Abstract—In this paper, we investigate the energy efficiency of downlink transmissions in heterogeneous networks (HetNets). Our objective is to satisfy the rate requirements of users while optimizing the energy efficiency of the network. We employ the fractional frequency reuse (FFR) scheme to increase the energy efficiency of downlink transmissions and to eliminate outages for the cell-edge users. The problem is decomposed into sequentially solved subproblems in the frequency and power domains. Max-Min Fair scheduler is incorporated to satisfy rate requirements of users. We propose an algorithm that is based on the Levenberg-Marquardt method combined with Lagrange relaxation to solve the power control subproblem. In order to make the base stations further reduce intercell interference, interference pricing is applied. This scheme penalizes the utility of a base station with the interference it creates. The performance of the proposed algorithm is simulated in a Long Term Evolution (LTE) network. Our numerical results reveal that significant gains in terms of energy efficiency can be achieved with proposed algorithm. The outage probability is also reduced compared to the no power control and non-cooperative power control algorithms.

Index Terms—Energy efficiency, heterogeneous cellular networks, Levenberg-Marquardt algorithm, Long Term Evolution (LTE), power control, resource allocation, scheduling.

I. INTRODUCTION

With the proliferation of the mobile devices such as smartphones, tablets, and laptops, ubiquitous, fast, and reliable wireless connections are needed. A recent study in [2] estimates that the number of mobile devices will reach 10 billion by 2018. According to the same source, this will be accompanied with an increase in mobile data traffic volume reaching a compounded annual growth rate of 61 percent between 2013 and 2018. Motivated by the financial and ecological concerns, network operators are pursuing energy-efficient solutions to keep their energy consumption at reasonable levels while satisfying this demand. These solutions have been studied in the literature under the general description “Green Communication,” see, e.g., [1] and the references therein. In this paper, we study energy efficiency maximization of heterogeneous networks (HetNets) with rate constraints using the fractional frequency reuse (FFR) scheme and propose a Levenberg-Marquardt method-based approach to solve the resource allocation problem. The dual decomposition techniques are used for minimum rate constraints of users.

Resource allocation problem is an important research area in the field of game theory with wide applications in wireless communication networks, see, e.g., [2]–[7]. This problem arises when multiple agents or players with conflicting interests compete for the same resources. For example, resource allocation in HetNets reflects a case such that each base station tries to maximize its utility by increasing its transmit power, while this creates an excessive interference in the system for the neighboring cells. In this paper, we employ interference pricing, which is introduced in [3]–[5]. Each base station is penalized according to the interference it creates. The penalty factor increases if the interference leads to outage of users in other sectors. The pricing algorithm defines the utility as the energy efficiency of a sector minus the created interference to other sectors. This makes Macrocell Base Stations (MBSs) decrease their transmission power, and the energy efficiency of the network is further improved. In addition, due to the drop of the interference level, the outage probability of users decreases.

Energy-efficient wireless networks have been widely investigated in the literature, see, e.g., [8]–[12] and the references therein. An iterative energy-efficient link adaptation framework is presented in [8]. This work considers the circuit power and transmit power jointly to determine the optimal solution that maximizes the energy efficiency in a non-cooperative manner. A similar link adaptation algorithm that considers interference to determine the transmit power levels is proposed in [9]. Both of these works employ universal frequency reuse (FR) and do not consider HetNet deployments. Our proposed algorithms determine the optimal MBS transmit power levels considering picocell transmissions. In advanced networks such as Long Term Evolution (LTE), FFR has been identified as an efficient, and at the same time, a low-complexity method to mitigate intercell interference [13]. Although the FFR scheme has been studied in the literature for single-layer networks, see, e.g., [14] and [15], its performance has not been investigated to its potential in multi-tier networks. Recently, a novel multi-tier FFR scheme has been proposed in [16] and [17]. In [16], the performance of this multi-tier FFR scheme is investigated in terms of throughput and outage probability. The spectral efficiency of this FFR scheme is investigated for constant cell-
center boundaries in [17]. In this paper, we will also employ this FFR scheme, but with dynamic cell-center boundaries to maximize the energy efficiency.

The problems of energy efficiency maximization and satisfying rate constraints do not always coincide. On the contrary, these problems contradict with each other. In order to satisfy rate requirement of users, base stations may need to increase their transmission power to the energy inefficient levels. In the literature, these two problems are studied in [8], [11], [12]. However, none of these works considers intercell interference conditions that increase the complexity of the problem. In this paper, we address this problem in a multicell environment in which the intercell interference is very critical. We have previously proposed a gradient ascent based power allocation method to solve the energy efficiency maximization problem without considering rate constraints of users in [18]. In this paper, we will first propose a Levenberg-Marquardt method-based approach to provide faster convergence and extend the framework to consider Quality of Service (QoS) constraints.

The remainder of this paper is organized as follows. Section II introduces the system model and base station power consumption models. Section III formulates the energy-efficient resource allocation problem in HetNets. The proposed solution methods are presented in Section IV. Simulation results are presented to evaluate the performance gains in Section V and concluding remarks are presented in Section VI.

II. SYSTEM MODEL

In this section, we first present the system model and the multi-tier FFR scheme. We then study a linearized base station power consumption model that will be later used to formulate the energy-efficient resource allocation problem in Section III.

Consider a cellular layout of 19 hexagonal cells as depicted in Fig. 1. As is commonly done in the literature, we will use this layout together with the wrap-around technique, e.g., [18], [19], to model a cellular network of infinite dimensions. Assume that macrocells employ 3-sector antennas and pico-eNBs have omni-directional antennas. The spectrum allocation in the macrocell and picocell tiers is fundamental to determine the interference conditions and user rates at each tier. In densely deployed networks, intercell interference becomes a significant problem, limiting the system performance. The FFR scheme provides a solution by assigning the frequency resources in a coordinated manner such that high interference conditions are avoided. In this paper, we employ the FFR scheme depicted in Fig. 1. We refer to the macrocell-associated users as MUEs and picocell-associated users as PUEs. The MUEs can be categorized into cell-center and cell-edge users depending on factors such as their received reference power, path loss, or traffic load within the sector [18]. In this process, the variable \( r_{th,s} \) determines the cell-center region boundary of sector \( s \). In Section IV-A, we elaborate on the selection process of \( r_{th,s} \) in detail. In terms of spectrum allocation, the MBSs can allocate subcarriers in subband \( A \) to their cell-center users, while the cell-edge MUEs are assigned to either one of the remaining three subbands. For example, in Sector 1, subband \( B \) is allocated to the cell-edge users. Pico-eNBs in the cell-center region are assigned to orthogonal channels with respect to the subbands that MBS operates at. This is to reduce the cross-tier interference as the cell-center pico-eNBs are close to the MBS. For example, cell-center pico-eNBs operate at subbands \( C \) and \( D \). Note that, in a typical LTE deployment, the MBSs and pico-eNBs have around 16 dB transmit power difference [20]. This would have detrimental effects for the cell-center PUEs in the downlink if they were not assigned to orthogonal channels with the MBS. For the cell-edge pico-eNBs, subband \( A \) can also be reused in order to increase the throughput. For example, cell-edge pico-eNBs operate at subbands \( A, C, \) and \( D \) in Sector 1. Note that the multi-tier FFR scheme depicted in Fig. 1 favors the cell-center PUEs and cell-edge MUEs that would have been exposed to severe interference if, for example, universal FR had been employed. This also enables operation at a lower interference region such that the energy efficiency of the downlink transmissions can be significantly increased. This fact is demonstrated later in Section V through numerical simulations. We test our findings in an LTE scenario. In LTE, the smallest scheduling granularity is per resource block (RB) in which an RB consists of 12 subcarriers [20].

In this paper, we employ constant power allocation across subbands. Let \( N_A, N_B, N_C, \) and \( N_D \) denote the total number of subcarriers in subbands \( A, B, C, \) and \( D \), respectively. We introduce two variables, \( \varepsilon \) and \( \beta \), to determine transmission power levels of MBSs. The parameter \( \varepsilon \) denotes the ratio of the downlink transmissions of cell-edge MUEs to cell-center MUEs. This parameter is introduced to favor either cell-
center or cell-edge users that do not satisfy their minimum rate requirements. The variable $\beta$ scales the MBS transmissions. This parameter introduces energy savings into the system. The corresponding spectrum and power utilization scheme is illustrated in Fig. 2. The downlink transmission per subcarrier of an MBS for cell-center MUEs in Sector 1, $P_M$, is given by

$$P_M = \frac{\beta P_{\text{max},M}}{N_A + \varepsilon N_B},$$  \hspace{1cm} (1)$$

where $P_{\text{max},M}$ is the maximum transmit power of an MBS. Similarly, for the cell-edge MUEs, the MBS transmit power per subcarrier is $\varepsilon P_M$. It is straightforward to obtain the expressions for Sectors 2 and 3 by replacing $N_B$ with $N_C$ and $N_D$, respectively. In the picocell tier, the downlink transmission per subcarrier can be obtained by simply replacing the values of $P_M$, $\varepsilon P_M$, where $\varepsilon$ and $P_M$ represent the respective subband values.

The maximum transmit power of a pico-eNB, $P_P^c$, in Sector 1 can be expressed as

$$P_P^c = \frac{P_{\text{max},P}}{N_C + N_D}$$
and

$$P_P^p = \frac{P_{\text{max},P}}{N_A + N_C + N_D},$$ \hspace{1cm} (2)$$

where $P_P^c$ and $P_P^p$ denote the transmit power of a cell-center and a cell-edge pico-eNB per subcarrier, respectively. The maximum transmit power of a pico-eNB represented by $P_{\text{max},P}$. For pico-eNBs in Sectors 2 and 3, a similar expression can be obtained by simply replacing the values of $N_C$ and $N_D$ with the respective subband values.

The aggregate throughput of a sector $s$ can be expressed as

$$R_s(\varepsilon_s, \beta_s) = \sum_{k \in \mathcal{K}_{s}^{M}, n \in \mathcal{N}_{M,k}} r_1^{(k,n)} + \sum_{k \in \mathcal{K}_{s}^{E}, n \in \mathcal{N}_{M,k}} r_2^{(k,n)}$$
$$+ \sum_{k \in \mathcal{K}_{s}^{C}, n \in \mathcal{N}_{P,k}} r_3^{(k,n)} + \sum_{k \in \mathcal{K}_{s}^{E}, n \in \mathcal{N}_{P,k}} r_4^{(k,n)},$$ \hspace{1cm} (3)$$

where $\mathcal{K}_{s}^{M}, \mathcal{K}_{s}^{E}, \mathcal{K}_{s}^{C}, \mathcal{K}_{s}^{E}$, and $\mathcal{K}_{s}^{E}$ denote the sets of MUEs in the cell-center and cell-edge regions, and the sets of PUEs in the cell-center and cell-edge regions, respectively. The subcarriers assigned to an MUE $k$ and a PUE $m$ are denoted by $N_{M,k}$ and $N_{P,k}$, respectively. The throughput of a cell-edge PUE $k$ is studied in two segments, one to denote the throughput of users assigned to the subcarriers in Band $j$, the throughput of the cell-center, and $r_4^{(k,n)}$ is the throughput of the cell-edge PUEs. Note that for a given user $k$, only one of the $r_j^{(k,n)}$, $j = 1, 2, 3, 4$, is employed in (3). The channel gain between user $k$ and MBS $m$ and between user $k$ and pico-eNB $p$ on subcarrier $n$ are denoted by $g_{k,m}^{(n)}$ and $g_{k,p}^{(n)}$, respectively. The interference incurred by user $k$ on subcarrier $n$ is denoted by $I_j^{(k,n)}$ in (4), where $j = 1, 2, 3, 4$. The value of $I_j^{(k,n)}$ can be calculated as follows

$$I_j^{(k,n)} = \sum_{m \in \mathcal{B}_{m}^{(n)}} P_m^{(n)} g_{k,m}^{(n)} + \sum_{p \in \mathcal{B}_{p}^{(n)}} P_p^{(n)} g_{k,p}^{(n)} \text{ for } j = 1, 2,$$
$$I_j^{(k,n)} = \sum_{m' \in \mathcal{B}_{m}^{(n)}} P_m^{(n)} g_{k,m'}^{(n)} + \sum_{p' \in \mathcal{B}_{p}^{(n)}} P_p^{(n)} g_{k,p'}^{(n)} \text{ for } j = 3, 4$$ \hspace{1cm} (5)$$

where $\mathcal{B}_{m}^{(n)}$ and $\mathcal{B}_{p}^{(n)}$ denote the sets of MBSs and pico-eNBs that transmit on subcarrier $n$, respectively. The sets of interfering MBSs and pico-eNBs differ based on the sector, associated tier, and whether the UE is in the cell-center or cell-edge region. The transmit power of an MBS and a pico-eNB are represented by $P_m^{(n)}$ and $P_p^{(n)}$, respectively. Likewise, these power levels depend on the sector and transmission band. The thermal noise power per Hz and bandwidth of a subcarrier are represented by $N_0$ and $\Delta f$, $\Delta f = 15$ kHz for LTE systems, respectively [20]. Notice that for the cell-edge PUEs that are assigned to Band $A$, the interference term in $r_4^{(k,n)}$ includes the intra-sector interference contributions from the MBS to a cell-edge PUE $k$ in the same sector, that is $P_M g_{k,m}^{(n)}$. Including these terms enables us to balance the cross-tier interference while maximizing the sector energy efficiency. In the proposed algorithm, the expressions in (4) are used to obtain the derivatives of variables such as $P_M$ with respect to $\varepsilon_s$ and $\beta_s$.

Several base station power consumption models are proposed in the literature, see, e.g., [21]–[23]. These models help characterize the power consumption of different equipment such as the baseband unit, radio frequency transceiver, power amplifier, power supply unit, and cooling devices. In this paper, as in [18], we consider the model in [21]. The power consumption at an MBS is given by

$$P_{\text{MBS}} = \frac{N_T^{\text{RX}} (P_{0,M} + \Delta_M \beta P_{\text{max},M})}{N_T^{\text{RX}} P_{\text{sleep},M}},$$ \hspace{1cm} (6)$$

where $P_{\text{MBS}}$ and $P_{\text{MBS}}^{\text{TX}}$ are the total power consumption at an MBS and RF transmit power, respectively. $N_T^{\text{RX}}$ is the number of transceiver chains and $P_{0,M}$ is the power consumption at the minimum non-zero output power at an MBS. The slope of the load-dependent power consumption at an MBS is denoted by $\Delta_M$. When an MBS does not transmit, it is considered to be in the sleep mode and its power consumption is captured in $P_{\text{sleep},M}$. In this model, the total power consumption depends on the transmit power or the load. Therefore, it is referred to as the load-dependent power consumption model [21]. Similarly,
the power consumption of a pico-eNB is given by

\[ P_{\text{Pico}} = \begin{cases} N_{T_{\text{RX}}}^P (P_0 + \Delta_P P_{T_{\text{TX}}}), & \text{if } 0 < P_{T_{\text{TX}}}^P \leq P_{\text{max}}^P, \\ N_{T_{\text{RX}}}^P P_{\text{sleep}}, & \text{if } P_{T_{\text{TX}}}^P = 0, \end{cases} \]

(7)

where \( P_{\text{Pico}}, P_0, P_{\text{sleep}}, \) and \( P_{T_{\text{TX}}}^P \) denote the total power consumption, power consumption at the minimum non-zero output power, power consumption in sleep mode, and RF transmit power at a pico-eNB, respectively. The number of transceiver chains at the pico-eNBs is denoted by \( N_{T_{\text{RX}}} \). The slope of the load-dependent power consumption at pico-eNB is denoted by \( \Delta_P \). In Table I, we present the values of the linearized power consumption model parameters for MBSs and pico-eNBs. Using (6) and (7), the power consumed in sector \( s \) can be expressed as

\[ \psi_s(\beta_s) = P_{\text{MBS}} + N_{\text{pico}} P_{\text{Pico}}, \]

(8)

where \( N_{\text{pico}} \) is the number of pico-eNBs in sector \( s \).

Using the aggregate throughput and power consumption expressions in (3) and (8), the energy efficiency of sector \( s \), in bits/Joule, is defined as

\[ \eta_s(\varepsilon_s, \beta_s) = \frac{R_s(\varepsilon_s, \beta_s)}{\psi_s(\beta_s)}. \]

(9)

Note that the variables \((\varepsilon_s, \beta_s)\) determine the throughput of the cell-center and cell-edge MUEs, and cell-edge PUEs. The MBS transmissions on Band 4 determine the cross-tier interference for the cell-edge PUEs in the same sector as well as the intercell interference. However, as the downlink transmissions of the MBS and cell-center pico-eNBs in the same sector are orthogonal, these variables do not affect the throughput of cell-center PUEs in the same sector. In the next section, we formulate the energy efficiency maximization problem.

III. ENERGY-EFFICIENT RESOURCE ALLOCATION FRAMEWORK

In this section, we develop the framework for a utility-based resource allocation in which the objective is to maximize the energy efficiency while satisfying the rate requirement of MUEs. The energy efficiency maximization problem can be formulated as

\[
\begin{align*}
\max_{\mathbf{x}} & \quad \sum_{s \in S} \eta_s(\mathbf{x}) \\
\text{s.t.} \quad & \sum_{n \in \mathcal{N}_{M,s}} r_{1}^{(k,n)} \geq R_{\text{min},k}, \quad \text{for all } k \in \mathcal{K}_{M,s}, s \in S \\
& \sum_{n \in \mathcal{N}_{M,s}} r_{2}^{(k,n)} \geq R_{\text{min},k}, \quad \text{for all } k \in \mathcal{K}_{M,s}, s \in S \\
& \varepsilon \geq 0 \quad \text{and} \quad 0 \leq \beta \leq 1,
\end{align*}
\]

(10)

where \( R_{\text{min},k} \) is the minimum rate constraint of user \( k \). Throughout the rest of the paper, the vector \( \mathbf{x} \), will be used for the \((\varepsilon, \beta)\) couple and the vector \( \mathbf{x} \) will be used for the vector couple \((\varepsilon, \beta)\).

The Lagrangian of this problem can be written as

\[
\mathcal{L}(\mathbf{x}, \lambda, \nu, \tau, \rho) = \sum_{s \in S} \eta_s(\mathbf{x}) - \sum_{s \in S} \sum_{k \in \mathcal{K}_{M,s}} \lambda_{k,s}(R_{\text{min},k} - \sum_{n \in \mathcal{N}_{M,k}} r_{1}^{(k,n)}) - \sum_{s \in S} \sum_{k \in \mathcal{K}_{M,s}} \lambda_{k,s}(R_{\text{min},k} - \sum_{n \in \mathcal{N}_{M,k}} r_{2}^{(k,n)}) + \sum_{s \in S} (\nu_s \beta_s + \tau_s (1 - \beta_s) + \rho_s \varepsilon_s),
\]

(11)

where \( \lambda_{k,s}, \nu_s, \tau_s, \) and \( \rho_s \) denote the Lagrange multiplier associated with the rate constraint of MUE \( k \) in sector \( s \), the Lagrange multiplier associated with lower and upper bound on \( \beta_s \), and the lower bound on \( \varepsilon_s \), respectively. Then, the KKT conditions of the problem can be written as

\[
\nabla_x \mathcal{L}(\mathbf{x}^*, \lambda^*, \nu^*, \tau^*, \rho^*) = 0, \forall s \in S
\]

(12a)

\[
\lambda_{k,s}^* (R_{\text{min},k} - \sum_{n \in \mathcal{N}_{M,k}} r_{1}^{(k,n)}) = 0
\]

(12b)

\[
\lambda_{k,s}^* (R_{\text{min},k} - \sum_{n \in \mathcal{N}_{M,k}} r_{2}^{(k,n)}) = 0
\]

(12c)

\[
\rho_s^* \varepsilon_s^* = 0, \quad \nu_s^* \beta_s^* = 0, \quad \tau_s^* (1 - \beta_s^*) = 0, \quad \forall s \in S
\]

(12d)

The objective function of this problem is non-convex. Therefore, exhaustive search is required over all frequency and power domains to obtain the optimum solution. In [18], we showed that the energy efficiency function of a sector is quasiconcave when the interference conditions are constant. To benefit from that property, we divide this problem into \( |S| \) subproblems such that each sector maximizes its own energy efficiency while satisfying the rate requirements of its users, where the number \(|S|\) corresponds to the number of sectors in the network. During the maximization problem the received interference is assumed to be constant. Therefore, by using convex optimization techniques, the optimal \( \beta_s \) and \( \varepsilon_s \) that maximize the energy efficiency of the sector \( s \) and satisfy the rate constraints of users can be obtained. Due to the fact that each MBS tries to satisfy the rate requirement of its users, the transmission power of the MBSs may increase imprudently. This increases not only the reduction of the energy efficiency of the network, but also augments the intercell interference, and elevates the outage probabilities of the users in the other sectors. Therefore, we study a pricing mechanism to account for the interference caused to the users associated with other base stations. This pricing mechanism reduces the intercell interference by giving incentives to MBSs for decreasing their transmit powers.

The pricing function penalizes the utility of an MBS based
on the interference it creates. If the interference leads to outage of
of the users in other sector, the penalty factor increases. Let
\( \theta_s(x_s) \) denote the pricing function of sector \( s \). The energy
efficiency maximization problem with pricing function per
sector can be formulated as

\[
\max_{x_s} \; \eta_s(x_s) - \theta_s(x_s)
\]

s.t.
\[
\sum_{n \in N_{M,k}} x_1^{(k,n)} \geq R_{\min,k}, \quad \text{for all } k \in K_{M,s}^C
\]
\[
\sum_{n \in N_{M,k}} x_2^{(k,n)} \geq R_{\min,k}, \quad \text{for all } k \in K_{M,s}^E
\]
\[
\varepsilon_s \geq 1, \quad \text{and} \quad 0 \leq \beta_s \leq 1,
\]

where the objective is to maximize the energy efficiency
minus the pricing function. Several pricing functions have been
proposed in the literature. For example, in [6] and [7], the
authors propose to use \( \theta_s(x_s) = c_s \beta_s P_{\text{max}}^s \), where \( c_s \) is a constant. The cost function proposed in [6] and [7] penalizes
the utility of an MBS with the total amount of power it
transmits. In this paper, we pursue an alternative approach
and penalize the interference that an MBS creates. This type of
pricing function was first proposed in [3]–[5]. In this approach,
the pricing function is defined as

\[
\theta_s(x) = \sum_{n \in N_m} p_m^{(n)} \sum_{l \notin k, l \in K_{s}^{(n)} \setminus \{k\}} \frac{\partial \eta_s}{\partial p_m^{(n)}},
\]

where the transmit power of MBS \( m \) on subcarrier \( n \) is
denoted by \( p_m^{(n)} \). The set of subcarriers that MBS \( m \) allocates
for the cell-center and cell-edge regions is denoted by \( N_m \).
Let \( K_{s}^{(n)} \) denote the set of users that are assigned to subcarrier
\( n \). Then, the set of users that MBS \( m \) interferes on subcarrier
\( n \) are represented by \( l \in K_{s}^{(n)} \). Let a user \( l \) be in sector \( s' \), then
the energy efficiency of the sector \( s' \) is denoted by \( \eta_{s'}. \) The term \( \frac{\partial \eta_{s'}}{\partial p_m^{(n)}} \) denotes the derivative of energy efficiency
of sector \( s' \) with respect to the transmit power of MBS \( m \)
of sector \( s \). Thus, the penalty function (14) characterizes the
marginal change in the utility of a neighboring sector \( s' \) per
unit power change in MBS \( m \) of sector \( s \).

Since the downlink transmissions of an MBS \( m \) in sector \( s \),
\( p_m^{(n)} \), are characterized by \( \varepsilon_s \) and \( \beta_s \), we need to adopt (14).
In addition, we need to include the effect of interference to
the rate constraints of users in other sectors. Therefore,
the pricing function can be written as

\[
\theta_s(x_s) = x_s^T \sum_{s' \in S} \sum_{s' \neq s} \left( \nabla_{x_s} \eta_{s'}(x) + \sum_{k \in K_{C,s'}} \sum_{n \in N_{M,k}} \nabla_{x_s} x_1^{(k,n)} \right)
+ \sum_{k \in K_{M,s'}} \sum_{n \in N_{M,k}} \nabla_{x_s} x_2^{(k,n)}
\]

Hence, the pricing function in (15) reflects the marginal costs
of the variables \( \varepsilon \) and \( \beta \). The detailed expressions of the terms
in (15) are given in the Appendix.

Thus, the Lagrangian of the problem in (13) can be written
as

\[
\mathcal{L}(x_s, \lambda, \nu_s, \tau_s, \rho_s) = \eta_s(x_s) - \theta_s(x_s)
\]

\[
- \sum_{k \in K_{C,s}^C} \lambda_{k,s}(R_{\min,k} - \sum_{n \in N_{M,k}} x_1^{(k,n)})
\]

\[
- \sum_{k \in K_{M,s}^E} \lambda_{k,s}(R_{\min,k} - \sum_{n \in N_{M,k}} x_2^{(k,n)})
+ \nu_s \beta_s + \tau_s (1 - \beta_s) + \rho_s \varepsilon_s.
\]

For simplicity, we will use \( \mathcal{L}_s \) for the \( \mathcal{L}(x_s, \lambda, \nu_s, \tau_s, \rho_s) \)
throughout the rest of the paper due to space considerations.

In LTE standards, the X2 interface provides a fast and
reliable backhaul link between base stations [20]. In this paper,
we use this interface for two reasons: First, pico-eNBs send
the channel state information for their users to MBS where
these are processed. Second, for the pricing method, the same
interface distributes the interference prices between MBSs.

IV. PROPOSED SOLUTION

Our formulation in (13) enables us to develop an energy-
efficient resource allocation algorithm. Similar resource
allocation algorithms have been studied in the literature, see, e.g.,
[8], [9], and Chapter 3 of [2]. In our proposed algorithm, we
decouple the main problem into three sub-problems. First, we
start with setting the cell-center boundaries such that the cell-
center and cell-edge MUEs are selected and the corresponding
information is sent to the pico-eNBs, identifying their regions
and subbands. Then, we solve the frequency allocation and
power control problems consecutively. After each MBS solves
its power allocation subproblem, the interference prices and
power levels are distributed among the network. By using this
information, each MBS resolves the frequency allocation and
power control problems. In the sequel, we discuss each stage
of the proposed algorithm in detail.

A. The Cell-Center Region Boundaries

The first stage of the proposed algorithm determines the
cell-center region boundary of each sector. It is shown in [17]
that over 10% throughput improvement can be achieved by
proper selection of the constant cell-center region boundaries.
Further improvements can be obtained by selecting the bound-
daries dynamically in each sector. For single-layer networks,
the cell-center boundary categorizes its users into the cell-center
and cell-edge MUEs. MUEs can typically be distinguished
through base station and users, whereas the RSRQ measurement gives
the ratio of the reference signal to the interference. Since it
is shown in [24] that the performance of both schemes are
similar, in this paper, we also distinguish users into cell-center
and cell-edge users based on the RSRP measurements. If the
RSRP of a user is higher than a threshold, it is considered to
be in the cell-center region, and vice versa.
For two-tier networks, the cell-center boundary also determines the available RBs for each pico-eNB. For example, consider the multi-tier FFR scheme depicted in Figure 1. If a pico-eNB is located in the cell-center region, the number of available RBs for this pico-eNB reduces. This frequency allocation alleviates the cross-tier interference between the cell-center MUEs and cell-edge PUEs. On the other hand, if a pico-eNB is located in the cell-edge region, all subbands, except the subband which is used by cell-edge MUEs, are available to this pico-eNB. Therefore, when a pico-eNB is located in the cell-center region, the available RBs and correspondingly the total throughput of the pico-eNB significantly decreases.

In two-tier networks, due to the high power difference of the reference signal between the MBSs and pico-eNBs, PUEs are typically located close to the pico-eNB [25]. For this reason, PUEs typically have low path loss values, or equivalently, high channel gains to their serving pico-eNBs. When the number of subbands available to the pico-eNB is decreased, it can be expected that the total throughput of PUEs will be reduced. However, due to the reduction in the cross-tier interference, the signal-to-interference-plus-noise ratio (SINR) of the cell-center MUEs increases subsequently. In [18], we proposed two different cell-center selection algorithms. The first algorithm is proposed to maximize the throughput and the energy efficiency of the network at the cost of the fairness. On the other hand, the second one maximizes the fairness among users at the cost of throughput. In the first algorithm, the MUE that is closest to the MeNB is selected to be in the cell-center region, while the rest of the MUEs are in the cell-edge region. The first algorithm may lead to the starvation of some users, due to the fact that resources are not shared fairly among cell-center and cell-edge users. Even some users do not get any resources, when the network is crowded. Therefore, this cell-center selection algorithm is not suitable for a network where users have rate requirements. On the other hand, the second cell-center selection algorithm chooses the cell-center radius in a way that the number of resource blocks and users will be proportional. This algorithm performs well when the rate requirement of users are similar. However, when the rate requirements of users vary, the performance of this algorithm drops. Therefore, we propose another cell-center selection algorithm in this paper to include the rate constraints of users in the cell-center radius selection process. The proposed algorithm divides the cell into cell-center and cell-edge regions such that the summation of the demand of the users in these two regions will be proportional with the number of resource blocks assigned to these regions. We refer to this algorithm as Cell-Center Selection Algorithm 1 (CCSA1). Note that, when the rate requirement of users is same for all users, the performance of this algorithm is not different than the fair cell-center radius selection algorithm in [18].

B. Frequency Assignment Problem

In the second stage of the problem, we determine the frequency assignments. There are many scheduling methods discussed in the literature, see, e.g., [20], [26]. Each scheduler has an efficiency and fairness tradeoff. In general, both of these utilities cannot be increased at the same time. In the LTE radio protocol stack, scheduling is handled by the medium access control (MAC) layer [20]. The scheduler in each base station is responsible for the distribution of frequency resources and this is left to the network operator for implementation. In the proposed algorithm, we investigate the max-min fair (MMF) scheduler, that is studied in detail in [27]. We study this scheduler to share resources fairly among users and decrease the probability of outages, and satisfy the rate requirements of users.

**MMF Scheduler:** In the MMF scheduler, RBs are distributed among users to maximize the minimum throughput. Note that, we run this algorithm twice in the MBSs, one for the cell-center MUEs and one for the cell-edge users, due to the fact that these users do not compete for the same RB sets. The proposed scheduler is the adoption of the algorithm that has been described in [27]. The primary difference between these two algorithms is the scheduler granularity. The one described in [27] allocates subchannels to users, however the smallest granularity in LTE standard is RB. Therefore we need to adopt the algorithm in [27] to work at resource block level. In addition, in our work, we try to satisfy the rate requirement of all users in the sectors. Maximizing the minimum throughput may lead to outages of the users who have large rate demands, and users who have small rate demands may end up with extra resources. This needs to be considered during the resource allocation process. In the proposed scheduling algorithm, each user is assigned the RB with the best average channel gain. We remove the user from the user set and the resource block from the resource block set. This process continues until one of the sets is empty. If the user set is empty first, then the users are sorted according to the difference between the rate requirement and the actual rate of the user with current assignment. Then the resource block that has the best channel is assigned to the first user and users are resorted according to updated rates. This process continues until all resource blocks are assigned.

C. Power Control Problem

The third stage of the proposed solution determines the power assignments such that the optimal power levels are assigned to each subband in order to maximize the energy efficiency and satisfy the rate requirements in the sector. Given the frequency assignments from the previous stage, it remains to solve this power control problem. First, we make the following observations: For a constant \( \varepsilon \), increasing \( \beta \) increases both the total throughput of MUEs and power consumption of an MBS, while it decreases the throughput of cell-edge PUEs as it creates more interference. On the other hand, for a constant \( \beta \), increasing \( \varepsilon \) reduces the throughput of cell-center MUEs, while it increases the throughput of cell-edge MUEs and cell-edge PUEs that are using Band A. In this paper, we try to exploit these tradeoffs by optimizing \( \varepsilon \) and \( \beta \).

The Levenberg-Marquardt method is a modification to the Newton method. The Newton method premultiplies the gradient ascent direction by the inverse of the Hessian matrix. The motivation of the Newton method is to find a suitable direction
based on the quadratic approximation of a function, whereas the gradient ascent method seeks to find a linear approximation of a function. Consider the Lagrangian function in (20). Its quadratic approximation evaluated at \( y_s^{(l)} = (\varepsilon_s^{(l)}, \beta_s^{(l)})^T \) can be expressed as
\[
g(y) = L_s + \nabla L_s^T(y - y_s^{(l)}) + \frac{1}{2}(y - y_s^{(l)})^T \nabla^2 L_s(y - y_s^{(l)}),
\]
where \( \nabla^2 L_s \) is the Hessian matrix of \( L \) evaluated at \( y_s^{(l)} \). Note that we are going to use \( y_s^{(l)} \) for \( (\varepsilon_s^{(l)}, \beta_s^{(l)}) \) pair for the Newton iteration \( l \) and the parameter \( x_s^{(l)} \) will be used for the same pair at time instant \( t \). The parameter updates that maximize \( g(y) \) are given by
\[
y_s^{(l+1)} = y_s^{(l)} - \mu_l(\nabla^2 L_s^{(l)})^{-1}\nabla L_s^{(l)},
\]
where the Newton search direction is \( d_s^{LM} = -\nabla^2 L_s^{(l)} - \lambda I \). In general, convergence of the Newton method is not guaranteed [28]. This is due to the fact that the Hessian can be singular or the search direction, \( d_s^{LM} \), may not even give an ascent direction. Therefore, even when the inverse of the Hessian matrix exists, it does not necessarily imply that \( L_s^{(l+1)} \) is greater than \( L_s^{(l)} \). However, when the starting point \( y_s^{(0)} \) is close enough to the optimal \( y^* \) such that \( \nabla^2 L_s^{(0)} = 0 \) and \( \nabla^2 L_s^{(0)} \) is full rank, then the Newton method converges to the optimal \( y^* \). In order to address the convergence problem of the Newton method, several methods have been proposed in the literature, see Chapter 8 of [28] and Chapter 5.2.4 of [29]. In this paper, we employ the Levenberg-Marquardt method due to its guarantee of convergence.

With the Levenberg-Marquardt method, the parameter updates are given by
\[
y_s^{(l+1)} = y_s^{(l)} - \mu_l(\nabla^2 L_s^{(l)} - \xi I)^{-1}\nabla L_s^{(l)},
\]
where \( d_s^{LM} = -\nabla^2 L_s^{(l)} - \lambda I \) is the search direction evaluated at \( y_s^{(l)} \) and \( I \) is the identity matrix. The constant \( \xi \) ensures all the eigenvalues of \( D = (\nabla^2 L_s^{(l)} - \lambda I) \) are negative such that \( D \) is negative definite. It is called as the damping or the Levenberg-Marquardt parameter [28]. If the largest eigenvalue of \( \nabla^2 L_s^{(l)} \) is negative, then \( \xi \) will be equal to zero and the Levenberg-Marquardt method reduces to the Newton method, that is \( d_s^{LM} = d_s^{N} \). Under this condition, quadratic convergence is achieved. If the largest eigenvalue of the Hessian is non-negative, then we take \( \xi = \omega_{\text{max}} + \sigma \), where \( \omega_{\text{max}} \) is the largest eigenvalue of \( \nabla^2 L_s^{(l)} \) and \( \sigma > 0 \) is a sufficiently small number. This operation forces the Hessian to be negative definite. In our simulations, this offset is taken as \( \sigma = 1 \). The parameter \( \alpha_{k,s} \) in Step 11 is the positive scalar step size. In addition, controlled increase mechanism is used to update power levels. When the power level of the MBSs changes largely between two consecutive time instants, the interference pricing mechanism does not accurately model the effect of the interference over the utilities of the other sectors [4]. The controlled increase mechanism prevents large changes of the power levels. The selection of small \( \zeta \) slows down the convergence of the algorithm. On the other hand, large \( \zeta \) may fail cause large changes in the power levels. Therefore, an adaptive algorithm is used to select \( \zeta \) in this paper. The parameter \( \zeta \) is equal to \( t/(2t+1) \) where \( t \) is the time instant [2]. When \( t \) goes to infinity \( \zeta \) will converge to \( 1/2 \).

Note that the Levenberg-Marquardt method guarantees convergence regardless of the starting point [28, p. 312]. The proposed approach using the interference pricing method is depicted under the heading Algorithm 1, where \( \ell_{\text{max}} \) is the maximum number of iterations and \( \epsilon \) is a sufficiently small positive number to determine when to exit the algorithm. Using the Levenberg-Marquardt method, the parameters are updated in Step 10. The expression \( \sqrt{(\nabla L_s^{(l)})^T d_s^{LM}} \) is called as the Newton increment [30]. It is used as a stopping criterion in iterative line search algorithms [30]. Note that this stopping criterion is also the condition to check whether a search direction \( d_s^{LM} \) is an ascent direction or not, that is, to check if \( \nabla^2 L_s^{(l)} d_s^{LM} > 0 \) is true. The loop terminates when the convergence condition is satisfied or the maximum number of iterations is reached.

In general, the main computational effort in Algorithm 1 is at Step 9 where the inverse of matrix \( \nabla^2 L_s^{(l)} - \lambda I \) is calculated [28]. Using classical approaches such as the Gauss-Jordan elimination method, the computation complexity of taking the inverse of an \( n \times n \) matrix is \( O(n^3) \). For large-scale problems, this operation becomes prohibitively complex. Consider the centralized resource allocation approach where we solve for 57 \((\varepsilon, \beta)\) pairs, one pair for each sector. Taking the inverse of

<table>
<thead>
<tr>
<th>Algorithm 1 Proposed Power Control Algorithm with Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Initialize: ( y_s^{(0)} = (\varepsilon_s^{(0)}, \beta_s^{(0)})^T ) and set ( l = 0 )</td>
</tr>
<tr>
<td>2: % Each sector solves (20) by using the Levenberg-Marquardt Method</td>
</tr>
<tr>
<td>3: for ( l := 1 ) to ( \ell_{\text{max}} ) do</td>
</tr>
<tr>
<td>4: if ( \omega_{\text{max}} = \max (\text{eig}(\nabla^2 L_s^{(l)})) &lt; 0 ) then</td>
</tr>
<tr>
<td>5: ( \xi = 0 ).</td>
</tr>
<tr>
<td>6: else</td>
</tr>
<tr>
<td>7: ( \xi = \omega_{\text{max}} + \sigma ).</td>
</tr>
<tr>
<td>8: end if</td>
</tr>
<tr>
<td>9: ( d_s^{LM} = -\nabla^2 L_s^{(l)} - \lambda I )</td>
</tr>
<tr>
<td>10: Update the power control parameters, ( y_s^{(l+1)} ), using</td>
</tr>
<tr>
<td>( y_s^{(l+1)} = y_s^{(l)} + \mu_l d_s^{LM} )</td>
</tr>
<tr>
<td>11: Update the Lagrange multiplier, ( \lambda_{k,s}^{(l+1)} ) for all ( k \in K_{M,s}^C ) and ( K_{E,s}^E ), using</td>
</tr>
<tr>
<td>( \lambda_{k,s}^{(l+1)} = \left( \lambda_{k,s}^{(l)} + \alpha_{k,s}(R_{\min,k} - \sum_{n \in N_{M,k}} r_{(k,n)}^{(l)}) \right) \forall i = 1, 2. )</td>
</tr>
<tr>
<td>12: if (</td>
</tr>
<tr>
<td>13: Break</td>
</tr>
<tr>
<td>14: end if</td>
</tr>
<tr>
<td>15: end for</td>
</tr>
<tr>
<td>16: ( x_s^{(l+1)} = (1 - \zeta)(x_s^{(l)} + \zeta y_s^{(l)}) )</td>
</tr>
<tr>
<td>17: Price Update: Each user calculates interference prices and feeds these values back to its base station.</td>
</tr>
<tr>
<td>18: Interference prices are distributed among base stations.</td>
</tr>
<tr>
<td>19: Go to Step 2 and repeat.</td>
</tr>
</tbody>
</table>
this big matrix would require \( O(57^3) \) floating point operations (flops). Fortunately, with the proposed distributed algorithm, we only need to take the inverse of \( \nabla^2 L_s^{(1)} - \xi I \), which is a \( 2 \times 2 \) matrix, and the calculation of its inverse is straightforward. Under these conditions, the computational complexity of both the gradient and Levenberg-Marquardt methods are on the same order per iteration step. Along with similar computational complexity, the Levenberg-Marquardt based-method has significantly faster convergence rate compared to the gradient-based method. The convergence properties of the algorithm are inherited from the detailed analysis in [18]. Also, note that the expressions of the gradient and Hessian of the energy efficiency function, \( \eta_s \), are presented in detail in the Appendix of [18].

In this paper, we are maximizing the energy efficiency of the network while satisfying the rate constraints of users. This particular approach is beneficial for the applications such as Voice-over-IP, video call, streaming, and real time gaming applications that requires minimum rate to perform properly. As we show in the simulation results, when we enforce higher rates, the energy efficiency of the network may decrease.

V. SIMULATION RESULTS

In this section, first, we evaluate the performance of the proposed algorithm in terms of energy efficiency for different rate constraints. Second, we compare the performance of the proposed algorithm with no power control and non cooperative power control algorithms in terms of outage probabilities. Third, we assess the effect of the scheduler on the energy efficiency and the outage probabilities. Fourth, we study the outage probabilities of the network with different cell-center region boundary selection algorithms.

For the FFR method, the spectrum allocation scheme is such that 14 RBs are assigned to subband A. The remaining 36 RBs are divided into three equal segments and assigned to subbands B, C, and D. The simulation layout is illustrated in Fig. 1. The simulation area consists of 19 hexagonal cells with wrap-around edges. Single antenna transmission is considered, i.e., \( N_{TRX,M} = 1 \) and \( N_{TRX,P} = 1 \) for all sectors. Two pico-eNBs are randomly generated in each sector. Twenty users are generated in each sector. First, two users are generated within the radius of 40 meters for each pico-eNB. Then, the rest of the users are uniformly distributed in the sector area. The highest RSRP method is used for the cell-association [25].

The simulation parameters, distance constraints in generating new nodes, and the base station power consumption values are given in Table I [31]. Furthermore, the initial values are chosen as \((e_0, \beta_0) = (1, 1)\).

In the first part of the simulations, we assume that all users in the network have the same guaranteed-bit-rate (GBR) requirements, i.e., \( R_{min,k} = R_{min} \) for all \( k \in (K_{C,M,s} \cup K_{E,M,s}) \) and \( s \in S \). Four different rate constraints are considered, \( R_{min} \) is equal to 64, 128, 256, and 512 kbits/sec (kbps). In the second part of the simulations, we show the decrease in outage probabilities with the proposed algorithm compared to no power control and non-cooperative power control algorithms. The non-cooperative power control algorithm seeks to maximize the energy efficiency of the network while satisfying the users minimum rate constraint by considering only the intra-cell interference. This power control algorithm is described in [18]. In the third part of the simulation, we compare the performance of the proposed scheduler with the Equal Bandwidth (EBW) scheduler for no rate constraint and \( R_{min} \) is equal to 512 kbps. The EBW scheduler distributes RBs equally among users. Assume that there are \( K \) users and \( N_{RB} \) RBs. We assign \( \lfloor N_{RB}/K \rfloor + 1 \) RBs to \( K_h = \text{mod}(N_{RB}, K) \) users and \( \lfloor N_{RB}/K \rfloor \) RBs to \( K_i = K - K_h \) users. This scheduler is discussed in the LTE proposals to calibrate system level simulations in [31]. In the fourth part of the simulation, users have different rate requirements. We compare the performance of the proposed cell-center radius selection algorithm with the ones that are described in [18].

Fig. 3(a) shows the energy efficiency of the network for the proposed algorithm, when \( R_{min} \) is equal to 0, 64, 128, 256, and 512 kbps. When more strict rate constraints are enforced, the energy efficiency of the network decreases. For the case without rate constraints, the energy efficiency of the network is 13.6%, 30.2%, 53.4%, and 122.1% higher than the cases with \( R_{min} \) is equal to 64, 128, 256, and 512 kbps, respectively. In Fig. 3(b), the outage probabilities of the network using the proposed algorithm are shown for the same rate constraints. As expected, the outage probabilities increase for higher rate requirements. After 30 time instants, the outage probabilities become 8.3%, 1.3%, 0.9%, and 0.4% for GBR requirements in the same order as before. When the rate constraints are increased, the energy efficiency of the network decreases in each iteration to satisfy the rate requirements of the users, while the outage probabilities decrease. This points out to the tradeoff between the network energy efficiency and outage probability, which becomes more severe as the GBR requirement increases.

Fig. 4 illustrates the cumulative distribution function of user rates of MUEs for no power control, non-cooperative power

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Total number of RBs</td>
<td>50 RBs</td>
</tr>
<tr>
<td>Freq. selective channel model (CM)</td>
<td>Extended Typical Urban CM</td>
</tr>
<tr>
<td>UE to MBS PL model</td>
<td>128.1 + 37.6 log_{10}(d)</td>
</tr>
<tr>
<td>UE to Pico eNB PL model</td>
<td>140.7 + 36.7 log_{10}(d)</td>
</tr>
<tr>
<td>Effective thermal noise power, ( N_0 )</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>UE noise figures</td>
<td>9 dB</td>
</tr>
<tr>
<td>MBS and Pico eNB antenna gain</td>
<td>14 dBi and 5 dBi</td>
</tr>
<tr>
<td>UE antenna gain</td>
<td>0 dBi</td>
</tr>
<tr>
<td>Antenna horizontal pattern, ( A(\theta) )</td>
<td>( -\min[(12/\theta)^2, A_{\infty}] )</td>
</tr>
<tr>
<td>( A_{\infty} ) and ( \theta_{\infty} )</td>
<td>20 dB and 70°</td>
</tr>
<tr>
<td>Penetration loss</td>
<td>20 dB</td>
</tr>
<tr>
<td>Macrocell and picocell shadowing</td>
<td>8 dB and 10 dB</td>
</tr>
<tr>
<td>Inter-site distance</td>
<td>500 m</td>
</tr>
<tr>
<td>Minimum MBS to user distance</td>
<td>50 m</td>
</tr>
<tr>
<td>Minimum pico-eNB to user distance</td>
<td>10 m</td>
</tr>
<tr>
<td>Minimum pico-eNB to MBS distance</td>
<td>75 m</td>
</tr>
<tr>
<td>Minimum pico-eNB to pico-eNB distance</td>
<td>40 m</td>
</tr>
<tr>
<td>Traffic model</td>
<td>Full buffer</td>
</tr>
<tr>
<td>Power Consumption Parameters</td>
<td>MBS: (130W, 75W, 20W, 4.7)</td>
</tr>
<tr>
<td>(( P_b, P_{sleep}, P_{max}, \Delta ))</td>
<td>Pico-eNB: (56W, 39W, 6.3W, 2.6)</td>
</tr>
</tbody>
</table>
Throughput (kbits/sec)

Cumulative Distribution Function

260
280
300
0
5
10
15
20
25
30

0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18

0 500 1000 1500 2000 2500 3000 3500 4000 4500

GBR: 512 kbps, Full Power, MMF
GBR: 512 kbps, Non-Cooperative, MMF
GBR: 512 kbps, Proposed Algorithm, MMF

Throughput (kbits/sec)

Cumulative Distribution Function

260
280
300
0
5
10
15
20
25
30

0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18

0 500 1000 1500 2000 2500 3000 3500 4000 4500

GBR: 512 kbps, Full Power, EBW
GBR: 512 kbps, Non-Cooperative, EBW
GBR: 512 kbps, Proposed Algorithm, EBW

control, and the proposed algorithm with the MMF schedulers, when $R_{\min}$ is equal to 512 kbps. The outage probabilities of the network are 14% and 34% for the full power transmission and non-cooperative power control algorithms, respectively. Due to the fact that the non-cooperative power control algorithm does not consider intercell interference conditions during the resource allocation process, it leads to a large number of users being at outage. The proposed power control algorithm decreases the network outage probability to 8%. Although the outage probabilities of the full power transmission and proposed algorithm are close, the energy efficiency of the proposed algorithm is 42% better than the case with full power transmission.

Fig. 5(a) shows the average energy efficiency per sector for No GBR requirement and GBR requirement of 256 kbps with EBW and MMF schedulers. When no GBR requirement is enforced, the EBW scheduler performs 5.1% better than the MMF scheduler. The EBW scheduler disregards the rate requirement of users and the channel quality between user and the base station during the resource allocation process. Each user obtains equal amount of RBs. Therefore, users with better channel qualities get significantly better rates than the other users. On the other hand, the MMF scheduler assigns most of the resources to the users with worse channel conditions to maximize the minimum throughput. Therefore, when there are no GBR requirements, the throughput and corresponding energy efficiency of the EBW scheduler are better than the ones for the MMF scheduler. However, if users have minimum rate requirements, higher power transmission levels are necessary for the EBW scheduler to satisfy the rate requirements of users that are in outage. Therefore, the energy efficiency of the sector decreases. In addition, the intercell interference becomes a significant problem because
of these high transmission levels. Due to the fact that MMF scheduler assigns more resources to the user with worse channel conditions, less power might suffice to satisfy the rate requirement of these users. Therefore, the energy efficiency of the MMF scheduler is 56.5% better than EBW scheduler. In Fig. 5(b), we compare the outage probabilities of the network when the GBR requirement is 256 kbps. Due to the fact that MBSs need to transmit in higher transmission power levels to satisfy the rate requirements, the intercell interference becomes a significant problem for EBW scheduler. Due to the higher intercell interference and fewer resource block allocation to users with worse channel conditions, the outage probability of the EBW scheduler becomes significantly higher than the one for the MMF scheduler. The outage probability of the EBW scheduler is 14.6%, whereas the one for the MMF scheduler is only 1.3%. These results indicate the significance of the scheduler selection.

Next, in Fig. 6, we investigate the effect of the cell-center radius selection algorithms on outage probability. The proposed cell-center radius selection algorithm (referred to as CCSA2 in Fig. 6) is compared with the ones that are proposed in [18]. The first cell-center selection algorithm in [18] is referred to as CCSA1. The second algorithm determines the cell-center radius selection process. This gain may become even more significant, when the variance of the rate requirement of users increases.

VI. CONCLUSION

In this paper, we studied the energy efficiency of HetNets. We have proposed an energy-efficient resource allocation algorithm in which the power allocation and scheduling problems are decoupled. The proposed algorithm maximizes the sector energy efficiency while satisfying the rate requirement of users. The interference pricing mechanism is introduced to prevent selfish behaviour of base stations. The proposed algorithms employ the Levenberg-Marquardt method which has faster convergence than the algorithm proposed in [18]. Furthermore, the effect of cell-center radius is investigated. Based on our simulation results, we demonstrate that significant energy savings can be achieved, while the outage probability is also reduced.
where $\gamma$ to the cell-edge region.

For the FFR scheme in Fig. 1, it takes the value on the subband. For the FFR scheme in Fig. 1, it takes the region, whereas it is $n$

Note that the MBS transmit power on subcarrier $n$ depends on the subband. For the FFR scheme in Fig. 1, it takes the value $p_{m}^{(n)} = P_M$ for the subcarriers allocated to the cell-center region, whereas it is $p_{m}^{(n)} = \varepsilon P_M$ for the subcarriers allocated to the cell-edge region.

\begin{align}
\frac{\partial \mathcal{L}_{s'}}{\partial \Omega_{l,n}} &= \left(\lambda_{l,s'} + \frac{1}{\psi_{s'}}\right) \Delta f l \ln(2) \frac{\gamma_l}{1 + \gamma_l} \tag{22}
\end{align}

\begin{align}
\theta_s(\varepsilon, \beta) &= \sum_{n \in N_M} \varepsilon_s \sum_{\ell \neq k, \ell \in K^{(n)}} \gamma^{(n)}_{k,l} \beta_{l,m} \frac{p_m^{(n)}}{\partial \varepsilon_s} + \sum_{n \in N_M} \beta_s \sum_{\ell \neq k, \ell \in K^{(n)}} \gamma^{(n)}_{k,l} \beta_{l,m} \frac{p_m^{(n)}}{\partial \beta_s}. \tag{23}
\end{align}

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