

# Fear of the Unknown: Familiarity and Economic Decisions\*

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**Abstract.** Evidence indicates that people fear change and the unknown. We model this behavior as familiarity bias in which individuals focus on adverse scenarios in evaluating defections from the status quo. The model explains portfolio underdiversification, home and local biases. More importantly, equilibrium stock prices reflect an unfamiliarity premium. In an international setting, our model predicts that while the standard CAPM fails to hold with respect to the world market portfolio, a modified CAPM holds wherein the market portfolio is replaced with a portfolio of the stock holdings of investors not subject to familiarity bias.

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## 1. Introduction

People fear change and the unknown. Experimental evidence on judgment and decisionmaking documents that individuals prefer familiar goods, status quo choices, and gambles which seem unambiguous and that individuals feel competent to evaluate. These effects are also manifested in capital markets. Individuals favor investments that they are more familiar with, and that are geographically and linguistically proximate (familiarity, local, or home bias); investors are reluctant to trade away from their current ownership positions, and are biased in favor of choice options made salient as default choices (*status quo* bias); and hold strongly to past choices or investments that they currently hold (inertia, endowment effect).

We provide a model that captures a range of experimental phenomena and capital markets evidence based upon two psychological forces. The first is the tendency for individuals to use a focal choice alternative as a benchmark for comparison in

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evaluating other possible choices. An alternative becomes focal by being familiar, cognitively simple to analyze, salient, and assigned by default. We refer to such a focal choice option as the *status quo*. The other force is the tendency to evaluate skeptically choice alternatives that deviate from the status quo. This force reflects fear of change and of the unfamiliar.

We model these forces as arising from egocentrically pessimistic guesses about how the world works in the presence of uncertainty. In our approach individuals do *not* penalize the status quo choice option for the uncertainty associated with its outcomes. Pessimistic beliefs are primed only by contemplation of an action that deviates from the status quo choice. This linkage between contemplated action and pessimism captures fear of change and of the unfamiliar—*familiarity bias*.

The decisionmaker acts as if he thinks that any choice that deviates from the status quo is likely to be countered by a structure of the world that minimizes his welfare. In other words, we model an inclination of individuals who are faced with uncertainty to focus on worst-case (or at least, bad-case) scenarios when contemplating deviations from the status quo. An individual selects a strategy over the status quo only if the strategy provides higher expected utility over a sufficiently large probability mass of possible models of the world.<sup>1</sup>

We explore the implications of the model taking the status quo as a consumer or investor's current position. Familiarity bias induces anomalies relating to the unwillingness to trade or to shift investment policy. An investor evaluates purchases (sales) under a probability distribution that is adverse to buying (selling) (i.e., one in which the expected utility from the good or security is low (high)). This gives rise to the difference between the willingness to pay and the willingness to accept (e.g., Thaler, 1980). In a capital budgeting context, our model can explain the use by managers of excessively high hurdle rates in investment choices, and also reluctance to terminate on the part of managers in their existing investments (e.g., Poterba and Summers, 1995; Graham and Harvey, 2001).

When investors are initially endowed with portfolios that include only a subset of available goods or securities, then pessimism about trades provides a quantifiable model of various puzzles of non-participation in securities markets and investors' limited diversification across stocks and asset classes. In calibration analysis we find that with modest levels of uncertainty about the mean stock returns, our model implies portfolio underdiversification and a home bias comparable to the observed magnitude.

<sup>1</sup> Individuals' choices in our model depend upon salient benchmarks, but in a fashion different from prospect theory (Kahneman and Tversky, 1979). In our approach decisionmakers fear deviations from a salient *choice alternative*. In contrast, under prospect theory, individuals evaluate outcomes in terms of gains and losses relative to a benchmark *payoff* level. Further, the key assumption in our setting is investors' pessimistic beliefs rather than the shape of the utility function as in the prospect theory.

Furthermore, we derive implications of familiarity bias for equilibrium asset pricing. We consider stock markets in two countries, each populated by both rational and familiarity-biased investors. Given an endowed portfolio, there is an interval of prices within which the familiarity-biased investors do not trade. In equilibrium, familiarity-biased investors' trade depends on the uncertainty about expected stock returns. When the degree of uncertainty is either very low or very high for both countries, the effects of familiarity bias on stock demand and supply offset. In these cases, equilibrium asset prices are not affected by the presence of familiarity-biased investors, and the standard CAPM relation holds with respect to the world market portfolio, although no one holds the world market portfolio.

In contrast, when investors' uncertainty is moderate so that it does not completely deter familiarity-biased investors from participating the stock markets in both countries, but high enough to affect these investors' demand or supply of stocks, familiarity-bias influences equilibrium stock prices. The difference in the stock prices between the two economies without and with familiarity-biased investors captures an unfamiliarity premium—an extra return to compensate familiarity-biased investors for deviating from their endowment positions. The unfamiliarity premium increases with the fraction of familiarity-biased investors and decreases with the degree of uncertainty.

Under this circumstance, the standard CAPM with respect to the world market portfolio no longer holds. The absolute pricing errors of the standard CAPM in both countries increase with the fraction of familiarity-biased investors. Since familiarity-biased investors are more likely to hold only domestic equity, the absolute pricing errors of the standard CAPM are predicted to be positively correlated with the amount of home bias. Nonetheless, we show that a modified CAPM holds when the world market portfolio is replaced by the aggregate stock holdings of the rational investors.

Our analysis offers a new approach for testing the international CAPM with respect to the aggregate stock portfolio of rational investors, given measures for the degree of uncertainty and the fraction of rational investors. For example, Anderson, Ghysels and Juergens (2009) use the data on professional forecasters to extract a measure of uncertainty, while the fraction of rational investors can be proxied by the fraction of investors participating in foreign (world) stock markets. With these measures, a proxy can be formed for the aggregate portfolio of rational investors, which permits testing the familiarity-based version of the CAPM using the modified market portfolio.

Our paper differs in several ways from previous studies relating ambiguity aversion and model uncertainty to underdiversification and home bias (e.g., Dow and Werlang, 1992; Uppal and Wang, 2003; Epstein and Miao, 2003; Cao, Wang, and Zhang, 2005). The fundamental difference is that in our approach pessimism about the uncertainty associated with a choice is triggered by the

deviation of the individual's contemplated decision from some familiar default choice or status quo. Thus, in our model the aversion to uncertainty is *conditional*. In most models investors dislike uncertainty of all choice options symmetrically, so that there is no special role in the status quo.<sup>2</sup> In contrast, we consider agents who are influenced by their status quo options (typically, their initial endowments).

This basic difference in motivation and modeling leads to different implications. First, some of the important findings of previous studies are driven by the assumption that some assets have greater uncertainty than others—an assumption that is often reasonable. However, our main findings apply even when investors are equally uncertain about the returns of different assets. For example, home bias in our model does not derive from differences in uncertainty, but from the fact that the pessimism induced by a given level of uncertainty is greater in an unfamiliar asset than in a familiar one.

Second, in previous models of home bias, investors always hold non-zero quantities of stocks (including foreign stocks). In our model, some investors can reach an autarkic outcome; if they start out not holding some asset class such as foreign stocks, they continue to hold zero of that asset class. Thus, our model explains not just home bias, but frequent zero holdings in foreign stocks by many investors.<sup>3</sup>

Third, our model explicitly derives an equilibrium asset pricing model based upon familiarity bias, and conditions under which the standard CAPM or else a CAPM with respect to a modified market portfolio obtains. Fourth, we suggest in the conclusion that policy implications of our approach differ from those of previous work in which initial endowment has no effect on future decisions, because in our approach endowment plays a critical role.

## 2. Motivating Evidence

We begin by summarizing the evidence relating to human attitudes toward the familiar and toward deviations from salient benchmark choice alternatives.

One of the rationality axioms underlying von-Neumann Morgenstern expected utility is the Independence of Irrelevant Alternatives, which says that whether

<sup>2</sup> This leads to important differences. For example, suppose that uncertainty increases for all risky assets, but the increase is greater for the status quo choice than for alternative choices. In previous models which make no distinction between status quo versus other choices, investors will reduce their holdings of the choice whose uncertainty has increased the most. In contrast, in our approach the individual sticks *more strongly* to the status quo choice, since he evaluates the alternatives more pessimistically.

<sup>3</sup> In reality, holding positive amounts of the domestic stock market and zero foreign stocks is quite common for many investors in many countries and time periods.

choice A or B is better should not be changed by the availability of an irrelevant third alternative. However, extensive experimental evidence shows that individuals usually focus attention on an irrelevant alternative as a benchmark for evaluating other alternatives.<sup>4</sup>

Often, the focal choice alternative is to do nothing, and remain at the status quo. An individual who is subject to the status quo bias prefers either the current state or some choice alternative that has been made salient as the default option that will apply should no alternative be selected explicitly (Samuelson and Zeckhauser, 1988; Fox and Tversky, 1995). For example, in a set of experiments on portfolio choices following a hypothetical inheritance, Samuelson and Zeckhauser (1988) find that an option becomes significantly more popular when it is designated as the status quo while others are designated as alternatives.

When neither choice alternative is made salient as a passive or default choice, sometimes the focal choice alternative is the one that is easier to process. The greater comfort that individuals have with easily processed choice alternatives probably lies in the fact that people prefer choices about which they can feel expert and competent. As shown experimentally by Heath and Tversky (1991), individuals prefer to bet in a decision domain within which they feel expert than on another gamble with an identical distribution of payoff outcomes.

Another principle that emerges from the experimental studies is that when there is a single clearcut focal choice alternative, people evaluate skeptically the possible outcomes of choice alternatives that deviate from the focal choice. For example, when the focal choice is the status quo, individuals tend to dislike risks that derive from active choices more than risks that result from remaining passive. Psychologists have referred to this as the omission bias (Ritov and Baron, 1990; Josephs et al., 1992). For example, individuals are reluctant to take seemingly risky actions such as getting vaccinated, often preferring to bear the much bigger risks associated with remaining passive.

There is a great deal of evidence suggesting that these two psychological forces—the tendency to evaluate choices relative to a focal choice, and the tendency to be unduly skeptical about the non-focal choice alternatives relative to the focal one—operate in capital market decisions as well.

In stock investments, individuals prefer familiar choices. Experimentally, Ackert et al. (2005) find that investors have a greater perceived familiarity with local and domestic securities and, in turn, invest more in such securities. Empirical evidence show that investors tend to concentrate holdings in stocks to which they are geographically, linguistically or professionally close or that they have held for a long period (e.g., Coval and Moskowitz, 1999; Grinblatt and Keloharju, 2001;

<sup>4</sup> See, e.g., Tversky, 1972; Huber, Payne, and Puto, 1982; Simonson, 1989; Simonson and Tversky, 1992.

Huberman, 2001; Massa and Simonov, 2006).<sup>5</sup> In pension fund investments, many people invest a significant fraction of their discretionary contributions in their own company stock (e.g., Mitchell and Utkus, 2002; Benartzi, 2001; Meulbroek, 2002).

In international financial markets, investors tend to hold domestic assets instead of diversifying across countries, a puzzle known as home bias (e.g., French and Poterba, 1991). A related phenomenon is that firms tend to cross list their stocks in countries where investors are more familiar with the firms to be listed (e.g., Pagano, Roell, and Zechner, 2002; Sarkissian and Schill, 2004). Guiso, Sapienza, and Zingales (2009) find that closer culture match (e.g., religious and genetic similarities) toward citizens of a country lead to higher portfolio allocation to assets in that country.

The preference for the familiar goes above and beyond motivations based upon lower true risk or higher returns. Both individuals and portfolio managers have more pessimistic expectations about foreign stocks than about domestic stocks (Shiller, Konya, and Tsutsui, 1996; Strong and Xu, 2003; Kilka and Weber, 2000). This is consistent with our modeling approach.

### 3. The Model

To highlight the intuition of the model, we consider a two-date setting in which investment decisions are made at date 0, and consumption takes place at date 1. We consider a preference relation that reflects aspects of the preferences described by Gilboa-Schmeidler (1989), but which emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action.

The unique subjective probability distribution used in standard expected utility calculation is replaced by a set of probability distributions  $\mathcal{P}$  which capture investors' uncertainty about the distribution of asset payoffs or returns. A larger set  $\mathcal{P}$  corresponds to a higher degree of uncertainty.

Each individual has a twice differentiable and concave utility function  $U(W)$  defined over the end-of-period wealth,  $W$ . Let  $W(x)$  denote the wealth random variable for an investor following a given strategy  $x$ . The following definition describes a preference relation that captures fear of change and unfamiliar choices.

#### Definition 1 Status Quo Deviation Aversion

*Let  $x$  be a feasible strategy and  $s$  be the status quo strategy. Then  $x$  is strictly preferred to  $s$  if and only if the certainty equivalent value of  $x$  is higher than the certainty equivalent value of  $s$  under any probability distribution  $Q$  in  $\mathcal{P}$ .*

$$x \succ s \text{ iff } \min_{Q \in \mathcal{P}} \{U^{-1}(E^Q[U(W(x))]) - U^{-1}(E^Q[U(W(s))])\} > 0. \quad (1)$$

<sup>5</sup> Seasholes and Zhu (2005) find that investors prefer local stocks despite a lack of information about these stocks.

Status Quo Deviation Aversion (SQDA) gives a privileged position to the status quo strategy. A strategy is preferred to the status quo strategy only if it provides higher certainty equivalent value under all probability models in  $\mathcal{P}$ .<sup>6</sup>

Status quo deviation aversion is an incomplete preference relation, as it does not specify how to compare two non-status-quo alternatives. The following definition gives one way to complete the preference ordering:

**Definition 2 Strong Status Quo Deviation Aversion**

*Let  $x$  and  $y$  be any two strategies and  $s$  be the status quo strategy. Then*

$$\begin{aligned} x > y & \text{ iff } \min_{Q \in \mathcal{P}} \{U^{-1}(E^Q[U(W(x))]) - U^{-1}(E^Q[U(W(s))])\} \\ & > \min_{Q \in \mathcal{P}} \{U^{-1}(E^Q[U(W(y))]) - U^{-1}(E^Q[U(W(s))])\}. \end{aligned} \quad (2)$$

It is straightforward to show that SSQDA implies SQDA.

Status Quo Deviation Aversion, both in its basic form and its strong form, assigns a privileged role to a status quo alternative. This familiar option is chosen unless there exists an alternative that is preferred for all possible beliefs within the set  $\mathcal{P}$ . Thus, a familiar choice option acts as an anchor from which deviations are pessimistically considered. When there is uncertainty, deviations from more familiar choices will be scrutinized with skepticism and suspicion. This results in a tendency to prefer more familiar choices, or choices that seem to preserve the status quo.

SSQDA implies that when there are choices that dominate the status quo option, the investor chooses among them according to a procedure similar to that described by Gilboa and Schmeidler (1989), i.e., the investor evaluates each strategy under the scenario that is most adverse to that strategy. Thus, if the status quo action is dominated by an alternative strategy  $x$ , then strategy  $x$  is evaluated according to the minimum gains in certainty equivalent value, and the alternative strategy with the highest minimum gains in certainty equivalent value is selected.

When specifying the set  $\mathcal{P}$ , we consider a reference distribution  $P$  (e.g., an investor might use an empirical estimate of probability) and form the set  $\mathcal{P}$  around  $P$  based on the log likelihood ratio. We define  $\mathcal{P}$  as the collection of all probability distributions  $Q$  satisfying  $E^Q[-\ln(dQ/dP)] < \beta$  for a preselected positive value  $\beta$  which measures the amount of investor's uncertainty. Intuitively,  $\mathcal{P}$  can be viewed as a confidence region around  $P$ .

For analytical tractability, we assume a constant absolute risk aversion (CARA) utility function  $U(W) = -e^{-\lambda W}$  and normally distributed stock payoffs or returns. Furthermore, investors are assumed to have precise knowledge of the variances

<sup>6</sup> The certainty equivalence principle served an important role in developing rational expectations models. Hansen and Sargent (2004) show how the certainty equivalence principle also pertains to settings with model uncertainty.

and covariances of stocks but do not know their means with certainty.<sup>7</sup> When  $\mathcal{P}$  is the set of normal distributions with a common known variance-covariance matrix  $\Sigma$ , Kogan and Wang (2002) show that the confidence region of the mean can be described by a set of quadratic inequalities. Specifically, it takes the form of  $\mu + v$ , where  $v$ , the adjustment made to  $\mu$ , the vector of sample averages estimated from historical stock data of length  $T$ , satisfies

$$v^\top \Sigma^{-1} v \leq \alpha^2, \quad (3)$$

where  $\alpha = \beta/\sqrt{T}$ . The higher is  $\alpha$ , the wider is the range for the mean. Thus, in our model, fear of the unfamiliar derives from aversion to model uncertainty about the mean payoffs or returns of unfamiliar choice alternatives. Investors will consider a set of probability distributions with different means when making their investment decisions.

In the approach above, investors who exhibit familiarity bias focus on the worst case scenarios associated with contemplated deviations from status quo choices. Similar results can be obtained under a less extreme assumption: investors focus on bad cases instead of worst cases. To define ‘bad cases,’ we consider an investor who is uncertain about which model of the world is valid. Let  $s$  be the status quo strategy and  $x$  be an alternative strategy that the investor is contemplating. We rank the probability distributions  $Q$  in  $\mathcal{P}$  by the difference in the certainty equivalent values  $U^{-1}(E^Q[U(W(x))]) - U^{-1}(E^Q[U(W(s))])$ . The individual may pessimistically select a probability distribution  $Q$  at the  $1 - \delta$  quantile of this ranking ( $\delta > 0.5$ ). This yields a quantile utility gain relative to the status quo choice, for any given degree of pessimism  $\delta$ .

We can then define status quo deviation aversion based on this more moderate skepticism about deviations. This condition is milder, making it easier for individuals to choose alternatives over the status quo. Even if a choice alternative could conceivably pay off worse than the status quo, the alternative might be preferred if this is sufficiently unlikely. Similar results of autarky on the part of individuals (the endowment effect), and quantification of the circumstances under which this occurs, can be derived under the more moderate familiarity bias described by this quantile approach.

#### 4. Familiarity Bias and Individual Decisions

In this section we examine the implications of familiarity bias for individuals’ decisionmaking. We demonstrate that familiarity bias can induce the endowment

<sup>7</sup> This is motivated by the econometric fact that by increasing observation frequency it is much easier to obtain accurate estimates of variances and covariances than the expected values.



effect, the underdiversification in risky asset holdings, and the home bias. We also quantify the magnitude of the effect of familiarity bias.

4.1 THE ENDOWMENT EFFECT

It has been well documented that people often demand a higher price to give up an object than they would be willing to pay to acquire it (e.g., Knetsch and Sinden, 1984). This so called endowment effect is commonly interpreted as the result of loss aversion (e.g., Kahneman, Knetsch, and Thaler, 1991). In our approach, the endowment effect arises without loss aversion. Instead, it derives from skepticism about the desirability of giving up the object by virtue of the fact that retaining the object is the focal, status quo choice.

Consider the case of acquiring more shares of a stock whose random payoff is denoted  $r$ . We assume that the individual perceives making no trade as the default or status quo choice option. Let  $W_0$  denote the initial wealth in the risk-free bond,  $e$  denote the endowment in the stock,  $c$  denote the dollar amount the individual pays for the additional shares of the stock, and  $d$  denote the dollar amount the individual receives for giving up the additional shares of the stock. For small additional shares in the stock  $\Delta e$ , let  $\Delta C_P$  denote the greatest amount of cash an investor would be willing to give up in exchange for the additional quantity of the asset,

$$\Delta C_P \equiv \sup_c \{c \mid \min_{Q \in \mathcal{P}} U^{-1}(E^Q[U(W_0 + (e + \Delta e)r - c)]) - U^{-1}(E^Q[U(W_0 + er)]) > 0\}. \tag{4}$$

Similarly, we let  $\Delta C_A$  denote the least amount of cash required to induce an individual to give up a small amount of the stock,

$$\Delta C_A \equiv \inf_d \{d \mid \min_{Q \in \mathcal{P}} U^{-1}(E^Q[U(W_0 + (e - \Delta e)r + d)]) - U^{-1}(E^Q[U(W_0 + er)]) > 0\}. \tag{5}$$

The willingness to accept (WTA) and willingness to pay (WTP) are defined as

$$WTA = \lim_{\Delta e \rightarrow 0} \frac{\Delta C_A}{\Delta e}, \quad WTP = \lim_{\Delta e \rightarrow 0} \frac{\Delta C_P}{\Delta e}. \tag{6}$$

**Proposition 1.** *Under our model of familiarity-bias, willingness to accept (WTA) is greater than willingness to pay (WTP). The difference between WTP and WTA increases with the amount of model uncertainty and the degree of risk.*

$$WTA - WTP = 2\alpha\sigma. \tag{7}$$

The disparity in WTA and WTP comes from the difference in perceived outcome distribution. When an investor purchases a share of stock, he considers the scenario

that is most adverse to buying, and when he sells a share of stock, he considers the scenario that is most adverse to selling.

4.2 PORTFOLIO CHOICE UNDER FAMILIARITY BIAS

We now consider the optimal risky portfolio of an investor who has CARA utility but is subject to Strong Status Quo Deviation Aversion (SSQDA). There are two stocks whose returns are normally distributed. The following proposition summarizes the optimal risky portfolio choice under familiarity bias. We use superscript “*b*” to refer to a familiarity-biased investor and superscript “*R*” to refer to a rational investor.

**Proposition 2.** *Suppose the familiarity-biased investor’s initial endowed equity portfolio is  $e \equiv (\omega, 1 - \omega)^\top$ . Then his optimal risky portfolio  $(\omega^b, 1 - \omega^b)^\top$  is*

$$\omega^b = \begin{cases} \frac{\mu_1 - \mu_2 - v_m - \gamma u^\top \Sigma (1-u)/2}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ \omega & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \leq v_m \\ \frac{\mu_1 - \mu_2 + v_m - \gamma u^\top \Sigma (1-u)/2}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m, \end{cases}$$

where  $u \equiv (1, -1)^\top$  and  $v_m = \alpha \sqrt{u^\top \Sigma u}$ .

Proposition 2 implies that the familiarity-biased investor’s optimal trade from endowed equity position satisfies

$$\Delta D = \begin{cases} \frac{\mu_1 - \mu_2 - v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ 0 & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \leq v_m \\ \frac{\mu_1 - \mu_2 + v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m. \end{cases} \quad (8)$$

Intuitively, in the first case of (8), the difference between the expected returns of stocks 1 and 2 is sufficiently high to overcome investor’s fear of change and uncertainty, so that he increases the weight on stock 1 relative to the endowment. On the other hand, in the third case of (8), stock 2 is sufficiently more attractive so that the investor buys more of stock 2 and sell some shares of stock 1. The second case of (8) corresponds to the region of no trade, which occurs when the degree of uncertainty is sufficiently high.

When there is no familiarity bias (i.e.,  $\alpha = 0$ ), Proposition 2 specializes to the optimal risky portfolio  $(\omega^R, 1 - \omega^R)$  for a rational investor with standard CARA utility. Unlike the rational investor’s optimal risky portfolio which is determined by the expected returns of stocks and their covariances, the familiarity-biased investor’s equity portfolio also depends on his endowment and the degree of uncertainty about expected stock returns. For a given degree of uncertainty and endowment portfolio  $e = (\omega, 1 - \omega)^\top$ , the difference between the optimal equity portfolio of a familiarity

biased investor and an otherwise identical rational investor is

$$\omega^b - \omega^R = \begin{cases} \frac{-v_m}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ \omega - \omega^R & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \leq v_m \\ \frac{v_m}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m. \end{cases}$$

Thus, even when the familiarity-biased investor trades away from his endowment in the direction of the stock having superior risk-return tradeoff, he is more conservative than the rational investor as he underweights the more attractive stock. This is a direct consequence of the familiarity-biased investor’s pessimism when deviating from the status quo.

Similar approach used to establish Proposition 2 can be applied to solve the portfolio choice of a risky stock and a riskfree asset, which is a special case of Proposition 2 when the variance of one of the stock returns degenerates to zero. In particular, for every initial stock endowment  $e$ , there is a price interval  $[\mu - \gamma\sigma^2 e - \alpha\sigma, \mu - \gamma\sigma^2 e + \alpha\sigma]$  within which the investor does not deviate from his endowed position. In this sense, the effect of familiarity bias on an investor’s portfolio choice resembles that of transaction cost. In the case of a single stock with 50% sample standard deviation based on  $T = 100$  observations,  $\beta = 1$  (thus  $\alpha = \beta/\sqrt{T} = 0.1$ ), and a risk aversion  $\gamma = 1$ , the effect of familiarity bias on the investor’s portfolio choice is similar to a setting in which there is a 5% proportional transaction cost without familiarity bias.

### 4.3 APPLICATIONS

Next, we apply the model to explain two widely documented empirical puzzles: underdiversification of investors’ portfolios, and the home bias puzzle. Calibration analysis shows that these phenomena occur in our model under plausible parameter values. Although various explanations for the home bias puzzle have been offered, none has been shown to explain the magnitude of observed home bias (e.g., Lewis (1999)). Further, our model implies that investors who is endowed with only domestic stocks may hold zero of foreign stocks, whereas in previous models of home bias, investors always hold non-zero quantities of foreign stocks.

#### 4.3.1 Underdiversification

Blume and Friend (1975) find that investors hold highly underdiversified portfolios. Using more recent data from a major discount brokerage firm, Barber and Odean (2000) find that investors, on average, hold 4.3 stocks at this brokerage firm, with the median being only 2.6 stocks. This phenomenon is in sharp contrast to the recommendation of standard portfolio theory, and especially puzzling prior to the rise of mutual funds in recent decades. We illustrate here that when deviating

from the status quo choice triggers investor aversion to uncertainty, investors may remain at poorly diversified initial endowment positions and do not perceive further diversification to be beneficial.

Consider the case of  $N$  stocks with identically distributed returns. Assume that asset returns are jointly normally distributed, with variance  $\sigma^2$  and correlation  $\rho$ . The amount of uncertainty is assumed to be the same for each stock. We define a portfolio  $p_e$  as *undominated* if a familiarity-biased investor who starts with  $p_e$  as his status quo prefers to hold  $p_e$ . Thus, a portfolio  $p_e$  is undominated if, for any arbitrary portfolio  $p$ ,

$$\min_{Q \in \mathcal{P}} \{U^{-1}(E^Q[U(W(p))]) - U^{-1}(E^Q[U(W(p_e))])\} \leq 0.$$

Given the symmetry of the model, all risk-averse investors would hold equal-weighted portfolios. The next proposition gives the minimum number of stocks in an undominated status quo portfolio.

**Proposition 3.** *The minimum number of stocks  $K$  in a familiarity-biased investor's status quo portfolio such that he would not deviate is*

$$K = 1 + \text{Int} \left[ \frac{\gamma^2 \sigma^2 (1 - \rho) N}{N \alpha^2 + \gamma^2 \sigma^2 (1 - \rho)} \right],$$

where  $\text{Int}[x]$  represents the largest integer less than or equal to  $x$ .

When there is no familiarity bias, i.e.,  $\alpha = 0$ , the investor holds all  $N$  stocks. However, when  $\alpha > 0$ , a familiarity-biased investor holds on to portfolios with a much fewer number of stocks. To see if our model generates investor's portfolio underdiversification for reasonable parameter values, we examine the following example. Let  $N = 500$ ,  $\rho = 0.5$ ,  $\gamma = 1$ ,  $\sigma = 0.5$ . With the model uncertainty associated with deviation from the status quo at  $\alpha = 0.2$ , an equally weighted portfolio with only four stocks ( $K = 4$ ) is undominated. The amount of uncertainty in this case implies that the investor adjusts the sample mean stock return up and down by, at most, one-fifth of the standard deviation. Thus, our model can generate empirically observed underdiversification with reasonable parameter values.

Figure 1 plots the minimum number of stocks needed to construct an undominated portfolio for different degree of uncertainty. It illustrates the tradeoff between the benefit of risk reduction through diversification and the fear of the uncertainty associated with deviating from the initial endowment. As uncertainty increases, the minimum number of stocks needed to construct an undominated portfolio decreases monotonically, reflecting the investor's desire to reduce the overall uncertainty in his portfolio. Furthermore, as the investor's risk aversion increases, the gains from diversification are higher and the investor increasingly desires to hold a well diversified portfolio. Holding fixed the amount of uncertainty, the number of stocks

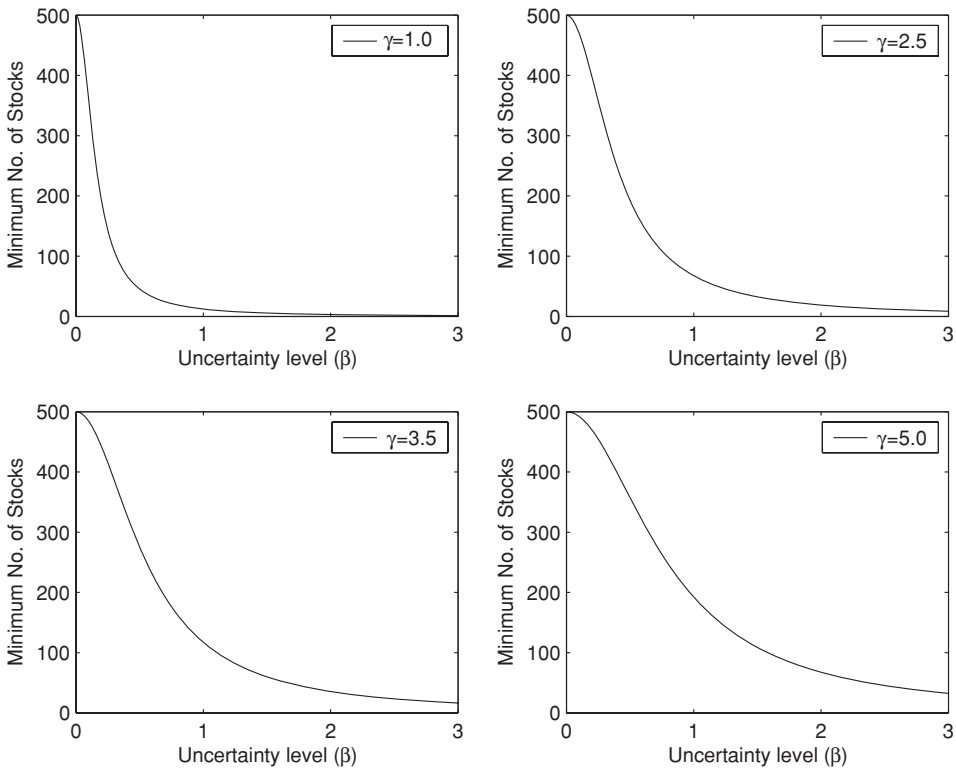


Fig. 1. Underdiversification and uncertainty.

This figure plots the minimum number of stocks in an investor’s portfolio when defection from the endowment induces aversion to uncertainty, for various risk aversion coefficients. There are 500 stocks with identically distributed returns. Assume the investor uses  $T = 100$  data points and estimates that the annual standard deviation of stock return is  $\sigma = 0.3$ , and the pairwise correlation is 0.5.

in an investor’s undominated portfolio is uniformly larger for higher values of risk aversion.

Our finding that limited diversification occurs due to fear of unfamiliar choice options suggests that mutual funds (and especially index funds) provide a social benefit for a reason different from the standard argument that mutual funds reduce the transaction cost needed for investors to diversify. For a long-term buy-and-hold investor, it is not really all that costly to form a reasonably diversified portfolio on an individual account.

In our model, investors stop adding stocks to their portfolios because a large diversification gain is needed to offset the aversion to buying an unfamiliar stock. A mutual fund can address this issue in two ways. First, the individual needs to

add just a single new asset to his portfolio, the mutual fund. Second, by focusing on marketing to investors, mutual funds can make their products more familiar to investors. In other words, where corporations specialize in making profits, mutual funds can specialize in being invested in. Our approach suggests that there is a socially valuable complementarity between being good at marketing that assuages investor fears about stocks, and providing a diversified portfolio of securities in which individuals can invest.<sup>8</sup>

#### 4.3.2 *Home bias*

A well known puzzle in international finance is that investors in aggregate tend to hold mostly the assets of the country they reside in, rather than diversifying internationally—home bias (e.g., French and Poterba (1991)). Since domestic assets start out being owned by domestic investors (i.e., firms that are born in a given country are typically owned first by domestic entrepreneurs), domestic stocks tend to initially be part of the endowment of domestic individuals. Thus, home bias could be viewed as a more general version of the endowment effect. For historical reasons, domestic stocks start out domestically held, and there is a reluctance to shift from this initial position.<sup>9</sup>

For our calibration analysis, we use the results in Proposition 2 on the optimal portfolio holding of familiarity-biased investors. Specifically, for each country we calculate the optimal holdings of the domestic portfolio and the world equity portfolio for the familiarity-biased investor at different levels of model uncertainty.

To pin down the parameters, we calibrate the model to the data for four countries, including Germany, Japan, United Kingdom, and the United States. Table 1 shows the summary statistics of annual stock market returns for the four countries and the world market portfolio, based on data from 1975–2006.<sup>10</sup> To facilitate comparison, we use value-weighted dollar returns for all four countries and the world market portfolio. Investor initial endowment is assumed to be 100 percent in domestic stock

<sup>8</sup> Cronqvist (2006) finds that funds took advantage of investor familiarity in their advertisements (e.g., Absolut Strategi Fund associated itself with the Vodka brand Absolut). Fund advertising is shown to affect investors' portfolio choices, although it provides little information. In particular, advertising induces more home bias.

<sup>9</sup> Of course, in a dynamic setting with heterogeneous investors, there can be movement over time to a situation in which some investors hold foreign stocks and some do not. Those investors who become familiar with the foreign asset class may become more willing to increase their investments in the future. Nevertheless, the basic fact that domestic assets start out domestically owned suggests that home bias may be the result of fear of change rather than an active effort to sell off foreign stocks. This is the possibility that our analysis captures.

<sup>10</sup> We thank Kenneth French for making available on his webpage <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french> the data used in our analysis.

*Table 1.* Summary statistics of annual stock market returns for various countries

The reported statistics are for the annual value-weighted dollar returns from January 1975 to December 2006. “Correlation” measures the sample correlation between the stock market return in each country and the return on the world market portfolio. The original datasets are obtained from Kenneth French’s website: <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french>.

	Mean	Std. Dev.	Correlation
Germany	0.1356	0.2431	0.5679
Japan	0.1434	0.3017	0.8508
UK	0.1890	0.2504	0.6076
US	0.1478	0.1569	0.5004
World	0.1495	0.2084	1.0000

market. This offers the highest level of certainty equivalent gains for diversifying into the world equity market. It therefore creates the most challenging situation for home bias. The risk aversion is set at  $\gamma = 2$ .

The portfolio chosen by investors in each country reflects the fear of unfamiliar associated with defecting from the initial endowment in order to invest more globally. Figure 2 plots the optimal domestic equity proportion in investor’s total portfolio as a function of the perceived uncertainty for the four countries. At low levels of model uncertainty, the optimal weights of domestic equity for investors in all four countries fall below their respective initial domestic endowments, suggesting that it is beneficial for these investors to shift from entirely domestic equity to the world market portfolio.<sup>11</sup> On the other hand, with sufficiently high levels of uncertainty about stock returns, familiarity-biased investors in all four countries perceive their endowment (which is 100 percent domestic equity) as optimal.<sup>12</sup> This is consistent with empirically observed home bias.

As shown in Proposition 2, the effect of familiarity bias on investors’ portfolio choices increases with the level of uncertainty. Figure 2 shows that familiarity-biased investors hold more foreign stocks when they perceive less uncertainty

<sup>11</sup> Based on the sample estimates, the optimal weight on the Japanese stock market is negative when the degree of uncertainty is low. Since it is costly to sell short in international markets, the weight is set to be zero in such cases in Figure 2.

<sup>12</sup> We show that these results hold in equilibrium analysis. Specifically, using the equilibrium prices and investors’ holdings of the two-country model in Section 5, we measure the equilibrium home bias for domestic investors as the ratio of their domestic holdings in the total risky portfolio relative to the weight of the market value of domestic stock in the world market portfolio. We find that the equilibrium home bias ratio initially increases rapidly with the degree of uncertainty. At sufficiently high levels of uncertainty, familiarity-biased investors choose not to trade and the home bias ratio reaches a peak level.

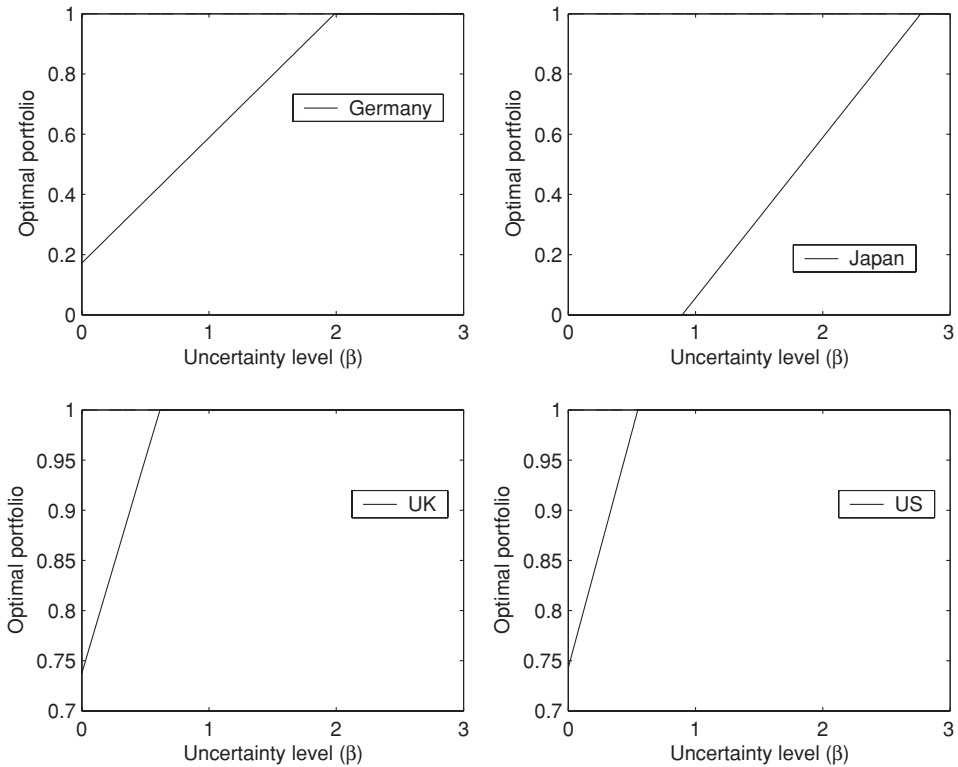


Fig. 2. Optimal weight on domestic equity for familiarity biased investors.

This figure plots the perceived optimal domestic equity proportion as a function of the level of uncertainty ( $\beta$ ) for Germany, Japan, United Kingdom, and the United States. Investors are allowed to hold their domestic market portfolio and the world market portfolio. Their initial endowment is 100% in domestic equity. The adjustments  $v$  for the mean stock returns for familiarity biased investors satisfies  $v^T \Sigma^{-1} v \leq \beta^2 / T$ , where  $\Sigma$  is the covariance matrix of return of a country with the world market using the annual return data from January 1975 to December 2006 ( $T = 32$ ). The risk aversion coefficient is set to  $\gamma = 2$  for all four panels.

about mean stock returns. Consistent with this prediction, Graham, Harvey, and Huang (2006) find that investors who feel competent about investing have more internationally diversified portfolios. Investors who have a strong feeling of general competence are likely to perceive less model uncertainty about stock return distributions.

There is also cross-sectional variation in the amount of uncertainty needed to induce investors to hold on to their endowed portfolio. In Germany, the uncertainty parameter needs to be above two. In Japan, the required uncertainty is slightly under three before investors find it unattractive to add world stock market exposure to



their portfolio. In contrast, UK and U.S. investors stop diversifying into world stock market at much lower levels of uncertainty (about one-half).

### 5. Capital Market Equilibrium with Familiarity Bias

We have analyzed the portfolio choices perceived to be optimal by investors who have familiarity bias. We now turn to the question of how familiarity bias affects stock prices in an endogenously determined market equilibrium. We assume that there are two stock markets, domestic and foreign.<sup>13</sup> The population size of each country is normalized to one, and the proportion of rational investors in each country is denoted  $m$ . Thus, there are four groups of investors: domestic and foreign rational investors, as well as domestic and foreign familiarity-biased investors. All investors have CARA utility function with risk aversion coefficient  $\gamma$ . We use subscript “ $d$ ” to denote home country and subscript “ $f$ ” to denote foreign country.

The payoffs  $V$  of the stocks in the two countries are assumed to be joint normally distributed with mean vector  $\mu = (\mu_d, \mu_f)^\top$ . The variance-covariance matrix of the payoffs,  $\Sigma$ , has diagonal elements of  $\sigma_d^2$  and  $\sigma_f^2$ . The correlation of stock payoffs is  $\rho$ .  $\Sigma$  is known to all investors. The per capita supplies of the domestic and foreign stocks are denoted  $x_d$  and  $x_f$ , respectively. We assume that the entire supply of domestic stocks is initially endowed among domestic investors evenly, while the entire supply of foreign stocks is endowed among foreign investors evenly. Besides the risky stocks, there is a risk-free asset in zero net supply with zero rate of return.

The following proposition describes equilibrium stock returns.

**Proposition 4.** (1) *When  $\alpha < \min\{(1 - \rho)\gamma\sigma_d x_d/2, (1 - \rho)\gamma\sigma_f x_f/2\}$ , rational and familiarity-biased investors trade internationally. Equilibrium stock returns satisfy*

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \left(\frac{\gamma}{2}\right) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}. \tag{9}$$

(2) *When  $\alpha \geq \max\{(1 - \rho)\gamma\sigma_d x_d/2, (1 - \rho)\gamma\sigma_f x_f/2\}$ , rational investors trade internationally, whereas familiarity-biased investors remain at their endowment positions. Equilibrium stock returns are also given by (9).*

(3) *When  $\alpha$  is between  $(1 - \rho)\gamma\sigma_d x_d/2$  and  $(1 - \rho)\gamma\sigma_f x_f/2$ , rational investors trade internationally, while familiarity-biased investors from the country with a higher uncertainty threshold  $((1 - \rho)\gamma\sigma x/2)$  invest in their home market, and familiarity-biased investors from the other country remain at their endowment positions. If the domestic country has a higher uncertainty threshold, then equilibrium*

<sup>13</sup> Although we consider international stock markets, our results also apply to a cross-section of stocks in one market, e.g., to settings where investors have preferred habitats or styles.

stock returns are given by

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \Sigma \left( \begin{pmatrix} \frac{1}{1+m} \gamma x_d - \frac{(1-m)\alpha}{(1+m)(1-\rho^2)\sigma_d} \\ \frac{\gamma x_f}{2} \end{pmatrix} \right). \quad (10)$$

The case in which the foreign country has a higher uncertainty threshold is symmetric.

Case (2) here is the equilibrium analog of the no-trade case in Proposition 2. Further, even when a familiarity-biased investor trades away from his endowment position, he does not move all the way to the rational optimal position. The equilibrium holdings of a familiarity-biased investor differ more from those of a rational investor when the uncertainty is higher, and when the correlation between domestic and foreign stock payoffs is higher.

Equilibrium expected stock returns in Cases (1) and (2) of Proposition 4 coincide with those when all investors are rational but for different reasons. In Case (1), the effects of familiarity bias on domestic and foreign investors offset each other, leaving the rational investors holding the same optimal portfolios as when there are no familiarity-biased investors. In Case (2), uncertainty is so high that only rational investors participate in the markets and determine the prices. Familiarity-biased investors stay at their endowment positions and do not affect the equilibrium prices.

To better understand the effect of familiarity bias on equilibrium asset prices in Case (3), without loss of generality, suppose that the domestic uncertainty threshold is higher than foreign uncertainty threshold. Proposition 4 implies that the equilibrium price for domestic stock  $P_d$  is lower than the fully rational price  $P_d^R$ :

$$P_d - P_d^R = \left( \frac{1-m}{1+m} \right) [\alpha/(1-\rho^2) - \gamma\sigma_d x_d/2] \sigma_d < 0. \quad (11)$$

This occurs because in equilibrium domestic familiarity-biased investors sell some domestic shares, but foreign familiarity-biased investors do not buy domestic shares. To clear the market, rational investors have to hold more domestic stock than the optimal amount when all investors are rational. Thus, the equilibrium price  $P_d$  has to be lower relative to the rational benchmark  $P_d^R$  to induce risk-averse rational investors to hold more shares. The equilibrium expected return for the domestic stock is higher when there are familiarity-biased investors than when all investors are rational. The difference increases with  $1-m$ , the fraction of familiarity-biased investors.

Assuming that each investor is exogenously informed on only a subset of stocks and only trades stocks that he is aware of, Merton (1987) shows that stock's price is reduced more and its expected return is increased more when a greater fraction of investors are not aware of the stock. Familiarity bias provides a possible justification for the non-participation assumed in Merton (1987). In our model a

familiarity-biased investor knows about all stocks, and endogenously *chooses* not to participate in an unfamiliar stock when the uncertainty is sufficiently high. By endogenizing the decision to participate, our model provides new empirical implications about the effects of uncertainty, the prevalence of familiarity bias, and other parameters on trading and prices.

The difference in the stock price between a fully rational market and that with familiarity-biased investors ( $P_d^R - P_d$ ) captures an unfamiliarity premium. Correspondingly, the expected stock return  $\mu_d - P_d$  in our model can be decomposed into two components: the rational risk premium and the unfamiliarity premium. Equation (11) shows that the unfamiliarity premium increases with the fraction of familiarity-biased investors ( $1 - m$ ). It also decreases with the degree of uncertainty ( $\alpha$ ). Intuitively, when uncertainty is higher, domestic familiarity-biased investors sell less of domestic stock because the perceived gains of deviating from their endowment positions are smaller. This leads to reduced supply of shares in the domestic market, and thus a higher equilibrium price and a lower unfamiliarity premium.

The stock market of the country with low uncertainty threshold is less affected by familiarity bias. When  $(1 - \rho)\gamma\sigma_f x_f/2 < \alpha < (1 - \rho)\gamma\sigma_d x_d/2$ , only rational investors participate in the foreign stock market. Familiarity bias affects foreign stock price only indirectly through its correlation with the domestic stock:

$$P_f^R - P_f = \left( \frac{1 - m}{1 + m} \right) [\gamma\sigma_d x_d/2 - \alpha/(1 - \rho^2)]\rho\sigma_f. \quad (12)$$

The foreign stock price is lower (higher) relative to the rational benchmark if the domestic and foreign stock returns are positively (negatively) correlated. When the domestic and the foreign stock markets are uncorrelated, the stock market with low uncertainty threshold is unaffected by familiarity bias.

Since familiarity bias affects the expected equity premium, it affects the CAPM which characterizes the relation between expected stock returns and the systematic risk of stocks. Our next proposition concerns the validity of international CAPM when some investors are subject to familiarity bias. In Cases (1) and (2) of Proposition 4, equilibrium stock returns are the same as in the case when all investors are rational. It is not surprising that the CAPM holds in these cases. What is interesting is that a modified CAPM holds even when no one holds the market portfolio.<sup>14</sup> In Case (3), familiarity bias affects stock returns, and the standard CAPM fails. However, in this case, a modified CAPM holds with respect to the rational investors' aggregate stock portfolio rather than the world market portfolio.

<sup>14</sup> Using evidence from large-scale experimental financial markets, Bossaerts and Plott (2004) find that financial assets are priced by the CAPM even though the subjects participating in the experiments do not hold the market portfolio.

**Proposition 5.** (1) *When the uncertainty is either sufficiently low or sufficiently high ( $\alpha < \min\{(1 - \rho)\gamma\sigma_d x_d/2, (1 - \rho)\gamma\sigma_f x_f/2\}$  or  $\alpha \geq \max\{(1 - \rho)\gamma\sigma_d x_d/2, (1 - \rho)\gamma\sigma_f x_f/2\}$ ),*

$$E[r_i] = \beta_i E[r_M], \tag{13}$$

where  $r_i$  and  $r_M$  are the return of country  $i$ 's stock market ( $i = d$  or  $f$ ) and the value-weighted world market portfolio  $M$ ,  $\beta_i$  is the beta of stock  $i$ 's return with respect to the world market return.

(2) *When  $\alpha$  is between  $(1 - \rho)\gamma\sigma_d x_d/2$  and  $(1 - \rho)\gamma\sigma_f x_f/2$ ,*

$$E[r_i] = \tau_i \beta_i E[r_M], \tag{14}$$

where  $\tau_i$  is greater (smaller) than one for the country with the high (low) uncertainty threshold. The absolute pricing error of the standard CAPM with respect to the world market portfolio increases with the fraction of familiarity-biased investors. Furthermore, a modified CAPM holds:

$$E[r_i] = \beta'_i E[r_{M'}], \tag{15}$$

where  $M'$  is the rational investors' aggregate stock portfolio,  $\beta'_i$  is the beta of stock  $i$ 's return ( $i = d$  or  $f$ ) with respect to  $M'$ . Suppose  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$ . Then the portfolio  $M'$  consists of  $n_1 x_d$  shares of the domestic stock and  $n_2 x_f$  shares of the foreign stock, where

$$n_1 = \frac{1}{1+m} - \left( \frac{1-m}{1+m} \right) \frac{\alpha}{(1-\rho^2)\gamma\sigma_d x_d}, \quad n_2 = \frac{1}{2}.$$

Proposition 5 suggests that the failure of the empirical testing of the international CAPM may be caused by familiarity bias on the part of some investors. We find that the absolute pricing error of the standard CAPM increases with the fraction of familiarity-biased investors. Since familiarity-biased investors are more likely to hold only domestic equity, the absolute pricing error of the standard CAPM is expected to be positively correlated with the amount of home bias.

Proposition 5 presents a testable hypothesis on the modified international CAPM. Given measures for the degree of uncertainty and the fraction of rational investors, we can construct the aggregate stock portfolio held by rational investors as in Proposition 5. In practice, the uncertainty can be measured according to Anderson, Ghysels and Juergens (2009) using the data on professional forecasters. The fraction of investors that participate in foreign (world) stock markets can serve as a proxy for the fraction of rational investors. The empirical test of the modified international CAPM is left for future studies.

## 6. Discussion and Conclusions

We propose here that the emotions of fear and suspicion directed to the unfamiliar and toward potential change can explain several puzzles in economic and financial decisions. We model an inclination of individuals who are faced with uncertainty to focus on worst-case scenarios when contemplating deviations from the status quo. The endowment effect arises endogenously in our setting. The model also offers an explanation for limited diversification of investors across stocks and asset classes, including the underdiversification puzzle, and the home bias puzzle. For plausible parameter values, investors settle for very undiversified portfolios because defection-induced fear of uncertainty deters individuals from diversifying further. In calibration analysis, we find that the observed magnitude of home bias is consistent with a reasonable level of uncertainty.

Our approach offers different policy implications from that of previous studies. For instance, in our approach, if investors can be induced to purchase a new asset class, this asset class will become more familiar and will be more likely to remain in the portfolio.<sup>15</sup> In contrast, in previous models where endowment does not matter, such a change has no lasting effect. Similarly, our analysis suggests that privatizations of government owned firms in which shares are allocated to individual investors can permanently increase their stockholdings by making the holding of these shares (and to some extent, the holding of stocks in general) more familiar.

More importantly, our analysis provides implications on the effect of familiarity bias on equilibrium asset prices and returns, and the circumstances under which these effects are stronger, weaker, or nonexistent. In a two countries setting, when the degree of uncertainty is either low or high enough, the effects of familiarity bias on domestic and foreign investors offset each other. Equilibrium expected stock returns coincide with those when all investors are rational. However, when the degree of uncertainty is not too high to completely deter familiarity-biased investors in both countries from participating but high enough to influence the stock demand and supply of some familiarity-biased investors, equilibrium stock price is lower than that in a market with full rational investors. In this case, the standard CAPM fails. However, a modified CAPM holds when the world market portfolio is replaced by the aggregate stock holdings of the rational investors.

Our findings on the pricing effects of familiarity bias are related to the asset pricing implications of the incomplete information model of Merton (1987).

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<sup>15</sup> For example, if an employer pays an employee with stock, the stock becomes familiar as part of the portfolio, so the employee may voluntarily retain a lot of own-company stock even though, if he had not been paid in stock, he would not have purchased it. Or, if a stock market boom and heavy media publicity increase the familiarity of stock investing and induce individuals to start participating in the stock market, they are likely to continue to hold stocks later even if favorable publicity about holding stocks declines.

Assuming that each investor is ‘uninformed’ about a subset of stocks (i.e., exogenously unable to take a position in these stocks), Merton shows that for a given stock, the price discount increases with the fraction of uninformed investors on this stock. In our model, a familiarity-biased investor is ‘informed’ about all stocks (there is no exogenous constraint on participation), but he may choose not to participate in an unfamiliar stock when the uncertainty is sufficiently high. From that perspective, the fraction of familiarity-biased investors in our model endogenizes the fraction of uninformed investors in Merton’s model. By doing so, our analysis provides stronger predictions about the form of deviations from the CAPM and suggests the construction of a modified market portfolio with respect to which the CAPM relation still holds.

The analysis in this paper is static. While there are empirical evidence suggests that investors in reality often persistently eschew certain assets or asset classes for long periods of time (such as local and home biases), changes in participation do occur through time. For instance, there is increased participation in the stock market over a period of decades that has accompanied the rise of mutual funds and defined contribution retirement plans, and increased interest in investments in international stocks and in commodities. These suggest that investors’ status quo evolves over time which may require dynamic models to fully understand the behavior.

In dynamic extensions, different possible specifications of the status quo can be used. One possible dynamic status quo specification is to utilize the most recent choice as the status quo for the next period. More generally, the status quo could also reflect a weighted average of previous holdings that may have been sold but the decisionmakers are still familiar. In general, as long as the decisionmaker optimizes period by period under Status Quo Deviation Aversion preference, the results in this paper still apply in the dynamic setting.<sup>16</sup>

Such extensions of the current model already have interesting conjectural implications. For example, suppose news arrives which increases the expected return of an asset that is not currently part of the status quo portfolio by enough to overcome the uncertainty associated with this unfamiliar asset. The investor therefore buys some of the asset, which becomes part of the new status quo. Repetitions of this process over time can lead to a gradual evolution toward more diversified portfolios.

Furthermore, after the investor buys an asset, it becomes more familiar and its perceived uncertainty is reduced. Thus, the investor is likely to buy more of this asset subsequent to the initial purchase even without additional favorable news. Such an effect is potentially testable, and would imply dynamic inconsistency (from the viewpoint of rational expected utility maximization).

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<sup>16</sup> However, dynamic modelling becomes technically challenging if the decisionmaker looks forward taking into account the implications of current choices for future shifts in the status quo.

An interesting further issue that comes up in a dynamic setting is the possibility that the arrival of new information will occasionally stimulate new uncertainty about the economic environment, thereby making individuals reluctant to trade. For example, it seems likely that extreme economic news could raise doubts among investors about whether their beliefs about how the world is structured are correct. In such circumstances of heightened uncertainty, familiarity bias effects could become especially strong, leading to reduction in trade.<sup>17</sup> Fear of the unfamiliar deserves further study as a possible explanation for the dynamics of market participation, liquidity, and prices.

## Appendices

### A1. Proof of Proposition 1

For the CARA utility  $U(W) = -e^{-\lambda W}$ , the certainty equivalent value of a random variable  $Y$  under a probability measure  $Q$  is

$$U^{-1}(E^Q[U(Y)]) = E^Q[Y] - \frac{\gamma}{2}\text{Var}(Y).$$

Under the assumption of normal distributions for the set  $\mathcal{P}$ , the worst distribution for holding additional shares of stock is a normal distribution with mean payoff  $\mu - \alpha\sigma$  (the mean stock payoff adjusted downward by  $-\alpha\sigma$ ). Based on these facts, it is straightforward to show that

$$\Delta C_P = \Delta e(\mu - \alpha\sigma) - \frac{\gamma\sigma^2}{2}[(e + \Delta e)^2 - e^2].$$

Letting  $\Delta e$  approach zero, we obtain the marginal willingness to pay:

$$\text{WTP} = \mu - \alpha\sigma - \gamma e\sigma^2.$$

Similarly, the worst case scenario for selling stock is a normal distribution with a mean payoff  $\mu + \alpha\sigma$  (the mean stock payoff adjusted upward by  $\alpha\sigma$ ). Therefore, the amount that an investor requires to reduce stock holding from  $e$  to  $e - \Delta e$  units is

$$\Delta C_A = \Delta e(\mu + \alpha\sigma) - \frac{\gamma\sigma^2}{2}[(e - \Delta e)^2 - e^2].$$

The marginal willingness to accept is  $\text{WTA} = \mu + \alpha\sigma - \gamma e\sigma^2$ . Thus, the difference between WTP and WTA is

$$\text{WTA} - \text{WTP} = 2\alpha\sigma.$$

<sup>17</sup> See Routledge and Zin (2003) on how ambiguity aversion can lead to fluctuations in liquidity, such as the extreme illiquidity and ‘flight to quality’ that occurred in international bond markets during the Russian debt crisis of August 1998.

**A2. Proof of Proposition 2**

Let  $e \equiv (\omega, 1 - \omega)^\top$  denote the investor’s endowed equity portfolio and  $(\omega + \Delta D, 1 - \omega - \Delta D)^\top$  be a contemplated new portfolio, where  $\Delta D$  is the investor’s trade in domestic stock. The investor’s initial wealth is normalized to one. Given the CARA utility considered here, this normalization does not affect the investor’s portfolio choice. Under the SSQDA preference, the perceived certainty-equivalent gain of the trade from the endowment portfolio to the contemplated new portfolio is

$$\begin{aligned}
 C(\Delta D, e) &\equiv \min_v \{ (\Delta Du)^\top (\mu + v) - \frac{\gamma}{2} [ (\Delta Du + e)^\top \Sigma (\Delta Du + e) - e^\top \Sigma e ] \} \\
 &= \Delta D [ u^\top \mu - \text{sign}(\Delta D) v_m ] - \frac{\gamma}{2} [ \Delta D^2 u^\top \Sigma u + 2 \Delta D u^\top \Sigma e ], \quad (16)
 \end{aligned}$$

where  $u \equiv (1, -1)^\top$ ,  $v$  satisfies (3) and

$$v_m \equiv - \min_{Q \in \mathcal{P}} u^\top v = \alpha \sqrt{u^\top \Sigma u}.$$

Given initial endowment  $e$ , the optimal trade  $\Delta D$  maximizes the certainty equivalent gain  $C(\Delta D, e)$ . The unconstrained first order condition is:

$$u^\top \mu - \text{sign}(\Delta D) v_m - \gamma \Delta D u^\top \Sigma u - \gamma u^\top \Sigma e = 0.$$

There are two scenarios: (1) No trading is perceived to be optimal, i.e.,  $\Delta D = 0$ ; (2) Trading is perceived to be optimal and satisfies the first order condition above, which implies

$$\Delta D = \frac{u^\top \mu - \text{sign}(\Delta D) v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}.$$

The no trade scenario occurs if and only if

$$-v_m < u^\top \mu - \gamma u^\top \Sigma e < v_m.$$

Otherwise,  $\Delta D$  is positive when  $u^\top \mu - \gamma u^\top \Sigma e > v_m$ , and is negative when  $u^\top \mu - \gamma u^\top \Sigma e < -v_m$ .

**A3. Proof of Proposition 3**

For any positive integer  $K$ , let  $e_K$  denote an equally weighted portfolio of  $K$  stocks. To prove Proposition 3, we examine the conditions under which a familiarity-biased investor endowed with  $e_K$  would not want to combine  $e_K$  with any  $e_{M-K}$ ,  $K < M \leq N$ , where  $e_{M-K}$  denotes an equal-weighted portfolio of  $M-K$  stocks not contained in  $e_K$ .

Let  $v_K$  and  $v_{M-K}$  be the adjustments to the mean returns of portfolios  $e_K$  and  $e_{M-K}$  due to uncertainty, respectively. As in Section 4.2, familiarity-biased



investors contemplating a trade make adjustments  $v = (v_K, v_{M-K})^\top$  to perceived mean portfolio returns satisfying

$$v^\top \Sigma_M^{-1} v \leq \alpha^2,$$

where  $\Sigma_M$ , the variance-covariance matrix of returns of  $e_K$  and  $e_{M-K}$ , is

$$\Sigma_M = \begin{pmatrix} \frac{1+(K-1)\rho}{K} & \rho \\ \rho & \frac{1+(M-K-1)\rho}{M-K} \end{pmatrix} \sigma^2.$$

Applying Proposition 2, familiarity-biased investors would hold onto their initial endowment portfolio  $e_K$  if the degree of uncertainty is sufficiently high as in the second case of (8):

$$v_M \geq \gamma u^\top \Sigma_M (1, 0)^\top, \tag{17}$$

where  $u = (1, -1)^\top$ , and  $v_M = \alpha \sqrt{u^\top \Sigma_M u} = \alpha \sigma \sqrt{\frac{M(1-\rho)}{K(M-K)}}$ . It is straightforward to show that (17) implies

$$K \geq \frac{\gamma^2 \sigma^2 (1 - \rho) M}{M \alpha^2 + \gamma^2 \sigma^2 (1 - \rho)}. \tag{18}$$

Thus, given the uncertainty about mean stock returns described by  $\alpha$ , a familiarity-biased investor who holds the portfolio  $e_K$  with  $K$  stocks will not want to diversify further, as long as (18) holds for all  $M$  such that  $K < M \leq N$ . Since the right-hand side of (18) increases with  $M$ , the minimum number of stocks  $K$  so that a familiarity-biased investor endowed with  $e_K$  will not diversify further is

$$K = 1 + \text{Int} \left[ \frac{\gamma^2 \sigma^2 (1 - \rho) N}{N \alpha^2 + \gamma^2 \sigma^2 (1 - \rho)} \right],$$

where  $\text{Int}[x]$  represents the largest integer below  $x$ .

**A4. Proof of Proposition 4**

Consider an investor’s optimal portfolio choice corresponding to a given price vector  $P = (P_d, P_f)^\top$ . Let  $W_0$  denote his initial wealth in the risk-free asset,  $e \equiv (e_d, e_f)^\top$  denote his initial share endowment in domestic and foreign stocks, and  $\Delta D \equiv (\Delta D_d, \Delta D_f)^\top$  denote trade from the initial endowment. The optimal portfolio holdings of domestic rational, domestic familiarity-biased, foreign rational, and foreign familiarity-biased investors are denoted respectively by  $D_{dr}$ ,  $D_{db}$ ,  $D_{fr}$ , and  $D_{fb}$ .

The rational investors maximize  $E[e^{-\gamma W_1}]$ , where  $W_1$  is the wealth next period,

$$W_1 = W_0 + (e + \Delta D)^\top V - (\Delta D)^\top P.$$

It follows that

$$D_{dr} = D_{fr} = \frac{1}{\gamma} \Sigma^{-1}(\mu - P). \tag{19}$$

The familiarity-biased investor’s optimal trade  $\Delta D^*$  can be computed in two steps. First, for each proposed demand deviation  $\Delta D$ , evaluate the certainty equivalent gains  $G(\Delta D, e)$  of deviating from endowment  $e$  by  $\Delta D$ . Second, choose  $\Delta D^*$  to maximize  $G(\Delta D, e)$  for a given endowment  $e$ .

Under SSQDA preferences, the perceived certainty equivalent gains of moving from endowment portfolio  $e$  to a portfolio  $e + \Delta D$  is

$$C(\Delta D, e) \equiv \min_v \left\{ \Delta D^\top (\mu + v - P) - \left(\frac{\gamma}{2}\right) [\Delta D^\top \Sigma \Delta D + 2\Delta D^\top \Sigma e] \right\},$$

where  $v$  is the adjustments to perceived mean stock payoffs  $v \in [-\alpha\sigma_d, \alpha\sigma_d] \times [-\alpha\sigma_f, \alpha\sigma_f]$ . The familiarity-biased investor evaluates any deviation in the worse case scenario among the possible probability distributions. For  $i = 1$  (corresponding to domestic stock) or  $i = 2$  (corresponding to foreign stock), if  $\Delta D_i > 0$  (buy more shares), the worse case scenario mean adjustment is  $-\alpha\sigma_i$ ; if  $\Delta D_i < 0$  (sell some shares), the worse case scenario mean adjustment is  $\alpha\sigma_i$ . Thus,

$$C(\Delta D, e) = \Delta D^\top [\mu - P - \text{sign}(\Delta D)v_m] + \left(\frac{\gamma}{2}\right) [\Delta D^\top \Sigma \Delta D + 2\Delta D^\top \Sigma e], \tag{20}$$

where  $\text{sign}(\Delta D)$  is a vector that gives the sign of each component of the vector  $\Delta D$ , and  $v_m$  is a vector defined as  $v_m \equiv \alpha(\sigma_d, \sigma_f)^\top$ .

The optimal trade  $\Delta D_b^*$  for a familiarity biased investor corresponding to a given endowment  $e$  maximizes  $G(\Delta D, e)$ . If it is nonzero, then it necessarily satisfies the first order condition derived from (20),

$$\mu - P - \text{sign}(\Delta D_b^*)v_m - \gamma \Sigma \Delta D_b^* - \gamma \Sigma e = 0, \tag{21}$$

which implies that familiarity biased investor’s optimal holding is

$$\Delta D_b^* + e = \left(\frac{1}{\gamma}\right) \Sigma^{-1}[\mu - P - \text{sign}(\Delta D_b^*)v_m]. \tag{22}$$

This applies to both domestic and foreign familiarity biased investors, with  $e = (x_d, 0)$  and  $e = (0, x_f)$  respectively.

There are several possibilities for the familiarity biased investors’ demand in equilibrium. In the first case when the amount of uncertainty is sufficiently low in both countries, we will show that familiarity biased investors would sell some of their own country’s stock and buy some of the other country’s stock. In the second case when the amount of uncertainty is sufficiently high in both country, the familiarity biased investors would keep their endowment. We also consider a third case where the amount of uncertainty is too high in only one country.

Case (1): when uncertainty is low for both domestic and foreign stock markets, so that familiarity biased investors in both countries sell some of their own country's stock and buy some of the other country's stock. Then by (22), the optimal demand of domestic familiarity biased investor is

$$D_{db} = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d + \alpha\sigma_d \\ \mu_f - P_f - \alpha\sigma_f \end{pmatrix}, \tag{23}$$

and the optimal demand by the foreign familiarity biased investor is

$$D_{fb} = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d - \alpha\sigma_d \\ \mu_f - P_f + \alpha\sigma_f \end{pmatrix}. \tag{24}$$

Aggregating the rational investors' demand in (19) and familiarity biased investors' demand in (23) and (24), the market clearing condition is

$$\begin{aligned} &\left(\frac{2m}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + \left(\frac{1-m}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d + \alpha\sigma_d \\ \mu_f - P_f - \alpha\sigma_f \end{pmatrix} \\ &+ \left(\frac{1-m}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d - \alpha\sigma_d \\ \mu_f - P_f + \alpha\sigma_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix}. \end{aligned}$$

This simplifies to

$$\left(\frac{2}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix},$$

which implies that the equilibrium stock prices in the first case satisfy

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \left(\frac{\gamma}{2}\right) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix},$$

just as claimed in Case (1) of Proposition 4 (see Equation (9)). The equilibrium stock prices in Case 1 coincides with the equilibrium stock prices when all investors are rational.

We need to check that familiarity biased investors in both countries sell some of their own country's stock and buy some of the other country's stock. For this to obtain the model parameters must satisfy:

$$\begin{aligned} \frac{1}{2}x_d + \frac{\alpha}{(1-\rho)\gamma\sigma_d} < x_d, & \quad \frac{1}{2}x_d - \frac{\alpha}{(1-\rho)\gamma\sigma_d} > 0, \\ \frac{1}{2}x_f - \frac{\alpha}{(1-\rho)\gamma\sigma_f} > 0, & \quad \frac{1}{2}x_f + \frac{\alpha}{(1-\rho)\gamma\sigma_f} < x_f. \end{aligned}$$

The necessary and sufficient condition for the above to hold is

$$\alpha < \min \left\{ \left(\frac{1-\rho}{2}\right) \gamma\sigma_d x_d, \left(\frac{1-\rho}{2}\right) \gamma\sigma_f x_f \right\}.$$

Case (2):  $\alpha > \max\{(\frac{1-\rho}{2})\gamma\sigma_d x_d, (\frac{1-\rho}{2})\gamma\sigma_f x_f\}$ . In this case, both domestic and foreign familiarity biased investors choose to stay at the endowment because the perceived amount of uncertainty is too high. The market clearing condition is

$$\left(\frac{2m}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + (1-m) \begin{pmatrix} x_d \\ 0 \end{pmatrix} + (1-m) \begin{pmatrix} 0 \\ x_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix},$$

which implies that the equilibrium stock prices satisfy (9).

Case (3): The amount of uncertainty is too high in one country but not the other. Without loss of generality, assume the parameters are such that  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$ . In this case, the domestic familiarity biased investor sells some of his endowment but he does not buy any shares of the foreign stock. The foreign familiarity biased investor stays at his endowed foreign stock shares and does not invest in the domestic stock. The market clearing condition is

$$\begin{aligned} &\left(\frac{2m}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + \begin{pmatrix} 1-m \\ (1-\rho^2)\gamma\sigma_d^2\sigma_f^2 \end{pmatrix} \\ &\times \begin{pmatrix} (\mu_d - P_d + \alpha\sigma_d)\sigma_f^2 - \rho\sigma_d\sigma_f(\mu_f - P_f) \\ 0 \end{pmatrix} + (1-m) \begin{pmatrix} 0 \\ x_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix}. \end{aligned}$$

This is equivalent to the following system of linear equations for  $\mu_d - P_d$  and  $\mu_f - P_f$ :

$$\begin{pmatrix} 1 \\ (1-\rho^2)\sigma_d^2\sigma_f^2 \end{pmatrix} \begin{pmatrix} \sigma_f^2 & -\rho\sigma_d\sigma_f \\ -\rho\sigma_d\sigma_f & \sigma_d^2 \end{pmatrix} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}.$$

But

$$\Sigma^{-1} = \frac{1}{(1-\rho^2)\sigma_d^2\sigma_f^2} \begin{pmatrix} \sigma_f^2 & -\rho\sigma_d\sigma_f \\ -\rho\sigma_d\sigma_f & \sigma_d^2 \end{pmatrix},$$

so the equilibrium stock prices satisfy (10) as claimed in the case (3) of Proposition 4.

**A5. Proof of Proposition 5**

The world stock market  $M$  consists of  $x_d$  shares of the domestic stock and  $x_f$  share of the foreign stock. Its payoff next period is normally distributed as

$$V_M \sim \mathcal{N}\left(x_d\mu_d + x_f\mu_f, (x_d \ x_f)\Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}\right).$$

The value of the world stock market  $P_M$  is  $x_d P_d + x_f P_f$ . Stock returns are

$$r_d = \frac{V_d - P_d}{P_d}, \quad r_f = \frac{V_f - P_f}{P_f}.$$

$$r_M = \frac{V_M - P_M}{P_M} = \frac{x_d(V_d - P_d) + x_f(V_f - P_f)}{P_M}.$$

It follows that

$$\begin{aligned} E[r_M] &= \left(\frac{1}{P_M}\right) (x_d \ x_f) \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix}, \\ \text{Var}(r_M) &= \left(\frac{1}{P_M^2}\right) (x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}, \\ \text{Cov}(r_i, r_M) &= \left(\frac{1}{P_i P_M}\right) \left[ \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix} \right]_i, \end{aligned}$$

where  $[\cdot]_i$  denotes the  $i$ th component of a vector,  $i = 1$  (respectively  $i = 2$ ) corresponds to the domestic (foreign) stock. Thus, the beta of the domestic (respectively foreign) stock return with respect to the world market return  $\beta_d$  (respectively  $\beta_f$ ) is

$$\beta_d = \left(\frac{P_M}{P_d}\right) \frac{\left[ \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix} \right]_1}{(x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}}, \quad \beta_f = \left(\frac{P_M}{P_f}\right) \frac{\left[ \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix} \right]_2}{(x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}}.$$

It follows that

$$\beta_i E[r_M] = \left(\frac{\beta_i}{P_M}\right) (x_d \ x_f) \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix}, \quad i = d \text{ or } f. \tag{32}$$

CAPM holds if and only  $E[r_i] = \beta_i E[r_M]$  in the equilibrium (the riskfree rate is zero in our economy).

For Case 1 and Case 2 of Proposition 4, equilibrium prices  $P_d$  and  $P_f$  satisfy (9). Substituting (9) into (25),

$$\begin{aligned} \beta_d E[r_M] &= \left(\frac{\gamma \beta_d}{P_M}\right) (x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix} \\ &= \left(\frac{\gamma}{2P_d}\right) \left[ \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix} \right]_1 \\ &= \frac{\mu_d - P_d}{P_d} \\ &= E[r_d] \end{aligned}$$

Thus, CAPM holds for the domestic stock. Similarly, CAPM holds for the foreign stock in these cases as well.

For Case 3 of Proposition 4, to conserve space we consider only the case  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$  (the case  $(\frac{1-\rho}{2})\gamma\sigma_d x_d < \alpha < (\frac{1-\rho}{2})\gamma\sigma_f x_f$  can be dealt with in the same manner.) The equilibrium prices  $P_d$  and  $P_f$  satisfy (10). It follows that

$$\begin{aligned} \beta_d E[r_M] &= \left(\frac{1}{P_d}\right) \frac{\left[\Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_1}{(x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}} (x_d \ x_f) \Sigma \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix} \\ &= k_d E[r_d], \end{aligned}$$

where

$$k_1 = \frac{(x_d \ x_f) \Sigma \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}}{(x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}} \frac{\left[\Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_1}{\left[\Sigma \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}\right]_1}.$$

Similarly, for the foreign stock market,

$$\beta_f E[r_M] = k_f E[r_f],$$

where

$$k_2 = \frac{(x_d \ x_f) \Sigma \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}}{(x_d \ x_f) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}} \frac{\left[\Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_2}{\left[\Sigma \begin{pmatrix} \frac{1}{1+m} \left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}\right]_2}.$$

The constants  $\tau_i$ 's in the Proposition 5 are

$$\tau_d = 1/k_d, \quad \tau_f = 1/k_f.$$

They are not equal to one in general. Thus, the CAPM does not hold when the degree of uncertainty is between the uncertainty thresholds of the two countries. In fact,  $\tau_d > 1$  and  $\tau_f < 1$  when  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$ . It is straightforward to verify that  $\tau_d - 1$  can be expressed as a fraction whose denominator is positive, with numerator

$$\left(\frac{1-m}{1+m}\right) \sigma_d \sigma_f^2 x_f^2 [(1-\rho^2)\gamma\sigma_d x_d / 2 - \alpha].$$

The numerator of  $\tau_d - 1$  is also positive because  $\alpha < (1 - \rho)\gamma\sigma_d x_d/2$ , and  $1 + \rho > 1$ . Similarly,  $\tau_d - 1$  can be expressed as a fraction whose denominator is positive, with numerator

$$-\left(\frac{1 - m}{1 + m}\right) \sigma_d \sigma_f^2 x_d x_f [(1 - \rho^2)\gamma\sigma_d x_d/2 - \alpha].$$

The numerator of  $\tau_f - 1$  is negative, thus  $\tau_f < 1$  when  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$ . The expected pricing errors of the standard CAPM under our model are given by  $(\tau_d - 1)E[r_M]$  and  $(\tau_f - 1)E[r_M]$ . The absolute pricing errors are proportional to  $|\tau_i - 1|$ , which increase with  $1 - m$ , the fraction of familiarity-biased investors.

Finally, we show that a modified version of CAPM holds. Suppose  $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$ . The rational investors' optimal holdings are

$$\left(\frac{1}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix}$$

Substituting the equilibrium stock returns given by (10), the rational investors' portfolio  $M'$  consist of  $n_1 x_d$  shares of the domestic stock and  $n_2 x_f$  shares of the foreign stock, where

$$n_1 = \frac{1}{1 + m} - \left(\frac{1 - m}{1 + m}\right) \frac{\alpha}{(1 - \rho^2)\gamma\sigma_d x_d}, \quad n_2 = \frac{1}{2}.$$

Note that  $n_1 > n_2$ , and the difference increases with the amount of uncertainty  $\alpha$ .

The expected return of the portfolio  $M'$  is:

$$E[r_{M'}] = \left(\frac{1}{P_{M'}}\right) (n_1 x_d \ n_2 x_f) \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix}.$$

The beta of stock  $i$  with respect to the portfolio  $M'$  is ( $i = 1$  for the domestic stock,  $i = 2$  for the foreign stock)

$$\beta_i = \left(\frac{P_{M'}}{P_i}\right) \frac{\left[\Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}\right]_i}{(n_1 x_d \ n_2 x_f) \Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}}.$$

By the definition of  $n_1$  and  $n_2$ , and the equilibrium return relation (10),

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}.$$

Using Equations (6) and (6), it follows that for the domestic stock,

$$\begin{aligned}\beta_d E[r_{M'}] &= \left(\frac{1}{P_d}\right) \frac{\left[\Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}\right]_1}{(n_1 x_d \ n_2 x_f) \Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}} (n_1 x_d \ n_2 x_f) \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} \\ &= \left(\frac{1}{P_d}\right) \left[\Sigma \begin{pmatrix} n_1 x_d \\ n_2 x_f \end{pmatrix}\right]_1 \\ &= \frac{\mu_d - P_d}{P_d} \\ &= E[r_d].\end{aligned}$$

Thus, the CAPM holds for the domestic stock with respect to the modified market portfolio  $M'$ . The case for the foreign stock is similar.

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