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First-grade predictors of mathematical learning disability: A latent class trajectory analysis

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ABSTRACT

Kindergarten to third grade mathematics achievement scores from a prospective study of mathematical development ($n = 306$) were subjected to latent growth trajectory analyses. The four corresponding classes included children with mathematical learning disability (MLD, 6% of sample), and low (LA, 50%), typically (TA, 39%) and high (HA, 5%) achieving children. The groups were administered a battery of intelligence (IQ), working memory, and mathematical-cognition measures in first grade. The children with MLD had general deficits in working memory and IQ and potentially more specific deficits on measures of number sense. The LA children did not have working memory or IQ deficits but showed moderate deficits on these number sense measures and for addition fact retrieval. The distinguishing features of the HA children were a strong visuospatial working memory, a strong number sense, and frequent use of memory-based processes to solve addition problems. Implications for the early identification of children at risk for poor mathematics achievement are considered.

About 7% of children and adolescents will experience a substantive learning deficit in at least one area of mathematics (MLD) before graduating from high school (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen 2005; Lewis, Hitch, & Walker, 1994; Ostad, 1998; Shalev, Manor, & Gross-Tsur, 2005). They are accompanied by another 5–10% of children and adolescents, and perhaps more, with learning difficulties (for reviews see Berch & Mazzocco, 2007; Dowker, 2005). The latter students have

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specific difficulties in one or more areas of mathematics that are independent of cognitive ability and reading achievement and in this sense might be considered to have a moderate learning disability in mathematics. It is not known if the factors underlying their difficulties are simply less pervasive or severe than those that appear to underlie MLD or are qualitatively different (e.g., due to poor instruction; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Murphy, Mazzocco, Hanich, & Early, 2007). Thus, we distinguished the two groups and classified the children with moderate difficulties as low achieving (LA), to be consistent with recent studies (Murphy et al., 2007). Other unresolved issues concern the extent of the grade-to-grade stability of a child's classification as MLD or LA (Silver, Pennett, Black, Fair, & Balise, 1999) and identification of the early risk factors for long-term inclusion in one or the other of these groups (Gersten, Jordan, & Flojo, 2005).

We addressed each of these issues using data from a longitudinal, prospective study of children's mathematical learning and learning disability (Geary, in press). Using latent growth trajectory analyses applied to mathematics achievement scores from kindergarten to third grade, we identified stable groups of MLD and LA children. We compared and contrasted the performance of these groups to groups of typically achieving (TA) and high achieving (HA) children on a battery of mathematical cognition, intelligence (IQ), and working memory tasks administered in first grade. The approach allowed us to identify early risk for persistent MLD and LA and to determine if the cognitive factors contributing to inclusion in these groups represented lower and higher points, respectively, of the same underlying ability continuum or represented qualitatively different patterns of deficit.

1. Mathematical cognition

The mathematical tasks were chosen based on areas in which children with MLD or LA children have been found to have deficits or difficulties in earlier studies (Berch & Mazzocco, 2007; Geary, 1993, 2004). The areas assessed include children's early number sense, their implicit knowledge of counting principles, and the mix of strategies they use to solve addition problems.

1.1. Number sense

Children's number sense includes a non-verbal and implicit understanding of the absolute and relative magnitude of sets of small numbers of objects and of symbols (e.g., Arabic numerals) that represent these quantities. This implicit knowledge is manifested in their ability to apprehend the quantity of sets of 3–4 objects or actions without counting (Starkey & Cooper, 1980; Strauss & Curtis, 1984; Wynn, Bloom, & Chiang, 2002); use of non-verbal processes or counting to quantity small sets of objects and to add and subtract small quantities to and from these sets (Case and Okamoto, 1996; Gelman & Gallistel, 1978; Levine, Jordan, & Huttenlocher, 1992; Starkey, 1992); and proficiency at estimating the magnitude of sets of objects and the results of simple numerical operations (Dehaene, 1997). We use two measures to capture children's number sense—the speed and accuracy of identifying and processing number sets and the ability to represent quantity along a mathematical number line.

1.2. Number sets

Children with MLD have potential deficits in the core non-verbal ability to apprehend the quantity of small sets of objects and in the conceptual insight that numbers are composed of sets of smaller numbers (Butterworth & Reigosa, 2007; Geary et al., 2007; Geary, Bailey, & Hoard, 2009; Koontz & Berch, 1996). Koontz and Berch assessed the ability of third and fourth grade children with MLD and TA children to apprehend, without counting, the quantity of small sets of items or Arabic numeral representations of these sets. As an example, the children were asked to determine if combinations of Arabic numerals and number sets were the same (e.g., 2-■■) or different (e.g. 3-■■). Reaction time patterns for the TA children indicated fast access to representations of quantities of two and three, regardless of whether the code was an Arabic numeral or number set. Children with MLD showed fast access to numerosity representations for the quantity of two but appeared to rely on counting to determine quantities of three. These results suggest that some children with MLD might not have an inherent non-verbal representation for numerosities of three or more, implying the representational system for three does not allow for reliable discrimination of two from three.

Using the *Number Sets Test* (described below), we have also found evidence for less fluent (slow and inaccurate) processing of number-set information by children with MLD and, to a lesser extent, LA children (Geary et al., 2007, 2009). The test items require children to match groups of objects and Arabic numerals (e.g., •• 3) to a target number, such as 5. Signal detection methods can be applied to the corresponding number of hits and false alarms to generate sensitivity (d') and response bias (C) variables (MacMillan, 2002). The d' variable represents the child's sensitivity to quantities represented in task items and the C variable represents the child's tendency to respond to task items, whether they match the target number or not. Children who correctly identify many target quantities and commit few false alarms will have high d' and low C scores, whereas children who have as many hits as false alarms will have low d' and high C scores. In the latter case, the high number of correct items is due to the child's bias to respond and not to sensitivity to quantity. We found that higher d' scores in first grade were associated with higher mathematics achievement scores through third grade [rs = .49–.58 (p's < .001)], and that children at risk for MLD by 3rd grade—as determined by achievement scores less than the 15th national percentile ranking-have lower d' scores in first grade (Geary et al., 2009).

1.3. Number line

The mathematical number line (in which the difference between two consecutive numbers is identical regardless of position on the number line) is a core component of many aspects of mathematics and a critical part of basic education in mathematics. Children's competence with the number line is also theoretically interesting because magnitude representations, including those that support the mathematical number line, may be based on the potentially inherent number-magnitude system that contributes to performance on the *Number Sets Test*; this system is represented by specific areas in the parietal cortices (Isaacs, Edmonds, Lucas, & Gadian, 2001; Kadosh et al., 2007). Making placements on a physical number line that are based on use of the inherent number-magnitude system results in a pattern that conforms to the natural logarithm (ln) of the number (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Siegler & Booth, 2004; Siegler & Opfer, 2003). More precisely, use of this representation results in placements that are compressed for larger magnitudes such that the perceived distance between 92 and 93 is smaller than the perceived distance between 2 and 3. With instruction on the mathematical number line, children's placements become linear.

If children with MLD have deficits in the number-magnitude representational system (Koontz & Berch, 1996), then their number line placements might not conform to the natural log model or might show more compression (closer placements) for smaller numbers. We have found several patterns in the number line placements of children with MLD and LA children that suggest these types of deficits (Geary et al., 2007). We found that children with MLD were more heavily reliant on the natural number-magnitude representational system to make their number line placements than were LA and TA children; MLD children were learning the linear mathematical number line more slowly than these other children (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Even when they made placements consistent with use of the natural number-magnitude system, the placements of children with MLD and their LA peers did not conform as precisely to the predicted natural-log pattern as those of the TA children early in first grade, before much if any formal instruction on the number line. By the end of second grade, LA children caught up with their TA peers, but the MLD children, though they improved, lagged behind the other children. One possibility is that children with MLD and LA children begin school with a less precise underlying system of natural number-magnitude representations. This system may quickly mature, yielding improved ability to discriminate between quantities of near equal value, in LA children and may not mature or do so at a slower rate for children with MLD (Halberda, Mazzocco, & Feigenson, 2008).

1.4. Counting

Most school children quickly learn to count by rote, and this in and of itself is not a useful indicator of MLD or LA status. Children's understanding of the core principles of counting, such as one-one correspondence, and their ability to actively apply counting to solve arithmetic problems are, however, potential sources of poor mathematics achievement. We have explored both of these facets using a task that assesses children's sensitivity to the violation of counting principles (Briars & Siegler, 1984; Gelman & Gallistel, 1978). More precisely, following Gelman and Meck (1983) and Briars and Siegler, we assess this knowledge by asking the child to monitor a puppet's counting of objects. If the puppet violates a basic counting principle and the child detects the miscount, we assume the child at least implicitly understands the principle. Children's knowledge of counting principles and sensitivity to violations of these principles emerge during the preschool years and mature during the early elementary-school years (LeFevre et al., 2006).

Our studies suggest that children with MLD and LA children understand most basic counting principles, but they are sometimes confused when counting deviates from the standard left to right counting of adjacent objects (Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, Byrd-Craven, & Desoto, 2004). A more consistent finding is that children with MLD, but not LA children, fail to detect errors when the puppet double counts the first object in an array of objects, that is, this single object is tagged "one", "two". They detect these double counts when they occur with the last item, indicating they understand one-one correspondence but have difficulty retaining a notation of the counting error in working memory during the count (Geary et al., 2004; Hoard, Geary, & Hamson, 1999). The forgetting of miscounts is potentially important for children's learning to use counting to solve arithmetic problems. Ohlsson and Rees (1991) predicted that children who are skilled at detecting counting errors would more readily learn to correct these miscounts and thus eventually commit fewer errors when using counting to solve arithmetic problems. The evidence for this prediction is mixed (Geary et al., 1992, 2004), but detection of these double-counting errors is still a good empirical indicator of risk for MLD (Geary et al., 2007; Gersten et al., 2005).

1.5. Arithmetic

Children use a mix of counting strategies and memory-based processes to solve addition problems (Ashcraft, 1982; Siegler & Shrager, 1984). The mix is initially dominated by finger and then verbal (e.g., out loud) counting. With both counting strategies, children tend to use the sum or min procedure (Fuson, 1982; Groen & Parkman, 1972); the former involves counting both addends starting from one; the latter involves stating the cardinal value of the larger addend (e.g., "five") and counting a number of times equal to the value of the smaller addend (e.g., "six, seven, eight" to solve 5 + 3). As a result of schooling and practice, the strategy mix changes such that children gradually use the min procedure more often and eventually rely primarily on memory-based processes. The latter include direct retrieval of the answer and decomposition. To solve 7 + 5, a child might decompose the 7 into 5 and 2, retrieve the answer to 5 + 5 and then add 2.

In comparison to their TA peers, children with MLD, and LA children to a lesser extent, are delayed in their ability to effectively use counting to solve addition problems. They rely on finger counting for more years, adopt the min procedure at a later age, and commit more counting errors (Geary, 1993; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Montani, 1997; Ostad, 1997). The most consistent finding is that children with MLD show a deficit in the ability to use retrieval-based processes (Barrouillet, Fayol, & Lathuliere, 1997; Geary, 1990; Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003a). It is not that these children never correctly retrieve answers. Rather, they show a persistent difference in the frequency with which they correctly retrieve basic facts, and they rarely use decomposition. A similar pattern is found with young LA children, but the grade-to-grade persistence of their retrieval deficit is not currently known.

2. Intelligence and working memory

Intelligence (IQ), working memory, and speed of processing are the central cognitive mechanisms that contribute to learning across academic domains (Baddeley, 1986; Engle, Tuholski, Laughlin, & Conway, 1999; Fry & Hale, 1996; Kail, 1991; Walberg, 1984), including mathematics (Bull, Johnston, & Roy, 1999; Geary et al., 2004; McLean & Hitch, 1999; Swanson, 1993; Swanson & Sachse-Lee, 2001). The contribution of speed of processing to performance on our mathematical tasks is not substantial, however, when we control for individual differences in working memory (Geary et al., 2007). Thus, our focus here is on IQ and working memory. These two constructs are highly related but capture independent components of ability. The important features of working memory appear to be the attentional and inhibitory control mechanisms of the central executive (Engle et al., 1999), whereas the important feature of IQ may be the ability to logically problem solve (Embretson, 1995).

Geary et al. (2008) found that individual differences in children's accuracy at making placements on a number line was related to IQ in first grade, but improvement in accuracy from first to second grade was more strongly related to the central executive than to IQ. The relation between IQ and number line performance in first grade is consistent with the ability to think logically and systematically as related to learning the logical structure of the mathematical number line. The attentional and inhibitory control aspects of the central executive may be important during the actual placement of the numbers on the number line and for the inhibition of the natural-magnitude representational system during these placements; access to this system is associated with less accurate placements. The visuospatial sketch pad and phonological loop may also contribute to mathematics learning but in more restricted domains; the visuospatial sketch pad, for instance, may contribute to performance on the *Number Sets Test* and the number line task.

2.1. The present study

The present study adds to our previous results from the longitudinal project in several ways. The use of four years of achievement scores and the estimation of latent growth trajectories allowed for the identification of stable groups of MLD, LA, TA, and HA children. The procedure obviates the need to use cutoff scores for group classification and thus avoids fluctuations in the diagnosis of MLD due to measurement error in the achievement tests; for instance, a child with scores at the 9th and 14th national percentile ranking for mathematics achievement might be classified as MLD one year and LA the next with no substantive change in cognitive competence. With four years of achievement test data and a battery of IQ, working memory, mathematical-cognition measures administered in first grade, we were able to identify the early cognitive predictors of longer-term growth in mathematical achievement, as well as identify potential cognitive mechanisms that contribute to achievement growth differences across groups.

The present approach does not address growth in the specific forms of mathematical cognition deficit (Chong & Siegel, 2008; Jordan et al., 2003a), but it does allow us to identify groups that are learning mathematics at substantially different rates during the early school years and the early cognitive profiles of children that will form these achievement groups. Finally, in contrast to our previous analyses from this project, we included the full range of IQ scores when determining achievement groups. The cost was the introduction of IQ as a potential confound to the classification of MLD and LA, but with the benefit of providing a more realistic assessment of achievement groups that emerge in school settings.

3. Method

3.1. Participants

All kindergarten children from 12 elementary schools were invited to participate in a longitudinal prospective study of MLD. Parental consent and child assent were received for 37% ($n = 311$) of these children (Geary et al., 2007), and the latent trajectory analysis included the 306 children (142 male) having at least one year of achievement data. The ethnic composition was mixed (68% White, 12% Black, 5% Asian, 4% Hispanic, 6% of mixed race; the remaining participants did not report ethnicity) and came from schools that served families from a wide range of socioeconomic levels, although schools that have yielded high numbers of children with MLD or LA children in previous studies were oversampled.

3.2. Standardized measures

3.2.1. Intelligence

In first grade, the children were administered the Vocabulary and Matrix Reasoning subtests of the *Wechsler Abbreviated Scale of Intelligence* (Wechsler, 1999), and these scores were used to estimate IQ

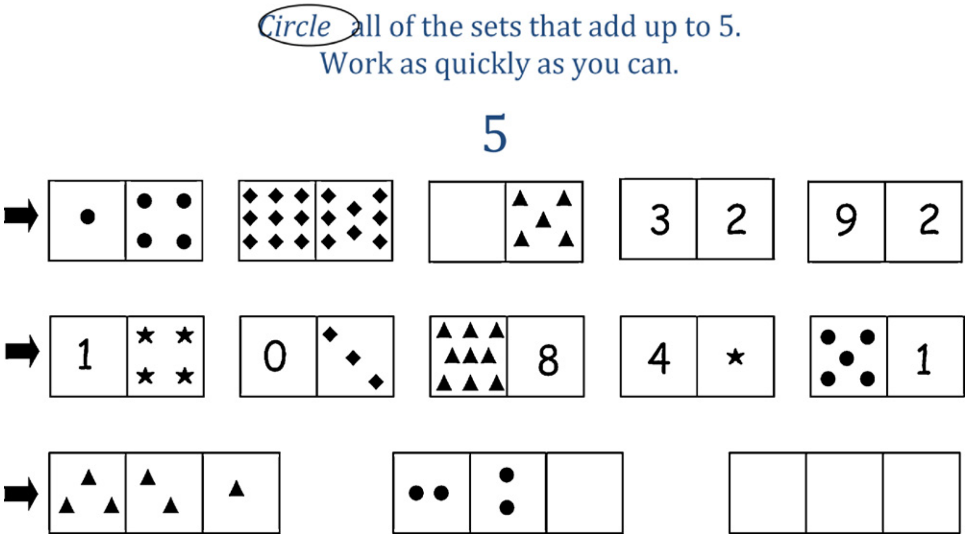

Fig. 1. Sample items from the *Number Sets Test*.

based on norms presented in the manual. For this study, we did not use scores for the *Raven's Coloured Progressive Matrices* (Raven, Court, & Raven, 1993), which was administered in kindergarten. The cost was slightly lower mean IQ scores (about 3 points) but the gain was more recent IQ norms that are calibrated with the achievement test norms.

3.2.2. Achievement

During the second semester of each grade, the children were administered the Numerical Operations and Word Reading subtests from the *Wechsler Individual Achievement Test-II-Abbreviated* (Wechsler, 2001). The Numerical Operations subtest assesses number discrimination, rote counting, number production, basic addition and subtraction, multi-digit addition and subtraction, and some multiplication and division. The Word Reading subtest includes matching and identifying letters, rhyming, beginning and ending sounds, phoneme blending, letter sounds, and word recognition.

3.3. Mathematical tasks

3.3.1. Number sets

This test was designed as a group-administered paper-and-pencil measure of the speed and accuracy with which children can identify the quantity of sets of objects and combine these with quantities represented by Arabic numerals (Geary et al., 2007). Fig. 1 illustrates several items from the measure. Two items matching a target number of 4 were first explained for practice. Next, using 3 as the target number, four lines of two sets of items were administered as practice. In these practice lines, one set was a match and one was clearly not a match (3 vs. 9, 10, or 11). Once it was determined that the child understood the task, the experimental items were administered. There are four pages of such items, and the child is instructed to move across each line of the page from left to right without skipping any and to "circle any groups that can be put together to make the top number, five (nine)" and to "work as fast as you can without making many mistakes." The child is given 60 s and 90 s per page for the targets 5 and 9, respectively, and is asked to stop at the time limit. We chose to time the task to avoid ceiling effects and because a timed measure should provide a better assessment of fluency in recognizing number combinations than an untimed measure.

The task yields numbers of hits, misses, correct rejections, and false alarms for each problem type and size. Geary et al. (2007) found that first graders' performance was consistent across target number and item content (e.g., whether the rectangle included Arabic numerals or shapes) and could thus be combined to create an overall frequency of hits (alpha, $\alpha = .88$), correct rejections ($\alpha = .85$), misses ($\alpha = .70$), and false alarms ($\alpha = .90$). Using a response operator characteristic curve analysis, Geary et al. (2009) derived a sensitivity measure, d', by subtracting each participant's z-score for misses from their z-score for hits (MacMillan, 2002). As noted, the value provides a measure of sensitivity to number while controlling for the child's response bias. We used the d' measure in the current analyses.

3.3.2. Number line estimation

A series of twenty-four 25 cm number lines containing a blank line with two endpoints (0 and 100) was presented, one at a time, to the child with a target number (e.g., 45) in a large font printed above the line. The child's was instructed to mark on the line where the target number should lie (for a detailed description see Siegler & Booth, 2004). Accuracy is defined as the absolute difference between the child's placement and the correct position of the number. For the number 45, placements of 35 and 55 produce difference scores of 10. The overall score is the mean of these differences across trials.

3.3.3. Counting knowledge

The child watched a puppet count a series of alternating red and blue chips at several different counting string lengths. Sometimes the puppet counted correctly and other times did not. The correct counts could be the standard left to right count or could be nonstandard (e.g., right to left), whereas the incorrect counts violated a basic counting principle (e.g., one-one correspondence; Gelman & Gallistel, 1978). The child's task was to tell the puppet if the way he counted was "OK" or "not OK", and thus assessed the child's awareness of counting principles and their understanding of whether variation from the standard left to right counting could still be correct (Briars & Siegler, 1984). Across a series of studies, we have found that children with MLD consistently miss trials on which the first chip in the sequence is double counted (Geary et al., 1992). Therefore, the variable used in this study was the percentage of trials (out of 3) on which the child correctly detected this particular counting error.

3.3.4. Addition strategy assessment

Fourteen simple (e.g., 3 + 6) and six complex (9 + 15) addition problems were horizontally presented in a large font (about 2 cm tall), one at a time, at the center of a "5 by 8" card. The child was asked to solve each problem (without the use of paper and pencil) as quickly as possible without making too many mistakes. It was emphasized that the child could use whatever strategy was easiest to get the answer, and the child was instructed to speak the answer out loud. Based on the child's description of how they got the answer and the experimenter's observations, the trial was classified into 1 of 6 strategies—counting fingers, fingers (i.e., holding up fingers and then retrieving an answer without counting), verbal counting, retrieval, decomposition, or other/mixed strategy (Geary et al., 2007; Siegler, 1987; Siegler & Shrager, 1984). A mixed trial was one in which the child started using one strategy but completed the problem using another. The four most commonly used strategies were counting fingers, verbal counting, retrieval, and decomposition.

For the current analyses, we used five variables from the strategy tasks. The first was the percentage of retrieval trials that were correct for simple problems (direct retrieval was uncommon for the complex problems), because this percentage is correlated with mathematics achievement scores (Geary, Bow-Thomas, Liu, & Siegler, 1996) and is an indicator of MLD (Geary, 2004). The second and third variables were based on Geary et al. (2007)—the raw number of problems correctly solved by a memory-based process (i.e., retrieval or decomposition) for simple (MemoryS) and complex problems (MemoryC). The two final variables captured the sophistication and accuracy of the children's use of backup strategies when they could not correctly use a retrieval-based process. If the number of retrieval errors was greater than the number of problems correctly solved by finger counting or verbal counting, then backup (BackupS for simple problems, BackupC for complex problems) was coded as [0 − number of retrieval errors]. If the number of problems solved correctly by finger counting or verbal counting was greater than the number of retrieval errors, then backup was coded [(2 × Correct Min counts) + (Correct Sum counts) − total counting errors]. Low scores indicated frequent guessing, whereas high scores indicated frequent use of the sophisticated min counting procedure.

3.4. Working memory

The Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001) consists of nine subtests that assess the central executive, phonological loop, and visuospatial sketchpad. All of the subtests have six items at each span level. Across subtests, the span levels rang from one to six to one to nine. Passing four items at one level moves the child to the next. At each span level, the number of items (e.g., words) to be remembered is increased by one. Failing three items at one span level terminates the subtest.

3.4.1. Central executive

The central executive is assessed using three dual-task subtests. Listening Recall requires the child to determine if a sentence is true or false and then recall the last word in a series of sentences. Counting Recall requires the child to count a set of 4, 5, 6, or 7 dots on a card and then to recall the number of counted dots at the end of a series of cards. Backward Digit Recall is a standard format backward digit span.

3.4.2. Phonological loop

Digit Recall, Word List Recall, and Nonword List Recall are standard span tasks with differing content stimuli; the child's task is to repeat words spoken by the experimenter in the same order as presented by the experimenter. In the Word List Matching task, a series of words, beginning with two words and adding one word at each successive level, is presented to the child. The same words, but possibly in a different order, are then presented again, and the child's task is to determine if the second list is in the same or different order than the first list.

3.4.3. Visuospatial sketchpad

Block Recall is another span task, but the stimuli consist of a board with nine raised blocks in what appears to the child as a "random" arrangement. The blocks have numbers on one side that can only be seen from the experimenter's perspective. The experimenter taps a block (or series of blocks), and the child's task is to duplicate the tapping in the same order as presented by the experimenter. In the Mazes Memory task, the child is presented a maze with more than one solution, and a picture of an identical maze with a path drawn for one solution. The picture is removed and the child's task is to duplicate in the path in the response booklet. At each level, the mazes get larger by one wall.

3.5. Procedure

All children were tested in the spring of their kindergarten year and in the fall and spring of first, second and third grade. The achievement tests were administered each spring and the first grade mathematical tasks were administered in the fall. The majority of children were tested in a quiet location at their school site and occasionally in a testing room on the university campus or in a mobile testing van if the child moved between assessments. The WMTB-C was administered in the testing van or on the university campus during first grade. The assessment times and corresponding sample sizes are shown in Table 1; as shown, attribution is low after the spring of first grade.

3.6. Results

The results are organized into three sections. In the first, we describe procedures for determining latent groups, estimating growth trajectories, and corresponding group differences on the achievement, IQ, working memory, and mathematical-cognition measures. In the second and third sections, we describe univariate and multivariate ratios for the odds of class membership using IQ, working memory, and the mathematical-cognition measures as predictors.

Table 1
Assessment schedule and sample sizes.

Measures	Assessment schedule							
	Kindergarten		First grade		Second grade		Third grade	
	Fall	Spring	Fall	Spring	Fall	Spring	Fall	Spring
Achievement		X		X		X		X
Intelligence				X				
Working memory			X	X				
Mathematical cognition			X					
Group	Sample size							
MLD		17	16	13		13		14
LA		154	141	125		125		123
TA		120	117	109		109		108
HA		15	15	14		14		14

Note: MLD = mathematical learning disability; LA = low achieving; TA = typically achieving; HA = high achieving.

Table 2
Fit statistics across numbers of classes.

No. classes	BLRT *p*-value	BIC	ABIC	Entropy	(%) in smallest class
2	<.0001	5171.10	5129.87	0.72	43
3	<.0001	5109.64	5052.55	0.78	6
4	**<.0001**	**5069.43**	**4996.48**	**0.82**	**5**
5	<.0001	5068.49	4979.69	0.82	2
6	<.0001	5065.67	4961.01	0.84	<1

Note: BLRT = Bootstrapped Likelihood Ratio Test, between n classes and $(n-1)$ classes; BIC = Bayesian Information Criterion; ABIC = Bayesian Information Criterion, adjusted for sample size.

3.7. Latent class models

Analyses were performed in Mplus, Version 5.1 (Muthen & Muthen, 2008). Growth mixture models and latent class growth analyses were considered (see Jung & Wickrama, 2007), but for various reasons, we chose a latent trajectory analysis solution.[1] With the latter, individuals were assigned to endogenous categories based on raw mathematics achievement scores at the 4 measurement occasions. The number of classes for the solution was determined based on the interpretability and likely replicability of the classification of individuals as well as model fit indexes. The latter were the Bootstrapped Likelihood Ratio Test, Bayesian Information Criterion (BIC), and the Adjusted Bayesian Information Criterion (ABIC; Nylund, Asparouhov, and Muthen, 2007), and are shown in Table 2. As can be seen, the overall fit of the 4-, 5-, and 6-class solutions is roughly similar. The 6-class solution, however, produced one class representing less than 1% of the sample and we therefore examined the 4- and 5-class solutions. The latter produced a slightly better fit than the former, but consisted of three high-achieving classes that were not clearly distinct from one another based on posterior probabilities. For this reason and because our objective is to identify lower-achieving children, the 4-class model was retained.

The entropy value for the 4-class model-bound between 0 and 1 (with values of 1 indicating a clear delineation between classes; Celeux & Soromenho, 1996) was 0.82. Values on the diagonal of the posterior probability matrix, which indicate the distinctness of the classification for each of the four respective groups, ranged from 0.88 to 0.94.

As shown in Fig. 2 (produced using the Sciplot package in R, Morales, 2008), the classes show distinct mathematics achievement scores and trajectories across measurement occasions. On the basis of mean

[1] Lower-achieving children's mathematics achievement scores show a floor effect in kindergarten and first grade. Therefore, classes are predicted to grow differently across measurement occasions with respect to an intercept, slope, and quadratic factors. With more measurement occasions, separate linear or even logistic functions might appropriately model growth across measurement occasions, but due to the low number of measurement occasions and the complex nature of observed growth across these occasions, a parsimonious LCA model was chosen.

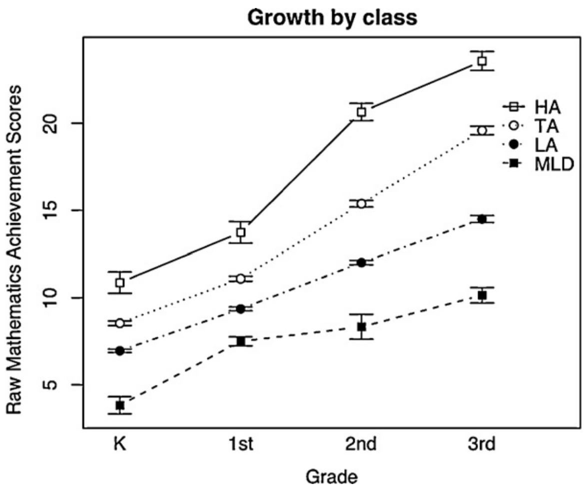

Fig. 2. Mathematical achievement raw score growth by class across measurement occasions. HA = High achieving, TA = typically achieving, LA = low achieving, MLD = mathematical learning disability.

Table 3
Mean IQ, working memory, and achievement scores for the four classes.

	Working memory								Mathematics								Reading							
	IQ		CE		PL		VSSP		K		First grade		Second grade		Third grade		K		First grade		Second grade		Third grade	
	M	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD
MLD	83	7	83	9	80	12	83	12	11	14	8	6	5	4	2	2	36	25	21	25	17	23	15	19
LA	95	12	95	13	98	15	97	14	42	20	22	15	20	13	18	12	66	19	51	29	52	28	47	23
TA	104	13	106	12	104	13	103	14	66	20	44	23	55	23	55	21	80	17	73	24	74	21	66	20
HA	115	17	116	13	107	13	113	13	85	20	74	24	94	8	84	14	88	12	89	15	86	15	81	16

Note: IQ = Intelligence, CE = central executive, PL = phonological loop, VSSP = visuospatial sketchpad. Achievement scores are national percentile rankings. MLD = mathematical learning disability, LA = low achieving, TA = typically achieving, HA = high achieving.

achievement scores shown in Table 3, we refer to these classes, from lowest to highest achieving, as MLD, LA, TA, and HA, with prevalence based on most likely class membership: 6% (n = 17), 50% (n = 154), 39% (n = 120), and 5% (n = 15) of the sample belonging to each respective class. The overall achievement trajectory of the LA group suggests a moderate learning difficulty in mathematics, but to be consistent with terms used in recent studies (Geary et al., 2007; Murphy et al., 2007) and because this is likely to be a heterogeneous group of children, we used the term low achieving rather than learning disabled.

Independent analyses of variance (ANOVAs) confirmed significant group differences (p's < 0.001) for IQ, $F(3, 257) = 23.63$, and each of the working memory variables; for CE, $F(3, 265) = 41.16$, VSSP, $F(3, 265) = 16.50$, and for PL, $F(3, 265) = 16.27$. Follow-up analyses revealed all pairwise comparisons of means to be significant at Tukey-adjusted alpha = 0.05, except for PL differences between HA and TA ($p = 0.87$) and between HA and LA [$p = 0.09$].

An analysis of covariance (ANCOVA) with mathematics and reading achievement as within-subjects variables and group as a between-subject variable, and IQ as the covariate yielded significant main effects for group and IQ [F's (1, 253); (3, 253) = 35.89, 94.43, p's < 0.0001], which were qualified by significant test by IQ, $F(1, 253) = 11.26$, $p < 0.001$, test by group, $F(3, 253) = 14.12$, $p < 0.0001$, time by group [Wilk's Lambda = 0.87, $F(9, 611.02) = 4.04$; $p < 0.001$], and test by time by group [Wilk's Lambda = 0.90, $F(9, 611.02) = 2.95$, $p < 0.005$] interactions. The critical tests confirmed that the class differences for mathematics achievement were not due to class differences in IQ at any grade [F's (3, 253) = 32.71, 40.48, 97.29, and 113.97, p's < 0.0001].

Table 4
Mean *z*-scores for the mathematical cognition measures for the four classes.

	d′		Number line		Count error		PCR		MemoryS		MemoryC		BackupS		BackupC	
	M	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD
MLD	−1.75	1.41	1.27	0.62	−1.81	1.31	−0.57	1.28	−0.63	0.19	−0.20	0.52	−1.37	0.67	−1.33	0.47
LA	−0.12	1.08	0.41	0.99	−0.35	0.80	−0.30	0.80	−0.38	0.65	−0.27	0.25	−0.18	1.05	−0.27	0.98
TA	0.36	0.44	−0.51	0.67	0.55	0.63	0.28	1.05	0.33	1.02	0.12	1.08	0.43	0.72	0.46	0.80
HA	0.35	0.37	−0.95	0.45	0.97	0.55	1.19	0.29	1.64	1.47	1.82	2.37	−0.03	0.78	0.42	0.73

Note: All variables were standardized with *M* = 0 and SD = 1. MLD = Mathematical learning disability, LA = low achieving, TA = typically achieving, HA = high achieving, *d*′ = sensitivity measure from *Number Sets Test*; Number line = mean of absolute error scores on the number line task, Count error = percentage of trials detecting double-counting error, PCR = percentage of correct retrieval trials for simple addition, MemoryS = frequency of correct retrieval and decomposition for simple addition, MemoryC = frequency of correct retrieval and decomposition for complex addition, problems, BackupS = backup strategies for simple addition problems, BackupC = backup strategies for complex addition problems.

3.8. Univariate class diagnoses

Because specific classes are clearly distinct (e.g., MLD vs. TA), individual children are not likely to be misclassified as members of non-adjacent classes; for example, a child with MLD is unlikely to be misclassified as TA. Some children, however, represent borderline cases between adjacent classes; for example, a child with MLD might be misclassified as LA. As a means to address these potentially ambiguous cases, we estimated the odds ratios that a child is a member of one of two adjacent classes for each of the key variables used in this study; specifically, the IQ and working memory measures and the mathematical-cognition measures shown in Table 4. Because these variables are continuous, the interpretation of each ratio is the increase in odds that an individual is a member of the first class for each one SD increase in the variable; each variable has been standardized (*M* = 0, SD = 1). Each of these values represents an odds ratio returned from a single variable diagnostic logistic regression model. Using the IQ variable to illustrate, given that a child is in the region where they could be classified as either HA or TA, a 1 SD increase in IQ was associated with a 1.98 fold increase in the odds of membership in the HA class (95% confidence interval = [1.10, 3.57]; since the interval does not include 1, it is significant at alpha = 0.05). As can be seen, the diagnostic utility of IQ increases as achievement decreases. In the case of a child who is either LA or MLD, a 1 SD increase in IQ is associated with a 30 fold increase in the odds that individual is LA.[2]

3.8.1. HA/TA

Examination of the odds ratios in Table 5 indicate that, among IQ and the working memory measures, higher levels of IQ, VSSP, and CE increased the odds of belonging to the HA group as opposed to the TA group. Regarding mathematical tasks, higher levels of *d*′, PCR, MemoryS, and MemoryC increased the odds of belonging to the HA group whereas higher levels of BackupS decreased the odds of belong to this group.

3.8.2. TA/LA

Higher levels of IQ, PL, VSSP, and CE increased the odds of belonging to the TA (compared to the LA) group. Among mathematical tasks, higher levels of *d*′, Count Error, PCR, MemoryS, MemoryC, BackupS, and BackupC increased the odds of belonging to the TA group; notably, a 1 SD increase in *d*′ resulted in a more than 17-fold increase in the odds of placement in the TA group. Higher error levels on the Number Line task resulted in roughly an 8.33 decrease in odds of belonging to the TA group.

[2] Odds ratio confidence intervals were estimated using robust estimates based on the Maximum Likelihood estimation function. Confidence intervals based on bootstrap draws for IQ yielded extremely similar results, which did not change any interpretations of statistical significance.

Table 5
Estimates and 95% confidence intervals for univariate odds ratios.

Variable	Class/Class		
	HA/TA	TA/LA	LA/MLD
Intelligence and working memory			
IQ	1.98 (1.10, 3.57)[a]	2.89 (1.82, 4.60)[a]	30.05 (9.18, 98.39)[a]
PL	1.14 (0.64, 2.02)	1.91 (1.36, 2.67)[a]	4.70 (2.21, 9.97)[a]
VSSP	2.30 (1.20, 4.42)[a]	1.74 (1.24, 2.43)[a]	4.66 (1.77, 12.28)[a]
CE	2.22 (1.15, 4.28)[a]	3.86 (2.28, 6.55)[a]	12.38 (4.18, 36.72)[a]
Mathematical tasks			
d'	3.37 (1.11, 10.24)[a]	17.36 (8.21, 36.69)[a]	6.46 (3.00, 13.94)[a]
Number line[b]	0.11 (0.005, 2.09)	0.12 (0.06, 0.24)[a]	0.32 (0.15, 0.68)[a]
Count error	0.99 (0.33, 2.93)	2.31 (1.41, 3.78)[a]	3.01 (1.62, 5.60)[a]
PCR	1.94 (1.19, 3.17)[a]	2.97 (1.81, 4.87)[a]	3.00 (0.86, 10.46)
MemoryS	2.69 (1.54, 4.71)[a]	4.74 (2.51, 8.96)[a]	5.23 (1.06, 25.92)[a]
MemoryC	2.02 (1.46, 2.81)[a]	3.50 (1.83, 6.56)[a]	0.51 (0.13, 2.01)
BackupS	0.45 (0.26, 0.77)[a]	2.81 (1.94, 4.07)[a]	5.67 (2.80, 11.50)[a]
BackupC	0.77 (0.43, 1.35)	3.18 (2.15, 4.71)[a]	6.23 (2.71, 14.34)[a]

Note: MLD = Mathematical learning disability, LA = low achieving, TA = typically achieving, HA = high achieving, d' = sensitivity measure from *Number Sets Test*; Number line = mean of absolute error scores on the number line task, Count error = percentage of trials detecting double-counting error, PCR = percentage of correct retrieval trials for simple addition, MemoryS = frequency of correct retrieval and decomposition for simple addition, MemoryC = frequency of correct retrieval and decomposition for complex addition, problems, BackupS = backup strategies for simple addition problems, BackupC = backup strategies for complex addition problems.

[a] Significant at alpha = 0.05.

[b] Lower number line deviation scores indicate more accurate placements and thus the odds ratios are lower than 1.

3.8.3. LA/MLD

Higher levels in all IQ and working memory variables increased the odds of LA group membership compared to MLD group membership. Higher levels in the mathematical tasks of d', Count Error, MemoryS, BackupS, and BackupC increased the odds of belonging to the LA group whereas higher error levels in the Number Line task resulted in decreased odds of belong to this group.

3.8.4. Gender

Although not an a priori aspect of our study, the sex (female coded as 0, male coded as 1) of the child was related to class membership. In all three cases, the odds ratio was significant, indicating different proportions of boys and girls across groups. Given that a child is LA or MLD, the odds of being MLD are 5 times higher for boys. Given that a child is TA or LA, a boy has more than twice the odds of being TA. We could not estimate the ratio for discriminating between the HA and TA classes, because there were no girls in the HA class; therefore, based on these data, girls are more likely to belong to the TA class.

3.9. Multiple regression analyses

To explore the constellation of strengths and deficits underlying class membership, cognitive variables were entered into logistic regressions predicting dichotomous class membership and linear regressions predicting continuous posterior probability variables; the latter are estimates of the probability of membership in each of the four classes. Models were selected using the backwards stepwise procedure with the AIC as a criterion for model fit (Akaike, 1974). Because each predictor is standardized, the interpretation of the regression weights for the logistic regressions is increase in the natural log (ln) odds of class membership given a one SD increase in the predictor, independent of other predictors. The corresponding odds ratios are provided by e^b, where b is the regression coefficient. The interpretation of the regression weights for the linear regressions is increase in the probability of class membership (compared to any other class) given a one SD increase in the predictor, independent of other predictors. Both sets of analyses were based on the 228 children, without any missing data—MLD (n = 12), LA (n = 101), TA (n = 103), HA (n = 12).

Table 6
Best fitting multivariate predictors from logistic regression.

	Estimate	Std. error	z-Value	Pr(>|z|)	Odds ratio (e^b)
Mathematical learning disability					
Intercept	−6.59	1.54	−4.27	0.000	–
Sex	1.31	0.94	1.39	0.163	3.71
IQ	−0.81	0.53	−1.52	0.129	0.44
PL	−0.84	0.52	−1.63	0.104	0.43
VSSP	−1.01	0.58	−1.76	0.078	0.36
Number line	1.62	0.66	2.45	0.014	5.05
Count error	−0.51	0.32	−1.60	0.110	0.60
Low achieving					
Intercept	0.12	0.21	0.56	0.576	–
Sex	−0.73	0.33	−2.23	0.026	0.48
CE	−0.54	0.22	−2.45	0.014	0.58
PL	0.31	0.20	1.59	0.126	1.36
d'	−0.45	0.24	−1.90	0.058	0.64
Number line	0.32	0.22	1.49	0.136	1.38
PCR	−0.39	0.20	−2.01	0.045	0.68
Typically achieving					
Intercept	−0.66	0.19	−3.41	0.000	–
CE	0.30	0.19	1.60	0.110	1.35
d'	1.00	0.29	3.47	0.001	2.72
Number line	−0.54	0.23	−2.30	0.021	0.58
Count error	0.62	0.27	2.29	0.022	1.86
High achieving					
Intercept	−8.39	2.01	−4.17	0.000	–
Sex	2.32	1.33	1.75	0.080	10.18
IQ	0.77	0.44	1.73	0.083	2.16
CE	1.13	0.72	1.56	0.119	3.10
VSSP	1.07	0.63	1.70	0.089	2.92
d'	1.25	0.85	1.47	0.143	3.49
PCR	1.18	0.46	2.59	0.010	3.25

3.9.1. Logistic regressions

The best fitting equations are shown in Table 6. The individual predictors may not be significant in any particular equation, due to collinearity, but the combination represents the best fitting constellation of variables for predicting membership in the four respective classes. Considering the first equation, it can be seen that being a boy results in a 3.71-fold increase in the odds of classification into the MLD group (vs. LA, TA, or HA class membership), independent of the other predictors. Notably, a 1 SD increase in the degree of error on the number line measure resulted in a 5-fold increase in the odds of MLD. Higher IQ, phonological loop, and visuospatial sketch pad scores were associated with a reduced likelihood of MLD classification. Or stated differently, a 1 SD decrease in visuospatial sketch pad scores, for instance, resulted in a 2.78 (1/.36)-fold increase in the odds of MLD.

A contrast of this first equation with that associated with the LA classification reveals similarities but more importantly many differences. The latter suggests different mechanisms underlying the poor mathematics achievement of these two groups of children. Being a girl is associated with a 2.08 (1/.48)-fold increase in the likelihood of LA classification. Cognitive differences emerged in terms of the importance of the central executive, the d' variable, and the percentage of correct retrieval of simple addition facts (i.e., PCR). A one SD increase in these respective competencies independently result in 1.72 (1/.58)-, 1.56 (1/.64)-, and 1.47 (1/.68)-fold decreases in the likelihood of LA classification.

Three of the four variables that emerged with the TA classification equation also emerged with the LA equation, suggesting the LA and TA groups may be largely lower and higher ends, respectively, of the same population. Increases in fluency in identifying number sets (d') and percentage of correct identification of double counting errors increased the likelihood of classification into the TA group, whereas decreased accuracy in number line placements decreased the likelihood of TA class member-

Table 7
Best fitting multivariate predictors from linear regression.

	Estimate	Std. error	*t*-Value	Pr(>|*t*|)
Mathematical learning disability				
Intercept	0.04	0.015	2.66	0.008
Sex	0.04	0.023	1.76	0.079
PL	−0.02	0.013	−1.67	0.097
VSSP	−0.01	0.013	−0.98	0.328
d'	−0.05	0.015	−3.33	0.001
Number line	0.03	0.016	1.63	0.105
Count error	−0.05	0.013	−3.75	0.000
Low achieving				
Intercept	0.53	0.035	15.14	0.000
Sex	−0.13	0.054	−2.34	0.020
CE	−0.09	0.031	−2.46	0.015
d'	−0.06	0.036	−1.72	0.087
Number line	0.08	0.044	2.27	0.024
PCR	−0.07	0.029	−2.45	0.015
Typically achieving				
Intercept	0.42	0.025	16.92	0.000
CE	0.06	0.030	2.18	0.030
d'	0.11	0.034	3.08	0.002
Number line	−0.13	0.034	−3.70	0.000
High achieving				
Intercept	0.03	0.017	1.72	0.086
Sex	0.05	0.026	2.04	0.042
IQ	0.03	0.014	2.38	0.018
PCR	0.07	0.013	5.19	0.000
VSSP	0.03	0.014	2.42	0.017

ship. The most notable feature of the final equation is the 10.18-fold increase in the likelihood of a HA classification for boys. Unlike the LA and TA classifications and in inverse to the relation to that found for MLD classification, the visuospatial sketch pad emerged as an important predictor for HA (odds ratio = 2.92). A one SD increase in the fluency of identifying number sets (d') and accuracy in retrieving addition facts from long-term memory independently resulted in more than a three-fold increase in the odds of HA classification.

3.9.2. Linear regressions

As can be seen in Table 7, the predictors in the linear regressions were largely the same as those that emerged in the logistic equations; overall model fits ranged from $R^2 = .24$ to .31, p's < .0001. As an example, independent of other predictors, a one SD decease in the fluency of identifying number sets (d') and in accuracy in detecting double counting errors respectively resulted in a 5% increase in the probability of an MLD classification. Again, there was considerable overlap in the predictors of the probability of LA and TA classification and, unlike these two groups, the visuospatial sketch pad emerged as a predictor of the probability of a HA classification.

4. Discussion

The central goals of the current study were to identify stable groups of MLD and LA children, to determine similarities and differences in the cognitive deficits contributing to group membership, and to identify early cognitive predictors of group membership. We consider each of these in turn.

In their prospective, longitudinal study of mathematics learning, Murphy et al. (2007) classified children as MLD if their mathematics achievement scores were below the 11th percentile of their sample for at least two years, from kindergarten to third grade, and as LA if their scores were between the 11th and 25th percentiles, inclusive, for at least two of these years. Children in these respective groups not only showed, by definition, lower and higher levels of mathematical performance, but different growth trajectories. The children with MLD started lower on the achievement test and showed more shallow growth, relative to their LA peers and a comparison group of TA children. The two latter groups did not differ in terms of growth in mathematics achievement, but their start points differed by design. Jordan and her colleagues have also identified classes of children with different initial levels of mathematical competence and sometimes with different growth rates (Jordan, Hanich, & Kaplan, 2003b; Jordan, Kaplan, Olah, & Locuniak, 2006).

Our results confirm Murphy et al.'s (2007) and Jordan et al.'s (2006) findings for the existence of a small number of classes of children with different initial levels of mathematical achievement, and Murphy et al.'s (2007) findings in terms of class differences in the growth trajectories of achievement scores; Jordan et al. (2006) also identified groups with different trajectories across the kindergarten year but focused on growth on mathematical-cognition tasks, not achievement measures. As with the MLD group identified by Murphy et al., our group of children with MLD showed some grade-to-grade growth on the mathematics test, but their below-average trajectory resulted in declining national percentile scores across grades. Our MLD class and that of Murphy et al. also showed below average IQ and reading achievement scores, but performance in both of these domains was above that found for mathematics achievement. As detailed below, whatever the contributions of IQ and reading competence to their mathematical development, children with MLD appear to have deficits specific to mathematics.

Murphy et al. also found that 2 out of 3 of the children in their MLD group were boys and 2 out of 3 children in their LA group were girls. Although their results were not statistically significant, the same pattern emerged in our study and suggests that low mathematics achievement may manifest differently in boys and girls (see also Barbaresi et al., 2005). In any case, our results add to accumulating evidence for the existence of an identifiable group of at least 5% of children who have a persistent, grade-to-grade learning disability in one or several areas of mathematics that cannot be entirely attributable to IQ or poor reading ability.

5. Cognitive profiles of MLD and LA groups

It has been demonstrated many times that children with MLD score below average on measures that assess one or several working memory systems (Bull et al., 1999; Geary et al., 2004; McLean & Hitch, 1999; Swanson, 1993) and have delayed procedural development in arithmetic and difficulty learning basic arithmetic facts (Chong & Siegel, 2008; Geary, 1993; Jordan et al., 2003a). However, the criterion used to define groups of children with MLD has varied from one study to the next and, as a result, many of these groups included both children with MLD and LA children, as defined here and by Murphy et al. (2007). The conflation of these groups has made it difficult to determine if the same or different mechanisms underly their poor mathematical development. There has also been a tendency in this literature to classify children with poor mathematics achievement based on whether the deficits are specific to mathematics (MD-only) or involve poor outcomes in both mathematics and reading (MD/RD; Geary et al., 2000; Geary, Hoard, Hamson, 1999; Hanich et al., 2001; Jordan & Montani, 1997). Children with MD-only and MD/RD appear to be similar to the LA and MLD groups, respectively, identified in this study and by Murphy et al.; thus the following discussion may also apply to the MD-only and MD/RD classifications.

Murphy et al. (2007) identified subtle differences in the working memory deficits of their MLD and LA groups, but there was no specific cognitive or mathematics variable that differentiated the groups. Rather, the groups differed in terms of more or less severe deficits in the same cognitive and mathematical domains. Geary et al. (2007), in contrast, identified a group of average-IQ children with MLD that had substantial working memory deficits and a group of average-IQ LA children with no such working memory deficits. We did not control for IQ in the current analyses, but the same group differences in working memory emerged. The best constellation of variables for predicting membership in the MLD class included IQ and the phonological loop and visuospatial sketch pad scores, the means for all of which were about 1 SD below average (Table 3). However, the central executive variable did not emerge as a significant predictor of MLD class membership in our current analyses, in contrast with Geary et al. (2007), which may have been due to the emergence of the count error variable in these equations and collinearity; specifically, the deficit of children with MLD on this counting variable appears to be mediated by their central executive deficits (Geary et al., 2007). In other words, the central executive deficits of the children with MLD were better captured by the count error variable than by the central executive measure per se.

The emergence of the central executive and phonological loop variables for prediction of LA class membership was not due to deficits in these components of working memory. In fact, controlling for other predictors, higher phonological loop scores were associated with higher odds of classification as LA, potentially due to the over-representation of girls in this class. Lower central executive scores were associated with higher odds of LA classification, despite scores in the average range (Table 3), due to the contrast with the above average central executive scores of the children in the TA and HA groups.

Independent of working memory and IQ scores, the best predictor of membership in the MLD class was the number line variable for the logistic regression and the d' and count error variables in the linear regression. As stated, the count error variable could be capturing variance in central executive competence, but the emergence of the number line and d' variables suggest these children have a poor number sense, in support of other findings (Butterworth & Reigosa, 2007; Geary et al., 2009; Koontz & Berch, 1996). The children with MLD also correctly retrieved fewer addition facts than did TA and HA children (Table 4), as is typically found, but this did not emerge in the regressions as their most serious deficit. The number line and d' measures also emerged as core predictors of membership in the LA class, as did their low frequency of correct addition-fact retrieval. Whether the number sense and retrieval deficits of the children with MLD and LA children represent different cut points along the same underlying ability continuums or different forms of deficit is not clear. For instance, the IQ and working memory deficits might contribute to the poor performance of the children with MLD on these tasks (Geary et al., 2008), but this cannot be the case for the LA children.

In fact, given the unexpectedly large percentage (50%) of children in our overall sample who were classified as LA, it is unlikely that all of them have number sense or fact retrieval deficits of the magnitude that would support a learning disability diagnosis. The poor performance of some proportion of these children may result from instructional practices or lack of early informal exposure to number information at home. Our planned follow-up analyses of the grade-to-grade changes on the mathematical-cognition measures may provide a means to better separate children in our LA class who have actual mathematics cognition deficits from those whose low achievement is the result of experiences that do not support full development of their potential in mathematics.

For this particular sample, the emergence of similar sets of predictors in the regressions and based on the pattern of mean scores, the LA and TA classes appear to represent lower and higher ends, respectively, of largely the same constellation of ability continuums. No doubt the children in the MLD and HA classes vary along the same continuums for individual competencies, but there were several potentially interesting differences within the constellations of variables that best predicted inclusion in these two classes. The children classified into the MLD group had across-the-board deficits in all three working memory systems, if we assume the count error variable is capturing poor central executive abilities. The other notable feature is that the mathematical tasks that emerged in the regressions assess their number sense, and not their competence in procedural execution or addition-fact retrieval. The children with MLD do, in fact, have difficulties in these two latter areas (Table 4), as found in most previous studies (Geary, 2004), but an earlier emerging and potentially more fundamental deficit may exist in their number sense (Butterworth & Reigosa, 2007; Koontz & Berch, 1996); potentially related to the inherent number-magnitude representational system.

In addition to the finding that they were all boys, the most distinct aspect of the regressions for predicting HA class membership was the importance of the visuospatial sketch pad. In an earlier study in which we identified HA children based on IQ scores (mean of 126 as compared to 115 for the current HA group), and not mathematics achievement scores, the key working memory mechanism that contributed to their advantages on most of the mathematical tasks was the central executive (Hoard, Geary, Byrd-Craven, & Nugent, 2008). The central executive is important for the current HA group as well, but the visuospatial sketch pad may contribute as much or more to their growth trajectory in mathematics as the central executive (see also Dark & Benbow, 1991).

6. Predictors of MLD and LA status

The results of this study are consistent with those of other studies of early potential predictors of poor outcomes in mathematics (Gersten et al., 2005; Mazzocco & Thompson, 2005) and confirm the importance of an early number sense for mathematical development (Case and Okamoto, 1996). More precisely, the *Number Sets Test*, the number line task, the double-count error items from the counting knowledge task, and assessment of the ability to correctly retrieve basic addition facts from long-term memory all emerged as important mathematical-cognition variables in the prediction MLD or LA class membership, independent of the contributions of IQ and working memory. The critical distinction between membership in the MLD or LA classes was the below average IQ and working memory competencies of the former group. The implication is that risk assessment for poor long-term outcomes in mathematics should include IQ and working memory measures, as well as measures that assess children's number sense, their ability to monitor the act of counting, and their knowledge of very simple addition and presumably subtraction facts.

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