CHAPTER 1 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Find the sum of the roots of the equation $\sqrt{x-1} + \sqrt{2x-1} = x$.
   
   (A) 1  (B) 2  (C) 4  (D) 5  (E) 6

2. Determine the set of positive values of $x$ that satisfy the following inequality:
   
   $$\frac{1}{x} - \frac{1}{x-1} > \frac{1}{x-2}$$

   (A) $(0, 1) \cup (\sqrt{2}, 2)$  (B) $(0, \frac{1}{2}) \cup (1, 2)$  (C) $(\frac{1}{2}, 1) \cup (\sqrt{2}, 2\sqrt{2})$
   (D) $(0, \sqrt{2}) \cup (\frac{3}{2}, 2)$  (E) $(1, \sqrt{2}) \cup (2, 2\sqrt{2})$

3. Solve for $x$: $|x+1| - |x| + 2|x - 1| = 2x - 1$

   (A) $x = -\frac{1}{2}, 1$  (B) $x = -\frac{1}{2}, 2$  (C) $x = 1, 2$  (D) $x = -\frac{1}{2}, 1, 2$  (E) $x \geq 1$

4. Let $f$ be a function such that $f(n + 1) = 1 - [f(n)]^2$ for all nonnegative integers $n$. Which of the following correctly expresses $f(n + 2)$ in terms of $f(n)$?

   (A) $2[f(n)]^2$  (B) $2f(n) - 2[f(n)]^2$  (C) $2f(n) + 2[f(n)]^2$
   (D) $2[f(n)]^2 - [f(n)]^4$  (E) $2[f(n)]^2 + [f(n)]^4$

5. Let $f$ be a real-valued function whose inverse is given by the equation:

   $$f^{-1}(x) = x(1 + x^2) + (1 - x^2)$$

   What’s the value of $f(f^{-1}(f(2)))$?

   (A) $-2$  (B) $-1$  (C) $1$  (D) $2$  (E) $7$
6. Let \( f, g, \) and \( h \) be real-valued functions defined for all positive \( x \) such that:

\[
(f \circ g)(x) = (g \circ h)(x)
\]

If \( f(x) = x + 1 \) and \( g(x) = \sqrt{x} \), what is \( h(x) \)?

(A) \( x^2 - 1 \)  
(B) \( x - 1 \)  
(C) \( \sqrt{x} + 2 \)  
(D) \( x\sqrt{x} + 1 \)  
(E) \( 1 + (\sqrt{x} + 2)\sqrt{x} \)

7. What's the equation of all points in the \( xy \)-plane that are equidistant from the points \((-1, 4)\) and \((5, -2)\)?

(A) \( 2x - y = 3 \)  
(B) \( x - y = 1 \)  
(C) \( x + y = 3 \)  
(D) \( y = x^2 - 4x + 1 \)  
(E) \( (x - 2)^2 + (y - 1)^2 = 18 \)

8. Which of the following best describes the graph of the equation \( x^2 + y^2 - 2x + 4y + 5 = 0 \) in the \( xy \)-plane?

(A) circle  
(B) parabola  
(C) ellipse  
(D) line  
(E) point

9. Let \( C \) be the curve in the \( xy \)-plane described by the equation \( x^2 + 4y^2 = 16 \). If every point \((x, y)\) on \( C \) is replaced by the point \( \left( \frac{x}{2}, y \right) \), what is the area enclosed by the resulting curve?

(A) \( 8 \)  
(B) \( 4\pi \)  
(C) \( 16 \)  
(D) \( 8\pi \)  
(E) \( 16\pi \)

10. Every point on the parabola \( y = \sqrt{2x - 1} \) is equidistant from the \( y \)-axis and which of the following points?

(A) \( \left( \frac{1}{2}, 0 \right) \)  
(B) \( (1, 0) \)  
(C) \( \left( \frac{3}{2}, 0 \right) \)  
(D) \( (2, 0) \)  
(E) \( \left( \frac{5}{2}, 0 \right) \)

11. One of the foci of the hyperbola \( y^2 = \left( \frac{x}{a} \right)^2 + 1 \) is the point \((0, \sqrt{2})\). Find \( a \).

(A) \( \frac{1}{2\sqrt{2}} \)  
(B) \( \frac{1}{\sqrt{2}} \)  
(C) \( \frac{1}{2} \)  
(D) \( 1 \)  
(E) \( \sqrt{3} \)

12. Which one of the following polynomials \( p(x) \) has the property that \( \sqrt{3} - \sqrt{2} \) is a root of the equation \( p(x) = 0 \)?

(A) \( 2x^2 + 6x + 3 \)  
(B) \( x^3 - 2x + 6 \)  
(C) \( x^4 + 2x^2 - 3 \)  
(D) \( x^4 - 10x^2 + 1 \)  
(E) \( x^4 - 5x^2 + 6 \)
13. When the polynomial \( p(x) \) is divided by \( x - 1 \), it leaves a remainder of 1, and when \( p(x) \) is divided by \( x + 1 \), it leaves a remainder of \(-1\). Find the remainder when \( p(x) \) is divided by \( x^2 - 1 \).

(A) \(-1\)  \quad (B) 0  \quad (C) \( x \)  \quad (D) \(-x \)  \quad (E) \( 2x \)

14. Given that \( p(x) \) is a real polynomial of degree \( \leq 4 \) such that one can find five distinct solutions to the equation \( p(x) = 5 \), what is the value of \( p(5) \)?

(A) 0  \quad (B) 1  \quad (C) 4  \quad (D) 5  \quad (E) Cannot be determined from the information given

15. If the roots of the equation \( x^4 + Bx + 1 = 0 \) are the squares of the roots of the equation \( x^2 + bx + 1 = 0 \), which of the following expresses \( B \) in terms of \( b \)?

(A) \( 2 - b^2 \)  \quad (B) \( 1 - b^2 \)  \quad (C) \( b^2 - 1 \)  \quad (D) \( b^2 \)  \quad (E) \( b^2 - 2 \)

16. Find the largest value of \( b \) such that \( 1 + bi \) satisfies the equation

\[
x^3 - 3x^2 + 6x - 4 = 0
\]

given that every root of this equation has the form \( 1 + bi \) (where \( b \) is real).

(A) 1  \quad (B) \( \sqrt{2} \)  \quad (C) \( \sqrt{3} \)  \quad (D) 2  \quad (E) 3

17. If \( a \) and \( x \) are positive numbers and \( A = a^2 \), express the following in its simplest form in terms of \( x \):

\[
g(\log a)^2 + (\log a)
\]

(A) \( 2x \)  \quad (B) \( x^2 \)  \quad (C) \( \sqrt{x} \)  \quad (D) \( x^3 \)  \quad (E) \( x \sqrt{x} \)

18. What are the roots of the following equation?

\[
(\log x)^2 = 2 \log x
\]

(A) \( 1, e^2 \)  \quad (B) \( 1, \sqrt{e} \)  \quad (C) \( 1, e^2 \)  \quad (D) all \( x > 0 \)  \quad (E) all real \( x \)

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19. The hyperbolic sine function, denoted sinh, is defined by the equation:

\[
\sinh x = \frac{e^x - e^{-x}}{2}
\]

Find a formula for \(\sinh^{-1}x\).

(A) \(\log(1 - \sqrt{x^2 + 1})\)  
(B) \(\log(1 + \sqrt{x^2 + 1})\)  
(C) \(\log(x - \sqrt{x^2 + 1})\)  
(D) \(\log(x + \sqrt{x^2 + 1})\)  
(E) \(\log(\sqrt{x^2 + 1} - x)\)

20. The hyperbolic cosine function, denoted cosh, is defined by the equation:

\[
\cosh x = \frac{e^x + e^{-x}}{2}
\]

If the hyperbolic tangent function, tanh, is defined by

\[
\tanh x = \frac{\sinh x}{\cosh x}
\]

find a formula for \(\tanh^{-1}x\).

(A) \(\frac{1}{2} \log 1 - \frac{x - 1}{x + 1}\)  
(B) \(\frac{1}{1} \log 1 - \frac{x - 1}{x + 1}\)  
(C) \(\frac{1}{1} \log 1 - \frac{x + 1}{x - 1}\)  
(D) \(\frac{1}{1} \log 1 + \frac{x + 1}{x - 1}\)  
(E) \(\frac{1}{1} \log 1 + \frac{x + 1}{1 - x}\)

21. Let \(x\) be the real number such that \(\sin(\sin x) = \frac{1}{2}\) and \(2 < x < 3\). What's the value of \(\cos(\sin x)\)?

(A) \(-\sqrt{1 - \left(\frac{\pi}{6}\right)^2}\)  
(B) \(\sqrt{1 - \left(\frac{\pi}{3}\right)^2}\)  
(C) \(\sqrt{1 - \left(\frac{\pi}{6}\right)^2}\)  
(D) \(-\frac{\sqrt{3}}{2}\)  
(E) \(\frac{\sqrt{3}}{2}\)

22. Which one of the following is in the domain of the function \(f(x) = \log(\sin x)\)? (You may use the fact that 1111 is just slightly greater than 353.64 \(\times \pi\).)

(A) 11  
(B) 111  
(C) 1111  
(D) 11,111  
(E) None of these

23. Simplify \(\tan(2 \arcsin \frac{1}{3})\).

(A) \(\frac{2\sqrt{2}}{9}\)  
(B) \(\frac{\sqrt{2}}{3}\)  
(C) \(\frac{3}{4}\)  
(D) \(\frac{4\sqrt{2}}{7}\)  
(E) \(\frac{6}{5}\)
24. Simplify \( \sqrt{\csc^2 \left( \arccot \frac{\pi}{4} \right) - 1} \).

(A) -1  (B) 1  (C) \( \frac{\pi^2}{16} \)  (D) \( \frac{\pi}{4} \)  (E) \( \frac{\sqrt{\pi}}{2} \)

25. Determine the exact value of the sum \( \arctan 1 + \arctan 2 + \arctan 3 \).

(A) \( \frac{\pi}{2} \)  (B) \( \pi \)  (C) \( \frac{3\pi}{2} \)  (D) \( \frac{\pi}{4} - 1 \)  (E) \( \frac{\pi}{2} - 1 \)