BURIDAN'S IMPETUS HYPOTHESIS

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<u>Abstract</u>*: I interpret the concept of* ***impetus*** *in the writings of the Paris Terminist, Jean Buridan (14th c.) as an accidental form that functions as a motive force according to Aristotle's fundamental law of motion. I suggest how Buridan may have come to develop his particular hypothesis. I defend the traditional view that Buridan's* ***impetus*** *is permanent, and in this respect a forerunner of Newtonian inertia, against the suggestion of Stillman Drake that Buridan's* ***impetus*** *is self-expending. In the last section I discuss whether Buridan anticipated Galileo's law of acceleration.*

<u>Outline:</u>

Buridan developed his *impetus* hypothesis in response to two very venerable puzzles in the Aristotelian account of motion. The puzzles had existed, unsolved, for a millenium and a half by the time Buridan took them up. Perhaps they are insoluble within the Aristotelian conceptual scheme. In any case, Buridan tried his hand at them, and the quality of his effort earned him a place in the medieval tradition. Without ultimately moving beyond the Aristotelian system himself, he helped pave the way for the massive shift in conceptual schemes we now refer to as the scientific revolution.

By now the story of that shift has had many tellings. Those written since the groundbreaking work of Pierre Duhem and Anneliese Maier typically include, as precursors to the great transition, at least two 14th c. developments: the mean speed theorem developed at Merton College, Oxford, and the impetus theory of Buridan and his colleagues at Paris. Historic connections from these schools forward to the pioneers of early modern science have been the object of intense scrutiny: on these matters there is a large and growing literature. Connections backward are the objects of less attention. It still is not known, for example, whether Buridan was familiar with the concept of *mail*, similar to *impetus*, in Arab writers like as-Farabi and Avicenna.

Many of the particular features of Buridan's *impetus* hypothesis and several of the empirical observations he brings forward against Aristotle's ideas on projectile motion and accelerated fall are found in earlier writings, all of which belong to a continuous tradition of critical commentary on Aristotle that stretches back almost to the Philosopher himself. Which of these authors Buridan had read is not known. What we find when we study this tradition is that certain points made by Philoponus, for example, recur in Buridan centuries later, just as many of the arguments brought against Aristotle by Buridan and Oresme recur in the writings of subsequent authors, up to and beyond Galileo, their sources unacknowledged. Anonymous examples and arguments get passed from author to author, like well-polished stones.

Impetus was certainly not a new idea with Buridan. Concepts of an intrinsic motive power had been introduced by Philoponus (6^{th} c. CE) and other Greek commentators of late antiquity to improve Aristotle's account of projectile motion. Arab commentators had also developed their own versions, several of which are reported and evaluated by Avicenna (d.1037) in his *Book of the Healing of the Soul.* In the view Avicenna favored, projectile motion continues because an inclination (*mail*) is impressed on the projectile by the moving force. This inclination was said to be permanent, so that the motion would continue in the absence of resistance, and to vary in strength with the weight of the body to which it is communicated. There is no direct textual evidence in Buridan's writings that he was acquainted with this hypothesis; the resemblances between the two are probably best seen as evidence that something like the *impetus* hypothesis is among the most reasonable ways to explain the relevant range of phenomena in an Aristotelian system. Another Arab commentator, Abu 'l-Barakat (d. 1164) had a similar theory, in which the impressed inclination was said to be self-expending rather than permanent.

Specific connections between the earlier Greek and Arab writers and medieval scholastics have not been established, but several 13^{th} c. schoolmen discussed theories of impressed forces and inclinations in their commentaries. Roger Bacon and Thomas Aquinas, for example, in their commentaries on the *Physics* and *De Caelo*, rejected the concept of an impressed force as contrary to the fundamental Aristotelian principle that violent motions of inanimate objects are extrinsically caused. They reasoned that positing an impressed force in the projectile would mean its continued motion resulted from an intrinsic principle, and this is contrary to the nature of violent motion.

Buridan and other writers of the 14th c. were still tinkering within the Aristotelian system. Some of their concepts anticipated later ones. Buridan's *impetus* resembles Galileo's *impeto* and Newton's *momentum,* since all three are defined as varying directly with mass and velocity. The relation is one of analogy, not homology, however. The *measure* of Buridan's *impetus* is the same as the *measure* of Newton's *momentum,* but as theoretical concepts in two different systems, they are not entirely commensurate. Similarly, *impetus* is often considered an anticipation of Newtonian inertia, though the latter is independent of velocity, insofar as both remove the need for an external cause of continuing uniform motion. But again, the two concepts are only loosely analogous. What are still for Buridan puzzles in a basically Aristotelian physics would become in the following centuries full-scale anomalies, signalling the need for a new science.

I. Aristotle and the chemistry of motion

The fundamental theoretical entities by which an Aristotelian like Buridan would attempt to explain such empirically observable phenomena as an iron poker getting hot in a fire, or an ox dying, are prime matter and substantial and accidental forms. Substantial forms explain how the matter of a thing is organized so as to be the kind of thing it is and function in the way proper to it. Accidental forms explain the states, functions and effects of a thing that it can exist without.

The process by which a certain form or state comes to be, however, requires an efficient cause. Every existence and occurrence, and thus every motion has an efficient cause. Everything that is moved is moved *by* something. Living things can be self-moved, because they unite in themselves a mover (the soul) and a moved (the body,) but in most cases, the mover and the thing moved are two different subjects. The moving cause must be in contact with the thing moved—there is no action at a spatial or temporal distance.

A few accidental forms function as efficient causes or active principles for Aristotle, however. These are the elemental qualities. In what seems an older, more primitive, part of Aristotle's thought, there is a kind of *chemistry* of motion based on the system of Empedocles (5th c. BCE). The elemental qualities are the intrinsic properties (hot, could, wet, dry,) of the 4 elements:

Fire

fiery: hot and dry

Air

aeriform: hot and moist

Water

liquid: cold and moist

Earth

solid: cold and dry

Two other active qualities are gravity and levity. Sometimes these are considered derivative from the four elemental qualities; elsewhere they are treated as equally primary. All naturally occurring bodies are believed to be *mixtures* of the four elements, so "pure earth" and "pure fire" function more as theoretical entities in models of motion than as descriptive terms. Pure fire is said to have levity absolutely, and pure earth to have weight or gravity absolutely. The two intermediate elements (air and water) are said to have levity or gravity relatively, depending on their locations. (Water when found in the clouds has gravity and a natural tendency to fall to its natural place above the earth. Subterranean water has levity and a natural tendency to rise.)

The general idea is that internal tendencies to motion are caused by differences in the composition of bodies out of the elements. To the question "Why does body X move naturally to place P?" Aristotle answers as follows:

> X is composed of the elements earth, air, fire, and water in the proportions p:q:r:s.
>
> The natural place of a body composed of the elements in proportions p:q:r:s is
>
> P(p,q,r,s).
>
> Bodies realize their natures most fully when they are in their natural places.
>
> Bodies in natural motion strive to realize their natures most fully. Hence,
>
> In natural motion, X moves to P.

Local movements of terrestrial bodies, composed of the four elements and their compounds, are governed by the natural tendencies of the elements to arrange themselves concentrically around the center of the world, with earth at the center and water, air and fire in successive layers outward. There is constant change as the elements are transformed into one another and different compounds are formed. The natural place of a body is determined by the element that predominates in its composition. When a body is away from its natural place, its form is a motive principle. In this way the elemental qualities become inorganic forces that cause motion. For example: the downward motion of a stone falling from the airy regions will have as its substantial efficient cause whatever generated it by giving it the substantial form of a stone; the accidental efficient cause of its motion will be its accidental form, gravity or weight; and the final cause specifies the direction of the movement. Other active accidental forms are said to cause motion indirectly; heat and cold, for example, can cause motion through condensation and rarefaction. Substantial forms are active through their accidents.

So if we ask what kinds of forces are available to explain motions in an Aristotelian account, there are organic forces (the muscle power of human beings and animals, and all the external instruments by which these are extended) and inorganic forces consis

ting of the six active or elemental qualities. Typical cases of motion are instances of pushing and pulling, like Socrates pushing a cart. But inorganic forces can cause motion, too. The heat of fire can make something else hot, cause it to expand, or cause a change in its magnitude and location. The dryness of air can dry wet clothes or shrink a plant by dessication.

Local motion is always against a resistance. Since causes act instantaneously, a motive force not opposed by some resistance would cause, not locomotion, which is successive by nature, but instantaneous change of place. (Aristotle argues further that since the concept of instantaneous motion is incoherent, all motive forces must meet some resistance, and therefore no vacuum exists in nature. *Physics* IV.8.215b.12-22) The basic principle relating motion to moving force and resistance is that the ratio of distance travelled to time elapsed varies directly with the motive force and inversely with the resistance. Aristotle spells this out in Bk. VII, Ch. 5 of the *Physics*: If mover A moves an object B through a distance C in the time period D, then the same force will move half of B in the time period D through twice the distance C, or it will move half of B in half of D through the whole distance C. Doubling the motive force or halving the resistance results in twice the velocity, *ceteris paribus.* (But as Aristotle recognizes in this passage, it is not always the case that halving the force or doubling the resistance will result in half the velocity. Motion only occurs if the force continues to be greater than the resistance after such changes have been made.) This Aristotelian law of motion is often expressed as **V F/R** where V = velocity, F = motive force, R = resistance, and the proportion symbol means "varies directly as."

How will projectile motion be treated in such a theory? An Aristotelian model of the motion of a projectile will make some abstractions, as all models must. It will treat the projectile simply as a heavy body, (although pure earth does not exist in nature,) thus abstracting from any levity it might have as a mixture. And it will lump together, additively, the resistance of the medium with resistance from the contrary inclination to downward motion of the moved object. While these particular abstractions could be regarded as damaging for the theory, the most serious problem has to do with the motive force. Since projectile motion is violent motion, its efficient cause is not some natural accident of the moveable object itself, like weight. It must be moved by a separate subject acting as an external mover, in constant contact with the moved.

So, what exactly keeps the projectile moving after it leaves the projector? Without an external push, a stone should either remain at rest or move in a straight line toward the center of the earth. If a man holds a stone in his hand and moves it forward slowly, then lets go, it drops to the ground. Why should its trajectory be different when he moves it forward more rapidly?

Aristotle's explanation was one of the weakest parts of his theory of motion. Buridan describes it as follows:

> Along with the projectile the projector moves the adjacent air, and that swiftly moved air has the power of moving the projectile… The air joined to the projector is moved by the projector, and that moved air moves another next to it and that other, up to a certain distance. Thus the first air moves the projectile to the second air and the second to the third and so on. Hence Aristotle says there is not one mover, but many, one after another. Hence he also says that the motion is not continuous, but is of contiguous beings.

The projectile is borne along by successive contiguous motions in the medium. Notice that the air is both moved, itself, and caused to have the power to act as a mover of the projectile and the neighboring air.

II. Buridan on projectile motion

In Book III Question 7 of his commentary on Aristotle's *Physics*, Buridan brings four objections against the Aristotelian account:

> 1) A turning millstone would continue to turn, once the hand is removed from it, even if by covering the millstone with a cloth the adjacent air were kept from contact with it.
>
> 2) However rapidly the air is moved, it is easily divisible; thus it is not obvious how it would sustain a stone weighing a thousand pounds projected from a sling or mechanical device.
>
> 3) One could move the adjacent air just as swiftly or more swiftly with his hand if he held nothing in the hand than if he held the stone in his hand which he wished to hurl. Therefore, if that air from the speed of its motion were of such a force that it could move the stone swiftly, it would seem that if one person should push the air against another equally swiftly, that air ought to push him forcefully and truly noticeably, and we do not perceive this.
>
> 4) If the continued motion of a projectile after it leaves the mover were sustained by the moved air adjacent to the projectile, it would follow that a feather would be projected farther than a stone, the less heavy object farther than the more heavy object of the same size and shape. But this is not what we observe.

Something interesting is going on in these objections to the Aristotelian explanation. The first objection is straightforward: the motion of a turning millstone does not seem to depend on the action of the surrounding medium. But the last three reject the air-as-motive-force explanation for reasons that have to do with force as a product of quantity-of-matter and velocity. They share an implicit assumption that when analyzing motions, moving force and resistance can be treated interchangeably, since the measure of both will vary with the weight and the velocity.

In Buridan's day there were few precise measures of force. The muscular energy of an animal and the intensity of an elemental quality like wetness do not lend themselves to quantitative analysis. In the paradigm example of *natural* motion, the relevant force (F) is the specific weight of the falling body, measured in terms of (total or average) velocity (V) and resistance (R) of the medium. This can be expressed in the simple formula, **V F/R**. Heavier objects of the same size and shape fall faster in the same medium. In *violent* motion forces are measured indirectly according to the principle that equal forces will cause equal weights (or equal quantities of matter) to move equal distances in equal times. (*Physics* VII.5.249b.30-250a.28) Thus, **F WD/T**

or **F WV** , where D is distance, W is weight or quantity of matter, and T is time. Weight functions both as the internal force of natural motions and as *resistance* to force, in violent motions. Weight is also a variable that could be measured independently of the Aristotelian laws of motion in which it functioned.

In objection #2, Buridan reasons that if the air is to be the cause of the projectile's continuing motion, it must act with a force greater than the total resistance. The resistance, in the case of a thousand-pound stone, will be the resistance of the medium, (however that would play out under the hypothesis in question,) and the weight of the stone. Buridan's intuition is that the air would have to act with a force greater than a thousand pounds to overcome the resistance of the stone's natural downward movement, and that this amounts to saying the air would have to offer resistance greater than the downward force of the stone's gravity. The terms *force* and *resistance* are interchangeable; what matters are the relative magnitudes. And since the resistance of a medium varies directly with its density, the air would not offer sufficient resistance to the stone's fall.

Buridan's argument in objection #3 is as follows: Assume *ex hypothesi* that in throwing a stone a man moves the stone and the surrounding air. Assume also that the force of impact of a moved object is determined by its velocity alone. Then, since by Aristotelian principles a man can move a lighter object faster than a heavy one, *ceteris paribus*, he should be able to produce a noticeable force of impact by throwing the air alone. But this is not what we observe.

The larger argument is a *reductio*. If force of impact is determined by velocity alone, then the most damaging projectiles are the lightest, since they can be thrown the fastest. But this is absurd. Again, Buridan is working with the role of quantity of matter in both force and resistance. He doesn't deny that, in principle, a man can move a light object faster than a heavy one, according to the Aristotelian law that in violent motion weight acts as resistance. But quantity of matter is also a factor in force. Air is less dense than rock, so air moving at the same velocity as the stone has less force than the stone does. Thus air cannot be the moving force for the stone's continued motion.

Notice the ambiguity at this point of the concept of the *force* the stone has while in motion. Buridan's argument is that because of its greater density, the stone has *more* of it than the air, which is supposed to be moving it, does. In Aristotelian categories he can say that the force (understood as weight or inclination to downward motion) of the stone is greater than the moving force of the air. Then this argument is the same as #2. But he is working toward another concept of a force internal to the projectile but different from weight. This line of thought continues in objection #4.

The fourth objection brings to the surface a very deep problem in the Aristotelian theory of motion. Aristotle's fundamental theorem, that the ratio of distance travelled to time elapsed varies directly with the motive force and inversely with the resistance, describes and unifies a wide range of empirical phenomena. In the large domain of violent motions, i.e.motions caused by external movers, the law that the same force moves a light object farther than a heavy one seems obvious: Socrates can carry a light load farther than a heavy one.

Yet as Buridan points out, the attempt to fit the motion of a projectile into the Aristotelian model of violent motion would require that the moving force should be able to move a light body farther than a heavy one of the same size and shape. "It would follow that you would project a feather farther than a stone, the less heavy farther than the more heavy, with the sizes and shapes the same; and this is observed to be false." But for a certain range of phenomena, just the reverse is true. We can throw a baseball farther than a safety pin. Air resistance is not the explanation, Buridan reasons, since we can generally throw a heavy object farther than a light one *of the same size and shape.* What is the explanation, then?

It is not easy to fit the phenomena of projectile motion into a single, Aristotelian model. If we move an object slowly, then let go, it falls to the ground. If we move it rapidly, then let go, it continues its motion in the same direction. Why is that? What relationship, between what variables, determines the critical velocity for "takeoff" in such cases? In the same way, while I can throw a stone farther than a feather, as Buridan says, I cannot throw a 100-lb. stone farther than a 5-lb. one. Why does the Aristotelian model for violent motion hold for part of the range of phenomena but not the rest? Why is it that while I can in general move a light object farther or faster than a heavy one, an object already in motion seems to move farther or faster if it is heavy?

Buridan had at his fingertips two models of motion from the Aristotelian theory. In the model for violent motion weight acts as a resistance, lowering velocity or distance travelled, or increasing time, *ceteris paribus*.

But in the model for natural motion, accidents like weight act as motive forces, not resistances. In the paradigm case of natural motion, weight increases the speed of a falling body, *ceteris paribus*. I suggest that Buridan saw that the simplest way to save all the phenomena, without contradicting fundamental Aristotelian principles, was to consider the motion of the projectile after it leaves the projector as a case of natural, rather than violent, motion. In the passages which follow, he does not explicitly refer to projectile motion as 'natural motion,' but by making *impetus* an accident that moves the subject in which it inheres, he accomplishes the same thing. *Impetus* joins weight, levity, heat, cold and the other active qualities included among the efficient causes of motion.

> And so it seems to me that what should be said is that the mover in moving what is moved impresses upon it a certain *impetus* or force that moves the moved thing in the direction the mover moved it… And the more swiftly the mover moves the moved thing, the stronger the *impetus* it impresses on it. The stone is moved by that *impetus* after the projector ceases to move it, but the *impetus* is continuously diminished by the resisting air and by the gravity of the stone inclining it against the direction the *impetus* inherently moves it. Hence the motion of that stone is made continuously slower, and finally the *impetus* is so diminished or corrupted that the gravity of the stone prevails over it and moves the stone down to its natural place. [commentary on Aristotle's *Physics,*Bk.VIII, Q.12]

The impressed force varies directly with the velocity of the initial motion. And since weight is understood to function as it does in natural motion, rather than as it does in violent motion, the *impetus* will vary directly with the weight of the moved body as well.

> If someone should ask how it is that I hurl a stone farther than a feather, and a hand-sized iron ball farther than the same sized wooden one, I reply that the reception of all natural forms and dispositions is in matter and by reason of the matter. Hence, howevermuch more there is of matter, by that much more can a body take on the *impetus* more intensely. Now other things being equal, there is more prime matter in a dense and heavy body than in a rarified and light one; hence the dense and heavy one receives more from that *impetus* and more intensely, just as iron can receive more from heat than wood or water of the same quantity. A feather, however, receives such an impetus so sparsely that such an *impetus* is at once corrupted by the resisting air. And so even if a light piece of wood and a heavy piece of iron of the same size and shape should be moved equally swiftly by the projector, the iron would be moved farther because the *impetus* would be impressed more intensely it it.

The important points to notice about Buridan's hypothesis are these:

1) The *impetus* imparted by the projector to the projectile acts in the same direction as the original mover was moving the projectile [rectilinear or circular.]

2) The *impetus* varies directly with the velocity of the projectile: "the more swiftly the mover moves the moved thing, the stronger the *impetus* it impresses on it."

3) Although the terms 'cause' and 'effect' are not used in the passage, it is clear that the *impetus,* as an impressed force, is both an effect of the original mover and a cause of the projectile's continued motion: "the mover in moving what is moved impresses upon it a certain *impetus* or force that moves the moved thing…"

4) The *impetus* is a natural form received in the matter of the projectile. The intensity of the form received varies directly with the quantity of matter in the projectile: "the dense and heavy body receives more from that *impetus* and more intensely…"

5) The *impetus* is a permanent quality distinct from the motion it causes. *Impetus* cannot simply

be the motion, since it "makes" or causes the motion, and "the same thing does not cause itself."

6) Impetus can be corrupted or impeded by resistance or a contrary inclination.

Perhaps the most obvious way in which Buridan's *impetus* hypothesis anticipates later developments in mechanics is by focusing exclusively on two magnitudes, out of the many possible variables in any instance of projectile motion. The quantity of *impetus*, Buridan says, varies directly with the velocity and quantity of matter of the moved body. (Thus the measure of *impetus* is the measure of momentum in Newtonian mechanics.)

In addition, since *impetus* is a permanent accident, a body which has acquired *impetus* will continue moving indefinitely if it meets no resistance. It is customary to see in this aspect of Buridan's *impetus* hypothesis an important step on the way to Newton's law of inertia: in Aristotle, the projectile's motion requires an external mover in constant contact with it; in Buridan, the moving force is internal to the projectile, eliminating the need for an external mover; and in classical mechanics, continuing uniform motion calls for no explanation at all. This interpretation may overstate the degree to which Buridan broke with the Aristotelian system, however. In an Aristotelian account, the generator of the essential form of a thing is regarded as the primary efficient cause of all the changes caused by the thing's accidental properties. For example, the primary efficient cause of the natural motion upward of a particular bit of fire is the earlier fire which produced it and conferred on it all its essential properties. The primary efficient cause of the natural motion of a heavy body when it falls is whatever agent originally produced the heavy body, with its natural tendency to fall to the earth. Similarly, in projectile motion, the projector remains the primary efficient cause of the projectile's ongoing movement, through the accidental form it has impressed.

Still, by making the impressed accident the proximate efficient cause of the motion, Buridan eliminated the need for things in motion to be physically accompanied by their movers. The celestial spheres, for example, could continue their motions without the aid of angelic intelligences.

> It might be said that when God created the world He moved each of the celestial orbs however He pleased; and in moving them He impressed an *impetus* which moves them without His moving them any more. …And those *impetuses* impressed upon the heavenly bodies were not afterwards lessened or corrupted because there was no inclination of the heavenly bodies to other motions nor was there the resistance which would corrupt or restrain that *impetus*…

Eliminating the need for contact may also have led to an important shift in the way motions were *perceived*. Toward the end of Bk.VIII,Q.12, Buridan talks about bouncing balls and vibrating strings and clangers of bells. He describes the motion of a vibrating string as one in which plucking the string gives it *impetus*, and the *impetus* is consumed in displacing the string to one side, against the resistance of the string's tension. The tension then carries the string back to the center, implanting increasing *impetus* along the way, so that at the center the *impetus* carries the string on to the opposite side, against the string's tension, and so on, back and forth in an ongoing process. Similarly, the clanger of a bell swings from one side to the other, ringing the bell for a long time after the man has stopped pulling the rope. From an Aristotelian point of view, this is simply a case of something falling. An Aristotelian analysis would have abstracted from all but the direct vertical distance by which the clanger eventually moved closer to the earth, (describing its velocity as vertical distance traversed over total time elapsed.) Buridan's *impetus* hypothesis would bring the perturbations from vertical motion into saliency.

To summarize, then: I have suggested that Buridan's key insight was to see the motion of a projectile on the analogy with the natural motion of a falling body. In rejecting the Aristotelian explanation, Buridan argues that the surrounding air can not be the motive force causing the projectile's continued motion, basically because it is less dense than the projectile. Then he proposes his own version of the impressed-force concept that had surfaced at various points in the Aristotelian tradition before his time, calling it "*impetus*." I believe the central insight which led him to this theory was the realization that in the motion of a projectile, weight or quantity of matter functions as a motive force, analogously to the role of weight in the Aristotelian model of natural motion, rather than as a resistance, as in the model of violent motion. I have suggested how he may have been led to this insight by reflecting on two aspects of the Aristotelian air-as-mover model: the problem of the relative densities of projectile and medium, and the interchangeability of force and resistance.

III. The application of the *impetus* hypothesis to the acceleration of a falling body

Buridan's *impetus* hypothesis for projectile motion had the virtue of explaining phenomena other than that for which it was constructed. The most important of these was the acceleration of a falling body.

There was no satisfying explanation of acceleration in Aristotle's physics. When Aristotle looked at a falling body, what he saw was a change in state. What was important in the analysis of the motion was the total distance travelled and total time elapsed, yielding average speed. The final cause of the motion was the natural place of the object. The primary efficient cause was the generating agent that produced the heavy object by giving some bit of prime matter its particular substantial form, including its essential accidents. Six accidental forms, including weight and levity, were internal motive forces. Whatever applied the cause of motion in particular cases, (by removing an obstacle to the object's downward motion, for example,) was an accidental mover. (*Physics*, Bk.VII) So the ultimate mover is the generating agent and the proximate mover is the substantial form acting through the accidental quality of gravity or weight. But once the body is falling, all of these causes, (natural place, generating agent, substantial form, gravity, remover of obstacle,) are constant. This presents a problem. Why does the object speed up?

Commentators on Aristotle had offered various explanations for the observed acceleration. Buridan discusses three of these in Bk II, Q. 12 of his commentary on Aristotle's *De Caelo.* The first was that the motion of the falling body produces heat, this heats the air and rarefies it, and, since rarified air is more easily divisible and less resistant, the speed increases. Buridan brings three empirical observations against this view. 1) Things don't fall more rapidly in summer than in winter, even though the air is hotter then. 2) If it is the body's moving through the air that heats the air, then it should be the air behind the falling body that is hot. But it is the air in front of the falling body which offers resistance to its fall. Therefore the resistance is not diminished by the heating. 3) If the movement of a body through the air heats the air, then anyone who in striking a man moves his hand as rapidly as a particular stone that falls on the same man, (the comparable speed being known by the comparable pain to the man,) should perceive with his hand the heating of the air. But this is not what we observe.

A second popular explanation was that the accelerated natural motion of a falling body arises from proximity to its natural place. The natural place of a body is a final cause of its movement toward that place. The earth moves the heavy body by a method of attraction, just as a magnet attracts iron. If place is the moving cause, then it can move the body more strongly when the body is nearer to it, for an agent acts more strongly on something near to it than on something far away from it. Thus a heavy body is moved more swiftly by the amount that it is nearer to its downward place. After appealing to authority, (to the effect that the explanation must be wrong because it would contradict the opinion of Aristotle and Averroes that if there were several worlds, the earth (i.e. dirt) of the other world would be moved to the middle of *this* world, not that one, in spite of the greater distance involved,) Buridan brings three empirical observations against the second explanation. 1) If the strength of the attraction of a stone toward its natural place were stronger the nearer it is to the earth, then it would be more difficult to lift a particular stone near the earth than to lift the same stone at a height some distance from the earth. But this is not what we observe. (Buridan adds that the hypothesis cannot be saved by positing a difference too small to perceive, because any cause proportional to the effect would be perceptible. A stone falling from a tower reaches a velocity, and causes an impact, several times as great as the velocity and strength of impact near the beginning of its fall. The effect of increased proximity to the earth on the difficulty of lifting the stone should be of similar magnitude. If it were, we would be able to detect it. But no such effect is observed.) 2) Let a stone begin to fall from a high place to the earth and another similar stone begin to fall from a low place to the earth. Then these stones, when they are at a distance of one foot from the earth, ought to be moved equally fast…if the greater velocity should arise only from nearness to their natural place. Yet this is not what we observe, for a body that has fallen from a high place will be moving much more quickly at one foot from the earth than a body that has fallen from a low place will be. 3) If two similar bodies fall ten feet, one at a high altitude and one at sea level, neither movement will be swifter than the other, though one occurs at greater proximity to the earth.

A third popular explanation was that as a body falls, there is less air beneath it, and this means less resistance. If the force of gravity remains the same and the resistance is decreased, the body ought to be moved more swiftly. Against this account Buridan repeats the third objection above, and adds the observation that since earth is more resistant than air, the resistance should be greater, not less, near the earth.

Buridan's solution differed from these in fundamental ways. He was convinced that the key variable in an explaining acceleration is not the distance to the center of the earth, or the properties of the surrounding medium, but the extent of the fall itself. Buridan's own explanation can be found in two places: at the end of his treatment of projectile motion, (BkVIII, Q.12, of his *Physics* commentary,) and in Bk. II, Q. 12 of his commentary on Aristotle's *De Caelo.* In the first he is applying the *impetus* concept he has just worked out in connection with the motion of projectiles.

And from this also appears the cause whereby the natural downward motion of a heavy thing is

> continuously speeded up, for at first only gravity moved it and so it moved more slowly; but in moving, *impetus* is impressed on that heavy thing, which *impetus* then moves it along with the gravity. Hence the motion becomes swift, and the swifter it goes, the more intense the *impetus* becomes. Hence the movement appears to become continuously swifter.

Impetus is not a force of attraction, like magnetism. The strength of *impetus* varies, not with proximity to a source, but with, as was commonly said, distance travelled:

> I conclude, therefore, that the accelerated natural movements of heavy and light bodies do not arise from greater proximity to their natural place, but from something else that is…varied by reason of the length [*longitudinis*] of the motion. Nor is the case of the magnet and the iron similar, because if the iron is nearer to the magnet, it immediately will begin to be moved more swiftly than if it were farther away. But such is not the case with a heavy body in relation to its natural place. …the greater velocity does not arise from greater proximity to the earth or because the body has less air beneath it, but from the fact that that moving body is moved from a longer distance and through a longer space. (*De caelo* commentary, Bk.II,Q.12.)

In the context of projectile motion the quantity of *impetus* was said to vary directly with quantity of matter and with velocity. Here, in the discussion of falling bodies, the quantity of *impetus* is correlated with distance travelled and velocity. Why the change?

Part of the answer is that quantity of matter falls out of the analysis in the process of idealization. It is true that for Buridan, as for Aristotle, a heavier body will fall faster than a lighter one, even apart from resistance of the medium, because gravity, as an accidental form, just *is* the weight of the body, and gravity is a motive force. But Buridan's question now is not "Why does a heavy body fall faster than a light one of the same size," but "Why does the velocity of a falling body increase over time?" To answer *this* question, Buridan needed to make some further simplifying assumptions. It was part of the Aristotelian view that natural movements upward due to levity were also accelerated, and that since all naturally occurring bodies were mixtures, gravity and levity would both be at work in many phenomena. The "chemistry" of the motion of mixed bodies could be quite complex. Buridan abstracts from these complicating factors to focus on the simple case of the fall of a (purely) heavy body. His goal is to bring that phenomenon under the general Aristotelian law of motion, **V F/R**. Resistance can normally include the resistance of the medium, resistance from an external force applied to an already moving object, the resistance of a contrary inclination (like that caused by admixtures of levity in a mostly heavy object,) and resistance due to the accidental quality by which a body tends to remain at rest in its natural place (encountered along with friction in the effort to move a heavy body horizontally, for example.) In the case of accelerated fall of a purely heavy body only the resistance of the medium is relevant. Buridan makes the further assumption that resistance is constant, (although, as we've seen, earlier explanations of acceleration had tried to correlate acceleration with changes in air resistance.)

In this suitably simplified situation, assuming a purely heavy body and constant air resistance, constant gravity might be thought to produce constant motion in the falling object.

> I suppose that if the moving body is the same, the total mover [gravity] is the same, and the resistance also is the same or similar, the movement will remain equally swift,since the proportion of mover tomoving body and to the resistance will remain [the same.]

But this is not what we observe. We observe instead that the motion is accelerated.

> In the movement downward of the heavy body the movement does not remain equally fast but continually becomes swifter. From these [suppositions] it is concluded that another moving force concurs in that movement beyond the natural gravity which was moving [the body] from the beginning and which remains always the same. (*De caelo* commentary, Bk.II,Q.12)

Since the motion downward due to gravity is constant, accounting for the observed acceleration requires a second cause.

> A heavy body not only acquires motion unto itself from its principal mover, i.e., its gravity, but it also acquires unto itself a certain *impetus* with that motion. This *impetus* has the power of moving the heavy body in conjunction with the permanent natural gravity. And because that *impetus* is acquired in common with motion, hence the swifter the motion is, the greater the *impetus* is. At the beginning the heavy body is moved by its natural gravity only; hence it is moved slowly. Afterwards it is moved by that same gravity and by the *impetus* at the same time; consequently, it is moved more swiftly. And because the movement becomes swifter, therefore the *impetus* also becomes greater and stronger, and thus the heavy body is moved by its natural gravity and by that greater *impetus* simultaneously, and so it will again be moved faster; and thus it will always and continually be accelerated to the end. (*De caelo* commentary, Bk.II,Q.12)

In the case of the projectile, the moving force was applied just once, so it caused the initial motion plus a single increment of *impetus*. The *impetus*, as impressed force, continued to move the projectile until corrupted by the resistance of the medium and the contrary inclination of the projectile downward, due to gravity. In the case of free fall, however, the force of gravity continues to act, imparting a new increment of *impetus* in each interval of time. One natural way to interpret this is as follows. The original moving force, the weight of the body, produces at each instant both the movement of the body and an increment of *impetus*. In the first instant the weight (**G**) produces an initial velocity (**V**) and an increment of *impetus* (**I**), which will act along with **G** in the second instant to produce an increment in the velocity, **V.** So in the second instant, gravity and *impetus* together produce the velocity **V + V** and another increment of *impetus*. In the third instant **G + 2I** produce a velocity of **V + 2V**, and so on. Gravity is producing two effects, velocity and *impetus*, in each interval of time. Since the amount of *impetus* added in each interval is constant, velocity increases uniformly with time.

By including *impetus* in the total applied force, we get the series of correlations,

G V

G+I V + V

G+2I V + 2 V

G+3I V + 3 V …

where force is proportional to velocity, as the Aristotelian law of motion requires, (and not to acceleration, as in classical Newtonian mechanics.)

IV. Is *impetus* permanent?

Clagett is expressing something of a consensus when he remarks in his commentary that "The exact nature of *impetus* as conceived by Buridan is difficult to pin down." We have only a few brief passages treating of the topic in Buridan's writings, and these raise several interesting questions. We would especially like to know whether *impetus* is permanent, since that is what makes it an anticipation of the Newtonian principle of inertia, on the standard view. Stillman Drake has pointed out that the word *permanens* had two meanings in medieval Latin: it was used as an antonym for *temporary*, but could also mean *being present all at once* as opposed to being successive. (Call these senses permanent-1 and permanent-2) He claims that it had the second sense, not the first, in Buridan's discussions of *impetus*.

> Buridan's use of the word 'permanent' as applied to *impetus* has generally been taken as meaning that he attached to *impetus* what was to become the essential kernel of the later concept of inertia, because the *impetus* would never leave the projectile except as a result of some external force counter to it, such as resistance of the medium or the striking of an obstacle to motion. I believe that this view is mistaken.

Drake bases his argument on the following passages excerpted from Buridan's question on projectile motion:

> The first [conclusion] is that the *impetus* is not the very local motion in which the projectile is moved, because that *impetus* moves the projectile and the mover produces motion. Therefore, the *impetus* produces that motion, and the same thing cannot produce itself. …
> The second conclusion is that the *impetus* is not a purely successive thing (*res,*) because motion is just such a thing and the definition of motion is fitting to it, as was stated elsewhere. And now it has just been affirmed that the *impetus* is not the local motion. Also, since a purely successive thing is continually corrupted and produced, it continually demands a producer. But there cannot be assigned a producer of that *impetus* which would continue to be simultaneous with it. …
> The third conclusion is that the *impetus* is a thing of permanent nature (*res nature permanentis,)* distinct from the local motion in which the projectile is moved. This is evident from the two aforesaid conclusions and from the preceding. And it is probable that the *impetus* is a quality naturally present and predisposed for moving a body in which it is impressed, just as it is said that a quality impressed in iron by a magnet moves the iron to the magnet. And it is also probable that just

> as that quality—the *impetus*—is impressed in the moving body along with the motion by the motor, so with the motion it is remitted, corrupted, or impeded by resistance or by a contrary inclination.

Drake's argument can be summarized as follows:
1) Interpreting *permanent* in the first sentence of the third conclusion as permanent-1 makes it sound as if the projectile has a certain *impetus* independently of the particular local motion undergone. But this would contradict the last clause of the same conclusion, which says that the *impetus*, while distinct from the motion, behaves just like the motion and is weakened etc. along with it. "Hence *impetus* does not, in any modern sense, reside permanently in the projectile whether it is in motion or at rest; it dwells there temporarily, only when there is motion, and in an amount proportioned in some way to the motion. (32-3)

2. *Impetus* is a property imparted by a mover, as magnetic attraction is a property imparted by a magnet. (33)

3. Impetus is a kind of force, and forces are not normally permanent-1. (33)

4. In the case of celestial motions, *impetus* is clearly said by Buridan to last indefinitely, (i.e. to be permanent-1,) but in this particular case he uses the phrase *in infinitum duraret,* rather than *permanet.* This supports the view that *permanet* does not mean permanent-1 elsewhere in Buridan's discussion. (33)

5. Buridan's *impetus* can be corrupted by contrary inclinations, (like the inclination downward of heavy bodies in cases of projectile motion,) and these are internal, not external, forces. This distinguishes *impetus* from later inertial concepts, since in Buridan's system, terrestrial projectiles were always undergoing a reduction of *impetus* that would eventually bring them to rest, even in the absence of external resistances. (34)

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6. Therefore Buridan's *impetus* is not permanent-1 (32), and

7. Buridan's *impetus* differs in an essential way from the classical Newtonian concept of *inertia*. (32)

We can dispose of (2), since nothing is said in the Buridan passages about whether magnetism is permanent-1. (One might reasonably assume that magnetic charge and *impetus* are both permanent-1 in just the same way.) In any case, the purpose of the comparison is simply to show, using magnetism, that there *are* such things as impressed qualities that act as motive forces.

(3) begs the question, since Buridan believes as Aristotle does that in addition to external forces there are internal qualities that act as motive forces, and while external forces are not normally permanent-1, internal qualities may be. *Impetus* is such a force, and whether it is permanent-1 is just the question at issue.

With regard to (4,) Drake is correct that the argument in the Buridan passage quoted above is about permanence in the sense of *being present all at once*. Buridan says that while motion is successive by definition, *impetus* is not identical with the motion it causes, (since nothing causes itself,) and it cannot itself be successive, since successive entities "continually demand a producer," and there is no continuing producer of the *impetus*. One doesn't need to construct an argument to the effect that for Buridan *impetus* is non-successive in nature, since Buridan says so himself. Our question is not about terminology. We want to know whether, given that he says it is non-successive, Buridan also conceives of *impetus* as the kind of thing that would last indefinitely in the absence of opposing forces. His use of the *impetus* hypothesis to explain the eternal motion of the celestial spheres clearly suggests that he does.

With regard to (5,) Drake is correct insofar as his claim is that *impetus* is not identical with Newtonian inertia. This follows from the fact that Aristotelian physics is not a mechanistic system in which all motive forces are external. But the fact that *impetus* can be corrupted by internal as well as external opposing forces follows from the plurality of such qualities that can act as motive forces in Aristotle's physics, and is irrelevant to the question of the permanence of each such quality in itself.

The real question about the nature of *impetus* is (1) above: how can *impetus* be a permanent quality proportional to the imparted velocity and quantity of matter of the body in which it inheres, and yet vary subsequently with changes in velocity, whatever their cause? Clearly *impetus* is not a force of the same sort as the initial projecting force. Ontologically, it is both an effect and a cause. As effect of the initial moving force that impresses it, it is the analog of classical Newtonian momentum: its measure varies with mass and velocity. Yet because in Aristotelian physics, continued motion is a new effect requiring a continuing cause, *impetus* must also function as a moving force. And in this causal role it seems not to be permanent-1, but to vary with the motion of the body it is moving.

In discussing the falling body case, Buridan's language is sometimes ambiguous as to whether the *impetus* is acquired from the gravity or from the motion of the body. This gives rise to Drakes' worry in (1) that the *impetus* seems impermanent in the sense that it is somehow dependent on the motion it is supposed to be helping to cause. Buridan says "A heavy body not only acquires motion unto itself from its principal mover, gravity, but it also acquires unto itself a certain *impetus* with that motion." Are the *impetus* and the motion correlated as two effects of gravity, as in the interpretation given in section III above, or is the motion of the falling body a cause of further *impetus*? Buridan's language is often ambiguous. He says, "And because the movement becomes swifter, *therefore* the *impetus* also becomes greater and stronger, and thus the heavy body is moved by its natural gravity and by that greater *impetus* simultaneously, and so it will again be moved faster, and thus it will always and continually be accelerated to the end." We can give "*therefore"* a weak interpretation, to mean there is a simple correlation between increasing velocity and increasing impetus, as two effects of a common cause. Then increments in velocity and in *impetus* are linear and parallel. Or we can give it a stronger interpretation, according to which changes in velocity *cause* changes in *impetus*, which in turn are a further cause of motion. Some historians have interpreted Buridan in this way. Passages like the following might suggest this second interpretation:

> The natural downward motion of a heavy thing is continuously speeded up, for at first only gravity moved it and so it moved more slowly; but in moving, *impetus* is impressed on that heavy thing, which *impetus* then moves it along with the gravity. Hence the motion becomes swift, and the swifter it goes, the more intense the *impetus* becomes.

On the second interpretation the thought here would be that gravity causes motion, which causes *impetus,* which causes more motion, which causes more *impetus*, etc. On this interpretation, both the velocity and the *impetus* would increase by increasing amounts per unit time. Buridan's contemporaries at Oxford distinguished between *uniformly difform* motion*,* defined as the acquisition of equal increments of *velocity* in any equal periods of time, (i.e. uniform acceleration,) and *uniformly difformly difform* motion, defined as the acquisition of equal increments of *acceleration* in any equal periods of time. On the second interpretation, the motion of a freely falling body would *be uniformly difformly difform* motion, not uniform acceleration.

There is reason to think this is not what Buridan meant. The relevant experimental measures of acceleration had not been made in his day, so he lacked the empirical reason for rejecting the second interpretation that was available after Galileo. But he had a theoretical reason to reject it. The second interpretation of *impetus* in free fall is inconsistent with Buridan's treatment of *impetus* in projectile motion. If the motion itself of a moving body caused further *impetus,* then a projectile would *accelerate* after losing contact with the motor. If, in free fall, the motion itself were causing *impetus*, (causing more motion and more *impetus,* etc*.,)* in a non-linear process that fed off itself, the same thing would happen with the projectile, minus some constant amount of motion representing the uniform motion caused by gravity in the former case.

Of course it is possible that Buridan overlooked this consequence of holding that *impetus* is causally dependent on the motion of the body, but in the absence of any reason to think he did, we ought to search for a more charitable interpretation of the passages in question.

I believe there is only one interpretation of the permanence of *impetus* consistent with Buridan's Aristotelian presuppositions. *Impetus* is a quality which, once impressed, is permanent, in the absence of other causal influences. It is a permanent accident. But as a quality determined by any moving force applied to the body in which it inheres, it will in most concrete situations be determined by a composition of forces. In the case of a projectile, the motor impresses a determinate initial motion and a determinate quantity of *impetus* on the projectile. In the absence of other causal influences, the *impetus would* remain constant, imparting a uniform motion to the projectile after contact is lost. The projectile's motion doesn't remain constant, however, because the projectile's weight is a second motive force at work in the situation. The weight of the projectile affects not only the motion, but the *impetus* as well. In general, there will be a composition of forces acting on both the motion and the *impetus,* so that as the motion changes, the *impetus* changes, as two effects of a common set of causes. In no case will the motion itself cause a change in the *impetus*, however, since motion, as successive, is not the right kind of thing to act as a moving cause.

In the case of a falling body, only weight and *impetus* are moving forces. The continually acting weight adds increments of *impetus;* uniform weight and uniformly increasing *impetus* compose to cause uniformly accelerated motion. The increasing velocity caused by the gravity and continually impressed *impetus* is a *measure* of the increased *impetus. Impetus* varies directly with the velocity it maintains. Weight and *impetus* are causes; *impetus* is cause and effect; motion is always an effect.

There is considerable textual evidence for this view. In the ambiguous passage cited above, we notice that it says "and because that *impetus* is <u>acquired in common with</u> motion, hence the swifter the motion is, the greater the *impetus* is." The *impetus* is acquired in common with the motion, from the common cause, gravity, rather than caused by the motion. And in his argument for the nonsuccessive nature of *impetus,* Buridan says that *impetus* cannot be successive, because successive entities require a continuing cause, and "there is no continuing producer" of the *impetus*. If motion itself were the cause of *impetus*, there would be such a continuing cause.

I conclude that the most plausible interpretation of the permanence of *impetus* in Buridan is that it is both non-successive and non-self-expending. *Impetus* is a permanent accident, causally determined by the composed motive forces acting on the body in which it inheres. It is added to the two active qualities, gravity and levity, which along with the four elemental qualities were counted among the canonical internal motive forces in an Aristotelian physics.

V. Does Buridan anticipate Galileo's law of acceleration?

Buridan clearly relates the velocity of a falling body to the extent of the fall itself, rather than to properties of the surrounding medium or to absolute distance from the earth. But he nowhere clearly distinguishes between velocity as varying directly with distance and velocity as varying directly with time. Since in classical Newtonian mechanics distance from rest is proportional to time squared while velocity is proportional to time, the typical language of Buridan and other medieval writers, which failed to distinguish between the two proportions, would seem to be an important confusion. But this assumes that the medieval writers used 'distance' to refer to total distance from rest. If by 'distance' Buridan and other writers in this tradition meant *incremental* distance, velocity's varying directly with distance and with time would be equivalent. It is really quite a simple idea that if velocity varies directly with time, the distance travelled in each interval will vary directly with time, since velocity is expressed as distance per unit time. I find it implausible that a thinker of the caliber of Buridan would have missed this obvious fact, as required by the attribution to him of the mistaken view, so I assume that he meant incremental distance in these contexts.

In any case, it seems implicit in Buridan's discussion of falling bodies that gravity impresses equal increments of *impetus* (and thus velocity and distance) in equal intervals of time, and this insight in combination with the mean speed theorem of Buridan's contemporaries is sufficient to derive the law of acceleration. Perhaps we should ask ourselves why Buridan did *not* arrive at the law of acceleration, in spite of the fact that he was familiar with the work of the Merton school, he had the idea of velocity varying directly with time, and (unlike his contemporaries at Oxford) he was interested in explaining changes in physical systems. Was it simply that Buridan's primary interests lay elsewhere, in logic, semantics, and metaphysics? Is it probable if he *had* devoted more time to physical theory he would have derived the law of acceleration from his *impetus* hypothesis? Or is there a systematic reason why Buridan would be unlikely to think of total distance as proportional to time squared?

We have one key example of the derivation of the time squared law from the definition of uniform acceleration in the case of Galileo. Writing a letter to a friend in 1639, Galileo described the process by which he had discovered the law of acceleration many years previously. Galileo said that he worked it out directly from the definition of uniform acceleration as the motion of a body that leaves from rest and goes with a speed always increasing uniformly with the growth of time.

> I assume nothing but the definition of that motion of which I wish to treat and whose properties I then demonstrate. …I prove the spaces passed by such a body to be in the squared ratio of the times. …I argue from supposition about motion defined in that manner, and hence even though the consequences might not correspond to the events of natural motion of falling heavy bodies, it would little matter to me. …But in this, I may say, I have been lucky; for the motion of heavy bodies, and the properties thereof, correspond point by point to the properties demonstrated by me.

Galileo's calculations relied on an ancient Greek theory of proportion that was not preserved in standard medieval translations of Euclid, and didn't become available until a better translation was made in 1543. Recent research confirms his claim that he derived the law of acceleration arithmetically, independently of experiment. His derivation was also independent of the mean speed theorem of the Merton school. Apparently Galileo began using a hypothesis of discrete integral accumulations of speed similar to Buridan's *impetus* theory, but switched to a concept of continuous acceleration on discovering what he considered problems in the older view.

In the case of the 14th c. mathematicians at Oxford, a mathematical treatment of the concept of uniform acceleration was developed from which the times-squared law can be derived directly. No one in the Merton school actually derived the law, however, and there was little concern for applying the mathematics to physical systems. Thomas Bradwardine, William Heytesbury, Richard Swineshead and John Dumbleton, working on the general problem of rate of change, developed ratio-based definitions of uniform acceleration. Taking the velocity at the mid-point in time as the mean speed, they hypothesized that the distance covered by uniformly accelerated motion from rest over a given period of time would be equivalent to the distance covered by uniform motion at the mean speed over the same time interval. It follows from this that in any uniformly accelerated motion from rest, one fourth the total distance is covered in the first half of the time, as compared to three fourths in the second. This means that in the sequence of numbers representing the distances travelled in successive intervals of time, the ratio of the first number to the second (1:3) will be the same as that between the first two numbers and the second two numbers, or the first three and the second three, etc., since the mean speed theorem must apply whatever unit of time we choose. If we combine this result with the Buridanian idea that acceleration adds an equal increment of velocity and thus distance in each interval of time, (so that the series of numbers representing the distances travelled in successive instants will be arithmetic,) we have two requirements the series must meet. The most obvious series of integers that satisfies both is the series of odd integers: it is an arithmetic series, and the ratio of 1 to 3 is the same as the ratio of (1+3) to (5+7), or (1+3+5) to (7+9+11), etc. This odd number rule was worked out as part of the mathematics of uniform acceleration at Merton College, and was proved geometrically by Oresme at Paris, shortly thereafter. And once the distances travelled in successive intervals of time are represented as 1,3,5,7,9,11… the fact that *total* distance from rest is proportional to time squared almost leaps from the page—one squared equals 1, two squared equals 1+3, three squared equals 1+3+5, etc.. Yet no one in the 14th c. drew this conclusion, and when Galileo did develop the time-squared law of acceleration, it seems he did so without drawing on the mean speed theorem.

There is no evidence that Buridan ever appealed to the mean speed theorem in connection with his *impetus* hypothesis. Without the mean speed theorem, or something equivalent, the most likely mathematization of uniform acceleration in a view like Buridan's is one which does not lead to the time-squared law. In section III the following schema was suggested.

G	**V**
G+I	**V + V**
G+2I	**V + 2V**
G+3I	**V + 3 …**

If we make the simplifying assumption that the incremental velocity caused by the *impetus* is of the same measure as the uniform velocity caused by gravity, we get the following correlations:

Time from rest

velocity

incremental distance

total distance from rest

1

1

1

1

2

2

2

3

3

3

3

6

4

4

4

10

5

5

5

15

If Buridan had something like this in mind, the correlation of total distance with time-squared would be obscured for him.

This raises the question *why* Buridan did not avail himself of the mean speed theorem. Perhaps there is sufficient reason in his moderate nominalism. For Buridan, mathematical formulas are abstract entities which cannot be identified with real physical systems in their concreteness. Following Aristotle, he distinguished between physical entities, which contain matter, and abstract entities, which do not. Buridan was suspicious of the mathematical concept of the infinite divisibility of a magnitude, for example; continuities might be useful in mathematics, but physically they are inadmissible. Drake has suggested that Buridan's analysis of the motion of falling bodies requires discrete increments of speed, since there has to be a first instant during which gravity, but not *impetus*, is operative, imparting a uniform velocity. Ascribing mathematical continuity to acceleration would require both *impetus* and weight to be present at the beginning of motion, and that would lead to uniform speed rather than acceleration. Similarly, the distinction between mathematical instants with no duration and physical instants was thought to be important for avoiding the paradoxes surrounding the beginning of motion.

> Medieval physicists were much concerned with the concept of a 'first instant' of motion. The last instant of rest, if identical with the first instant of motion, would violate the principle of non-contradiction, since it would imply an instant at which a body was both at rest and in motion. There was no way around this with mathematical instants; but a physical instant having any duration at all, however small, offered no problem. It was easy to distinguish the last physical instant in which no motion was present from the first physical instant in which motion is present.

Suppose, with Drake, that Buridan's explanation of the motion of projectiles and falling bodies requires an atomic theory of physical time, and that acceleration consists for Buridan in discrete increments of velocity, each maintained uniformly through an instant of some brief duration. It does not follow that the corollary of the mean speed theorem to the effect that the distances covered in successive instants are in the ratio 1:3 could not be applied to motion thus conceived. Insofar as the premises used in the original derivation of the theorem and its corollary were physical, not purely mathematical, Buridan might have found reason to incorporate them into his own account. *If* Buridan got as far as using some series of integers to represent the distances travelled in successive instants, his own hypothesis would require that the series be arithmetic. Having arrived at that point, he might have wondered how to decide among the possible arithmetic series, and in particular, how to conceive the proportion between the distance covered in the first instant, under the influence of gravity alone, and the distance covered in the second. At that point the mean speed theorem and its corollary might have held considerable interest.

We may never know the precise nature of Buridan's attitude toward the work of his contemporaries at Oxford. But I think we can answer the question posed above, whether there is some systematic reason an Aristotelian like Buridan would be unlikely to think of distance from rest as varying with time squared. One reason for the hiatus between medieval *impetus* theory and post-Galilean mechanics is that the former was developed within the framework of the basic Aristotelian law of motion, **V F/R.** The basic Aristotelian conviction is that velocity is proportional to force. A constant force produces uniform velocity, not uniform acceleration. It was argued in Section III above that Buridan's *impetus* is an effect of gravity and other motive forces acting on the body in which it inheres, not an effect of the motion itself. But as an impressed quality, this effect is also conceived of as a force. Looking back at the simple formalization of Buridan's *impetus* hypothesis for falling bodies,

G V

G+I V + V

G+2I V + 2V

G+3I V + 3 … ,

we notice that by conceiving *impetus* as a force, so that the total force acting on the body is the sum of gravity and *impetus,* Buridan avoids what would otherwise be an expression of constant gravity producing uniform acceleration.

G V

G V + V

G V + 2V

G V + 3 …

Buridan’s inclusion of *impetus* as a motive force reflects his commitment to Aristotelian principles which soon became otiose. The magnitudes of measureable variables like distances, times, and quantities of matter remained the same through the transition from an Aristotelian to a post-Aristotelian framework. For someone like Galileo, interested in precise descriptions of motions in terms of relations among these variables and *un*interested in Aristotelian causal principles, the mathematical correlations would move to the foreground, and the concept of *impetus* as a force would drop out. In the new framework *impetus* would be redundant.

Buridan, however, falls solidly on the Aristotelian side of the divide. So while it is true that a direct proportion between total distance and time-squared can be derived by fiddling with certain series of numbers that result from the correlation of incremental velocity and time implicit in Buridan’s definition of uniform acceleration with the mean speed theorem, this was a *mathematical* fact not likely to interest an Aristotelian looking for an explanation of real motions.