5. If \( f(x) = |x| + 3x^2 \) for all real \( x \), then \( f'(-1) \) is

(A) 7  (B) -5  (C) 5  (D) 7  (E) nonexistent

6. For what value of \( b \) is the value of \( \int_{b}^{b+1} (x^2 + x) \, dx \) a minimum?

(A) 0  (B) -1  (C) -2  (D) -3  (E) -4

7. In how many of the eight standard octants of \( xyz \)-space does the graph of \( z = e^{x+y} \) appear?

(A) One  (B) Two  (C) Three  (D) Four  (E) Eight

8. Suppose that the function \( f \) is defined on an interval by the formula \( f(x) = \sqrt{\tan^2 x - 1} \). If \( f \) is continuous, which of the following intervals could be its domain?

(A) \( \left( \frac{3\pi}{4}, \pi \right) \n
(B) \( \left( \frac{\pi}{4}, \frac{\pi}{2} \right) \n
(C) \( \left( \frac{\pi}{4}, \frac{3\pi}{4} \right) \n
(D) \( \left( -\frac{\pi}{4}, 0 \right) \n
(E) \( \left( -\frac{3\pi}{4}, -\frac{\pi}{4} \right) \n
GO ON TO THE NEXT PAGE.
9. \[ \int_{0}^{1} \frac{x}{2-x^2} \, dx = \]

(A) -\frac{1}{2}  \quad (B) \frac{5}{3}  \quad (C) \frac{\log 2 - e}{2}  \quad (D) -\frac{\log 2}{2}  \quad (E) \frac{\log 2}{2}

10. If \( f''(x) = f'(x) \) for all real \( x \), and if \( f(0) = 0 \) and \( f'(0) = -1 \), then \( f(x) = \)

(A) 1 - e^x  \quad (B) e^x - 1  \quad (C) e^{-x} - 1  \quad (D) e^{-x}  \quad (E) -e^x

11. If \( \phi(x, y, z) = x^2 + 2xy + xz^2 \), which of the following partial derivatives are identically zero?

I. \( \frac{\partial^2 \phi}{\partial y^2} \)

II. \( \frac{\partial^2 \phi}{\partial x \partial y} \)

III. \( \frac{\partial^2 \phi}{\partial z \partial y} \)

(A) III only  \quad (B) I and II only  \quad (C) I and III only  \quad (D) II and III only  \quad (E) I, II, and III

GO ON TO THE NEXT PAGE.
12. \[ \lim_{{x \to 0}} \frac{\sin 2x}{(1 + x)\log(1 + x)} = \]

(A) \(-2\) \hspace{1cm} (B) \(-\frac{1}{2}\) \hspace{1cm} (C) 0 \hspace{1cm} (D) \frac{1}{2} \hspace{1cm} (E) 2

13. \[ \lim_{{n \to \infty}} \int_1^n \frac{1}{x^n} \, dx = \]

(A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) \(\pi\) \hspace{1cm} (D) \(\pi\) \hspace{1cm} (E) \(+\infty\)

14. At a 15 percent annual inflation rate, the value of the dollar would decrease by approximately one-half every 5 years. At this inflation rate, in approximately how many years would the dollar be worth \(\frac{1}{1,000,000}\) of its present value?

(A) 25 \hspace{1cm} (B) 50 \hspace{1cm} (C) 75 \hspace{1cm} (D) 100 \hspace{1cm} (E) 125

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15. Let \( f(x) = \int_{1}^{x} \frac{1}{1 + t^2} \, dt \) for all real \( x \). An equation of the line tangent to the graph of \( f \) at the point \( (2, f(2)) \) is

(A) \( y - 1 = \frac{1}{5}(x - 2) \)  \hspace{1cm}  (B) \( y - \arctan 2 = \frac{1}{5}(x - 2) \)  \hspace{1cm}  (C) \( y - 1 = (\arctan 2)(x - 2) \)

(D) \( y - \arctan 2 + \frac{\pi}{4} = \frac{1}{5}(x - 2) \)  \hspace{1cm}  (E) \( y - \frac{\pi}{2} = (\arctan 2)(x - 2) \)

16. Let \( f(x) = e^{g(x)}h(x) \) and \( h'(x) = -g'(x)h(x) \) for all real \( x \). Which of the following must be true?

(A) \( f \) is a constant function.
(B) \( f \) is a linear nonconstant function.
(C) \( g \) is a constant function.
(D) \( g \) is a linear nonconstant function.
(E) None of the above

17. \( 1 - \sin^2\left(\arccos \frac{\pi}{12}\right) = \)

(A) \( \sqrt{\frac{1 - \cos \frac{\pi}{24}}{2}} \)  \hspace{1cm}  (B) \( \sqrt{\frac{1 - \cos \frac{\pi}{6}}{2}} \)  \hspace{1cm}  (C) \( \sqrt{\frac{1 + \cos \frac{\pi}{24}}{2}} \)  \hspace{1cm}  (D) \( \frac{\pi}{6} \)  \hspace{1cm}  (E) \( \frac{\pi^2}{144} \)

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18. If \( f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n} \) for all \( x \in (0, 1) \), then \( f''(x) = \)

(A) \( \sin x \)  
(B) \( \cos x \)  
(C) \( \frac{1}{1 + x^2} \)  
(D) \( \frac{-2x}{(1 + x^2)^2} \)  
(E) \( \frac{2x}{(1 - 2x)^2} \)

19. Which of the following is the general solution of the differential equation

\[
\frac{d^3y}{dt^3} - 3 \frac{d^2y}{dt^2} + 3 \frac{dy}{dt} - y = 0 \]

(A) \( c_1e^{t} + c_2te^{t} + c_3t^2e^{t} \)  
(B) \( c_1e^{-t} + c_2te^{-t} + c_3t^2e^{-t} \)  
(C) \( c_1e^{t} - c_2e^{-t} + c_3te^{t} \)  
(D) \( c_1e^{t} + c_2e^{2t} + c_3e^{3t} \)  
(E) \( c_1e^{2t} + c_2te^{-2t} \)

GO ON TO THE NEXT PAGE.
22. If \( b \) and \( c \) are elements in a group \( G \), and if \( b^4 = c^3 = e \), where \( e \) is the unit element of \( G \), then the inverse of \( b^2c^4e^2 \) must be

- (A) \( b^3c^2bc \)
- (B) \( b^4c^2b^2c \)
- (C) \( c^2b^4cb^2 \)
- (D) \( cb^2c^2b^4 \)
- (E) \( cbc^2b^3 \)

23. Let \( f \) be a real-valued function continuous on the closed interval \([0, 1]\) and differentiable on the open interval \((0, 1)\) with \( f(0) = 1 \) and \( f(1) = 0 \). Which of the following must be true?

I. There exists \( x \in (0, 1) \) such that \( f(x) = x \).
II. There exists \( x \in (0, 1) \) such that \( f'(x) = -1 \).
III. \( f(x) > 0 \) for all \( x \in [0, 1] \).

- (A) I only
- (B) II only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III

24. If \( A \) and \( B \) are events in a probability space such that \( 0 < P(A) = P(B) = P(A \cap B) < 1 \), which of the following CANNOT be true?

- (A) \( A \) and \( B \) are independent.
- (B) \( A \) is a proper subset of \( B \).
- (C) \( A \neq B \)
- (D) \( A \cap B = A \cup B \)
- (E) \( P(A)P(B) < P(A \cap B) \)

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25. Let $f$ be a real-valued function with domain $[0, 1]$. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all $x$ and $y$ in $[0, 1]$, which of the following must be true?

(A) $f$ is discontinuous at each point of $(0, 1)$.
(B) $f$ is not continuous on $(0, 1)$, but is discontinuous at only countably many points of $(0, 1)$.
(C) $f$ is continuous on $(0, 1)$, but is differentiable at only countably many points of $(0, 1)$.
(D) $f$ is continuous on $(0, 1)$, but may not be differentiable on $(0, 1)$.
(E) $f$ is differentiable on $(0, 1)$.

26. Let $i = (1, 0, 0)$, $j = (0, 1, 0)$, and $k = (0, 0, 1)$. The vectors $v_1$ and $v_2$ are orthogonal if $v_1 = i + j - k$ and $v_2 =$

(A) $i + j - k$
(B) $i - j + k$
(C) $i + k$
(D) $j - k$
(E) $i + j$

27. If the curve in the $yz$-plane with equation $z = f(y)$ is rotated around the $y$-axis, an equation of the resulting surface of revolution is

(A) $x^2 + z^2 = (f(y))^2$
(B) $x^2 + z^2 = f(y)$
(C) $x^2 + z^2 = |f(y)|$
(D) $y^2 + z^2 = |f(y)|$
(E) $y^2 + z^2 = |f(x)|^2$

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28. Let $A$ and $B$ be subspaces of a vector space $V$. Which of the following must be subspaces of $V$?

I. $A + B = \{a + b : a \in A \text{ and } b \in B\}$
II. $A \cup B$
III. $A \cap B$
IV. $\{x \in V : x \notin A\}$

(A) I and II only
(B) I and III only
(C) III and IV only
(D) I, II, and III only
(E) I, II, III, and IV

29. $\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2k}\right) =$

(A) 0  (B) 1  (C) 2  (D) 4  (E) $+\infty$

30. If $f(x_1, \ldots, x_n) = \sum_{1 \leq i < j \leq n} x_i x_j$, then $\frac{\partial f}{\partial x_n} =$

(A) $n!$  (B) $\sum_{1 \leq i < j < n} x_i x_j$  (C) $\sum_{1 \leq i < j < n} (x_i + x_j)$  (D) $\sum_{j=1}^{n} x_j$  (E) $\sum_{j=1}^{n-1} x_j$

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31. If \( f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } 0 \leq x \leq 1 \\ x - 1 & \text{for } 1 < x \leq 2, \end{cases} \)

then \( \int_0^2 f(x) \, dx \) is

(A) \( \frac{\pi}{2} \)

(B) \( \frac{\sqrt{2}}{2} \)

(C) \( \frac{1}{2} + \frac{\pi}{4} \)

(D) \( \frac{1}{2} + \frac{\pi}{2} \)

(E) undefined

32. Let \( R \) denote the field of real numbers, \( Q \) the field of rational numbers, and \( Z \) the ring of integers. Which of the following subsets \( F_i \) of \( R \), \( 1 \leq i \leq 4 \), are subfields of \( R \)?

\[
\begin{align*}
F_1 &= \{a/b: \ a, b \in Z \text{ and } b \text{ is odd}\} \\
F_2 &= \{a + b\sqrt{2}: \ a, b \in Z\} \\
F_3 &= \{a + b\sqrt{2}: \ a, b \in Q\} \\
F_4 &= \{a + b\sqrt{4/2}: \ a, b \in Q\}
\end{align*}
\]

(A) No \( F_i \) is a subfield of \( R \).

(B) \( F_3 \) only

(C) \( F_2 \) and \( F_3 \) only

(D) \( F_1, F_2, \) and \( F_3 \) only

(E) \( F_1, F_2, F_3, \) and \( F_4 \)

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33. If \( n \) apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

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<td>(A) ( \frac{1}{2} )</td>
<td>(B) ( \frac{1}{n} )</td>
<td>(C) ( \frac{1}{n!} )</td>
<td>(D) ( \frac{1}{2^n} )</td>
<td>(E) ( \left( \frac{1}{n} \right)^n )</td>
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34. \( \frac{d}{dx} \int_0^{x^2} e^{-t^2} \, dt = \)

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<td>(A) ( e^{-x^2} )</td>
<td>(B) ( 2e^{-x^2} )</td>
<td>(C) ( 2e^{-x^4} )</td>
<td>(D) ( x^2e^{-x^2} )</td>
<td>(E) ( 2xe^{-x^4} )</td>
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38. \[
\lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left[ \left( \frac{3i}{n} \right)^2 - \left( \frac{3i}{n} \right) \right] =
\]

(A) \(-\frac{1}{6}\) \hspace{1cm} (B) 0 \hspace{1cm} (C) 3 \hspace{1cm} (D) \frac{9}{2} \hspace{1cm} (E) \frac{31}{6}

39. For a real number \(x\), \(\log(1 + \sin 2\pi x)\) is not a real number if and only if \(x\) is

(A) an integer

(B) nonpositive

(C) equal to \(\frac{2n - 1}{2}\) for some integer \(n\)

(D) equal to \(\frac{4n - 1}{4}\) for some integer \(n\)

(E) any real number

40. If \(x\), \(y\), and \(z\) are selected independently and at random from the interval \([0, 1]\), then the probability that \(x \geq yz\) is

(A) \(\frac{3}{4}\) \hspace{1cm} (B) \(\frac{2}{3}\) \hspace{1cm} (C) \(\frac{1}{2}\) \hspace{1cm} (D) \(\frac{1}{3}\) \hspace{1cm} (E) \(\frac{1}{4}\)

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43. Let \( n \) be an integer greater than 1. Which of the following conditions guarantee that the equation
\[
x^n = \sum_{i=0}^{n-1} a_i x^i
\]
has at least one root in the interval \((0, 1)\)?

I. \( a_0 > 0 \) and \( \sum_{i=0}^{n-1} a_i < 1 \)
II. \( a_0 > 0 \) and \( \sum_{i=0}^{n-1} a_i > 1 \)
III. \( a_0 < 0 \) and \( \sum_{i=0}^{n-1} a_i > 1 \)

(A) None  
(B) I only  
(C) II only  
(D) III only  
(E) I and III

44. If \( x \) is a real number and \( P \) is a polynomial function, then \( \lim_{h \to 0} \frac{P(x + 3h) + P(x - 3h) - 2P(x)}{h^2} = \)

(A) 0  
(B) \( 6P'(x) \)  
(C) \( 3P''(x) \)  
(D) \( 9P''(x) \)  
(E) \( \infty \)

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