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# FUTURES TRADING, STORAGE, AND THE DIVISION OF RISK: A MULTIPERIOD ANALYSIS*

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The relationship of storage to futures trading was a favourite topic of the classical theorists of futures markets.[1] This paper presents an integrated framework for the analysis of the inter-related storage, lending and futures hedging decisions of the major classes of participants in commodity markets: by growers, storage firms, and outside 'speculators'. A distinctive and, as will be argued below, a crucial feature of the analysis is that futures trading individuals make decisions covering many harvests.[2] The model describes the joint equilibrium in spot and futures markets, with emphasis upon the determinants of 'futures price bias' – that is, the difference between the futures price and the expectation of the later spot price. (A positive difference or bias is termed 'contango'; if negative, the bias is termed 'backwardation'.)

The 'normal backwardation' theory of Keynes and Hicks was the starting point of modern treatments of the relation between storage and futures prices. In this scenario, suppliers of a commodity are supposed to be primarily concerned with the price risks they face as delivery date approaches. To hedge against price-induced variability in the value of their inventories, they take short positions in the futures markets.[3] As inducement to take the opposite long side of the futures positions held by hedgers, outside speculators receive a premium, taking the form of a downward futures price bias. Speculators on average profit from holding the futures contract, which compensates them for the associated risk they bear. Thus, on average the futures price should rise through the life of a contract until expiration. Cootner (1960, 1967) extended this argument to predict that expected futures price changes will vary depending on the aggregate level of stocks to be hedged.

The classical theme of storage as a source of downward bias has been formalised in two-date models by more recent writers such as Stoll (1979), and Berck and Cechetti (1985). These papers have made an important contribution in showing how hedging pressure affects commodity futures prices when many risky securities are traded. However, these papers are incomplete in other

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[1] E.g. Keynes (1927), Hicks (1939), Telser (1958), Cootner (1960, 1967).

[2] A number of authors have modelled multi-period storage decisions without futures trading (e.g., Danin *et al.* (1975), Newbery and Stiglitz (1981, pp. 421–38; 1982), Scheinkman and Schechtman (1983), and Newbery (1984)).

[3] In fact, under the assumption of no 'basis risk', i.e. that the futures price at expiration matches the spot price of the inventory to be sold, it is supposed to be feasible for storers to eliminate *all* risk, by taking short futures position equal to the amount being stored.

respects. First, the source of price fluctuations (supply versus demand shocks) in the spot market is left implicit. Second, they examine the futures trading behaviour of storers but not growers. Since storers are assumed to hold non-stochastic quantities of the commodity, these models allow for price risk but not quantity risk.

In a multi-period setting that includes traders who face quantity risk as well as price risk, this paper shows that *costless storage does not promote downward bias.* (And of course, some agents must face quantity risk, since stochastic price variability is primarily the result of output variability.) This is not to say that downward bias cannot occur when storage is costless but that, in those scenarios where it does, the bias would also be downward in a corresponding setting where no storage was possible. Storage neither causes nor is associated with downward bias; hence, in contrast with Cootner's theory, the model does not predict higher expected futures price changes when commodity stocks are high.

A number of papers combine features of both single-period and multi-period analysis. These hybrid models involve interlocking generations of traders each of whom survives a single period.[4] For a commodity that is not carried from one harvest to the next, the assumption of a single-period lifespan may be benign. However, carryover introduces serial interactions of risk that are not captured in a hybrid model. In the theory of portfolio choice, only under very special assumptions can multi-period optimisation problems be collapsed into sequences of standard single-period problems. (The induced single-period problems typically involve state-dependent utility functions.) Most notable of these assumptions is that the payoffs to investing in different assets be independent over time.[5]

Time independence of returns is impossible in the context of storable commodity hedging. The size of the crop at date $t+1$ determines the spot price at $t+1$, carryover into $t+2$, the distribution of price at $t+2$ and carryover into $t+3$, and so on. A rational supplier will take into account this entire series of effects, and not just foreseen date $t+1$ revenue outcomes. Furthermore, and related to the serial interaction of risks, the hybrid models artificially lead to hedging against shocks that redistribute wealth between cohorts.[6]

The assumption of long-lived traders dramatically changes some of the conclusions drawn from single-period and hybrid models. For example, a traditional prescription for hedging is that (with no bias in the futures price) a storage firm should go short by its entire stocks to obtain a perfect hedge. In a multi-period setting, the storer faces risk not only concerning the prospective revenues from current storage, but also the rents from storage to be undertaken in the future. Therefore, a full short hedge is in general not risk-minimising; sometimes a *long* position is required to reduce risk.

[4] Stein (1961) examined futures price determination when traders have exogenous expectations, and are faced with price but not output uncertainty. Turnovsky (1983), Kawai (1983) and Sarris (1984) have provided rational expectations models of spot and futures trading that include growers and storers.

[5] See Fama (1970), Merton (1971).

[6] Even under the special utility assumptions that can lead to 'myopic' decision rules being optimal (e.g. additive logarithmic utility; see Hakannson (1970)), this problem remains.

The main implications of the paper are developed in a basic example with two variations in Section III. In the key example, demand for the commodity is non-stochastic and unit elastic, storage is costless, and growers face perfectly correlated output shocks. Consider a vertically integrated case in which growers are able to store. Since identical growers will sell the same quantity to consumers, their revenues will be non-random from year to year. This eliminates the incentive to hedge, and so leads to a zero bias. If instead there is a separate group of storage firms, then this non-random revenue is divided into risky revenue distributions for storers and for growers which are perfectly negatively correlated (since when combined the joint activities are riskless). A futures market allows storers to shift their risk back to growers entirely, thereby cancelling growers' risks; and both groups will desire to make this trade at a futures price that is unbiased.

The conclusion of zero bias when demand is unitary elastic is consistent with single-period models of hedging and futures pricing without storage,[7] but not with the hybrid models or the normal backwardation theories of Keynes, Hicks and Cootner.[8] The source of the differing predictions of the hybrid models is the incentive to hedge against stochastic redistribution of wealth between generations.[9]

Two variations on the basic example introduce costs of storage and supply response. Variation 1 explores the incentive to hedge against future storage costs, the amount depending on the stochastic quantity to be stored in the future. Such a risk can only be contemplated in a multi-period model, in which agents who hedge today foresee a later storage decision to be made when the futures contract expires. In Variation 2, growers select planting levels each year, which leads to incentives to hedge against future planting costs.

The paper is structured as follows. Section I describes the economic setting. The futures/storage/lending decision problem is laid out in Section II. Then, market equilibrium in the basic example and the two variations discussed above are analysed in Section III. Section IV concludes the paper.

## I. THE ECONOMIC SETTING

Let there be $G$ identical growers, $H$ identical storage firms (or 'handlers'), and $S$ outside 'speculators'. All traders are risk-averse and agree on the distributions of all variables. Stochastic outputs are assumed to be perfectly correlated amongst growers. For most of the paper, the level of planting is not a decision variable (until Variation 2 in Section III). Growers produce a stochastic output of the commodity, and face a known demand curve. All traders are risk-

[7] E.g. J. Hirshleifer (1977), Newbery and Stiglitz (1981), and Anderson and Danthine (1983).

[8] Turnovsky (1983) and Sarris (1984) allow for both demand and output shocks. Both in their general models and in the special case of non-stochastic demand, the futures price is a biased predictor of the spot price with the direction of the bias depending on the current spot price; this is the case regardless of demand elasticity. Cootner (1967) allowed for the possibility of a positive, zero or negative bias in different seasonal periods, but demand elasticity played no role in his analysis.

[9] D. Hirshleifer (1988*a*) presents a hybrid example in which hedging by producers against inter-generational transfers biases the futures price, even though the aggregate revenues they receive are *non-stochastic*.

averse and have time-additive utility. Growers and storers are able to borrow or lend at a risk-free rate of return, to buy, store and sell the commodity, and to trade futures.[10] Growers and storers cannot issue equity, and so may wish to hedge against variable net revenues on the futures market.[11] Speculators are non-commercial participants in the futures market, and cannot buy the spot commodity. Both the futures and spot markets are assumed to be competitive. Notationally, a superscript of '$g$', '$h$', or '$s$' will refer to quantities specific to an individual trader (grower, storer, or speculator); the absence of a superscript indicates any of these types of individuals. In decision problems where the individual takes the behaviour of others as given, an uppercase $G$, $H$, or $S$ superscript will refer to specific quantities associated with any other individual grower, storer, or speculator.

At date 0 the initial output level $Q_0^G$ is known, information set $\theta_0$ is common knowledge, based on which producers select optimal storage levels, incur storage costs based on this choice, the remaining output is sold to consumers at spot price $P_0$, and producers and speculators take positions $\xi_0$ in a futures market with single-period contracts[12] at futures price $f_0$. Furthermore, traders select their level of lending $L_0$ at the risk-free rate for a single-period. Next year the stored commodity arrives (net of spoilage), along with new information $\theta_1$, and a new crop with output $Q_1^G(\theta_1)$ per grower is realised. In equilibrium, the new information determines a new spot price $P_1$ and futures price $f_1$ on a contract to be settled at date 2. The spot price realisation at date 1 determines the profit or loss on the futures positions taken at date 0, $\xi_0(P_1-f_0)$. Again, levels of storage are selected, costs of new storage are incurred, and that portion of the commodity available (either from current output or from past storage) that is not stored is sold to consumers. Lending from the previous date is repaid, and new lending is selected. This sequence is continued through date $T$, the final consumption date for all traders. Thus,

$\tilde{Q}_t$ = output by any trader at date $t$; $\tilde{Q}_t^g = \tilde{Q}_t^G$ since all growers are identical; and $Q_t^s = 0$ since storers are endowed with none of the commodity.

$S_t$ = stocks purchased by the trader at $t$ for resale at $t+1$.

$\gamma_t(S_t), \gamma_t', \gamma_t'' \geqslant 0$ is the storage cost as a deterministic function of the quantity stored, where the trader purchases the good at date $t$ and holds it until $t+1$.

$\omega_t$ = a constant waste or spoilage coefficient for storage, $0 < \omega_t \leqslant 1$; of every unit stored at $t$, $\omega_t$ arrives usably at $t+1$.

$P_t$ = spot price for the commodity at date $t$.

$D_t(P_t)$ = date $t$ demand as a function of price, $D_t' < 0$.[13]

[10] 'Futures trading' in the model is intended to represent both trading on organised exchanges, as well as the extensive individualised forward contracting that takes place between growers and intermediate firms in actual grain markets (Paul *et al.*, 1976).

[11] Moral hazard and adverse selection problems make equity issuance a costly means of diversifying risk. Most US grain output is grown by closely held farms, and much storage is by small grain elevators. Furthermore, even in widely held firms, optimal contracts that impose risk on managers may provide an incentive to hedge the firm's risk using futures.

[12] Most commodity futures contracts are liquid up to less than a year to expiration.

[13] This is the only way in which consumers enter the model; they do not store. I also assume that consumers do not hedge on the futures market against variable prices of consumption goods made from the commodity. This implicitly reflects fixed setup costs (possibly informational) of participating on the futures

$f_t$ = a one-period futures price quoted at date $t$.

$\xi_t$ = bushels of future (date-$t+1$) wheat purchased on the futures market at $t$.

$L_t$ = amount lent at date $t$ for a single period; it can be positive or negative.

$r = 1 +$ the real risk-free rate of return, assumed constant.

$C_t$ = consumption of a trader at date $t$.

$Y_0$ = initial wealth; for growers, $Y_0$ is taken to include initial sales revenues $P_0 Q_0$.

It is convenient to define $\tilde{Y}_t$, $t = 1, \ldots, T$ to be wealth at $t$ due to decisions made at $t-1$, where tildes indicate uncertainty about date-$t$ variables as of $t-1$,

$$\tilde{Y}_t \equiv (\tilde{Q}_t + \omega_{t-1} S_{t-1}) \tilde{P}_t + (\tilde{P}_t - f_{t-1}) \xi_{t-1} + rL_{t-1}, \quad t = 1, \ldots, T. \tag{1}$$

A producer's consumption is

$$\begin{aligned} C_0 &= Y_0 - S_0 P_0 - \gamma_0(S_0) - L_0 \\ \tilde{C}_t &= (\tilde{Q}_t + \omega_{t-1} S_{t-1} - \tilde{S}_t) \tilde{P}_t - \gamma_t(\tilde{S}_t) + (\tilde{P}_t - f_{t-1}) \xi_{t-1} + rL_{t-1} - \tilde{L}_t, \quad t = 1, \ldots, T \\ &= \tilde{Y}_t - \tilde{S}_t \tilde{P}_t - \gamma_t(\tilde{S}_t) - \tilde{L}_t, \quad t = 0, \ldots, T. \end{aligned} \tag{2}$$

The top equality shows that consumption at date 0 is initial wealth less (*i*) the cost of setting $S_0$ of the crop aside to store, (*ii*) storage costs $\gamma_0$, and (*iii*) lending $L_0$. In the second equality, the first term indicates net receipts from sales of the commodity (to consumers or to other producers). The quantity the producer sells (negative if he buys to store) is the sum of his output and stocks carried in from date $t-1$ less the quantity he carries over into the next crop year. The second term deducts storage costs (e.g. labour and equipment costs of holding the commodity), and the third term is the profit from the futures position taken at $t-1$. The last two terms are cashflows from past and current lending.

An element of the producer's decision problem not yet mentioned is the 'convenience yield' of holding the commodity. Brennan (1986) analyses the convenience yield, viewed as a benefit derived from having the commodity available for productive use in the event of a local shortage. The traditional assumption that the marginal convenience yield decreases with the inventory of the commodity can prevent stockouts (corner solutions with zero carryover). A nonlinear cost of storage function is explicitly considered in Variation 1 on the basic model in Section III; to accommodate a convenience yield, the function would be shifted downward so that at low levels of stocks, the cost of storage would be negative.[14]

Each of the variables in (1) and (2) are functions of the state of the world, which can be characterised extensively as the history of all the variables of the model up to date $t$. However, assuming rational expectations, the trader's strategy becomes a function of only the information history $\Theta_t \equiv (\theta_1, \theta_2, \ldots, \theta_t)$,

market. Breeden (1984) allows for consumer trading on the futures market; see also the discussion preceding Note 23.

[14] A different notion of the convenience yield, due to Newbery and Stiglitz (1981, p. 196) is based on the observation that storage offers a benefit of risk reduction to a producer whose revenue covaries inversely with the spot price. My model allows for such a convenience yield; however, I conjecture that the futures market, by offering an alternative means of shifting risk, would weaken the incentive to store as a hedge.

since the trader knows based on each information history what all the other traders do. In all that follows, date-$t$ expectations are with respect to distributions that are conditional on the history $\Theta_t$.

Speculators neither grow nor store the spot commodity. Their consumption constraint therefore takes the form

$$\tilde{C}_t = \tilde{N}_t + \xi_{t-1}(\tilde{P}_t - f_{t-1}) + rL_{t-1} - \tilde{L}_t, \quad t = 1, \ldots, T,$$
$$C_0 = Y_0 - L_0. \tag{3}$$

$\tilde{N}_t$ represents income that arises from sources not associated with the commodity market. In general, the pricing of a futures contract will depend on how the futures payoff covaries with $\tilde{N}_t$, but here we eliminate this effect in order to focus on the impact of producer hedging by assuming that $\tilde{N}_t$ is independent of the price $\tilde{P}_t$ and of past information $\Theta_{t-1}$.[15]

## II. THE DECISION PROBLEM

The decision problem at date 0 for growers and storers is to

$$\max_{\xi_t(\Theta_t),\, S_t(\Theta_t),\, L_t(\Theta_t),\, t=0,\ldots,T-1} \mathrm{E}_0\left[\sum_{t=0}^{T} U_t(\tilde{C}_t)\right] \text{subject to (2)}, \tag{4}$$

$$S_t \geqslant 0, \quad C_t > 0.^{16} \tag{5}$$

The utility function $U_t$ is assumed to be concave; the $t$ subscript allows for possible time preference. A rephrasing of this problem in terms of indirect utility as a function of wealth $Y_t$ is useful for the proofs, and is provided in the Appendix. Speculators solve a problem with the futures position and lending as the choice variables,

$$\max_{\xi_t(\Theta_t),\, L_t(\Theta_t),\, t=0,\ldots,T-1} \mathrm{E}_0\left[\sum_{t=0}^{T} U_t(\tilde{C}_t)\right] \text{subject to (3)}, \tag{6}$$

where again consumption must be positive with probability one.

### *Optimality Conditions for Growers, Storers, and Speculators*

In the problem for producers, the first order conditions that arise from optimising at a given date $t$ are found by differentiating the objective with respect to $\xi_t$, $L_t$, and $S_t$ respectively. If a grower cannot store, then $S_t$ is removed as a choice variable from his decision problem, so that only the first order conditions (7) and (8) below are applicable. (7) and (8) are also the first order conditions applicable for speculators.

$$\mathrm{E}_t[U'_{t+1} P_{t+1}] = f_t \mathrm{E}_t[U'_{t+1}] \tag{7}$$

[15] An alternative approach to the pricing of futures, the Capital Asset Pricing Model (CAPM), assumes that equity shares can be traded on all endowments (i.e. on both producers' revenues and on $\tilde{N}_t$ here). Stoll (1979) and Berck and Cecchetti (1985) examine equilibrium pricing of futures contracts in models where, instead of issuing equity, producers hedge their revenues on the futures market, while speculators trade other risky endowments on the equity market. The effects described here would also operate in a setting with many risky securities.

[16] It is assumed that preferences are such that the positive-consumption constraint is never binding.

$$U'_t = r\mathrm{E}_t[U'_{t+1}] \tag{8}$$

$$U'_t(P_t+\gamma'_t) \geqslant \omega_t\,\mathrm{E}_t[U'_{t+1}P_{t+1}], \quad = \text{ if } S_t \neq 0. \tag{9}$$

Condition (7) states that the futures position trades off the expected marginal disutility of paying the futures price at date $t+1$ with the expected utility of obtaining the value of the spot commodity. Condition (8) equates the disutility of sacrificing a dollar today against the expected marginal utility of an additional dollar at $t+1$. Condition (9), when storage is positive, equates the marginal disutility of sacrificing the value of a bushel of the commodity plus the marginal cost of storing the commodity against the expected marginal utility of obtaining the value of the net of spoilage commodity at $t+1$.[17]

For storage firms or storing growers, combining (7), (8), and (9) gives

$$(P_t+\gamma'_t)/\omega_t \geqslant f_t/r, \quad = \text{ if } S_t \neq 0, \quad t = 0, \ldots, T-1. \tag{10}$$

This 'arbitrage condition', which is well known from single-period models of storage and futures trading, indicates that storage and futures trading decisions are guided by the ratio of the current futures price ($f_t$) to the spot price ($P_t$), adjusting for the marginal storage costs ($\gamma'_t$ and $\omega_t$).[18] These optimality conditions characterise the trader's lifetime storage, futures trading and lending decisions.

Let futures price bias be defined as $B_t \equiv f_t - \mathrm{E}_t(\tilde{P}_{t+1})$, so that a negative (downward) bias indicates 'normal backwardation'. A preliminary result that directly generalises a corresponding result of single-period models is that in the absence of a bias, speculators do not trade futures (Newbery and Stiglitz (1981)). This is reasonable in terms of stochastic dominance, since a futures position then adds noise to consumption without affecting its mean. The proof is available on request.

Proposition 1. *If the futures price is unbiased,* $B_t = 0$, $t = 0, \ldots, T-1$ *then speculators optimally select null futures positions,* $\xi^S_t = 0$, $t = 0, \ldots, T-1$.

It should be noted that even if the bias is zero at a particular date, say $T-2$, futures positions may be non-zero if a futures price bias is expected to prevail at a later date $T-1$.[19]

[17] As an example of how these conditions implicitly reflect multiperiod considerations, substitute (8) applied at $T-1$ into (7) applied at $T-2$, which gives $\mathrm{E}_{T-2}\{P_{T-1}[\mathrm{E}_{T-1}(U'_T)]\} = f_{T-2}\,\mathrm{E}_{T-2}(U'_T)$. This shows that the futures position at $T-2$ is guided by how the spot price at $T-1$ covaries with news at $T-1$ affecting the later date-$T$ marginal utility. More generally, the first order conditions at date $t$ reflect the multiperiod tradeoffs involving the stochastically variable levels of consumption at all later dates.

[18] It may be deduced by comparing the payoffs to the alternative methods for obtaining a future bushel of wheat of storage versus a long futures position.

[19] For example, support that at $T-1$ a large crop (and low $P_{T-1}$) leads to a larger downward bias in futures price $f_{T-1}$. A downward bias at $T-1$ leads to a long speculative position; a larger bias makes the speculator better off by giving him a greater opportunity to profit from his long position. So the speculator is better off at $T-1$ when $P_{T-1}$ is low, and worse off when $P_{T-1}$ is high. To hedge this risk, or more precisely, to exploit the stochastic variation in his marginal utility of date $T-1$ wealth, he will in general take a non-zero futures position at $T-2$.

## III. MARKET EQUILIBRIUM

The heart of this paper consists of a basic example of market equilibrium and two variations that show how various economic factors (vertical integration of spot markets, spoilage and dollar storage costs, supply response of growers) affect hedging and the risk premium on futures contracts. Also, the stock-hedging risk premium theory of Cootner and more recent writers, and the single-period lifetime model of carryover that is uniform in the futures literature are shown to be seriously incomplete; a very different set of predictions arises in a multi-period equilibrium setting.

In the absence of carryover, random demand variability generally promotes short hedging by producers and downward bias in the futures price (Stiglitz, 1983). Several authors (cited in the introduction) have examined output variability and futures market equilibrium in two-date models. A conclusion of this literature is that the futures price bias depends on elasticity of demand, being toward downward bias in the usual case in which demand is inelastic, and upward bias if demand is elastic.[20] Therefore, to focus on the pure effect of *storage* and the level of inventories on hedging and price bias, a convenient benchmark case is where producers face stochastic output but non-stochastic demand, and where demand elasticity is unitary, since in the absence of carryover this case leads to zero bias. These assumptions, though special, are maintained in all the scenarios.[21] The scenarios explored yield results not previously obtained in the single-period literature; for this purpose working with special assumptions that could hold in reality is sufficient.

Before turning to the main analysis, the effect of varying two assumptions should be mentioned. First, if demand instead of output were stochastic, and if demand were stochastically independent over time, then I conjecture that downward bias would result, as occurs in single-period models. In such a case high demand would be good news for both growers and for storers, leading to an incentive to hedge short.[22] Second, the effect of allowing consumers to hedge would depend on demand elasticity, and on complementarity of preferences in consumption between different commodities (D. Hirshleifer, 1988*c*).[23]

[20] See, e.g. Anderson and Danthine (1983) and Britto (1984). The reason is that demand elasticity determines whether high output is good or bad news for producers' revenues. With inelastic demand, high output reduces the spot price *disproportionately*, reducing revenue. This creates a short-hedging incentive, and so downward bias. With elastic demand, the spot price falls less, so that revenue is high when output is high. This causes long hedging, and so upward bias.

[21] D. Hirshleifer (1988*a*) provided a third variation, which extended the two-date result relating demand elasticity to bias to a three-date setting with carryover.

[22] The simplifying assumption of additive logarithmic utility may be needed to limit hedging against intertemporal shifts in futures trading opportunities.

[23] Consider a single-period setting with full participation by consumers, and two complementary consumption goods. Assume that one good is in stochastic aggregate supply, and may be purchased at a prespecified price by taking a long futures position, and that the other is a numeraire commodity that is in non-stochastic supply. A high output of the stochastic commodity raises consumption, which raises the marginal utility of consuming more of the numeraire. Since this occurs when the spot price of the risky commodity is low, a positive risk premium on the futures contract is needed to compensate for the undesirable characteristic of paying off poorly when marginal utility is high.

*The Basic Example*: INTEGRATED VERSUS SEGREGATED PRODUCERS

*Part A. Vertically Integrated Producers*

Consider a market with identical producers who are faced with output uncertainty, and are able to carry the commodity from one harvest through the next. Although the commodity may spoil ($\omega_t < 1$), assume that the dollar cost of storage $\gamma(S_t)$ is zero.[24]

Let $R_t$ be sales revenue. With unit elastic demand, the market demand curve satisfies

$$\tilde{P}_t D_t(\tilde{P}_t) = R_t, \quad R_t \text{ non-stochastic.} \tag{11}$$

With $G$ identical producers, and suppressing $G$ superscripts, spot market clearing requires that demand at date $t$ equal supply,

$$D_t(\tilde{P}_t) = G(\tilde{Q}_t + \omega_{t-1} S_{t-1} - \tilde{S}_t), \tag{12}$$

so the grower's budget constraint, by (2) is

$$\tilde{C}_t^g = \frac{(\tilde{Q}_t^g + \omega_{t-1} S_{t-1}^g - \tilde{S}_t^g)}{(\tilde{Q}_t + \omega_{t-1} S_{t-1} - \tilde{S}_t)} \frac{R_t}{G} + (\tilde{P}_t - f_{t-1})\, \xi_{t-1}^g + rL_{t-1}^g - \tilde{L}_t^g, \quad t = 1, \ldots, T. \tag{13}$$

With identical growers, $\tilde{Q}_t^g = \tilde{Q}_t$, so substituting the optimal storage policy $\tilde{S}_t^g = \tilde{S}_t$ gives

$$\tilde{C}_t^g = \frac{R_t}{G} + (\tilde{P}_t - f_{t-1})\, \xi_{t-1}^g + rL_{t-1}^g - \tilde{L}_t^g, \quad t = 1, \ldots, T. \tag{14}$$

Revenue is non-stochastic, so all consumption variability may be eliminated by selecting a non-stochastic lending policy and zero futures positions for all later dates. Therefore, there is no hedging pressure toward a downward futures price bias. By Proposition 1, a zero bias also persuades speculators not to trade. So

PROPOSITION 2. *Assuming stochastic supply, non-stochastic and unit elastic demand, and that suppliers are vertically integrated, then the futures price is at all dates an unbiased predictor of the later spot price*, $f_t = \mathrm{E}_t(\tilde{P}_{t+1})$, $t = 0, \ldots, T-1$.

Under unitary demand elasticity and with identical suppliers, there is a perfect offset between the amount of the commodity sold to consumers and the spot price. In consequence, sales revenues are a non-stochastic flow in each season, regardless of output realisations and the amounts stored. Suppliers are perfectly hedged without resorting to the futures market, leading to a zero bias.

Naturally, in more complex scenarios there is no reason to expect perfect cancellation to take place. However, the example demonstrates that looking only at inventory hedging in determining the bias is capricious; the need to hedge random outputs may be equally important, and may act in an opposing direction.

[24] When storage costs are precisely linear (or zero), the payoffs from storage can be duplicated by lending and futures trading, implying the existence of an infinity of optimal policies that produce equivalent patterns of consumption. A convenient solving device is to assume that all members of an identical group of producers take the same actions; then the aggregate market-clearing condition pins down individual positions. It is best to view zero storage costs as a limiting case as identical storage costs approach zero, so that optimal positions are unique.

If demand rather than output were stochastic, then downward bias would typically occur, consistent with the prediction of normal backwardation theory in its simplest form. However, it is important to recognise that stochastic demand leads to downward bias even without storage, so there is no implication that storage is a *cause* of downward bias.

*Part B. Specialised Growers and Storers*

Cootner (1967) elaborated on the normal backwardation theory's predicting that short hedging of stocks promotes downward bias. He maintained that in seasonal periods when stocks were low, the bias could reverse in sign (becoming an upward bias), because processors should take long positions to hedge planned purchases of their inputs.[25] The unbiased futures price of Part A contrasts with the normal backwardation theory's prediction of a bias whose magnitude, and for Cootner, sign depends on the amount being stored. Based on an interlocking generations example, Anderson and Danthine (1983) maintain that '...the influence of storage companies contributes unambiguously toward backwardation.' Given the prominent role of short hedging by storage firms in traditional and recent accounts of futures price bias, it is interesting to compare Part A with a scenario in which the activities of storage and primary production are separated.

For notational simplicity, assume that pure growers do not have access to the storage technology. A grower's sales are therefore $\tilde{Q}_t$ at date $t$, while a storer's sales are $\omega_{t-1} S_{t-1} - \tilde{S}_t$. The aggregate quantity sold to consumers is therefore

$$G\tilde{Q}_t + H(\omega_{t-1} S_{t-1} - \tilde{S}_t) = G\left[\tilde{Q}_t + \left(\frac{H}{G}\right)(\omega_{t-1} S_{t-1} - \tilde{S}_t)\right] = \frac{R_t}{\tilde{P}_t}, \tag{15}$$

the last equality following by (11). If the grower were able to make his sales precisely proportionate to aggregate sales, he would achieve a riskless lifetime revenue stream. Equation (15) shows that he could do so, if only he could store $(H/G)\,\tilde{S}_t$, $t = 0, \ldots, T-1$.

Of course, in states where $S_t = 0$, the grower achieves this goal. In other states, by the arbitrage relation (10), he can simulate the payoff from storing $(H/G)\,S_t$ by lending $(H/G)\,\omega_t S_t P_t$ now and taking a long futures position of $\xi^g_t = (H/G)\,\omega_t S_t$. With such trades the grower's consumption is deterministic. Hence, if the futures price is unbiased, this hedging policy is optimal.[26]

Furthermore, if prices are unbiased, storers can eliminate all risk by shorting their net of spoilage stocks fully, $\xi^h_t = -\omega_t S_t$ period by period.[27] It remains to

[25] The second step of the reasoning is also incomplete, for two reasons: (i) A processor's profits depend not just on the price of his commodity input, but on the price of the finished good, and the costs incurred in processing. These quantities are all highly correlated, and while a high price of the input is bad, a high price of his output is good. (ii) It is essential to take into account the hedging incentives of the growers or other handlers who plan to sell to the processors. Weller and Yano (1988) and D. Hirshleifer (1988*b*) provide single-period models that address these issues.

[26] The optimal *lending* policy includes an additional, non-stochastic component to balance consumption through time, as well as the above-mentioned component to simulate storage.

[27] By the arbitrage relation, storers earn discounted net revenues of zero. This is reasonable, since with zero marginal storage costs, rents to storers are arbitraged down to zero. (With rising marginal costs, as in Variation 1 below, rents to storers vary stochastically.)

be verified only that these positions clear the futures market. Aggregate demand for futures contracts is

$$G\xi_t^G + H\xi_t^H = G(H/G)\,\omega_t S_t - H\omega_t S_t = 0, \quad t = 0, \ldots, T-1. \tag{16}$$

PROPOSITION 3. *Assuming stochastic supply, non-stochastic and unit elastic demand, and that identical pure growers and identical pure storers trade on the commodity market, then storers take full short positions; nevertheless, the futures price is at all dates an unbiased predictor of the later spot price,* $f_t = \mathrm{E}_t(\tilde{P}_{t+1})$, $t = 0, \ldots, T-1$.

Consistent with normal backwardation theory, storage firms are short fully here. Nevertheless, storage promotes neither upward nor downward bias. Thus, the analysis lends no support to the presumption that the expected trend in the futures price is largest when stocks are high. This is initially surprising in view of the downward pressure that short hedging should exert on the futures price. However, short-hedging by storers is offset by a long-hedging incentive of growers. This may be viewed in two ways. First, since the aggregate revenue each period is non-stochastic, growers' revenues must be perfectly negatively correlated with storers. So if a short hedge eliminates the storers' risks, a long hedge eliminates the risks of growers.

Second, and perhaps more informatively, the price risk borne by growers is reduced by the actions of storers, because storage acts as a buffer to reduce the price response to quantity shocks. Without storage, a high output is precisely offset by a low price, making revenue constant. With storage, a high output is only partly offset by a lesser fall in price, so growers do well when the price is low. Therefore a long futures position, which profits when the spot price is high, reduces growers' risks.

*Futures Trading as a Substitute for Vertical Integration*

In an industry with closely held producers, the basic example shows that vertical integration is a possible organisational response to risk. Vertically integrated producers are perfectly self-hedged by the offset between the quantity they sell to consumers and the spot price. Segregated producers, on the other hand, would (without a futures market) be forced to bear perfectly negatively correlated risks, because of the stochastic division of revenues between growers and storers.

However, instead of combining the productive activities under joint ownership, Part B demonstrates that producers can remain segregated and still perfectly diversify their negatively correlated risks in the futures market. Therefore, vertical integration and futures trading are substitutes. Vertical segregation in Part B promotes greater open interest in the futures market, and so improves the prospects for success of an exchange-traded futures contract. Conversely, the existence of a futures market reduces the need to integrate vertically, which may be costly. Regardless of the causal direction, a negative association of futures trading in a commodity and vertical integration is predicted.[28]

[28] Futures trading may also substitute for vertical integration in diversifying risk when demand is not unit elastic, as discussed by D. Hirshleifer (1988*a*, *b*).

*Variation* I: COSTLY STORAGE

To study the effects of a positive storage cost $\gamma_t(S_t)$, three further assumptions are made. First, let there be only three dates, 0, 1, and 2. Second, we rule out speculators entirely – equilibrium futures prices are set to force growers and storers to trade only amongst themselves, or if producers are integrated, to refrain from trading entirely. A familiar result of single-period models is that speculators are long and short according to whether the bias is upward or downward. Hence the main effect of including speculators would be to dilute the bias, by spreading producers' risks over a larger number of riskbearers.[29] Third, let output be stochastically independent over time.[30]

PROPOSITION 4. *Assume that output is stochastically independent over time, that demand is non-stochastic and unit elastic, and that there are three dates* (0, 1, *and* 2). *If suppliers are vertically integrated, and storage is costly, then the futures price at date* 0 *is downward biased,* $f_0 < E_0(\tilde{P}_1)$, *and the futures price at date* 1 *will be unbiased,* $f_1 = E_1(\tilde{P}_2)$.

Letting the term 'vertically segregated' refer to the case in which growers can store, but a group of pure storers cannot grow the commodity, we have the following Corollary.

COROLLARY. *If producers instead are vertically segregated, but have identical additive quadratic utility and constant marginal storage costs, then the futures price at date* 0 *is downward biased,* $f_0 < E_0(\tilde{P}_1)$, *and the futures price at date* 1 *will be unbiased,* $f_1 = E_1(\tilde{P}_2)$.

The intuition underlying these results is disjoint from that of the normal backwardation theory, in which short hedging of stocks leads to a bias. We saw in the basic example that short hedging of stocks in the current setting did not promote a bias. Here, the force promoting bias is hedging against the stochastic storage costs to be incurred in the future. The magnitude of the bias will be related to the amount of output uncertainty being resolved over the life of the futures contract.

The basic argument is that since storage is costly, even with unitary demand elasticity producers cannot eliminate all risk. It remains true that in equilibrium with producers storing identically, gross revenues are non-stochastic. However, by promoting storage a high output raises the total storage costs incurred at date 1, so a high $\tilde{Q}_1$ is bad news for producers. So at the optimum, the marginal indirect utility of date 1 wealth rises with $Q_1$. Therefore, the marginal utility of wealth is inversely ordered with the payoff provided by a long futures position, $\xi_0(\tilde{P}_1 - f_0)$. This creates an incentive at date 0 for producers to hedge short,

[29] In a multiperiod setting, matters are complicated by two types of second order effects. First are nuisance wealth effects on the degree of risk aversion, which result each time information arrives to redistribute wealth between speculators and hedgers. The second, which are influenced by the first, are due to the incentive for speculators to hedge against shifts in the investment opportunity set, i.e. shifts in bias. Results on bias with speculators in the market could be derived by assuming specific forms (such as quadratic) for the utility and cost functions.

[30] Allowing high output today to be associated with good news about next year's crop could lead to cases in which higher output inhibits storage. This would alter the returns that follow, but is probably an uncommon case.

which shifts wealth from good news states to bad news (low spot price) states. This short hedging pressure induces a downward bias. In the corollary, constant marginal storage costs are assumed to apply linearity (the Lemma in the proof), but the intuitive argument would seem to apply more generally.[31]

### *Hedging by Storers*

A venerable and widely accepted prescription for storage firms hedging in an unbiased futures market without basis risk is to take a full short position, i.e. sell short the entire inventory.[32] In a multi-period setting, full short hedging in general fails to minimise risk. Furthermore, a short position may be risk-*increasing*, so that a *long* hedge is called for.

The storers' sole source of risk in the conventional analysis is the resale value of stocks. A high aggregate output/low spot price would be bad news for a storer concerned only with the value of his stocks at date 1. This risk can be eliminated with a full short position. However, a storer at date 0 foresees a second period, and knows that a good crop, by reducing $P_1$, will raise the profitability of storage at date 1. Hence, his rents derived from the ability in the future to store are negatively correlated with the value of his current stocks. Indeed, if the crop at date 0 is small, so that he currently stores little, then it is future rents, not the value of current stocks that are the primary concern. In this event, the second effect predominates, so the optimal hedge is long.

PROPOSITION 5. *Suppose that there are three dates* (0, 1 *and* 2), *demand is non-stochastic and unit elastic, storers have rising marginal storage costs, and there is a positive probability of storage taking place at date* 1. *Then if the level of storage at date* 0 *is sufficiently close to zero, storers will hedge by taking long futures positions,* $\xi_0^H > 0$.

The Corollary to Proposition 4 for technical reasons required $\gamma'' = 0$. However, the conclusion of downward bias still obtains if the $\gamma(.)$ function is perturbed slightly so that $\gamma'' > 0$ as required by Proposition 5. In this case storers with low inventories take long positions, yet the futures price is downward biased. This occurs because the long hedging pressure of storers is offset by an even stronger incentive toward short hedging by growers.

### *Variation* 2. SUPPLY RESPONSE

In the preceding scenarios, the level of planting was assumed to be given (i.e. perfectly inelastic). More generally, hedging incentives are affected by the ability of growers to adjust planting in response to information they receive about current and future weather conditions. Supply response has been examined by Scheinkman and Schechtman (1983) in a fully multi-period model without futures trading, and by Turnovsky (1983), Kawai (1983) and Sarris (1984) in hybrid models with futures trading. We will see that the effect of supply response on the bias depends critically on the assumption of multi-period lifespans.

I simplify by assuming three dates, integrated producers, and that speculators

[31] With linearity, net hedging is determined by *aggregate* risks faced by producers.
[32] See, e.g. Telser (1958), Cootner (1960).

are removed from the futures market. Suppose a futures position is taken at date 0, and that at date 1 information arrives which determines the optimal level of planting at date 1 for later harvest at date 2.[33] For simplicity, the only information arriving at date 1 is the output realisation $Q_1$; this affects the incentive to plant at date 1 through its effect on storage $S_1$.[34] A high $Q_1$ reduces the benefit to planting, by raising carryover and so lowering $P_2$. It follows that although it will be assumed that weather conditions are independent over time, i.e. $\theta_t$ is independent of $\Theta_{t-1}$, now output will in equilibrium be autocorrelated.

To endogenise supply, let $x_t$ be the acreage planted at date $t$ and $V(x_t)$ be planting costs, $V' > 0$, $V'' > 0$. The output at date 2 depends on the level of planting and on the final state, $Q_2 = Q_2(x_1, \theta_2)$, where $\partial Q_2/\partial x_1 > 0$, and $\partial^2 Q_2/\partial x_1^2 \leqslant 0$. The consumption constraint for date 1 now deducts the cost of planting for harvest at date 2, $V(x_1)$,

$$C_1 = \frac{R_1}{G} - V(x_1) - \gamma_1(S_1) + rL_0 - L_1 + \xi_0(P_1 - f_0), \tag{17}$$

where $x_1 = g(Q_1)$. The first RHS term in (17) reflects the price/quantity offset of (11). Since no planting occurs at final date 2, the constraint for date 2 is precisely (2) applied at $t = T = 2$, and involves no revenue risk.

In this scenario, at date 0 producers will hedge against planting costs to be incurred at date 1, leading to upward bias (contango).

PROPOSITION 6. *Assume that demand is non-stochastic and unit elastic, and that there are three dates* (0, 1, *and* 2). *If planting for the date* 2 *harvest can respond to the output realisation at date* 1, *and if storage is costless, then at date* 0 *the futures price will be an upward biased predictor of the later spot price,* $f_0 > \mathrm{E}_0(\tilde{P}_1)$, *and the futures price at date* 1 *will be unbiased,* $f_1 = \mathrm{E}_1(\tilde{P}_2)$.

The proof of Proposition 6 follows similar lines to those of Proposition 4, and is omitted; see D. Hirshleifer (1988*a*).

Supply response creates hedging incentives that are the reverse of those caused by storage costs (Proposition 4 of Variation 1). As before, high $Q_1$ promotes storage, and so lowers $P_2$. But now the lower future price reduces the incentive to plant, lowering the planting costs incurred at date 1. So in equilibrium high output is good news for producers, leading to long hedging and contango.

The tendency for planting and the level of storage to be inversely related (noted by Scheinkman and Schechtman (1983)) will more generally lead to a partial offset between storage and planting costs. The direction of bias will depend on which of these costs predominates. High output stimulates storage, increasing the storage costs incurred; in the absence of planting costs, this is, in equilibrium, bad news. On the other hand, high output and storage reduces the

[33] There are several different possible ways to model the sequencing of planting, arrival of information and futures positions. I focus on futures trading in advance of planting, so that the hedger must take into account the supply response to new information.

[34] This rules out another important type of information, news about weather conditions influencing the yield at date 2. This would lead to a similar effect, since good news about date 2 output would depress storage and spot prices.

incentive to plant, reducing planting costs; without storage costs, this is, in equilibrium, good news. With both costs present, supply and storage elasticities will both be determinants of the bias.

## IV. CONCLUDING REMARKS

The analysis of commodity futures markets when there are long-lived individuals who face quantity risk as well as price risk leads to conclusions that differ from those of traditional and more recent studies. Contrary to the conventional view (beginning with Keynes), there is no presumption that costless carryover leads to a downward futures price bias, nor that the bias will be greater when a greater amount is stored. Although in the basic example of Section III storers have an incentive to hedge short, growers have a complementary risk position which leads them to hedge long. The correct way of studying futures markets and storage is to ask how risks are distributed and can be shifted, rather than looking at one or another group of producers in isolation. An interesting implication of the complementarity of risk positions is that vertical integration and futures trading can be substitute means of diversifying risk.

Two variants of the basic example show how other factors affect hedging and bias. First, in contrast to spoilage, which need not promote bias, dollar costs of storage induce short hedging and a downward bias. This is not due to hedging against variability in the value of stocks, as in the conventional accounts. Rather, it arises from hedging against fluctuations in total storage costs to be incurred, the variation in storage arising from stochastic output. Furthermore, with costly storage, storers with low inventories may wish to hedge long, contrary to conventional accounts. Second, when supply is flexible, hedging against future planting costs promotes upward bias (contango).

The examples of equilibrium provided in Section III demonstrated that multi-period analysis of the division of risk leads to implications that are dramatically different from previous models. However, they were developed under a number of very special assumptions. A fruitful avenue for future research may be to explore equilibrium storage, commodity pricing and welfare applying the more general multi-period decision framework for the analysis of storage, futures trading and lending provided here.

Empirically, the analysis suggests that high levels of inventories may not predict positive futures price changes, as maintained by Cootner (1960) and more recent writers. Rather, high inventories could be associated with either upward, downward or zero futures price bias. Some determinants of the bias suggested here are planting and storage costs and elasticities. Further study is called for to measure the impact of these and other factors

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## APPENDIX

*Proof of Proposition* 2. We show that an unbiased futures price, $f_t = \mathrm{E}(\tilde{P}_{t+1})$, $t = 0, \ldots, T-1$ clears the futures market at all times. First, by Proposition 1, speculators do not trade futures. Second, by (14) the problem of producers can be viewed as a special case of that of speculators with $\tilde{N}_t \equiv R_t/G$, so producers also select null futures positions, clearing the market.

### *An Envelope Condition*

The decision problem of growers and storers of (4) can be phrased in terms of an indirect utility function $J_t(Y_t;\, \Theta_t)$, defined recursively as follows. $J_T(Y_T;\, \Theta_T) \equiv U_T(C_T)$, and

$$J_t(Y_t;\, \Theta_t) \equiv \max_{\xi_t(Y_t;\,\Theta_t),\, S_t(Y_t;\,\Theta_t),\, L_t(Y_t;\,\Theta_t)} U_t(C_t) + \mathrm{E}_t[J_{t+1}(Y_{t+1};\, \Theta_{t+1})]. \tag{18}$$

It is assumed that optimal policy functions are differentiable with respect to own income. A standard result of multi-period consumption investment problems (Hakannson, 1970) is the envelope condition that

$$\frac{\partial J_t(Y_t;\Theta_t)}{\partial Y_t} = U_t'(C_t), \quad t = 0, \ldots, T. \tag{19}$$

(This obtains here despite the special feature of a non-negativity constraint on storage; see D. Hirshleifer (1988*a*).) At the optimum, the utility provided by an extra dollar consumed at $t$ is the same as the expected utility provided by investing that dollar either by lending or through storage.

*Proof of Proposition* 4. We will first put bounds on $S_1'$. We now view optimal date-1 policies to be functions $L_1(Q_1)$ and $S_1(Q_1)$ of the date-1 state of the world $Q_1$. For brevity, primes will denote parametric derivatives of policy functions, assumed differentiable, and of prices with respect to $Q_1$. Substituting for consumption from (2) into (8) and (9), differentiating parametrically with respect to $Q_1$ (for (9) only in the range of equality, $S_1 > 0$), and since integrated producers do not trade futures, $\xi_t' \equiv 0$,

$$-U_1''(L_1' + \gamma_1' S_1') = r^2 \mathrm{E}_1(U_2'' L_1'). \tag{20}$$

For the representative producer, substituting for $D_t$ in (11) from (12), at $t = 1, 2$ and parametrically differentiating with respect to $Q_1$ gives

$$P_1' = -\frac{P_1(1 - S_1')}{s_1}, \quad P_2' = -\frac{P_2\,\omega_1 S_1'}{s_2}, \tag{21}$$

where lowercase $s_t \equiv \omega_{t-1} S_{t-1} + Q_t - S_t$.

To determine optimal hedging at date 0 we examine how $C_1$ varies with outcome $Q_1$ at date 1. At date 1, the last decisions $L_1$, $S_1$, and $\xi_1 (= 0)$ are completed, and since $R_2$ is non-stochastic, so is $C_2$. It follows that $U_2'$ is a constant, so by (7) the futures price is unbiased, $f_1 = \mathrm{E}_1(\tilde{P}_2)$. Substituting for $f_1$ in (10) and differentiating with respect to $Q_1$ (in the range where $S_1 > 0$) gives

$$\frac{\omega_1}{r}\mathrm{E}_1(\tilde{P}_2') - P_1' = \gamma_1'' S_1'. \tag{22}$$

Substituting into (22) from (21) for $P_1'$ and $P_2'$ gives the following bounds on $S_1'$

$$0 < S_1' = \frac{P_1/s_1}{\gamma_1'' + \frac{\omega_1^2}{r} E_1\left(\frac{P_2}{s_2}\right) + \frac{P_1}{s_1}} < 1. \tag{23}$$

In the interior of the range where $S_1 = 0$, $S_1' = 0$.

By the optimality principle, the first order condition for $\xi_0$ is

$$0 = \left.\frac{dE_0 J_1(Y_1, \Theta_1)}{d\xi_0}\right|_{\xi_0=0} = E_0[(P_1 - f_0) J_1'] = \operatorname{cov}_0(J_1', P_1) - [E(J_1')] B_0, \tag{24}$$

where $B_0$ is the bias. Evaluating this derivative at the equilibrium futures position $\xi_0 = 0$ determines the equilibrium futures price. We next show that the covariance term is negative, from which the result that $B_0 < 0$ follows.

By (20), sign $(L_1') = -$sign $(S_1')$. So if $S_1 > 0$, then $S_1' > 0$, $L_1' < 0$. Otherwise, $S_1' = L_1' = 0$. Therefore, $Q_1$ is weakly similarly ordered with $-L_1$ (i.e. $Q_1 \simeq -L_1$) in the sense of Hardy *et al.* (1952) (as one goes up, so does the other). So by (1), higher $Q_1$ leads for each and every outcome $\theta_2$ to weakly lower consumption $C_2$, and so higher $U_2'(C_2)$. By (8) this implies that $Q_1 \simeq U_1'(C_1)$. By (19), at the optimum (with $\xi_0 \equiv 0$) marginal indirect utility of wealth $J'(Y_1; \Theta_1) \simeq Q_1$. By (21) and (23), $P_1' < 0$. Therefore $P_1 \simeq -J_1'$. It follows that $\operatorname{cov}_0(J_1', P_1) < 0$.

To prove the Corollary to Proposition 4, we will define a property of optimal policy functions, linearity, and state a lemma.[35] Let $\phi(Q_0, Q_1, \ldots, Q_T)$ denote the joint probability density function of an agent's outputs.

Definition. *Linearity of optimal policies obtains if, given endowments* $Y_0^i$, $\phi^i(Q_0, Q_1, \ldots, Q_T)$, *for agent* $i$, $i = a, b$ *and optimal policies* $S_t^i(\Theta_t)$, $L_t^i(\Theta_t)$, $\xi_t^i(\Theta_t)$, *an agent* $c$ *with endowment* $\tilde{Q}_t^c = \lambda \tilde{Q}_t^a + (1-\lambda)\tilde{Q}_t^b$, *and* $Y_0^c = \lambda Y_0^a + (1-\lambda) Y_0^b$, $\lambda < 1$ *has as optimal policies*

$$S_t^c = \lambda S_t^a + (1+\lambda) S_t^b, \quad L_t^c = \lambda L_t^a + (1-\lambda) L_t^b, \quad \xi_t^c = \lambda \xi_t^a + (1-\lambda)\xi_t^b \qquad t = 0, \ldots, T-1 \tag{25}$$

Linearity obtains with proportional storage costs if agents have the time-additive quadratic preferences

$$U_t(C_t) = \delta_t C_t - \frac{\alpha_t}{2} C_t^2, \quad \delta_t, \alpha_t > 0. \tag{26}$$

Lemma. *If growers and storage firms have identical, time-additive quadratic utility and identical proportional storage costs* $\gamma_t(S_t) = k_t S_t$, $k_t > 0$, *then optimal strategies are linear in endowments.*

*Proof of Corollary to Proposition* 4. Consider a market in which the $G$ growers and $H$ storers are replaced with $G+H$ integrated producers with output and income endowments of

$$\tilde{Q}^c = \left(\frac{G}{G+H}\right)\tilde{Q}_t, \quad Y_0^c = \left(\frac{G}{G+H}\right) Y_0^G + \left(\frac{H}{G+H}\right) Y_0^H.$$

[35] The Lemma that follows is a direct extension of the well known result in single-date models that optimal strategies are linear under quadratic utility (see, e.g. Newbery and Stiglitz (1981), pp. 74–6). The proof is available to interested readers on request.

Then by (25), defining $\lambda = G/(G+H)$, if prices are unchanged, aggregate storage and producers' aggregate futures positions are unchanged. Therefore, the same prices still clear spot and futures markets, so by Proposition 4, the futures price at date 0 is downward biased, and at date 1 is unbiased.

*Proof of Proposition* 5. To determine the incentives to hedge long or short, we show how $C_1$ for growers and storers varies with state $Q_1$. Since no storage costs are incurred at date 2, by the same reasoning as in the basic example, at date 1 growers and storers can mutually hedge perfectly with futures positions of $\xi_1^h = -\omega_1 S_1$, $\xi_1^g = (H/G)\,\omega_1 S_1$, giving unbiased futures prices $f_1 = \mathrm{E}_1(\tilde{P}_2)$.

Multiplying (15) by $P_t$ and differentiating with respect to $Q_1$ gives

$$P_1' = -\frac{P_1\left(1-\dfrac{H}{G}S_1'\right)}{s_1^*}, \quad P_2' = -\frac{P_2\,\omega_1\dfrac{H}{G}S_1'}{s_2^*}, \tag{27}$$

where $s_t^* = Q_t + (H/G)\,(\omega_{t-1} S_{t-1} - S_t)$. In the range where $S_1 > 0$, substituting into (22) gives the following bounds on $S_1'$:

$$0 < S_1' = \frac{G}{H}\frac{P_1/s_1^*}{\dfrac{G}{H}\gamma_1'' + \dfrac{\omega_1^2}{r}\mathrm{E}_1\left(\dfrac{P_2}{s_2^*}\right) + \dfrac{P_1}{s_1^*}} < \frac{G}{H}. \tag{28}$$

When storage is zero, $S_1' = 0$.

Recall that at date 1 a storer can offset the effects of his storage by taking a short futures position and borrowing $(f_1/r)\,\omega_1 S_1$. Let us define $\hat{L}_1^h = L_1^h + (f_1/r)\,\omega_1 S_1$.

1. *Ordering of Storers' Consumptions with Output*

For storers, by (8), (2), and substituting for $\xi_1^h = -\omega_1 S_1$,

$$U_1'[(\omega_0 S_0 - S_1)\,P_1 + (P_1 - f_0)\,\xi_0^h + rL_0^h - L_1^h - \gamma_1(S_1)] = rU_2'(r\hat{L}_1^h). \tag{29}$$

Differentiating with respect to $Q_1$, and by the definition of $\hat{L}_1^h$,

$$(\hat{L}_1^h)' = \frac{U_1''}{U_1'' + r^2 U_2''}\left[\omega_0 S_0 P_1' - S_1 P_1' - S_1' P_1 + P_1' \xi_0^h - \gamma_1' S_1' + \frac{\omega_1}{r}(f_1 S_1' + f_1' S_1)\right]. \tag{30}$$

The terms containing $S_1'$ cancel by the arbitrage condition when $S_1 > 0$. Since $S_1' > 0$ in this range, and recalling that $f_1' = \mathrm{E}_1(\tilde{P}_2')$, by (22) and (28), $\omega_1 f_1'/r - P_1' > 0$. We are free to choose $S_0$ to be sufficiently small to leave the sign of the term in brackets in (30) unchanged. So *if* $\xi_0^h \leqslant 0$, so that $P_1' \xi_0^h \geqslant 0$, then $(\hat{L}_1)' > 0$. It follows that $dC_2^h/dQ_1 > 0$, so $C_2^h \sim Q_1$. By (8), it follows that $C_1^h \sim Q_1$. So if $\xi_0^h \leqslant 0$, then high $Q_1$ is good news for storers.

2. *Ordering of Growers' Consumptions with Output*

For growers, substituting the optimal futures position $\xi_1^g = (H/G)\,S_1\,\omega_1$ into (2), and defining $\hat{L}_1^g = L_1^g - (H/G)\,S_1\,\omega_1(f_1/r)$ gives

$$C_2^g = P_2\left(Q_2 + \frac{H}{G}\omega_1 S_1\right) + r\hat{L}_1^g = \frac{R_2}{G} + r\hat{L}_1^g. \tag{31}$$

Therefore, by (8) and by unitary demand elasticity as reflected in (15),

$$U_1'\left(\frac{R_1}{G}-\frac{H}{G}(\omega_0 S_0-S_1)P_1+(P_1-f_0)\xi_0^g+rL_0^g-\hat{L}_1^g+\frac{H}{G}\frac{S_1\omega_1 f_1}{r}\right)=rU_2'\left(\frac{R_2}{G}+r\hat{L}_1^g\right). \tag{32}$$

Differentiating with respect to $Q_1$, solving for $(\hat{L}_1^g)'$, and by (10),

$$(\hat{L}_1^g)'=\frac{U_1''}{U_1''+r^2U_2''}\left[-\frac{H}{G}S_1'\gamma_1'-\frac{H}{G}\omega_0 S_0 P_1'+P_1'\xi_0^g+\frac{H}{G}S_1P_1'-\frac{H}{G}\frac{\omega}{r}f_1'S_1\right]. \tag{33}$$

Recall by (27) and (28) that $S_1'>0$ and $P_1'<0$. By (22), since $f_1=\mathrm{E}_1(\tilde{P}_2)$, the difference of the last two terms within the brackets is negative. Then if $\xi_0^g \geqslant 0$ and if $S_0$ is sufficiently small to leave the sign of the term within brackets unchanged, then $(\hat{L}^g)'<0$. It follows that $dC_2^g/dQ_1<0$, so $C_2^g \sim -Q_1$. By (8), it follows that $C_1^g \sim -Q_1$. If $S_0$ is sufficiently small and if $\xi_0^g \geqslant 0$, then high $Q_1$ is bad news for growers.

The proof now proceeds by contradiction. Suppose that $\xi_0^h \leqslant 0$, so that by market clearing $\xi_0^g \geqslant 0$. Then by (19), for storers $J_1^h \simeq -Q_1$, and for growers $J_1^g \simeq Q_1$, with a strong similar ordering over some range of outputs. Then applying (24) to storers implies $B>0$, while applying (24) to growers implies $B<0$, a contradiction.

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