CHAPTER 4 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Let \( y = f(x) \) be the solution of the equation

\[
\frac{dy}{dx} = \frac{x^2}{x^2 + 1}
\]

such that \( y = 0 \) when \( x = 0 \). What is the value of \( f(1) \)?

(A) \( 1 - \log 2 \)  (B) \( 1 + \log 2 \)  (C) \( 1 \)  (D) \( \log 2 \)  (E) \( \frac{1}{4} (4 - \pi) \)

2. A population of bacteria grows at a rate proportional to the number present. After two hours, the population has tripled. After two more hours elapse, the population will have increased by a factor of \( k \). What is the value of \( k \)?

(A) \( 6 \)  (B) \( 8 \)  (C) \( 9 \)  (D) \( 27 \)  (E) \( 81 \)

3. Every curve in a certain family, \( y = f(x, c) \), has the following property: the area of the region in the first quadrant bounded above by the curve from \( (0, 0) \) to \( (x, y) \) and bounded below by the \( x \)-axis is \( \frac{1}{3} \) the area of the rectangle with opposite vertices at \( (0, 0) \) and \( (x, y) \). Find \( f(x, c) \).

(A) \( cx^3 \)  (B) \( cx^3 + x \)  (C) \( cx^3 - x \)  (D) \( cx^2 \)  (E) \( c\sqrt{x} \)

4. Which of the following depicts integral curves of the differential equation \( \left( \frac{dy}{dx} \right)^2 = \frac{x}{y} \left( 2 \frac{dy}{dx} - \frac{x}{y} \right) \)?

(A)  
(B)  
(C)  
(D)  
(E)  

5. If \( a \) is a positive constant, let \( y = f(x) \) be the solution of the equation

\[
y'''' - ay'' + a^2 y' - a^3 y = 0
\]

such that \( f(0) = 1 \), \( f'(0) = 0 \), and \( f''(0) = a^2 \). How many positive values of \( x \) satisfy the equation \( f(x) = 0 \)?

(A) \( 0 \)  (B) \( 1 \)  (C) \( 2 \)  (D) \( 3 \)  (E) more than 3

178 ◆ CRACKING THE GRE MATHEMATICS SUBJECT TEST
6. Let \( g: \mathbb{R} \rightarrow \mathbb{R} \) be a differentiable and integrable function. The integral curve of the differential equation

\[
[y + g(x)] \, dx + [x - g(y)] \, dy = 0
\]

that passes through the point \((1, 1)\) must also pass through which of the following points?

(A) \((0, 0)\)  
(B) \((2, \frac{1}{2})\)  
(C) \((\frac{1}{2}, 2)\)  
(D) \((-1, -1)\)  
(E) \((0, 1)\)

7. Let \( y = f(x) \) be the solution of the equation

\[
\frac{dy}{dx} + \frac{y}{x} = \sin x
\]

such that \( f(\pi) = 1 \). What is the value of \( f(\frac{1}{2} \pi) \)?

(A) \( \frac{2}{\pi} - 1 \)  
(B) \( \frac{2}{\pi} \)  
(C) \( \frac{2}{\pi} + 1 \)  
(D) \( \frac{\pi}{2} \)  
(E) \( \frac{\pi}{2} + 1 \)

8. Let \( y = f(x) \) be the solution of the equation

\[
\frac{d^4 y}{dx^4} = \frac{d^2 y}{dx^2}
\]

such that \( f(0) = f'(0) = f''(0) = 0 \) and \( f'''(0) = -1 \). What is \( f(x) \)?

(A) \( x - \cosh x \)  
(B) \( x - \sinh x \)  
(C) \( x + \cosh x \)  
(D) \( x + \sinh x \)  
(E) \( \cosh x + \sinh x \)

9. What is the general solution of the differential equation

\[
2 \frac{d^3 x}{dt^3} + 3 \frac{d^2 x}{dt^2} + 3 \frac{dx}{dt} = 6?
\]

(A) \( x = 2t + c_1 e^t + c_2 e^{\frac{1}{2}t} + c_3 e^{3t} \)  
(B) \( x = 2 + c_1 e^t + c_2 e^{\frac{1}{2}t} + c_3 e^{3t} \)  
(C) \( x = t^2 + c_1 e^{\frac{1}{2}t} + c_2 e^{3t} \)  
(D) \( x = 2t + c_1 + c_2 e^{\frac{1}{2}t} + c_3 e^{3t} \)  
(E) \( x = 2 + c_1 + c_2 e^{\frac{1}{2}t} + c_3 e^{3t} \)

10. Given that the following differential equation has an integrating factor of the form \( \mu(x, y) = x^n y^r \), determine its general solution.

\[
(3xy^2 - 5y) \, dx + (2x^2y - 3x) \, dy = 0
\]

(A) \( x^y (\frac{1}{3} xy - 1) = c \)  
(B) \( x^y (xy - 1) = c \)  
(C) \( x^y (2xy - 1) = c \)

(D) \( x^y (\frac{1}{3} xy - 1) = c \)  
(E) \( x^y (2xy - 1) = c \)
11. At every point \((x, y)\) on a curve in the \(xy\)-plane, the slope is equal to:

\[
\frac{1 - 2xy}{x^2 + 3y^2 + 1}
\]

What is the equation of this curve, given that it passes through the point \((1, 1)\)?

(A) \(\frac{1}{3} x^3 + 3xy^2 + x + y - xy^2 = \frac{13}{3}\)  
(B) \(xy^2 + y^3 + x - y = 2\)
(C) \(\frac{1}{3} x^3 + 3xy^2 - x + y + xy^2 = \frac{13}{3}\)  
(D) \(x^2y + y^3 - x + y = 2\)
(E) \(x^2y^2 + xy^3 + x - y = 1\)

12. Find the general solution of the differential equation:

\[
\frac{dy}{dx} = \frac{x + y}{x}
\]

(A) \(e^{y/x} = cx\)  
(B) \(e^{y/x} = cy\)  
(C) \(e^{x/y} = cx\)  
(D) \(e^{x/y} = cy\)  
(E) \(e^{-x/y} = cx\)

13. Consider the family \(F\) of circles in the \(xy\)-plane, \((x - c)^2 + y^2 = c^2\), that are tangent to the \(y\)-axis at the origin. Which of the following gives the differential equation that is satisfied by the family of curves orthogonal to \(F\)?

(A) \(y' = \frac{x}{x - y}\)  
(B) \(y' = \frac{x}{y - x}\)  
(C) \(y' = \frac{xy}{x - y}\)  
(D) \(y' = \frac{2xy}{x^2 - y^2}\)  
(E) \(y' = \frac{2xy}{y^2 - x^2}\)

14. Let \(g(x, y)\) be the function defined for all \(x\) and all nonzero \(y\) such that the differential equation

\[
(sin\ xy)\ dx + g(x, y)\ dy = 0
\]

is exact and \(g(0, y) = 0\) for all \(y \neq 0\). What is \(g(x, 1)\)?

(A) \(\sin x + \cos x - 1\)  
(B) \(x \sin x + \cos x - 1\)  
(C) \(x \sin x - \cos x + 1\)  
(D) \(x \sin x + \cos x\)  
(E) \(\sin x - x \cos x + 1\)

15. If \(w = f(x, y)\) is a solution of the partial differential equation

\[
2\frac{\partial w}{\partial x} - 3\frac{\partial w}{\partial y} = 0
\]

then \(w\) could equal

(A) \((2x - 3y)^6\)  
(B) \(\sin[\log(3x - 2y)]\)  
(C) \(e^{\arctan(3x^2 y)}\)

(D) \([\arccos(2y - 3x)]^2\)  
(E) \(\sqrt{2x + 3y}\)