

**Section 4.2: The Owner Prefers a Higher Risk than the Manager**

**Section 4.2.1: Conflict of Interest.** We begin by illustrating the conflict of interest between manager and owner.

(1) **OP(Safe).** Suppose that the project is observed and that the owner wants to motivate choice of the safe project. Since there is no
uncertainty regarding the final outcome, the first-best solution is obtained through the forcing contract in which \( S_{FB} = \frac{1}{2} \) if \( X_M \) is observed, and otherwise the manager is penalized maximally. This is feasible, because 
\[
2\sqrt{S} - \nu_H = 0 \text{ if and only if } S = \frac{1}{4}.
\]
Thus, in this case, the owner's expected profit is 
\[
X_M - \frac{1}{4}. \tag{26}
\]

(2) \( OP(Risky) \). Suppose now that the project choice is observed, and the owner motivates choice of the risky project. The OP contract that motivates the risky project is the solution to the program (2) under the parameter values of this example. The owner therefore solves 
\[
\max_{S_L, S_M, S_H} \frac{1}{4}[X_L - S(X_L)] + \frac{1}{4}[X_M - S(X_M)] + \frac{1}{4}[X_H - S(X_H)]
\]
such that
\[
\frac{1}{u'(S_L)} = \lambda - 2\mu \]
\[
\frac{1}{u'(S_M)} = \lambda + \mu \]
\[
\frac{1}{u'(S_H)} = \lambda + \mu.
\]
Since \( \lambda > 0, \mu > 0 \), we have \( S_M = S_H > S_L \). Standard Lagrangian optimization gives
\[
S_L = 0, \quad S_M = \frac{9}{8}, \quad S_H = \frac{9}{8}. \tag{27}
\]
Thus, the owner's expected profits are 
\[
E[X - S|OP(Risky), a_H] = \frac{3}{8}(X_L + X_M + X_H) - \frac{3}{8}. \tag{28}
\]
Comparing with (26), the owner will prefer the risky project to the safe project if and only if
\[
\frac{3}{8}(X_L + X_M + X_H) - \frac{3}{8} > X_M - \frac{1}{4}. \tag{29}
\]
(3) The manager's decision when project choice is unobservable. We now examine whether the manager is willing to choose the risky project if the owner offers him the same contract unconditionalized with respect to project, the OP'(Risky) contract.

(i) If so, there is no efficiency loss due to the unobservability of project choice. Thus, (27) is the optimal contract when project choice is not observed.

(ii) If OP(Risky) does not motivate the manager to select the risky project, then it will be costly for the owner to motivate the risky project. The owner would have to modify the contract, in order to motivate choice of the risky project, to one which is less efficient at motivating effort given that the risky project is chosen over the OP(Risky) contract. The benefit of motivating the risk project may or may not outweigh the cost.

We now show that (ii) is the case here. Given (27), if the manager chooses the safe project and effort \( a_H \), his expected utility is

\[
E[H] = 2 \sqrt{\frac{9}{16}} - 1 = \frac{3}{2} - 1 > 0.
\]

This arises from the concavity of the manager's contract, which in turn results from the concavity of the likelihood ratio,

\[
1 - \frac{\Pr(X_H|a_L)}{\Pr(X_H|a_H)} = 1
\]
\[
1 - \frac{\Pr(X_M|a_L)}{\Pr(X_M|a_H)} = 1
\]
\[
1 - \frac{\Pr(X_L|a_L)}{\Pr(X_L|a_H)} = -2.
\]

Section 4.2.2: Distortion of Project Choice. The optimal UP(Safe) is just the project-unconditionalized OP(Safe), since the manager will be penalized if \( X_L \) or \( X_H \) is observed. So

\[
E[X - S|UP(Safe)] = E[X - S|OP(Safe)] = X_M - \frac{1}{4}.
\]

The UP contract corresponding to the risky project solves (4) under the parameter values of this example. To obtain UP(Risky), the owner solves

\[
\max_{S_L, S_M, S_H} \frac{1}{6}(X_L - S_L) + \frac{1}{6}(X_M - S_M) + \frac{1}{6}(X_H - S_H)
\]

such that

\[
\frac{1}{6}(2 \sqrt{S_L} + 2 \sqrt{S_M} + 2 \sqrt{S_H}) - 1 \geq 0
\]
\[ \frac{1}{3}(2 \sqrt{S_L} + 2 \sqrt{S_M} + 2 \sqrt{S_H}) - 1 \geq 2 \sqrt{S_L} \]  
(31)
\[ \frac{1}{3}(2 \sqrt{S_L} + 2 \sqrt{S_M} + 2 \sqrt{S_H}) - 1 \geq 2 \sqrt{S_M} - 1, \]  
(32)
where (32) represents the constraint that \( E[H(\text{Risky}, a_H)] \geq E[H(\text{Safe}, a_H)] \). Standard Lagrangian optimization gives the solution to this problem,

\[ S_L = 0, \quad S_M = \frac{1}{4}, \quad S_H = 1, \]  
(33)
which leads to expected profits for the owner of

\[ E[X - S|UP(\text{Risky})] = \frac{1}{3}(X_L + X_M + X_H) - \frac{\Delta}{2}. \]

For all \((X_L, X_M, X_H)\) such that

\[ \frac{1}{3}(X_M + X_H - \frac{\Delta}{2}) < X_M - \frac{1}{4} \leq \frac{1}{3}(X_L + X_M + X_H) - \frac{\Delta}{3}, \]

the equilibrium choice of the project will be distorted toward lower risk due to the unobservability of project choice.34

Section 4.2.3: Distortion of Managerial Effort. We provide an example to illustrate effort distortion. Consider the different possible contracts in this setting.

\begin{align*}
\text{OP(\text{Safe, } a_H)}: & \quad S = 2500 \\
\text{OP(\text{Safe, } a_L)}: & \quad S = 0 \\
\text{OP(\text{Risky, } a_H)}: & \quad S_L = 0, \quad S_M = 5625, \quad S_H = 5625 \\
\text{OP(\text{Risky, } a_L)}: & \quad S = 0 \\
\text{UP(\text{Safe, } a_H)}: & \quad S = 2500 \\
\text{UP(\text{Safe, } a_L)}: & \quad S = 0 \\
\text{UP(\text{Risky, } a_H)}: & \quad S_L = 0, \quad S_M = 2500, \quad S_H = 10,000 \\
\text{UP(\text{Risky, } a_L)}: & \quad S = 0.
\end{align*}

Therefore, the principal’s residuals in these cases are

\begin{align*}
\text{OP(\text{Safe, } a_H)}: & \quad X_M - 2500 \\
\text{OP(\text{Safe, } a_L)}: & \quad 0 \\
\text{OP(\text{Risky, } a_H)}: & \quad \frac{1}{3}(X_L + X_M + X_H) - 3750 \\
\text{OP(\text{Risky, } a_L)}: & \quad X_L \\
\text{UP(\text{Safe, } a_H)}: & \quad X_M - 2500 \\
\text{UP(\text{Safe, } a_L)}: & \quad 0 \\
\text{UP(\text{Risky, } a_H)}: & \quad \frac{1}{3}(X_L + X_M + X_H) - 4166\frac{2}{3} \\
\text{UP(\text{Risky, } a_L)}: & \quad X_L.
\end{align*}

34 A combination of parameter values for which this occurs is \( X_L = 0, X_M = 10, \) and \( X_H = 20.4375 \).
We derive parameter values such that \( \text{OP}(\text{Risky}, a_H) \) is optimal when project choice is observed, but \( \text{UP}(\text{Risky}, a_L) \) is optimal when project choice is not observed. This is the case if

\[
\frac{1}{3}(X_L + X_M + X_H) - 3750 > X_M - 2500, \quad (34)
\]

\[
\frac{1}{3}(X_L + X_M + X_H) - 3750 > X_L, \quad (35)
\]

\[
X_L > \frac{1}{3}(X_L + X_M + X_H) - 4166\frac{2}{3}, \quad (36)
\]

and

\[
X_L > X_M - 2500. \quad (37)
\]

Assuming \( X_L > 0 \), (35) also implies that \( \frac{1}{3}(X_L + X_M + X_H) - 3750 > 0 \). If \( X_M \geq 2500 \), (37) also implies that \( \text{UP}(\text{Risky}, a_L) > \text{UP}(\text{Safe}, a_L) \). A set of parameters values which satisfies (34), (35), (36), and (37) is

\[
X_L = 100, \quad X_M = 2500, \quad X_H = 9000.
\]

This shows that equilibrium effort can be distorted owing to the unobservability of project choice.

**Section 5: Compensation and Risky Growth Opportunities**

In order to solve for the \( \text{UP}(\text{HR}) \) contract, we first show that (10) is implied by (9) and (11). To see this, by (9) and (11), we have

\[
qu(S_L) + (1 - 2q)u(S_M) + qu(S_H) - V_H
\]

\[
\geq ru(S_L) + (1 - 2r)u(S_M) + ru(S_H) - V_H
\]

\[
\geq u(S_L) - V_L,
\]

which implies (10).

Letting \( \lambda, \mu, \) and \( \eta \) denote the Lagrange multiplier for (8), (9), and (11), respectively, the optimal contracts should satisfy

\[
\frac{1}{u'(S_M)} = \lambda + \mu \frac{r - 1}{r(1 - p) + pq} + \eta \frac{q - r}{r(1 - p) + pq}
\]

\[
\frac{1}{u'(S_M)} = \lambda + \mu \frac{1 - 2r}{(1 - p)(1 - 2r) + p(1 - 2q)}
\]

\[
- \eta \frac{2(q - r)}{(1 - p)(1 - 2r) + p(1 - 2q)}
\]
\[
\frac{1}{u'(S_H)} = \lambda + \mu \frac{r}{r(1 - p) + pq} + \eta \frac{q - r}{r(1 - p) + pq}.
\] (40)

**Lemma 1.** \( \mu > 0 \).

*Proof.* Since \( \mu \geq 0 \), assume the contrary; i.e., \( \mu = 0 \). Then, since \( \eta \geq 0 \), (38), (39), and (40) imply that

\[
u(S_H) = \nu(S_L) \geq \nu(S_M).
\]

Then

\[
u(S_L) + (1 - 2r)\nu(S_M) + ru(S_H) - \nu(S_L) - (V_H - V_L)
= (1 - 2r)[\nu(S_M) - \nu(S_L)] - (V_H - V_L) < 0,
\]

which is a contradiction to (9). ■

Note that \( \lambda > 0 \) for sufficiently large \( \overline{H} \). To see this, suppose the opposite; i.e., \( \lambda = 0 \). Then the owner will reduce the manager's compensation by \( \varepsilon > 0 \), thus providing a utility level of \( \overline{H} \).

**Lemma 2.** Suppose \( u = 2\sqrt{S} \). Then \( \eta > 0 \).

*Proof.* Assume that contrary; i.e., \( \eta = 0 \). Then when \( u = 2\sqrt{S} \), (38), (39), and (40) become

\[
u(S_L) = 2\left[ \lambda + \mu \frac{r - 1}{r(1 - p) + pq} \right],
\]

\[
u(S_M) = 2\left[ \lambda + \mu \frac{1 - 2r}{(1 - p)(1 - 2r) + p(1 - 2q)} \right],
\]

\[
u(S_H) = 2\left[ \lambda + \mu \frac{r}{r(1 - p) + pq} \right].
\]

Thus

\[
u(S_L) - 2\nu(S_M) + \nu(S_H)
= 2\left[ \lambda + \mu \frac{r - 1}{r(1 - p) + pq} - 2\lambda - 2\mu \frac{1 - 2r}{(1 - p)(1 - 2r) + p(1 - 2q)}
+ \lambda + \mu \frac{r}{r(1 - p) + pq} \right]
= 2\mu \left[ \frac{2r - 1}{r(1 - p) + pq} + \frac{2(2r - 1)}{(1 - p)(1 - 2r) + p(1 - 2q)} \right] < 0
\]
since $\mu > 0$ (by Lemma 1) and $r < \frac{1}{2}$. Thus,

$$(q - r)[u(S_L) - 2u(S_M) + u(S_H)] < 0$$

since $q > r$, which is a contradiction to (11). ■

The optimal UP(HR) contract in this setting is as follows.

**Lemma 3.** Suppose $u = 2\sqrt{S}$ and $p > 0$. Then

$$S_L = \left(\frac{H + V_L}{2}\right)^2$$
$$S_M = \left(\frac{H + V_H}{2}\right)^2$$
$$S_H = \left(\frac{H + 2V_H - V_L}{2}\right)^2.$$

**Proof.** Solving (8), (9), and (11) simultaneously yields the solution. ■

The optimal UP(LR) contract is as follows.

**Lemma 4.** Suppose $u = 2\sqrt{S}$ and $p = 0$; i.e., only the low-risk project is available. Then the optimal contract should satisfy

$$u(S_L) = H + V_L$$
$$u(S_M) = u(S_H) = H + \frac{V_H - rV_L}{1 - r}.$$

**Proof.** The optimal OP contracts should satisfy

$$\frac{1}{u'(S_L)} = \lambda - \mu(1 - r)$$
$$\frac{1}{u'(S_M)} = \lambda + \mu$$
$$\frac{1}{u'(S_H)} = \lambda + \mu,$$

which implies that $S_L < S_M - S_H$.

The solution is

$$u(S_L) - H + V_L$$
\[ u(S_M) = u(S_H) = \overline{H} + \frac{V_H - rV_L}{1 - r} \]

\[ S_L = \left( \frac{\overline{H} + V_L}{2} \right)^2 \]

\[ S_M = \left( \frac{\overline{H} + (V_H - rV_L)/(1 - r)}{2} \right)^2. \]

Since the contract is concave, the OP contract remains optimal even when the project choice is not observed. \[ \blacksquare \]

**Proof of Proposition 5.**

\[ E[X - S|UP, \text{High}] - E[X - S|UP, \text{Low}] \]

\[ = [(1 - p)r + pq](X_L - S_L) + [(1 - p)(1 - 2r) + p(1 - 2q)](X_M - S_M) + [(1 - p)r + pq](X_H - S_H) \]

\[ - \left\{ r \left( X_L - \frac{(H + V_L)^2}{2} \right) + (1 - 2r) \left[ X_M - \frac{1}{4} \left( \overline{H} + \frac{V_H - rV_L}{1 - r} \right)^2 \right] \right\} \]

\[ + r \left[ X_H - \frac{1}{4} \left( \overline{H} + \frac{V_H - rV_L}{1 - r} \right)^2 \right], \]

where

\[ S_L = \frac{1}{4}(\overline{H} + V_L)^2 \]

\[ S_M = \frac{1}{4}(\overline{H} + V_H)^2 \]

\[ S_H = \frac{1}{4}(\overline{H} + 2V_H - V_L)^2. \]

Rearranging the above term yields

\[ \Delta = E[X - S|UP, \text{High}] - E[X - S|OP, \text{Low}] \]

\[ = p(q - r)\Phi(X_L, X_M, X_H, \overline{H}, V_H, V_L) - \Pi(r, \overline{H}, V_H, V_L), \]

where

\[ \Phi(\cdot) = X_L - 2X_M + X_H \]

\[ - \frac{1}{4}(\overline{H} + V_L)^2 + \frac{1}{2}(\overline{H} + V_H)^2 - \frac{1}{4}(\overline{H} + 2V_H - V_L)^2 \]

and
\[ \Pi(\cdot) = \frac{1}{4} (1 - 2r)[H + V_H]^2 + \frac{1}{4} r[H + 2V_H - V_L]^2 \]
\[ - \frac{1}{4} (1 - r) \left( H + \frac{V_H - rV_L}{1 - r} \right)^2. \]

Without loss of generality, let \( H = 0 \), \( V_H = 1 \), and \( V_L = 0 \). Then
\[ \Delta = \rho(q - r) \left( X_L - 2X_M + \frac{X_H - 1}{2} \right) - \frac{r(1 - 2r)}{4(1 - r)}. \] (41)

Let \( \bar{X}_H \equiv \frac{1}{2} X_L + 2X_M \). Then for all \( \rho \) and for any \( X_H \leq \bar{X}_H \), \( \Delta < 0 \). Thus, when \( X_H \leq \bar{X}_H \), the owner will choose the low-risk projects.

Suppose \( X_H > \bar{X}_H \). Then there exists a value \( 0 < \hat{\rho} < 1 \) such that \( \Delta = 0 \), where
\[ \hat{\rho} = \frac{r(1 - 2r)}{4(q - r)(X_L - 2X_M + X_H - \frac{1}{2})(1 - r)}. \] (42)

In this case, \( \Delta > (\leq) 0 \) if \( \rho > (\leq) \hat{\rho} \). Thus, the owner will choose the high-risk project when \( \rho > \hat{\rho} \) and the low-risk project when \( \rho \leq \hat{\rho} \). Hence, the likelihood of the owner’s motivating the high-risk project (by means of a convex contract) is increasing in \( X_H \) and \( \rho \).

REFERENCES


HIRSHLEIFER AND SUH


