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Determinants of Hedging and Risk Premia in Commodity Futures Markets

David Hirshleifer*

Abstract

This paper examines the determinants of commodity futures hedging and of risk premia arising from covariation of the futures price with stock market returns, and with the revenues of producers. Owing to supply shocks that stochastically redistribute real wealth (surplus) between producers and consumers, and to limited participation in the futures market, the total risk premium in the model is not proportional to the contract's covariance with aggregate consumption. Stock market variability interacts with the incentive to hedge, causing the producer hedging component of the risk premium to increase (decrease) with income elasticity, for a normal (inferior) good. Production costs that depend on output raise the premium. We argue that output and demand shocks will typically be positively correlated, raising the premium. High supply elasticity reduces the absolute hedging premium by reducing the variability of spot price and revenue.

I. Introduction

In the Mayers (1972) capital asset pricing model, the risk premium of a security such as a futures contract is composed of a term proportional to its covariance with traded assets, and a term proportional to its covariance with non-marketable (indivisible) risks. Nonmarketable risks are potentially important for futures pricing in commodity markets in which producers or handlers use a futures contract to hedge their risky net revenues. The issuance of widely-held equity claims by suppliers in a number of commodity markets is minimal, possibly owing to problems of adverse selection and moral hazard.¹ Recognizing the importance of undiversified commodity hedgers, Stoll (1979) extended Mayers'

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¹ The bulk of planted acreage of grains in the U.S. is held by noncorporate farms, and even some major processing firms are closely held (such as Cargill, Continental Grain, and Dreyfus). Furthermore, even in widely-held firms, optimal contracts that impose risks on managers may provide an incentive to hedge the firm's risk using futures (see Diamond and Verrechia (1982)).

model to analyze how hedging pressure causes deviations in futures risk premia from the prediction based on stock market covariance.²

In a 1980 paper, Breeden applied the intertemporal consumption-based CAPM to examine the characteristics of commodity spot markets that determine the futures risk premia. He related consumption betas and, therefore, risk premia to output and demand shocks, and to the elasticities of supply and demand for the commodity. Aggregate consumption in a single-period single-good setting is the sum of the terminal values of the stock market and of nonmarketable endowments. However, Breeden's analysis focused primarily on factors affecting the covariance of the futures price with the consumption of outside investors, rather than the nonmarketable endowments of commodity hedgers.

Several writers³ have focused on hedging pressure by assuming that the only risky assets are the privately-held businesses belonging to commodity market suppliers, and that the only tradable security is a futures contract. With the stock market eliminated, the futures price bias is primarily determined by producers' hedging of their nonmarketable risks.

This paper includes both a stock market and nonmarketable endowments of commodity suppliers. In this respect, the analysis complements Breeden's (1980) analysis by emphasizing the producer hedging component as well as the stock market risk component of the premium. We show that stock market variability interacts with the hedging pressure of producers in determining the futures risk premium. This is despite the additive separation between the stock market covariance and the hedging components of the premium described by Stoll (1979). The pathway of interaction is income elasticity. Good or bad news in the stock market, reflecting real wealth changes, affects the demands for different commodities. The stochastic demand for a given commodity affects both suppliers' profits and the payoffs on a futures contract. Hence, stock market risk affects the covariation between producers' profits and the futures payoff, and so their incentive to hedge.

This paper also allows for a second form of market imperfection not present in the models of Stoll and Breeden: barriers to participation in futures markets.⁴ Owing to incomplete participation, the premium will be determined not in relation to the futures contract's covariation with society-wide aggregate consumption, but only with the consumptions of futures trading individuals. The futures trading incentives of commodity suppliers and outsiders differ greatly; for example, shocks to corn production through shifts in price can lead to negatively

² Consistent with Stoll's paper and the model presented here, Chang (1985) provides evidence that commodity futures price changes are predicted by hedging positions taken previously by producers. In contrast, the evidence for the traditional CAPM in the commodities context is at best mixed (Dusak (1976) and Bodie and Rosansky (1980)). See also Carter, Rausser, and Schmitz (1983) and the comment of Marcus (1984).

³ See, e.g., Newbery and Stiglitz (1981), Anderson and Danthine (1983), Britto (1984), and D. Hirshleifer (1988b).

⁴ Few investors participate in commodity futures markets, either directly or through financial intermediaries such as mutual or pension funds. This may arise from a reluctance of uninformed individuals to trade futures, or from regulatory or moral hazard constraints to commodity trading by institutional investors. Despite a recent proliferation of futures mutual funds, due to their very active management based on technical analysis, they have not provided a convenient means for uninformed investors to diversify.

correlated shifts in consumer surplus versus producer surplus. Consequently, it is necessary to analyze the risks faced by producers in particular, rather than consumption risk in the aggregate, to understand fully the sources of commodity futures risk premia.

D. Hirshleifer (1988a) examined the relationship between futures risk premia and market-model residual risk when futures market participation is limited. Here the analysis focuses on the features of the particular commodity market that determine the distribution of futures returns.

Three further determinants of risk premia that have received limited attention in previous research are examined. First, most previous work has assumed that the production cost for the crop is sunk prior to the opening of futures positions. An exception, Stoll (1979), introduces randomness in storage cost that is exogenous and assumed to be unrelated to the level of output. We examine the effect on the futures premium of constant unit costs of production, which cause total cost to be stochastic and correlated with the crop size since a large crop costs more to harvest and bring to market.

Second, although a number of the pure hedging-pressure papers have analyzed the effects of either supply or of demand shocks in isolation, in actual markets, both elements are present. We discuss here why supply and demand shocks will typically covary, and show how their covariation affects the risk premium.

Third, we examine the effect on premia of producers' supply response to news about weather conditions or demand. Breeden's (1980) analysis has implications for the effect of supply response on the risk premium that arise from the covariance of futures prices with the stock market. We examine further this stock market risk premium, and also how supply response affects the hedging premium that arises from covariance with the risks of closely held producers.

The plan for the remainder of the paper is as follows. Section II gives the economic setting of the model. Section III solves for market equilibrium and for general hedging and pricing relationships. In Section IV, we relate the futures risk premium to specific characteristics of the commodity market. Section V concludes the paper.

II. The Economic Setting

A two-date mean-variance model is employed.⁵ Three groups of competitive individuals, G producers (growers or handlers of the commodity), M consumers, and N outside investors called "speculators," make decisions affecting their consumption at the final date.⁶ Beliefs concerning the distributions of all variables are homogeneous and rational in the sense that beliefs match the underlying distributions of the model. Consumption takes place entirely at date 1, so the focus is on risk diversification between stock and future, but not intertempo-

⁵ With only two dates, the distinction between daily resettlement (futures contracts) and expiration date settlement (forward contracts) vanishes. With many dates, divergences between futures and forward prices can arise when daily interest rates are stochastic; e.g., Cox, Ingersoll, and Ross (1981). Kamara (1988) analyzes liquidity-induced differences between futures and forward prices.

⁶ We do not examine carryover here; because of serial interactions of risk from year to year, the analysis of carryover requires a more complex multiperiod setting.

ral consumption choice.⁷ Producers and speculators maximize a common mean-variance objective function.⁸

$$(1) \quad U = E[\tilde{C}] - \left(\frac{\alpha}{2}\right) \text{Var}(\tilde{C}).$$

Here \tilde{C} is consumption at date 1, and α is absolute risk aversion.

Consumers are represented indirectly at date 1 via a market demand curve for the spot commodity,

$$(2) \quad \tilde{Q} = M\tilde{\delta}(\tilde{P})^\eta, \quad \eta < 0,$$

where \tilde{Q} is aggregate demand for wheat, $\tilde{\delta}$ is a random multiplicative demand disturbance, \tilde{P} is the spot price at date 1, and η is demand price elasticity.⁹ Two assets are traded at date 0, a futures contract (an uncontingent claim to the commodity), and another risky asset that is suggestively termed the "stock market portfolio." This is meant to represent endowed risks that may be divided into equity shares and traded costlessly. The model deals with real rather than nominal futures contracts, abstracting from inflationary effects (see Grauer and Litztenberger (1979)).

A fixed setup cost of t for trading in futures is assumed to deter some speculators but no producers from the futures market.¹⁰ Setup costs will, in general, differ across individuals; in the analysis that follows, t should be viewed as the transaction cost of a marginal (price-setting) participant. Therefore, the number of speculators actually trading futures is $\hat{N} \leq N$. We also define

- f = the futures price of wheat set at date 0;
- ξ = the number of contracts held by a producer or speculator;
- \tilde{q} = the endowed output distribution of a producer¹¹ (for speculators, $q \equiv 0$);

⁷ All results would be essentially unchanged in a setting with consumption at date 0 as well, and a risk-free asset.

⁸ Such an expected utility function obtains under the assumptions of normally distributed consumption and Constant Absolute Risk Aversion preferences. However, normality of price and output implies that revenue, the product of these two variables, is not normal. Newbery (1987) estimates the errors of the mean-variance approximation in commodity markets to be negligible under reasonable parameter values for calculation of futures hedging positions and the benefits of risk reduction.

⁹ Postulating a given demand curve is standard in partial equilibrium models, but involves an implicit assumption that consumers' demand for the commodity is unaffected by their random profits or losses on the futures market. The nonparticipation of most consumers in commodity futures markets provides a possible justification for this. J. Hirshleifer (1977), Grauer and Litztenberger (1979), Breeden (1980), Richard and Sundaresan (1981), Stiglitz (1983), and Britto (1984) provide models that directly examine futures trading by individuals who consume many goods.

¹⁰ This is intended to reflect the fact that when there are many speculators relative to hedgers, each speculator in equilibrium takes only a very small futures position. So, it takes only a small fixed transaction cost to drive many speculators from the market. (The setup cost may be viewed as an investment in learning needed to avoid trading at an informational disadvantage in the futures market.) A hedger, on the other hand, has a nontrivial risk-reducing incentive to trade futures, so, relatively few hedgers are deterred by a small fixed cost.

¹¹ It is not difficult to extend the model to producers with diverse output distributions in which case all the results that follow still obtain taking \tilde{q} to be the output of a representative producer, $\tilde{q} \equiv (\sum_{g=1}^G \tilde{q}^g)/G$, where \tilde{q}^g is a single grower's output.

- W = initial wealth for a futures trading agent. W could differ across agents without altering the results;
 S = the size of the position in the noncommodity risky asset (the stock market portfolio);
 \tilde{R}_M = 1 plus the return on the stock market portfolio, i.e., a position of size S receives the random total payoff of $S\tilde{R}_M > 0$;
 \tilde{C} = consumption at date 1; and
 h = unit cost to a grower of harvesting or marketing the commodity (similar results would follow if h were to differ among producers).

Trading opportunities are described by

$$(3) \quad \tilde{C} = \begin{cases} W - t + (\tilde{P} - h)\tilde{q} + (\tilde{P} - f)\xi + S\tilde{R}_M, & \text{if trade futures,} \\ W + (\tilde{P} - h)\tilde{q} + S\tilde{R}_M, & \text{otherwise.} \end{cases}$$

For each futures trading individual, terminal consumption is endowed wealth less the transaction cost plus the value of the endowed output realization ($\tilde{q} \equiv 0$ for speculators) net of harvesting costs, plus the gain on the futures position and the payoff on the stock position.¹² As consumption takes place only at date 1, there is no intertemporal consumption tradeoff, so the futures and stock market trading decision is based on the impact of the selected positions on risk and expected consumption. Note that shares in endowed net revenue $(\tilde{P} - h)\tilde{q}$ cannot be sold, creating an incentive to take a futures position with inversely correlated payoff.

III. Spot and Futures Market Equilibrium

Let the risk premium $\pi \equiv E[\tilde{P} - f]$, and let $\tilde{\pi} \equiv \tilde{P} - f$. The premium is the negative of the bias in the futures price as a predictor of the later spot price. Table 1 summarizes the predictions concerning the premium that will be derived here.

A. Spot Market Equilibrium

In equilibrium, aggregate demand and supply for the spot commodity are equated, so

$$G\tilde{q} = M\tilde{\delta}(\tilde{P})^\eta.$$

Rearranging terms gives

$$(4) \quad \tilde{P} = \left(\frac{G\tilde{q}}{M\tilde{\delta}} \right)^{\frac{1}{\eta}} \equiv k \left(\frac{\tilde{q}}{\tilde{\delta}} \right)^{\frac{1}{\eta}},$$

¹² While participation in the futures market is taken to be costly, for analytic simplicity, trading in the stock market is assumed costless. This is to focus on the effect of excluding traders from the futures market. This assumption also may not be unrealistic in that: (1) far more investors take positions in the stock market (including investors in mutual and pension funds) than in commodity futures; and (2) for stocks, the setup costs (if viewed as an informational investment) may largely be avoided by investing in a passively managed mutual fund.

with k a positive constant. It follows that P rises with the demand shock, and declines with the output of the representative producer.

B. The Futures Hedging Problem

Let a j superscript indicate a particular individual (either producer or speculator). The futures trading problem is to maximize U over the futures position ξ^j and the stock position S^j , subject to (3). Concavity of the objective (1) in terms of the choice variables ensures that the optimal stock and futures positions by differentiation satisfy the first order condition

$$(5) \quad \begin{aligned} \pi &= \alpha \text{Cov}(\tilde{\pi}, \tilde{C}^j), \\ \bar{R}_M &= \alpha \text{Cov}(\tilde{R}_M, \tilde{C}^j). \end{aligned}$$

Let $\tilde{C}^* \equiv \Sigma \tilde{C}^j / (\hat{N} + G)$ be consumption averaged across all futures trading individuals. Then by linearity of the covariance operator, (5) applies, replacing \tilde{C}^j with \tilde{C}^* as well. In the spirit of the consumption CAPM, the futures risk premium is proportional to the covariance of an average of consumption with the date 1 spot price. However, the average is taken only over those individuals who trade futures.

We may solve for the futures position ξ , which is contained in \tilde{C}^j in (5), using the consumption constraint (3). This gives

$$(6) \quad \xi = \frac{\frac{\pi}{\alpha} - \text{Cov}(\tilde{\pi}, [\tilde{P} - h] \tilde{q} + S \tilde{R}_M)}{\text{Var}(\tilde{\pi})}.$$

A special case of the optimal hedge is that of speculators, $\tilde{q} \equiv 0$. For speculators, if the spot price is uncorrelated with the stock market return, the covariance term in the numerator vanishes, in which case speculators only trade if there is a premium. In the absence of a premium, if $S > 0$, positive (negative) correlation of the spot price with the market return leads to a short (long) futures position to diversify with respect to moves in the other risky asset, the stock market. Speculators decide whether to trade in futures based on whether the strategy of paying t and trading at date 0 to the position given by (6) generates higher or lower utility than refraining ($\xi = 0$).

C. Futures Market Equilibrium

Given the number of speculators \hat{N} who hold futures positions, we may find the equilibrium risk premium by summing the individual demands for futures in (6) over all $\hat{N} + G$ participants, and imposing the market clearing condition that

$$\sum_{j=1}^{\hat{N} + G} \xi^j = 0.$$

Let $b \equiv G / (G + \hat{N})$, and let $\bar{S} \equiv (1 - b)S^n + bS$, where an “ n ” superscript distin-

guishes the position of speculators from that of growers. Then the futures premium is

$$(7) \quad \pi = \alpha \bar{S} \text{Cov}(\tilde{R}_M, \tilde{P}) + \alpha b \text{Cov}([\tilde{P} - h] \tilde{q}, \tilde{P}).$$

The Mayers CAPM leads us to expect that the futures risk premium will be determined by the covariance of the futures contract payoff with the return on the stock market portfolio and with net revenues from production. In Stoll's Equation (18), the producer hedging covariance is further separated into a term reflecting storers' gross revenues, and a term reflecting costs of storage. The formula above implicitly reflects a transaction cost in b , the fraction of producers participating in the futures market. The main task of the paper will be to explore the underlying determinants in the commodity market of these covariances.

IV. Determinants of Futures Risk Premia

Some terminology prepares for the analysis of how the bias or premium relates to a number of determining factors (see Table 1). Let the stock market risk component of the risk premium be denoted π^M , and let the hedging component be π^H . These are the first and second terms, respectively, in the right-hand side of Equation (7). To relate the risk premium to the exogenous parameters, we substitute from (4) to obtain

$$(8) \quad \pi = \alpha k \bar{S} \text{Cov}\left(\tilde{R}_m, \tilde{q}^{\frac{1}{\eta}} \tilde{\delta}^{-\frac{1}{\eta}}\right) + \alpha b k^2 \text{Cov}\left(\tilde{q}^{1+\frac{1}{\eta}} \tilde{\delta}^{-\frac{1}{\eta}} - h k^{-1} \tilde{q}, \tilde{q}^{\frac{1}{\eta}} \tilde{\delta}^{-\frac{1}{\eta}}\right).$$

A. Demand Uncertainty

Let us first consider the special case of nonstochastic output q . Stiglitz (1983) has shown that the effect of pure demand shocks, in a model that excluded stock market risk, is toward a downward bias in the futures price (a positive risk premium).¹³ In this case, producers' revenues covary positively with the return on the futures contract, leading to a hedging supply of futures and, thus, to a positive risk premium in order to induce speculators to take the opposite long side of these contracts. Stoll's (1979) model, which assumes stochastic spot price but nonstochastic output, implicitly reflects demand shocks. He points out that the tendency toward a positive risk premium brought about by hedging pressure could be outweighed by a negative covariance of the futures payoff with the stock market.

¹³ This may be seen here by setting q constant and assuming the demand shocks to be independent of the stock market return, so that by (8),

$$\pi = \alpha b k^2 q^{1+\frac{2}{\eta}} \text{Var}\left(\tilde{\delta}^{-\frac{1}{\eta}}\right).$$

Stock market variability interacts with commodity shocks in determining both components of the risk premium, π^M and π^H . A path of influence of market return on the commodity market is that good news about wealth (high R_M) will raise or lower demand for a superior or inferior commodity. So, the two covariances in (7) are influenced by the effect of the stock market return R_M on the demand shock δ and, thus, on the spot price P .

Let ϵ be income elasticity of demand for the commodity, and consider for now the special functional form of

$$(9) \quad \tilde{\delta} = a(\tilde{R}_M)^\epsilon, \quad a > 0,$$

for the demand shock. This form reflects the fact that higher stock market return corresponds to higher wealth for consumers, so that demand is higher or lower for the good by an amount depending on how superior or inferior is the good.

In the covariances in (7), we may substitute for \tilde{P} from (2), where the output of the representative producer is $q = Q/G$, and substitute for $\tilde{\delta}$ in terms of \tilde{R}_M to obtain

$$(10) \quad \pi = \alpha \bar{S} k \left(\frac{q}{a}\right)^{\frac{1}{\eta}} \text{Cov}\left(\tilde{R}_M, \tilde{R}_M^{-\epsilon/\eta}\right) + \alpha b k^2 q \left(\frac{q}{a}\right)^{\frac{2}{\eta}} \text{Var}\left(\tilde{R}_M^{-\epsilon/\eta}\right).$$

π^M is proportional to a covariance of \tilde{R}_M with a power of \tilde{R}_M whose exponent is signed according to the sign of income elasticity ϵ . On the other hand, π^H , the variance term, is necessarily nonnegative; in other words, the hedging effect is always toward a higher premium. Proposition 1 follows (see Appendix for proofs).

Proposition 1. When demand shocks for the commodity are induced by stock market outcomes, and output is nonstochastic, the hedging component of the premium $\pi^H > 0$; the stock market risk component of the premium π^M is positive if the commodity is superior, and negative if it is inferior.

Although stock market risk and hedging both promote a positive premium for superior commodities, their effects are opposed for inferior commodities (such as potatoes), for which $\pi^M < 0$ and $\pi^H > 0$. π^M and π^H are graphed in relation to income elasticity in Figure 1. Breeden (1980) has described how higher income elasticity tends to raise the premium through its positive effect on the covariance between the spot price and aggregate consumption. This effect is reflected here in the π^M term. For a good with high income elasticity, good macroeconomic news (high R^M) has a greater impact on demand (δ), causing the spot price (P) to be especially high.

In addition, a second effect arises in the current model due to the interaction of stock market risk with hedging by producers. Basically, high versus low demand affects both price and revenue in the same direction, so that higher dispersion of demand increases the revenue covariance. Specifically, higher absolute income elasticity ϵ makes δ more sensitive to variation in R^M . This raises dispersion of δ and, thus, the premium. For a superior good, $\epsilon > 0$, so that for luxury commodities such as meats, the premium is predicted to be high. In contrast, ϵ is lower for rye, a lower quality substitute for wheat. For an inferior good ($\epsilon < 0$), a

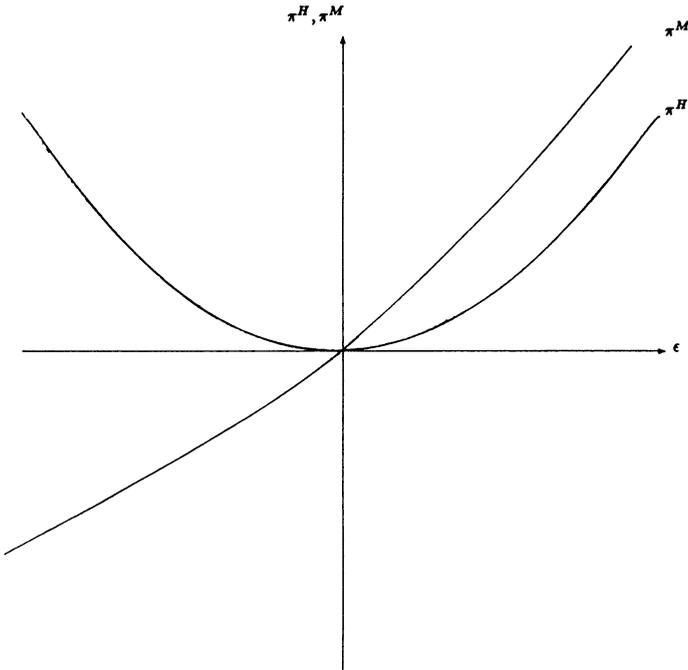


FIGURE 1

Market Risk and Hedging Components of the Risk Premium

higher (less negative) income elasticity will *reduce* the premium. These effects are summarized in Row 1 of Table 1.

B. Output Uncertainty, Harvest Costs, and Distribution Risk

This section examines the effect of harvest costs on the premium when output is uncertain and demand is certain. This is useful for clarifying how and why the predictions of models with nonparticipation differ from those of perfect markets asset pricing models.

Proposition 2 may be derived by letting δ be constant in (8).

Proposition 2: Harvest Costs. If output is random, demand is certain, the unit production cost h is positive, and demand is either unit elastic or inelastic, then the hedging premium π^H is positive. Regardless of demand elasticity, the premium is increasing with h .

Proposition 2 shows that variable costs of harvesting or marketing the commodity promote a positive hedging premium. To see why, consider a representative producer who knows the demand curve with certainty, but faces stochastic output (due to the weather, for instance). The total harvest cost is greater for a large crop. So, under unitary demand elasticity, under which gross revenue is nonrandom, *net* revenue for a typical producer will be highest when the spot price is high. Therefore, profit covaries positively with the payoff on a futures contract, so a short futures position hedges the grower. Variable harvest costs are, therefore, a force toward a higher premium (line 2 of Table 1).

In addition to this cost effect, a revenue effect on the risk premium described by the previous literature on hedging pressure should be mentioned. In the absence of harvest costs, this effect brings about a positive or negative hedging premium when demand is elastic or inelastic, respectively (see, e.g., Anderson and Danthine (1983) and Britto (1984)).

A careful look at Proposition 2 suggests that fixed costs of participation, by changing the relative importance of hedging by producers versus consumers of the commodity, can affect the magnitude and the sign of the risk premium. The source of this difference is *distribution risk* (stochastic wealth transfers between investors). Variations in producers' revenues often result from redistribution between consumers and producers, rather than aggregate social risk.¹⁴ This leads to opposite hedging incentives on the futures market for producers and consumers, so that the asymmetric participation by these two groups can bias the futures price.¹⁵ In particular, the nonparticipation of consumers, who would otherwise take futures positions to share risk with producers, biases the futures price so as to reduce the profitability of hedging by producers.

C. Correlated Commodity Shocks

Correlation in shocks to output and demand has received little attention in analyses of futures risk premia. The impact of covarying shocks on the premium differs from the effects of demand or supply shocks considered in isolation. Furthermore, it is realistic to expect that demand shifts and output shocks for a commodity will typically be correlated. A shift in demand for a commodity will often result from the output realization of a substitute; for example, corn and soybeans are both used as feeds, and wheat and rye to make bread. When outputs for two commodities are both influenced by a common weather factor, the output of a commodity will be correlated with its demand shock.

We now relax the special functional form of (9) for demand shocks. Assuming that the dispersions of demand and output shocks are small, the premium may be approximated by a two-variable Taylor expansion in δ and q about their means. For simplicity, let the harvest cost $h = 0$. Then Proposition 3 follows.

Proposition 3: Correlated Commodity Shocks. For low dispersion of output and demand shocks (so that third and higher order moments may be neglected), and if demand is not extremely price elastic ($\eta > -2$), then:

- (1) a rise in $\text{corr}(q, R_M)$ reduces the stock market risk premium π^M , and a rise in $\text{corr}(\delta, R_M)$ raises the stock market risk premium;
- (2) if $\text{corr}(\delta, R_M) > 0$ (< 0), then π^M rises (falls) with $\text{var}(\delta)$; if $\text{corr}(q, R_M) > 0$ (< 0), then π^M falls (rises) with $\text{var}(q)$;
- (3) the hedging premium π^H decreases with $\text{corr}(\tilde{\delta}, \tilde{q})$; π^M is unaffected;
- (4) if demand and output shocks are positively/negatively correlated and demand is elastic/inelastic, then the hedging component of the premium rises/falls with $\text{var}(\tilde{q})$; and

¹⁴ For example, under unit elastic demand, which implies that aggregate gross revenue is constant, a crop failure tends to be good news for growers (as it leads to lower harvest costs), yet is clearly bad on the social level.

¹⁵ An explicit analysis of consumption decisions and the pricing effect of distribution risk is provided by D. Hirshleifer (forthcoming).

(5) if $\text{corr}(\tilde{\delta}, \tilde{q}) \leq 0$, then the hedging component of the premium rises with $\text{var}(\tilde{\delta})$.

Proposition 3 shows that negative correlation between demand and output shocks raises the hedging premium. Conversely, positive correlation of \tilde{q} and $\tilde{\delta}$ decreases the hedging premium; in fact, even with inelastic demand (which, with pure output shocks, causes a negative hedging premium), if \tilde{q} and $\tilde{\delta}$ are positively correlated, the hedging component can turn negative.

Part (1) follows because the greater the extent to which output covaries with the market, the more the spot price (which is negatively related to output) tends to covary against the market, reducing the premium. Similarly, the more demand covaries with the market (as we expect if stock market risk affects demand), the more the futures payoff covaries with the market.

Part (2) results because if a certain type shock raises (lowers) the covariance of the spot price with the stock market, raising the dispersion of that shock holding its correlation with the market constant increases (decreases) that covariance further. Parts (1) and (2) are jointly summarized by the right-hand entries of Rows 3 and 4 of Table 1.

For Part (3), recall that a high demand shock, *ceteris paribus*, raises the spot price, thereby raising both the revenues of producers and the payoff on the futures contract. With output constant, a positive revenue covariance occurs, implying a positive premium. Here, however, a high demand may be associated with high output, which tends to reduce the spot price, offsetting or even reversing the effect of the demand shock. So, correlation tends to reduce or even reverse the positive covariance of the spot price with revenue¹⁶ (see Row 5 of Table 1).

To see Part (4), suppose shocks are negatively correlated and demand is inelastic. Then high output tends to be associated with low revenues because, for moves along an inelastic demand curve, high output causes price to drop more than in proportion (and also, because with negative correlation, the demand curve tends to be low). Now, depending on how dispersed are demand shocks, high output might be associated with either high or low price. However, as the *dispersion* of output rises, *ceteris paribus*, the moves along the demand curve become more important compared to shifts in the demand curve, creating a greater tendency for price to be low when output is high. This promotes a more positive or less negative covariance between revenue and the futures payoff, and so a larger premium (see middle entry of Row 4, Table 1).

The intuition for Part (5) is that if the correlation is negative, then high demand corresponds to high price, both because demand is high and output is low; and whether high demand corresponds to high *revenues* depends on the relative amount of shifting in the demand curve (dispersion of δ) versus shifts along the demand curve (dispersion of q). Then an increase in the dispersion of δ tends to increase the effect of shifts in the demand curve, so that high δ is more likely

¹⁶ For enormously elastic demand ($\eta < -2$), this effect could be reversed by the effect of positive correlation of shocks on *revenue*. On the one hand, higher correlation of demand with output means that high demand, being associated with higher output, goes with higher revenue, raising the premium. On the other hand, lower price tends to reduce the revenue, but if demand is highly elastic price does not fall much. The remaining discussion rules out extremely high demand elasticity.

to correspond to high revenues. Thus, greater dispersion of δ promotes a greater covariance between revenue and futures payoff, and a larger premium. (See middle entry of Row 3, Table 1.)

The correlation between output and demand considered in Part (3) is likely to be present for many commodities. Demand and output shocks will often be negatively correlated. Wheat and rye, for example, are substitutes that are affected by common weather influences. Favorable weather will at the same time raise wheat output but, by increasing the supply of a substitute commodity, rye, will usually induce a downward shift in the demand curve for wheat. Part (3) predicts that the high correlation of outputs and the substitutability of these commodities tends to increase the premium relative to what would be expected based on covariance with the stock market. Corn and soybeans are also substitutes (used as feeds) but since they have very different seasonal growth patterns, their outputs will be less highly correlated, which attenuates this effect.

On the other hand, for a pair of complementary commodities, the correlation between output and demand shocks will be positive, leading to less downward bias or even to contango. Alternatively, if the outputs of two substitute commodities are affected in opposite ways by the same weather (e.g., one crop needs rain, the other sunshine), this also will lead to a positive correlation of output and demand shocks. The reason this can lead to contango is as follows. If demand and output are positively correlated, and output fluctuations are wide, price tends to move inversely with demand shifts. To hedge against revenue/demand variability, it is then necessary to be long rather than short in futures, which promotes contango instead of backwardation.

A final point worth noting is that the effect of correlation of shocks and dispersion of shocks on the premium can be seen even with large shocks by focusing on the special case of unitary demand elasticity. Recall that with output shocks alone, and zero harvest costs, unitary demand elasticity leads to a zero bias (see Section IV.B). Hence, this case serves as a baseline with which to display the effect of adding a correlated demand shift. Setting $\eta = -1$ in (4), we have $\tilde{P}\tilde{q} = k\tilde{\delta}$. Combining this with (7) gives

$$(11) \quad \pi^H = \alpha b k^2 \text{Cov}(\tilde{\delta}, \tilde{\delta} / \tilde{q}) .$$

It follows that the hedging premium will be positive when q and δ are negatively or weakly correlated; when they are positively correlated, δ must be sufficiently disperse relative to q . Hence, not only does positive correlation of \tilde{q} and $\tilde{\delta}$ decrease the hedging premium, but with \tilde{q} sufficiently disperse relative to $\tilde{\delta}$, a negative hedging premium results.¹⁷

D. Supply Response

Producers often receive important information about demand or weather conditions prior to committing themselves to a level of production. Several authors, such as Kawai (1983) and Turnovsky (1983), have examined supply re-

¹⁷ Under independence of supply and demand shocks, it is not hard to show by (8) that borderline inelastic demand leads to positive π^H . The possible occurrence of $\pi^H < 0$ arises from the correlation between output and demand shocks.

sponse, primarily in relation to price volatility, welfare, and storage-induced changes in the expected level of spot prices. Here, we focus on determinants of the risk premium in a single period model. The option to adjust the production level affects how profits covary with the payoff on a futures contract. This suggests that supply as well as demand elasticity will determine the risk premium when output is flexible.

In order to endogenize production level, rather than a constant unit production cost, we now assume the case of constant elasticity cost curves. Let $d'(\tilde{q})$ be the marginal cost of producing output \tilde{q} , ω be a positive constant, and let \tilde{B} be a positive weather shock that affects the cost of producing a given level of output, the same for all producers, as in

$$(12) \quad d'(\tilde{q}) = \tilde{B}\tilde{q}^{\frac{1}{\omega}}.$$

In contrast to the harvest cost analyzed previously, the cost $d(\tilde{q})$ is incurred at the time of planting, rather than at harvest time; however, as the amount planted, \tilde{q} , is random, the level of costs is still stochastic here.

The two scenarios are as follows. At date 0, a futures position is taken. Then at date 1, either a demand or a weather shock is revealed, the production level is selected, costs are incurred, and output is completed without any further shock.¹⁸ Producers set output so as to equate price with marginal costs,

$$(13) \quad \tilde{P} = \tilde{B}\tilde{q}^{\frac{1}{\omega}}.$$

It follows that $\omega = (dq/dP)(P/q)$ is supply elasticity. Substituting for the equilibrium spot price from (4) gives output in terms of the shocks \tilde{B} and $\tilde{\delta}$, and the constant k as

$$(14) \quad \tilde{q} = k \frac{-\eta\omega}{\omega-\eta} \frac{\omega}{\tilde{\delta}^{\omega-\eta}} \tilde{B}^{\omega-\eta}.$$

The risk premium may be evaluated using the general formula (5) applied to \tilde{C}^* , average consumption across participants. Total production cost is found by integrating (12), which gives

$$\tilde{B} \int_0^{\tilde{q}} \frac{1}{s^\omega} ds = \tilde{B} \left(1 + \frac{1}{\omega}\right)^{-1} \tilde{q}^{1+\frac{1}{\omega}}.$$

So, the consumption constraint (3) is modified to subtract production cost,

$$(15) \quad \tilde{C} = W - t + S\tilde{R}_M + \tilde{P}\tilde{q} - \tilde{B} \left(1 + \frac{1}{\omega}\right)^{-1} \tilde{q}^{1+\frac{1}{\omega}} + (\tilde{P}-f)\xi.$$

The risk premium with supply response is described by the following proposition.

¹⁸ The assumption that futures positions are taken before *any* commitment to output level is extreme; the purpose of the model is to focus on the effects of having at least *some* flexibility in the choice of inputs affecting the output level subsequent to taking a hedging position.

Proposition 4: Supply Response. With pure demand shocks and flexible supply, the hedging component of the premium is positive. With pure weather shocks and flexible supply, the hedging premium is positive or negative as demand is inelastic or elastic. In general, with joint demand and weather shocks, the futures risk premium approaches the stock market risk component as supply elasticity $\omega \rightarrow \infty$. Furthermore, if the source of commodity shocks is entirely on the demand side, then both components approach zero.

Proposition 4 shows that in a market with flexible supply, pure demand shocks still imply a positive hedging premium, and that weather shocks lead to a positive or negative hedging premium according to demand elasticity. The latter result is a surprising extension of the result from the literature with inflexible supply that the degree of offsetting between price and quantity risk in determining revenue depends on demand elasticity, with unitary elasticity at the borderline leading to zero risk (see Section IV.B). Here, with supply response, there is not only an offsetting of price and quantity risk on the revenue side, but an offsetting of effects on the cost side as well. Poor weather (high B) raises the entire cost function, but reduces the optimal output level q (a shift along the cost function toward lower cost) by an amount related to demand elasticity. Hence, when demand elasticity is unity, the lower cost of producing a given output when weather is fair is precisely offset by the associated increase in planting, so that total cost is constant.

Intuitively, the shape of the supply curve affects the hedging premium in the following way. For simplicity, consider pure demand shocks. A higher supply elasticity implies that a given change in demand will cause a smaller variation in the spot price. This tends to reduce the dispersion of both the spot price and producers' revenues and, thus, the covariance between the two. In consequence, the hedging premium vanishes with high supply elasticity.

Supply elasticity also has an effect on the premium that operates through the stock market covariance term, rather than the hedging term, described by Breeden (1980).¹⁹ His result is based on income effects on demand for the commodity. For a superior good, this effect suggests that less elastic supply will tend to raise the market covariance and, so, the premium. But for an inferior good, the argument reverses, so that less elastic supply would tend to reduce the premium.

Proposition 4 describes an effect of supply elasticity on the premium that operates through the revenue risks of producers, rather than through income effects and return covariance with the stock market. The effect on the hedging premium does not depend on income elasticity and, so, obtains for inferior and normal goods with unchanged direction. The effect of high supply elasticity on the stock market risk premium when growers face pure demand shocks arises from the buffering effect that high supply elasticity has on the impact of demand shocks on the spot price. In the limit, this reduces the covariance of the spot price with the return on the stock market to zero. When weather shocks are

¹⁹ In his model, good news about aggregate consumption leads, for a normal good, to high demand, which, in turn, leads to a higher spot price. How much higher depends on how elastic the supply of the commodity is. So, the covariance of the spot price with the stock market return is related to supply elasticity.

present, this need not be the case. These implications are summarized in Row 6 of Table 1.

TABLE 1
Determinants of Futures Risk Premia

Determinant	Direction of Effect on Hedging Premium	Direction of Effect on Stock Market Risk Premium
Stock Market Variability: Nonstochastic Output	upward for nonzero income elasticity; effect vanishes for a borderline-inferior good	upward if demand is superior; downward if demand is inferior
Harvest Cost	upward	—
Demand Variability	upward if demand is independent or negatively correlated with output	upward/downward if demand is positively/negatively correlated with stock market return
Output Variability	related to correlation of demand and output, dispersion of shocks and demand elasticity	downward/upward if output is positively/negative correlated with stock market return
Correlation of Demand and Output	downward	—
Supply Response	toward zero	toward zero under pure demand shocks

V. Conclusion

This paper has explored a number of factors influencing risk premia in commodity futures markets, and has related risk premia to harvest costs and to the price and income elasticities of supply and demand for the spot commodity. Despite additive separation of risk premia into a stock market risk component and a hedging pressure component, the two sorts of risk interact; stock market variability, via income elasticity, affects the incentive to hedge nonmarketable risks and, therefore, affects the hedging-pressure component of the premium.

The covariation of demand and output shocks leads to effects that cannot be understood by analyzing either in isolation. This is because the covariation between demand and output shocks affects the joint distribution of the revenues to be hedged and the payoff on a futures contract. It is argued that the two types of shocks typically do covary. Furthermore, the ability of growers to adjust their supply in response to news about demand conditions tends to reduce the risk premium toward zero.

The predictions described above differ from models in which individuals participate fully in the futures market. When there is incomplete participation, distribution risk (risk of stochastic wealth redistributions between producers and consumers) shifts the risk premium adversely to the average remaining participant in the futures market. The absence of consumers increases the magnitude of the producer-hedging component of the premium, which can cause the risk premium to be opposite in sign to that predicted based on the covariance of the futures payoff with aggregate consumption.

Like Breeden (1980), the focus of this paper has been on the determinants of cross-commodity variations in risk premia, rather than on time-series patterns.

However, given the seasonal pattern of information arrival in most commodity markets, the most important extension of this work would be to analyze time variations in the hedging premium and the stock market risk premium in relation to the rate of arrival of news about output and demand. Such an analysis could be used both to reexamine traditional theories of seasonally varying risk premia (see, e.g., Cootner (1960)), and to develop new testable predictions about how seasonal patterns of risk premia are related to futures price change variances, supply and demand elasticities, and storage costs.

Appendix

Proof of Proposition 1. To sign the market risk component, consider the case in which the commodity is a superior good, $\epsilon > 0$. R_M is similarly ordered with $(R_M)^{-\epsilon/\eta}$ in the sense of Hardy, Littlewood, and Polya (1952) (as one goes up, so does the other), so the first covariance in (10) is positive. If the commodity is an inferior good, $\epsilon < 0$, then R_M is inversely ordered with $(R_M)^{-\epsilon/\eta}$, so the covariance is negative. \square

Proof of Proposition 2. By (8),

$$(16) \quad \pi^H = \alpha b k^2 \delta^{-\frac{2}{\eta}} \text{Cov}\left(\tilde{q}^{1+\frac{1}{\eta}}, \tilde{q}^{\frac{1}{\eta}}\right) - h \alpha b k \delta^{-\frac{1}{\eta}} \text{Cov}\left(\tilde{q}, \tilde{q}^{\frac{1}{\eta}}\right).$$

If $\eta > -1$, then $q^{1+1/\eta}$ and $q^{1/\eta}$ are similarly ordered, so the first covariance is positive. If $\eta = -1$, the first covariance is zero. The second covariance is positive by the inverse ordering of q and $q^{1/\eta}$. The derivative of the second term with respect to h is also positive. The stock market risk term does not contain h .²⁰ \square

Proof of Proposition 3. For brevity, only a sketch of the proof is provided. We focus on points (3), (4), and (5) concerning the hedging premium; results (1) and (2) concerning stock market risk are proved in a similar fashion using a two-term Taylor expansion in δ and q . Let

$$\begin{aligned} f(\delta, q) &\equiv q^{1+1/\eta} \delta^{-1/\eta}, \\ g(\delta, q) &\equiv q^{1/\eta} \delta^{-1/\eta}, \text{ and} \\ h(\delta, q) &\equiv q^{1+2/\eta} \delta^{-2/\eta}. \end{aligned}$$

By (8) in the text, the hedging premium is proportional to

$$(17) \quad \pi^H \propto E[h] - (E[g])(E[f]).$$

²⁰ In general, b , which is a function of the number of speculators participating, will vary as h rises. However, the total derivative dx^H/dh allowing b to vary has the same sign as the partial derivative holding b fixed. The reason for this is that the incentive to enter rises with the premium. π^H could fall only if speculators entered to further spread the risk. But with a lower premium, the benefit to entry of the marginal speculator at the initial value of h would be *negative*, implying exit, not entry. This contradicts the premise that π^H falls.

The next step is to calculate these expectations by two-variable Taylor expansions in δ and q about their means. For example,

$$\begin{aligned}
 E[f] &\approx \bar{q}^{-1+1/\eta} \bar{\delta}^{-1/\eta} + \frac{1}{2}(1/\eta)(1/\eta+1)\bar{q}^{-1+1/\eta} \bar{\delta}^{-(1/\eta)-2} \sigma_{\delta}^2 \\
 (18) \quad &+ (1+1/\eta)(-1/\eta)\bar{q}^{-1/\eta} \bar{\delta}^{-(1/\eta)-1} \text{Cov}(\delta, q) \\
 &+ \frac{1}{2}(1/\eta)(1/\eta+1)\bar{q}^{-1/\eta-1} \bar{\delta}^{-(1/\eta)} \sigma_q^2.
 \end{aligned}$$

We calculate $(E[g])(E[f])$ up to second order terms,

$$\begin{aligned}
 (E[g])(E[f]) &\approx \bar{q}^{-1+2/\eta} \bar{\delta}^{-2/\eta} + (1/\eta)(1/\eta+1)\bar{q}^{-1+2/\eta} \bar{\delta}^{-(2/\eta)-2} \sigma_{\delta}^2 \\
 (19) \quad &- ([1/\eta] + [2/\eta^2])\bar{q}^{-2/\eta} \bar{\delta}^{-(2/\eta)-1} \text{Cov}(\delta, q) \\
 &+ (1/\eta^2)\bar{q}^{-2/\eta-1} \bar{\delta}^{-2/\eta} \sigma_q^2.
 \end{aligned}$$

It follows that

$$\begin{aligned}
 E[h] - (E[g])(E[f]) &\approx (1/\eta^2)\bar{q}^{-1+2/\eta} \bar{\delta}^{-(2/\eta)-2} \sigma_{\delta}^2 \\
 (20) \quad &- ([1/\eta] + [2/\eta^2])\bar{q}^{-2/\eta} \bar{\delta}^{-(2/\eta)-1} \text{Cov}(\delta, q) \\
 &+ [(1/\eta) + (1/\eta^2)]\bar{q}^{-2/\eta-1} \bar{\delta}^{-2/\eta} \sigma_q^2.
 \end{aligned}$$

The σ_{δ}^2 term is always positive, the sign of the $\text{Cov}(\delta, q)$ term depends on whether $\eta \geq -2$, and the sign of the σ_q^2 term depends on whether $\eta \geq -1$. Since $\eta > -2$, the $\text{Cov}(\delta, q)$ term has a sign that is opposite to the covariance. Differentiating this term with respect to $\text{corr}(\delta, q)$ gives Part (3) (bearing in mind that, as in the proof of Proposition 2, the derivative of the premium with respect to a parameter holding constant the level of participation has the same sign as the total derivative allowing participation to vary). Parts (4) and (5) also follow directly. \square

Proof of Proposition 4. By (5) and (15), the futures risk premium is

$$\pi^H = \alpha \text{Cov}(\tilde{P}, b\tilde{P}\tilde{q} - b\tilde{B}(1+1/\omega)^{-1}\tilde{q}^{1+1/\omega}).$$

Then by (13), this simplifies to

$$(21) \quad \pi^H = \frac{\alpha b}{1+\omega} \text{Cov}(\tilde{B}\tilde{q}^{1+1/\omega}, \tilde{B}\tilde{q}^{1/\omega}).$$

We may substitute for \tilde{q} from (14) into (21) to rewrite π^H in terms of the exogenous shock \tilde{B} and $\tilde{\delta}$, which gives

$$(22) \quad \pi^H = \frac{\alpha b}{1+\omega} k^{-\frac{\eta(\omega+2)}{\omega+\eta}} \text{Cov}\left(\tilde{B}^{\frac{\omega(1+\eta)}{\omega-\eta}} \tilde{\delta}^{\frac{\omega+1}{\omega-\eta}}, \tilde{B}^{\frac{\omega}{\omega-\eta}} \tilde{\delta}^{\frac{1}{\omega-\eta}}\right).$$

With pure demand shocks ($B = \text{a constant}$), the covariance and, so, the premium is always positive, by the similar ordering of positive powers of δ . With pure weather shocks ($\delta = \text{a constant}$), the covariance is signed by whether $\eta \geq -1$, since this determines whether the exponents of B are of the same or opposite sign. Lastly, under joint variability of B and δ , suppose that supply elasticity (ω) increases. As $\omega \rightarrow \infty$, $\pi^H \rightarrow 0 \times k^{-\eta} \times \text{Cov}(\delta \tilde{B}^{\eta+1}, \tilde{B}) = 0$. That is, high supply elasticity reduces the hedging premium.

To show how ω affects the stock market risk premium $\pi^M = \alpha \bar{S} \text{Cov}(P, R_M)$, similarly substitute for P from (13) and for q from (14) to obtain

$$(23) \quad \pi^M = \alpha \bar{S} k^{-\frac{\eta}{\omega-\eta}} \text{Cov} \left(\tilde{B}^{\frac{\omega}{\omega-\eta}} \tilde{\delta}^{\frac{1}{\omega-\eta}}, R_M \right).$$

As $\omega \rightarrow \infty$, $\pi^M \rightarrow \alpha \bar{S} \text{Cov}(\tilde{B}, R_M)$, which, in general, is a nonzero quantity. In the case of pure demand shocks, i.e., B nonstochastic, this quantity is zero. \square

References

- Anderson, R., and J. P. Danthine. "Hedger Diversity in Futures Markets." *The Economic Journal*, 93 (June 1983), 370–389.
- Bodie, Z., and V. I. Rosansky. "Risk and Return in Commodity Futures." *Financial Analysts Journal*, 36 (May–June 1980), 27–39.
- Breedon, D. T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics*, 7 (Sept. 1979), 265–296.
- _____. "Consumption Risk in Futures Markets." *Journal of Finance*, 35 (May 1980), 503–520.
- Britto, R. "The Simultaneous Determination of Spot and Futures Prices in a Simple Model with Production Risk." *Quarterly Journal of Economics*, 99 (May 1984), 351–365.
- Carter, C.; G. Rausser; and A. Schmitz. "Efficient Asset Portfolios and the Theory of Normal Backwardation." *Journal of Political Economy*, 91 (April 1983), 319–331.
- Chang, E. "Returns to Speculators and the Theory of Normal Backwardation." *Journal of Finance*, 40 (March 1985), 193–208.
- Cootner, P. H. "Returns to Speculators: Telser versus Keynes." *Journal of Political Economy*, 68 (Aug. 1960), 396–404.
- Cox, J.; J. Ingersoll; and S. Ross. "The Relation between Forward Prices and Futures Prices." *Journal of Financial Economics*, 9 (Dec. 1981), 321–346.
- Diamond, D. W., and R. E. Verrechia. "Optimal Managerial Contracts and Equilibrium Security Prices." *Journal of Finance*, 37 (May 1982), 275–287.
- Dusak, C. "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums." *Journal of Political Economy*, 23 (Nov.–Dec. 1973), 1387–1406.
- Grauer, F. L. A., and R. H. Litzenberger. "The Pricing of Commodity Futures Contracts Nominal Bond and Other Risky Assets under Commodity Price Uncertainty." *Journal of Finance*, 34 (March 1979), 69–83.
- Gray, R. W. "The Characteristic Bias in Some Thin Futures Markets." *Food Research Institute Studies*, 2 (1961), 296–312.
- Hardy, G. H.; J. E. Littlewood; and G. Polya. *Inequalities*, second ed. Cambridge, MA: Cambridge University Press (1952).
- Hershleifer, D. "Residual Risk, Trading Costs, and Commodity Futures Risk Premia." *Review of Financial Studies*, 1 (Summer 1988a), 173–193.
- _____. "Risk, Futures Pricing, and the Organization of Production in Commodity Markets." *Journal of Political Economy*, 96 (Dec. 1988b), 1206–1220.
- _____. "Hedging Pressure and Futures Price Movements in a General Equilibrium Model." *Econometrica* 1 (Winter 1988), 357–376.
- Hershleifer, J. "The Theory of Speculation under Alternative Regimes of Markets." *Journal of Finance*, 32 (Sept. 1977), 975–999.
- Kamara, A. "Market Trading Structures and Asset Pricing: Evidence from the Treasury Bill Markets." *Review of Financial Studies*, 1 (Winter 1988), 357–376.
- Kawai, M. "Spot and Futures Prices of Nonstorable Commodities under Rational Expectations." *Quarterly Journal of Economics*, 98 (May 1983), 235–254.
- Marcus, A. "Efficient Asset Portfolios and the Theory of Normal Backwardation: A Comment." *Journal of Political Economy*, 92 (1984), 162–164.
- Mayers, D. "Non-Marketable Assets and Capital Market Equilibrium under Uncertainty." In *Studies in the Theory of Capital Markets*, M. Jensen, ed. New York: Praeger (1972).
- Newbery, D. M. G., and J. E. Stiglitz. *The Theory of Commodity Price Stabilization: A Study in the Economics of Risk*. Oxford: Clarendon (1981).
- Newbery, D. M. G. "On the Accuracy of the Mean-Variance Approximation for Futures Markets." Working Paper, Univ. of California, Berkeley (1987).
- Richard, S., and S. Sundaresan. "A Continuous Time Equilibrium Model of Forward Prices and Futures Prices in a Multigood Economy." *Journal of Financial Economics*, 9 (Dec. 1981), 347–371.
- Stiglitz, J. E. "Futures Markets and Risk: A General Equilibrium Approach." In *Futures Markets*, M. Streit, ed. London: Basil Blackwell (1983).
- Stoll, H. R. "Commodity Futures and Spot Price Determination and Hedging in Capital Market Equilibrium." *Journal of Financial and Quantitative Analysis*, 14 (Nov. 1979), 873–894.
- Turnovsky, S. J. "The Determination of Spot and Futures Prices with Storable Commodities." *Econometrica*, 51 (Sept. 1983), 1363–1387.