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Investor Psychology and Tests of Factor Pricing Models

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We provide a model with overconfident risk neutral investors, and therefore no risk premia, in which a price-based portfolio such as HML earns positive expected returns and loads on fundamental macroeconomic variables. Furthermore, loadings on such portfolios are proxies for mispricing, and therefore forecast cross-sectional returns, even after controlling for characteristics such as book-to-market. Thus, an empirical finding that covariances incrementally predict returns does not distinguish rational factor pricing from a setting with no risk premia. The analysis reconciles the high risk (market betas) of low book-to-market firms with their low expected returns, and offers new empirical implications to distinguish alternative theories.

1 Introduction

In a seminal paper, Fama and French (1993) provide a three-factor asset pricing model where, in addition to the market index, factors are constructed as portfolios based upon two price-related variables: size (market value) and book-to-market ratios. Despite some criticisms, the three factor approach, at times augmented with momentum (Carhart (1997)) or other characteristics, has become a virtually canonical method in the finance field for controlling for risk in event studies and studies of portfolio trading strategies, and thereby distinguishing the effects of risk from market inefficiencies.[1]

Since rational asset pricing theory is in large part about covariance risk, the attempt to capture covariance risk parsimoniously is of fundamental significance. However, the Fama and French (1993) factors are extracted from security prices, not from fundamental macroeconomic news about cash flows.[2] Moreover, the portfolio factors are formed based upon characteristics already known to predict future returns. The question naturally arises as to whether these loadings actually represent rationally priced risks. In other words, does the ability of the Fama-French factor loadings to predict returns derive from a combination of investor risk aversion and covariance of returns with the marginal utility of consumption (as called for in the CAPM and its extensions)? Or, alternatively, is there a reason why loadings on return-predicting portfolios would themselves predict returns even if risk aversion is not the source of return predictability?

[1] A small sampling of papers that apply the 3-factor model as a risk-control include Brennan, Chordia, and Subrahmanyam (1998), Boehme and Sorescu (2002), Mitchell and Stafford (2000), and Naranjo, Nimalendran, and Ryngaert (1998). The model has also been applied to ascertain whether the factors are global or country-specific (Griffin (2002)), and whether they are related to fundamental macroeconomic variables such as GDP and the term spread (Vassalou (2003), Liew and Vassalou (2000), and Petkova (2003). Several empirical papers test the hypothesis (see Fama and French (1998)) that the Fama and French factors reflect distress risk (Dichev (1998), Griffin and Lemmon (2002), Vassalou and Xing (2004) and Ferguson and Shockley (2003)).

[2] In contrast, Chen, Roll, and Ross (1986) include factors based upon news about fundamentals, such as inflation surprises.

Intuitively, a portfolio has a high expected return because the securities it weights heavily have low prices relative to expected future dividends. The Fama-French procedure effectively selects portfolios on this basis, using size or book-to-market as indicators of low price and high expected returns. A loading of a security on a portfolio such as Fama and French's HML factor identifies commonality in the movement of that security with the portfolio. If factor comovement in a set of securities is a cause of their low prices, then a high loading on a portfolio of such securities will tend to identify other stocks that are influenced by the same source of low price and high returns.

For example, under rational factor pricing, it is risk in the form of comovement with an aggregate priced factor that reduces the price of a stock. So if a security has a high loading on a characteristics-based (e.g., HML) portfolio that was selected to have high expected return, the security will tend to have high loading on the underlying priced factor. The security will therefore also have high risk and high expected return.

However, another possible reason for a portfolio to have a low price relative to expected dividends is that it places heavy weight on stocks that are sensitive to a fundamental factor that is undervalued by investors. For example, if investors are, at a point in time, overly pessimistic about energy prices, then a portfolio of low-price firms will disproportionately include firms that are undervalued owing to their sensitivity to energy prices. A high loading on an HML portfolio tends to identify other stocks that are influenced heavily by the underpriced factor. Thus, when there are commonalities in mispricing, we show that a high HML loading predicts high future returns.

An empirical means that has been used to address the concern that HML loadings may be proxies for mispricing is to examine whether the loadings do any better at predicting returns than do the characteristics they are based upon. In a frictionless asset pricing model based on covariance risk, only the loadings should be priced. Therefore,

characteristics should predict returns only to the extent to which they correlate with the loadings. An empirical literature evaluates the Fama/French model based upon such tests (Daniel and Titman (1997), Davis, Fama, and French (2000), Daniel, Titman, and Wei (2001)). This literature is based upon the notion that if covariances do better than characteristics, the evidence supports the rational asset pricing theory and opposes an irrational psychology-based theory.

It is surprising, given the large and growing empirical literature on the 3-factor model and on characteristics versus covariances tests, that there has been little theoretical analysis of whether this presumption is correct. If a standard psychology-based approach also predicts that covariances should be stronger return predictors than characteristics, then any evidence that this is indeed the case does not support one theory over the other. On the other hand, if such an approach implies that covariances are much weaker predictors than characteristics, then such tests can distinguish hypotheses sharply.

In this paper, we provide a psychology-based asset pricing model that clarifies the extent to which evidence that loadings on characteristics-based factor portfolios such as Fama and French's SMB or HML predict returns supports rational versus psychology-based approaches. Our model is similar in spirit of some previous psychology-based asset pricing models, but makes the simplifying assumption that investors are risk neutral, so that there are no risk premia. In consequence, the model provides a sharp benchmark for comparing the implications of behavioral versus rational asset pricing theories.

In our model, investor overconfidence causes mispricing and return predictability, as in Daniel, Hirshleifer, and Subrahmanyam (1998, 2001). Security cash flows follow a linear factor structure, and overconfident investors overestimate the precision of their signals about both the factors and the firm-specific component of security values. Consistent with some previous behavioral models (see, e.g., Daniel, Hirshleifer, and

Subrahmanyam (2001) and Barberis and Huang (2001)), we find that price-containing characteristics such as book-to-market or size are proxies for misvaluation, and therefore forecast abnormal returns.

Our model further implies that the loadings on characteristics-based factor portfolios such as HML are also proxies for misvaluation. In consequence, a high loading on such factors is a positive predictor of future returns. Thus, even though there are no risk premia, the model is consistent with existing empirical findings that characteristics predicting returns, and that loadings predict returns in empirical factor models such as that of Fama and French (1992). So a standard (though simplified) behavioral asset pricing model has implications that are qualitatively similar in some important ways to those of rational asset pricing models. However, as discussed below, there are tests which can potentially distinguish the two approaches.

Our framework focuses on a single characteristics-generated factor, which without loss of generality can be referred to as the HML factor.[3] An implication of our model is that the HML portfolio is undervalued, and therefore generates positive expected returns. Intuitively, such a portfolio loads positively on factors with negative signals and negatively on factors with positive signals. Since investors overreact to these signals, the portfolio places positive weight on undervalued factors and negative weight on overvalued factors. Since mispricing corrects itself by way of a reversal of overconfident beliefs, holding an HML portfolio yields a positive expected return.

Conditional on factor realizations, in our model the HML portfolio has non-zero loadings on fundamental macroeconomic factors.[4] For example, a factor with an unex-

[3]The HML portfolio of Fama and French has been shown to be the most significant of the three factors they employ, and has attracted the most attention in subsequent research. The average return spread in portfolios with extreme values of HML is much greater than that across small and large firms (Fama and French (1993)). Fama and French (1992) show that the book-to-market effect is stronger in the cross-section than beta and size effects.

[4]In our model, returns (more precisely, price changes) are linear functions of the macroeconomic

pected positive shock during a time-period will load positively on HML. Furthermore, the model implies that a stock's loading on the HML portfolio is a proxy for the stock's degree of undervaluation by the market. Intuitively, a high loading of a stock on HML implies that the stock loads strongly on an undervalued factor. Thus, covariances with HML predict returns, even if there are no risk premia.

We further find that in a test of characteristics versus covariances, covariance with the characteristics-based factor portfolio has incremental power to predict returns, even after controlling for the characteristic. The reason is that characteristics capture both idiosyncratic mispricing (mispricing of signals about firm-specific payoff components) and factor mispricing, whereas covariances capture only factor mispricing. To the extent that factor mispricing differs from idiosyncratic mispricing, including the covariance helps disentangle the two.[5]

When investors are overconfident, in an extreme case in which there is no idiosyncratic mispricing, covariances dominate characteristics as return predictors, and thus characteristics versus covariance tests alone do not distinguish between rational factor pricing and behavioral theories. However, with idiosyncratic mispricing, characteristics should be positive predictors of returns even after controlling for covariances. Thus, in general a characteristics versus covariances test distinguishes between alternative hypotheses.

For similar reasons, the two approaches also offer distinct predictions about the sizes of the premia that factor loadings should earn in univariate versus in multivariate regressions on both loadings and characteristics. In a rational factor pricing model, adding

variables, so the HML portfolio also has non-zero loadings on return factors.

[5]Even when the tests assets are characteristic portfolios rather than individual securities, idiosyncratic mispricing does not average away, because each of the firms in the portfolios are post-selected on the basis of price, implying similar idiosyncratic mispricings. For example, a high book-to-market portfolio places high weights on stocks with low prices, which correlates with underpricing of idiosyncratic payoff components.

the characteristic as an additional regressor should not affect the coefficient in the univariate regression of returns on the covariance. But in behavioral approaches, in general including the characteristic reduces the coefficient on the covariance. Furthermore, as discussed in the conclusion of the paper, there are also other kinds of tests which can further distinguish alternative approaches.

Our findings indicate that care must be taken in interpreting tests of characteristics-based empirical factor models such as that of Fama and French. An empirical failure of loadings to predict returns would be a defeat for rational factor pricing models. It would also be a defeat for the hypothesis that there is misvaluation of common economics factors. However, it would not be inconsistent with idiosyncratic mispricing. Some incremental success of loadings in predicting returns would be consistent with a psychology-based approach in which there is sufficiently strong factor (and perhaps also idiosyncratic) mispricing. But a complete dominance of loadings over characteristics would only occur in the extreme case where there is no idiosyncratic mispricing.

A rational explanation for the empirical finding that low book-to-market firms earn low returns is that such firms have growth options, and under certain circumstances this can correlate with low systematic risk (Berk, Green, and Naik (1999)). However, empirically in the post-1963 period firms with low book-to-market have high market betas (Ang and Chen (2005), Campbell and Vuolteenaho (2005)). In our model, firms with growth opportunities tend to have low book-to-market but *high* market betas. Owing to investor overconfidence, firms with low book-to-market tend to be overpriced, and therefore subsequently earn low returns. Thus, our analysis reconciles the high risk (market betas) of low book-to-market firms with their low expected returns.

2 The Economic Setting

We assume that cash flows for security i ($i = 1, \ldots, N$) at date 2 follow the linear 2-factor structure

$$\theta_i = \bar{\theta}_i + \beta_i f_1 + \gamma_i f_2 + \eta_i, \tag{1}$$

where the $\bar{\theta}$'s, β's, and γ's are non-stochastic. The f's denote factor realizations, whereas the η's are the firm-specific component of the payoffs. We assume that the factors are rotated to be mutually independent. All the random variables in (1) are normally distributed and mutually uncorrelated with zero mean. Each of the factors has a variance v_f, whereas v_η represents the common variance of the firm-specific components of the payoff. As is standard with factor models, we specify without loss of generality that $E[f_k] = 0$, $E[f_1 f_2] = 0$, $E[\eta_i] = 0$, $E[\eta_i f_k] = 0$ for all i, k. The values of $\bar{\theta}_i$, β_i, and γ_i are common knowledge, but the realizations of f_1, f_2, and η_i are not revealed until date 2.

We assume that a finite mass of overconfident investors receives common signals $s_k \equiv f_k + e_k$, $k = 1, 2$ about each of the factors at date 1, where $E[e_k] = 0$, the e_k's are independently normally distributed, all signal error terms have a common variance v_e.[6] Investors under-assess the variance of each signal, $v_c < v_e$.

The price of a security P_i is equal to the expected value of its final payoff, conditional on observed signals,

$$P_i = E(\theta_i | s_k) = \bar{\theta}_i + \left(\frac{v_f}{v_f + v_c} \right) (\beta_i s_1 + \gamma_i s_2). \tag{2}$$

In this simple setting, owing to risk neutrality, in equilibrium security prices do not receive a risk premium. Nevertheless, we will see that returns are predictable and that

[6] Some previous models with common private signals include Grossman and Stiglitz (1980), Admati and Pfleiderer (1988), and Hirshleifer, Subrahmanyam, and Titman (1994). If some analysts and investors use the same information sources to assess security values, and interpret them in similar ways, the error terms in their signals will be correlated. For simplicity, we assume this correlation is unity; however, similar results would obtain under imperfect (but nonzero) correlation in signal noise terms.

expected returns are related to loadings on HML.

To begin, consider the unconditional mean $\bar{\theta}_i$ to be the book value of firm i. (We later allow for growth opportunities, so that the book value $B_i < \bar{\theta}_i$.) Since our model is based upon normal distributions and linearity of conditional expectations, we measure the book-to-market 'ratio' as the difference $\bar{\theta}_i - P_i$.

It is convenient to form an HML portfolio by weighting each security according to

$$w_i = N^{-1}\left[(\bar{\theta}_i - P_i) - (1/N)\sum_{i=1}^{N}(\bar{\theta}_i - P_i)\right], \tag{3}$$

where w_i denotes the weight of security i. The first term in this scheme places heavier weight on high book-to-market stocks.[7] The subtracted term in the above weighting scheme ensures that the weights sum to zero. It is evident that the return on the HML portfolio is stochastic, and that the portfolio has a finite, non-zero return variance. The weights on HML are conditional upon market prices, and therefore depend on the information signals that are used to form market prices.

3 Characteristics- and Covariances-Based Return Predictability

In this section, to illustrate the basic idea in a minimal way, we consider a special case of the model in which there are two firms or "industry sectors" $i = 1, 2$, and a single factor, so that security payoffs are

$$\theta_i = \bar{\theta}_i + \beta_i f + \eta_i,$$

where we refer to f as the factor and the η_i as idiosyncratic components. The results we derive here generalize in later sections in which there are many securities and two

[7] This weighting scheme is especially tractable, but results similar to those we derive here would apply if we were to put weights $+1$ and -1 on the top and bottom quintiles of stocks sorted by the book-to-market ratio as in Fama and French (1993).

factors.[8]

The true expected return of security i is

$$\begin{aligned} E[R_i|s] &= E[\theta_1 - P_1|s] \\ &= E[\beta_i f - \left(\frac{v_f}{v_f + v_c}\right)\beta_i s - P_1|s] \\ &= \beta_i s \left[\frac{v_f(v_c - v_e)}{(v_f + v_e)(v_f + v_c)}\right]. \end{aligned} \tag{4}$$

If investors are overconfident, so that $v_c < v_e$, the term in brackets is negative. If a security loads positively on a factor, then the more negative the signal is about the factor, the more underpriced the security is today, and therefore the more positive is the security's expected return. Intuitively, the security inherits mispricing from the factor in proportion to its loading on the factor. When investors overreact to a negative signal about the factor, the factor becomes underpriced. Securities that load positively on the factor also become underpriced, and subsequently earn high returns; securities that loading negatively on the factor subsequently earn low returns.

We next show that the HML portfolio goes long on an underpriced factor and short on an overpriced factor, and therefore is underpriced. We measure the expected return on the HML portfolio (formed according to equation (3) in price changes because of normality of prices. This expected return, $E[R_{HML}]$, is

$$E[R_{HML}] = E[w_1(\theta_1 - P_1) + w_2(\theta_2 - P_2)].$$

Substituting for the various quantities above from (1), (2), and (3), we obtain:

Proposition 1 *The expected return on the HML portfolio,*

$$0.25(\beta_1 - \beta_2)^2 \left[\frac{v_f^2}{(v_f + v_c)^2}\right](v_e - v_c)\,, \tag{5}$$

[8]The assumption of a single factor implies that the HML portfolio that we construct will be either perfectly positively or perfectly negatively correlated with the market. However, in later sections when we generalize the results to a setting with two factors this need not be the case; indeed the HML portfolio can have zero correlation with the market.

is positive if investors are overconfident ($v_c < v_e$).

Under full rationality ($v_e = v_c$), since investors are risk neutral the quantity in (5) is zero, so the HML portfolio yields no excess returns. When investors are overconfident, HML is associated with a higher expected return even though risk is not priced. This higher expected return is a misvaluation premium.

Recalling that $s \equiv f + e$ denotes the signal about the payoff on the factor, the covariance of the HML portfolios return with the fundamental factor is

$$\text{cov}[f, (w_1\theta_1 + w_2\theta_2)|f + e] = 0.25 \;\; \text{cov}[f, w_1\theta_1 + w_2\theta_2|f + e]. \tag{6}$$

Substituting for portfolio weights from (3), we obtain the HML portfolio's loading on the fundamental factor,

$$\beta_{HML,f} = -(\beta_2 - \beta_1)^2 \Pi s, \tag{7}$$

where Π is a positive constant which depends on the variances of the random variables in the model (including the assessed variance of the signal error term by the overconfident, i.e., v_c). This expression indicates that the loading of HML on a factor is positive if and only if the signal is negative. Hence, the HML portfolio loads positively on an underpriced factor and negatively on an overpriced factor. This occurs because investors overreact to a signal about a common factors driving security values. In contrast, with only firm-specific signals, price changes would be uncorrelated with a fundamental factors, so there would be no relation between HML and underlying macroeconomic factors.

For convenience, we now suppress signal conditioning notation. We denote sector i's conditional loading on HML by β_{iHML}. To calculate this loading, we start with the covariance between the HML portfolio and sector 1,

$$cov(R_{HML}, R_1) = 0.25 \;\; \text{cov}[\theta_1 - P_1, w_1(\theta_1 - P_1) + w_2(\theta_2 - P_2)].$$

Since $cov(R_{HML}, R_1) = \beta_{1HML}\text{var}(R_{HML})$, substituting for the various terms from (1) and (2) and dividing by $\text{var}(R_{HML})$, the above covariance reduces to

$$\beta_{1HML} = -\frac{0.25 \ \ [\beta_1 s(\beta_1 - \beta_2)^2] \left[\frac{v_f^2 v_e}{(v_f+v_c)(v_f+v_e)}\right]}{\text{var}(R_{HML})}. \tag{8}$$

Thus, the loading of sectors on the HML factor depends on whether the factor signal is positive or negative, and on whether the sector loads positively on the factor.

We can now derive the relationship between a sector's loading on HML and its subsequent return $R_1 = \theta_1 - P_1$. For Sector 1, this is

$$\text{cov}(R_1, \beta_{1HML}) =$$

$$\text{cov}\left(\theta_1 - P_1, -0.25\left[\beta_1 s(\beta_1 - \beta_2)^2\right]\left[\frac{v_f^2 v_e}{(v_f + v_c)(v_f + v_e)}\right]\right) / \text{var}(R_{HML}).$$

Let

$$\rho_1 \equiv \frac{\left[\beta_1^2(\beta_1 - \beta_2)^2\right] v_f v_e}{4(v_f + v_e)}.$$

Then the above covariance can be written as

$$\text{cov}(R_1, \beta_{1HML}) = \frac{\rho_1 v_f}{\text{var}(R_{HML})(v_f + v_c)}(v_e - v_c). \tag{9}$$

The following proposition obtains directly from the above expressions:

Proposition 2 *The covariance of a sector's return on its HML loading is*

$$\text{cov}(\theta_1 - P_1, \beta_{1HML}) = \left(\frac{\chi v_f}{v_f + v_c}\right)(v_e - v_c), \tag{10}$$

where χ is a positive constant. Thus, if investors are overconfident ($v_c < v_e$), the regression of the sector return on the HML loading has a positive coefficient.

This finding in a model without risk premia suggests that regressions of expected returns on loadings on a price-characteristics-based factor such as HML can convey the

impression that an HML 'risk factor' is priced in the cross-section, even if investors are not risk averse.

Intuitively, just like the book-to-market and size characteristics, HML factor loadings are proxies for security misvaluation. A high HML loading implies that the sector loads heavily on an undervalued factor. We analyze later the circumstances under which characteristics or covariances are better misvaluation proxies. We will show that when there is both firm-specific and systematic mispricing, it is not the case that characteristics are always better misvaluation proxies than loadings.

4 Characteristics and Covariances with Many Securities

We now extend the framework in Section 3 to consider multiple securities. This allows us to consider the effects of firm-specific versus factor mispricing, and how they affect the relative ability of characteristics versus covariances to predict returns. We then consider the relation of volatility of a security to its return.

4.1 Factor Loadings and Expected Returns with Multiple Securities

We first examine the relation of characteristics and covariances to expected returns. In analogy with equation (5), simple calculations show that with N securities and a single factor (with factor loadings denoted by β_i for security i), the expected return of the HML portfolio is

$$\left[\frac{v_f^2}{N(v_f+v_c)^2}\right](v_e-v_c)\left[\frac{N\sum_{i=1}^N\beta_i^2-\left(\sum_{i=1}^N\beta_i\right)^2}{N}\right]. \tag{11}$$

It is evident from (11) that the expected return is a function of the cross-sectional dispersion in the factor loadings of securities on the fundamental factors, and is always positive under overconfidence ($v_e > v_c$). For notational convenience, we define

$$Q \equiv \frac{N \sum_{i=1}^{N} \beta_i^2 - \left(\sum_{i=1}^{N} \beta_i\right)^2}{N^2}.$$

For large enough N, Q will be positive, and henceforth, we will assume that $Q > 0$. The loading of HML on the factor (the equivalent of (7)) is

$$\beta_{HML} = -Q\Pi' s, \tag{12}$$

where Π' is a positive constant. Furthermore, the expression (9) continues to hold, with ρ_1 redefined as

$$\rho_1 \equiv \frac{Q\beta_1^2 v_f v_e}{v_f + v_e}. \tag{13}$$

Thus, all the expressions from the two-security model generalize in a straightforward way, and our basic results continue to obtain.

To test for risk effects, Fama and French (1993) sort half the stocks in their sample into an HML portfolio, and then using the loadings of the other half on HML to explain average returns in that half. In our setting investor overconfidence about systematic factors implies a loading/return relationship, and it does not matter which subset of securities is chosen as the pool to draw from in forming the HML portfolio. For example, suppose that there are N securities and that half of them (subset A) are chosen at random to form such a portfolio. Then, loadings of securities in subset B on the HML portfolio formed from subset A will be positively associated with returns of securities in subset B.

To see this, assume that N is even for convenience. Then (9) still obtains with ρ for

a security 1 in subset B being redefined as

$$\rho_1 \equiv \frac{\beta_1^2 Q v_f v_e}{v_f + v_e}.$$

where β_1 is the factor loading of the chosen security on the factor, and the constant Q is calculated using the factor loadings of $N/2$ securities in subset A. Thus, the results of the Fama and French (1993) procedure, although consistent with a setting in which the HML portfolio captures priced risk factors, is equally consistent with a setting in which there are no risk premia.

4.2 The Relationship of Volatility to Expected Return

We briefly examine the relationship between the desirability of a security as an investment and its volatility within our setting. By (1) and (2), return volatility, $V \equiv \text{var}(\theta_i - P_i)$, is

$$V = v_\eta + \left(\sum_{j=1}^{K} \beta_{ij}^2\right) \left[\frac{v_f(v_c^2 + v_f v_e)}{(v_f + v_c)^2}\right], \tag{14}$$

which is decreasing in v_c, the variance assessed by the overconfident investors, so long as $v_c < v_e$ (so long as investors are overconfident). Thus, return volatility is increasing in investor overconfidence. More volatile stocks have greater absolute returns on average. Because of this, it is easy to show that more volatile stocks whose prices are below their unconditional means yield greater expected returns than less volatile stocks with this property. For example, consider the parameter values $v_f = 1$, $v_\eta = 1$, $v_e = 1$, $\bar{\theta}_i = 1$, suppose that there is one factor, and that securities' loading on the factor equals unity. Then, in the case where $v_c = 0.4$, the volatility $V = 1.26$, while the conditional expected return, $E[R] \equiv E(\theta_i - P_i | P_i < \bar{\theta}_i)$ equals 1.09. On the other hand, when $v_c = 0.1$, $V = 1.36$ and $E[R] = 1.22$. This implies that volatility is associated with increased attractiveness of an investment to a rational investor who is not very risk averse, because high volatility is associated with irrational investors trading more

actively. Specifically, in our framework, given two securities whose prices are below their unconditional means, the one with greater volatility will be more attractive to the investor. This finding provides a counterpoint to the intuition that volatility is an undesirable attribute of an asset because it is correlated with higher risk.

5 Characteristics versus Loadings on Characteristics-Based Factors

In this section, we analyze, when there are many securities, how well factor loadings versus characteristics do in predicting the cross-section of stock returns both individually and in bivariate regression (or two-way sorts). Our analysis focuses on the case of one characteristic (e.g., book-to-market), and a factor created based upon that characteristic (e.g., the HML factor loadings). It would be algebraically messy, but not conceptually difficult, to include 3 or more characteristics and their corresponding factors. For example, Daniel, Hirshleifer, and Subrahmanyam (2001) provide a model in which multiple characteristics-based factors as well as the market portfolio predict returns in a multivariate regression; they do not, however, examine loadings on characteristics-based portfolios. The main insights provided here would carry over to multiple characteristics/portfolio tests.

To analyze the relation of characteristics and covariances with returns, it becomes important to allow for N securities, and for a firm-specific signal for stock i, denoted by $t_i \equiv \eta_i + \nu_i$, where the variance of ν_i is denoted by v_ν. Overconfident investors under-assess the variance of v_ν to be $v_c' < v_\nu$.

Let

$$
\begin{aligned}
g &\equiv \frac{v_f v_\eta v_\nu}{(v_f + v_c)(v_\eta + v_\nu)} \\
h &\equiv \frac{v_\eta^2 v_\nu}{(v_\eta + v_c')(v_\eta + v_\nu)}.
\end{aligned}
$$

A straightforward generalization of (8) yields the result that the loading of security 1 on the HML portfolio, denoted b_1, is

$$
b_1 \equiv -\frac{Qs\beta_1 v_f^2}{(v_f + v_e)(v_f + v_c)} + N^{-1}\left[gs\left(-\beta_1 + \sum \beta_i/N\right) + h\left(-t_1 + \sum t_i/N\right)\right], \quad (15)
$$

where the second term captures the effect of firm-specific mispricing. In a large sample, provided security 1 is randomly selected, this second term will approaches zero.[9]

On the other hand, by (1) and (2), the book-to-market characteristic for security 1, denoted BM_1, is simply

$$
BM_1 \equiv \bar{\theta}_1 - P_1 = -s\beta_1\left(\frac{v_f}{v_f + v_c}\right) - t_1\left(\frac{v_\eta}{v_\eta + v_c'}\right), \quad (16)
$$

which reflects both idiosyncratic (the t_1 term) and factor (the s term) mispricing.

We assume that $\sum \beta_i^2/N$ is bounded as $N \to \infty$. Let this sum be denoted by W. Then the loading of security 1 on HML becomes

$$
-\frac{sW\beta_1 v_f^2}{(v_f + v_e)(v_f + v_c)},
$$

which reflects only factor mispricing.

[9] As in previous analyses in the paper, the loadings on HML are computed conditional on the signals, consistent with the notion that to form an HML factor, the researcher needs knowledge of the market prices. When the number of securities is large, conditioning on market prices is equivalent to conditioning on the full set of firm-specific and factor signals, since market prices of large portfolios reflect systematic factor realizations, which then allows the calculation of firm-specific signals from individual security prices.

5.1 Bivariate Regressions on Both Characteristics and Covariances

We first examine the bivariate relation between characteristics and loadings with returns. For a given security i and its loading β on the fundamental factor, it is routine to calculate the coefficients when its return is regressed on either its characteristic or its HML loading. However, in a regression across securities on characteristics or HML loadings, an econometrician must recognize that any given level of the characteristic or of the HML loading is compatible with different levels of the loading on the fundamental factor. Thus, to obtain cross-sectional regression coefficients on the characteristic and the HML loading, we need to take the expectation of the security-specific covariances and variances over the different possible fundamental loadings.

Straightforward calculations (confined to the appendix) yield:

Proposition 3 *The coefficients in a multiple regression of returns on HML loadings and the BM characteristic are respectively:*

$$y_l = \frac{(v_c' - v_\nu)(v_f + v_e) - (v_c - v_e)(v_\eta + v_\nu)}{W v_f v_e (v_\eta + v_\nu)}, \tag{17}$$

$$y_c = \frac{v_\nu - v_c'}{v_\eta + v_\nu}. \tag{18}$$

Both coefficients are in general non-zero.

Proposition 3 indicates that both loadings and characteristics help forecast future returns. Under full rationality, with $v_e = v_c$ and $v_\nu = v_c'$, both of the above coefficients reduce to zero. By (18), whether characteristics or covariances dominate in a multivariate regression depends on the nature of the mispricing. The characteristic picks up firm-specific mispricing. As factor mispricing increases (i.e., v_c decreases), the coefficient on the loading increases. When idiosyncratic mispricing is small (i.e., v_ν is very close to

v_c'), but factor mispricing is large (v_e is large relative to v_c), then the coefficient on the loading is expected to be larger than the coefficient on the characteristic.

Intuitively, in a bivariate regression of returns on characteristics and covariances, the coefficient on the characteristic (book-to-market) is determined by the idiosyncratic signal term (the coefficient y_c in the above proposition involves only the parameters related to idiosyncratic mispricing). The coefficient on the characteristic implicitly forces the same coefficient on the systematic component of the characteristic as on the idiosyncratic component. The coefficient on the covariance term can be viewed as providing whatever appropriate adjustment is needed to the coefficient on the systematic mispricing to optimally predict returns.[10]

For the purpose of distinguishing between rational factor pricing versus a behavioral approach, the problem with regressing returns on characteristics-based factor loadings goes beyond the fact that the regression coefficient fails to uniquely identify risk rather than mispricing effects. Under some circumstances mispricing can imply a larger coefficient on the loading than on the characteristic.

Consider, for example, the case in which there is overconfidence about factors but no overconfidence about the idiosyncratic components of firms' payoffs, so that $v_\nu = v_c'$. Then by the reasoning above and as indicated in Proposition 3, the coefficient on the characteristic is zero, whereas the coefficient on the loading is positive. Again, this is because a firm's loading and its characteristic both reflect factor mispricing, but the characteristic contains additional irrelevant variation owing to idiosyncratic signal realizations unrelated to mispricing. Thus, even when investors are risk neutral, under

[10] When book value measures fundamentals with noise, and the noise is systematic, then we conjecture that just as in Daniel, Hirshleifer, and Subrahmanyam (2001), there would be a role for a size (SMB) factor, i.e., a factor based on the returns of market capitalization-sorted portfolios, that would be similar to, and beyond the role of the HML factor in our model. That is, the SMB factor would pick up the portion of the systematic misvaluation that the HML factor cannot due to noise in book value.

some circumstances covariances can beat characteristics as return predictors.

However, so long as there is at least some overconfidence about (and therefore mispricing of) idiosyncratic components, a characteristics versus covariance test does distinguish the rational factor pricing theory from our behavioral theory. Under rational factor pricing, in a regression on both characteristics and covariances, covariances (if measured perfectly) should capture all variation in return so the coefficient on the characteristic should be zero. In contrast, in our behavioral approach, so long as there is any idiosyncratic mispricing, the characteristic helps predict returns.

Indeed, if idiosyncratic mispricing is as strong as factor mispricing, in our model loadings lose their predictive power. The numerator in (17) is equal to zero if and only if

$$\frac{v_c' - v_\nu}{v_\eta + v_\nu} = \frac{v_c - v_e}{v_f + v_e}. \tag{19}$$

The LHS is a measure of overconfidence about and therefore overreaction to the idiosyncratic components of firm payoffs. The RHS is a measure of overconfidence about and overreaction to the factor component. We therefore have:

Corollary 1 *If factor and idiosyncratic mispricing are equally strong in the sense of equation (19), then characteristics dominate covariances as return predictors, i.e., the coefficient on covariance in a bivariate regression of returns on characteristics and covariances is zero.*

5.2 Univariate Regressions on Characteristics or Covariances

We now examine the univariate relation between characteristics or covariances with returns. In univariate regressions, each of the two variables (the loading and the characteristic) enters with a positive coefficient. Let $E(\beta^2)$ denote the cross-security expectation of the square of the factor loading; when the number of securities N is large,

$E(\beta^2) = W$ (the average of the squared security betas, as defined earlier). Then, it follows from equations (15) and (16) that the corresponding univariate coefficients x_l and x_c are

$$\begin{aligned} x_l &= \frac{v_e - v_c}{W v_f v_e} \\ x_c &= \frac{W v_f^2 (v_\eta + v_c')^2 (v_e - v_c) + v_\eta^2 (v_f + v_c)^2 (v_\nu - v_c')}{W v_f^2 (v_\eta + v_c')^2 (v_f + v_e) + v_\eta^2 (v_f + v_c)^2 (v_\eta + v_\nu)}. \end{aligned} \tag{20}$$

When investors are overconfident, both of the these coefficients are positive. The equations in (20) indicate that in univariate regressions, the loading captures only the factor component of mispricing, whereas the characteristic captures both sources of misvaluation. As firm-specific mispricing dominates (v_c' becomes small relative to v_ν, and v_c increases to approach v_e), the univariate coefficient on the characteristic increases, whereas the univariate coefficient on the loading goes to zero.

Our analysis offers a new empirical implication about the relative size of the coefficient on loadings in a univariate regression of returns on loading, versus the marginal coefficient on loadings in a bivariate characteristics versus covariances regression. Comparing equation (20) with equation (17), we obtain:

Proposition 4 *In a single-fundamental-factor model with overconfidence, so long as there is any idiosyncratic mispricing, the univariate regression coefficient of returns on loadings exceeds the marginal coefficient on loadings in a bivariate regression on loadings and on characteristics. The difference in the coefficients is*

$$x_l - y_l = \frac{(v_c' - v_\nu)(v_f + v_e)}{W v_f v_e (v_\eta + v_\nu)} \tag{21}$$

This differential is increasing in the tendency toward market overreaction to information about firm-specific payoff components, as reflected by $v_c' - v_\nu$.

In contrast, in rational multifactor pricing models, loadings fully capture risk premia, so the inclusion of characteristics should not the coefficients on loadings as compared with to a regression that only includes loadings. Thus, the empirical implication of Proposition 4 distinguishes rational factor pricing models from our behavioral theory.[11]

5.3 Multiple Factors

The conclusion that in a regression on both characteristics and covariances, the characteristic fully captures firm-specific mispricing is sensitive to the assumption of one factor. Suppose now that there are two fundamental factors, and that there is a signal available about each of the factors. The signals and signal noise terms are mutually independent, and independent of the firm-specific signal. Furthermore, for convenience we assume that the factor signals have the same variance, denoted, as earlier, by v_f, and the same signal noises, v_e. In addition, we assume equal degrees of overconfidence, so that the common assessed signal noise variance is v_c.

Let security i's loading on the second factor be denoted by γ_i. Furthermore, let $W' \equiv \lim_{N\to\infty} \sum \gamma_i^2/N$. If the number of securities is large, this must be equal to $E(\gamma^2)$, the cross-security expectation of the squared loading on the second factor. Then the following proposition (proved in the appendix) obtains.

Proposition 5 *With two fundamental factors, the coefficients on the loading and the characteristic are*

$$y_l = \frac{[v_f(v_c' - v_\nu) + v_\eta(v_e - v_c) + v_c' v_e - v_c v_\nu][v_\eta^2(v_f + v_c)^2(W^2 + (W')^2)]}{D v_f v_e}, \tag{22}$$

[11] Of course, if there is measurement error in loadings, the prediction of rational factor pricing that measured loadings are all that is needed to predict returns could be weakened. To the extent that the range of possible outcomes of the rational approach is widened, the two approaches become harder to distinguish empirically .

and

$$\begin{aligned} y_c &= D^{-1}\left[WW'v_f^2(W-W')^2(v_\eta+v_c')^2(v_e-v_c)\right. \\ &+ \left.(v_f+v_c)^2v_\eta^2\{W^3+(W')^3\}(v_\nu-v_c')\right], \end{aligned} \tag{23}$$

where

$$D \equiv WW'v_f^2(W-W')^2(v_\eta+v_c')^2(v_f+v_e)+(v_f+v_c)^2v_\eta^2\{W^3+(W')^3\}(v_\eta+v_\nu).$$

Both coefficients are again non-zero in general.

In contrast with Proposition 3 in the single-factor case, the coefficient on the characteristic depends on both factor and firm-specific mispricing. However, the conclusion of Proposition 3 is restored in the special case in which $W' = W$, i.e., when $\lim_{N\to\infty}\sum\gamma_i^2/N = \lim_{N\to\infty}\sum\beta_i^2/N$. In this case, it follows directly from Proposition 5 that

Corollary 2 *With two fundamental factors, if $W' = W$, the coefficients on the loading and the characteristic are given by the same expressions as in the single-factor case of Proposition 3.*

The same intuition applies— the coefficient on the characteristic (book-to-market) is determined by the idiosyncratic mispricing, and the coefficient on the characteristic implicitly forces the same coefficient on the systematic component of the characteristic as on the idiosyncratic component. Once again, the coefficient on the covariance term can be viewed as providing the appropriate adjustment to the coefficient on the systematic mispricing to optimally predict returns. It similarly follows that if factor and idiosyncratic mispricing are equally strong in the sense of equation (19), the characteristic dominates the covariance.

Allowing for general W and W', the univariate coefficients in the regression of returns on either loadings or on characteristics when there are two fundamental factors are

$$x_l = \frac{(v_e - v_c)\left[W^2 + (W')^2\right]}{v_f v_e \left[W^3 + (W')^3\right]}, \text{ and} \tag{24}$$

$$x_c = \frac{(v_e - v_c)[(W + W')v_f^2(v_\eta + v_c')^2] + v_\eta^2(v_f + v_c)^2(v_\nu - v_c')}{(W + W')v_f^2(v_\eta + v_c')^2(v_f + v_e) + v_\eta^2(v_\nu + v_c')^2(v_\eta - v_\nu)}. \tag{25}$$

The loading still captures systematic mispricing, while the characteristic again captures both sources of mispricing. However, the coefficient of the characteristic depends on the loadings on both factors.

If $W = W'$, this simplifies to

$$x_l = \frac{(v_e - v_c)}{W v_f v_e}, \text{ and} \tag{26}$$

$$x_c = \frac{(v_e - v_c)[2W v_f^2(v_\eta + v_c')^2] + v_\eta^2(v_f + v_c)^2(v_\nu - v_c')}{2W v_f^2(v_\eta + v_c')^2(v_f + v_e) + v_\eta^2(v_\nu + v_c')^2(v_\eta - v_\nu)}. \tag{27}$$

The coefficient x_l is the same as when there is only a single fundamental factor. Once again, we see that as firm-specific mispricing dominates (v_c' becomes small relative to v_ν, and v_c increases to approach v_e), the univariate coefficient on the characteristic increases and the univariate coefficient on the loading approaches zero.

6 Opportunities for Future Growth

In this section we examine the relation between opportunities for new investment and price-based characteristics such as book-to-market, systematic risk, and the relation between corporate growth and characteristics-based loadings such as HML loadings.

Recent research has tried to explain the ability of the book-to-market characteristic to predict returns by modelling growth opportunities (see Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004)). In the model of Berk, Green, and Naik

(1999), low book-to-market firms are growth firms, and therefore have low systematic risk. However, growth opportunities can instead be associated with high betas (as we show below). Thus, in evaluating alternative explanations, it is important to turn to the empirical evidence about how to book-to-market is related to systematic risk. Empirically, in the post-1963 time period, firms with low book-to-market ratios have tended to have high market β's (Ang and Chen (2005), Campbell and Vuolteenaho (2005)). Thus, although the Berk, Green, and Naik analysis can explain the low returns of 'growth' (low book-to-market) stocks, it is not consistent with the high empirical β's of these stocks.[12] Of course, it is always possible that low book-to-market firms have low loadings with respect to some other factor. However, it is not clear why the Berk, Green, and Naik intuition would apply to some factors but not to the market factor. We provide an analysis which considers the relation between growth, loadings, and characteristics. Our theory is consistent with the fact that firms with low book-to-market earn low returns but have high market β's, and with the fact that stocks with low HML loadings on average earn low returns.

We allow for many firms, but begin by considering a 'mature' firm with book value B date 1 price P_m and payoff θ_m. We interpret B as the initial investment at date 0. It generates an expected payoff of $\bar{\theta}_m > 0$. We assume that $\bar{\theta}_m > B$, so that there is a positive expected return on investment.

Consider now a corresponding 'growth' firm with the same book value that has the opportunity to do a scale expansion of the mature firm. This yields a terminal payoff that will include an extra $\mu\theta_m$, but which subtracts the extra future investment μB.

[12]Evidence from the pre-63 period also provides no comfort to this explanation. In a double sort on BM and market beta, there is zero premium to market beta— for stocks with similar book-to-market ratios but different betas, there is no significant difference in mean returns (see Daniel and Titman (2005)). This does not support an explanation based upon market risk.

The firm will always take such an opportunity, leading to a cash flow of

$$\begin{aligned} \theta_g &= \theta_m + \mu(\theta_m - B) \\ &= (1+\mu)\theta_m - \mu B. \end{aligned} \tag{28}$$

So the date 0 prices (i.e., the prices before signal realizations) of the mature and growth firms are respectively

$$\begin{aligned} P_{m0} &= \bar{\theta}_m \\ P_{g0} &= (1+\mu)\bar{\theta}_m - \mu B \\ &= \bar{\theta}_m + \mu(\bar{\theta}_m - B) \end{aligned} \tag{29}$$

Thus, the final payoff of a growth firm is like a $1+\mu$ upward scaling of the mature firm with the extra investment of μB netted out.

If $B - P$ is the book-to-price difference for the mature firm, then since $B_g = B$ and by (29), the book-to-price difference for the growth firm is

$$B_g - P_{g0} = (1+\mu)(B - P_{m0}). \tag{30}$$

So the book-to-price difference for the growth firm is a scale expansion of the book-to-price difference of the mature firm. If investments have positive NPV, then typically $B - P < 0$ (as occurs, for example, in the realization where all signal values happened to be equal to zero). This means that the initial investment in the mature firm, B, on average generates a higher expected cash flow, P_{m0}. So the scaled up $B - P$ of the growth firm means that typically its $B - P$ is more negative than that of the mature firm. This resulting $B - P$ is the same as if the growth firm were simply a scaled up mature firm with all the investment expended in advance.[13]

[13] Intuitively, if the extra investment expenditure is deferred, then in calculating the book-minus-price value of the growth firm, we can treat the firm as a simple $(1+\mu)$ scale replica of the mature firm, except that we subtract the needed future investment of μB both from the book value of the scale replica (because this portion is not yet spent), and from the current market price (since price is an expectation which reflects the prospect of $-\mu B$ being incurred in the future).

The growth firm also has proportionately stretched factor loadings, since the stochastic portion of its payoff is $1+\mu$ times the payoff on the mature firm. Specifically, for the mature and growth firms firm, we can reduce the notation to

$$\begin{aligned} \theta_m &= \bar{\theta}_m + \sum_{k=1}^{K} \beta_k f_k + \eta \\ \theta_g &= \bar{\theta}_g + \sum_{k=1}^{K} \beta_{gk} f_k + \eta_g. \end{aligned} \tag{31}$$

By (28),

$$\theta_g = (1+\mu)\bar{\theta}_m + \sum_{k=1}^{K} (1+\mu)\beta_k f_k + (1+\mu)\eta. \tag{32}$$

This confirms the growth/loading relation

$$\beta_{gk} = (1+\mu)\beta_{mk},$$

i.e., the loadings of the growth firm on fundamental factors are more extreme (farther from zero) than the loading of the mature firm, where μ is a measure of how rapidly growing the firm is.[14]

From (2), the market price of the mature firm is

$$\begin{aligned} P_m &= E[\theta_m|\mathbf{s}] \\ &= \bar{\theta}_m + \sum_{k=1}^{K} \beta_{mk}\rho_k, \end{aligned} \tag{33}$$

where $\rho_k \equiv [v_f/(v_f+v_c)]f_k$. Similarly, for the growth firm

$$\begin{aligned} P_g &= E[\theta_g|\mathbf{s}] \\ &= E[(1+\mu)\theta_m - \mu B|\mathbf{s}] \\ &= (1+\mu)\bar{\theta}_m + \sum_{k=1}^{K} (1+\mu)\beta_{mk}\rho_k. \end{aligned} \tag{34}$$

[14] In a setting with risk aversion, the growth firm would also have a higher risk premium, because it is just like an upward scaling of the mature firm except that μB is subtracted both from its terminal payoff and from its initial price.

We next compare the average book-to-price difference of the growth firm to that of the mature firm. For the growth firm, by (34),

$$B - P_g \;=\; (1+\mu)(B - \bar{\theta}_m) - \sum_{k=1}^{K}(1+\mu)\beta_{mk}\rho_k,$$

where $B - \bar{\theta}_m < 0$. The mature firm is the special case where $\mu = 0$,

$$B - P \;=\; (B - \bar{\theta}_m) - \sum_{k=1}^{K}\beta_{mk}\rho_k$$

So the difference is

$$(B - P_g) - (B - P_m) \;=\; \mu(B - \bar{\theta}_m) - \sum_{k=1}^{K}\mu\beta_{mk}\rho_k.$$

Taking expected values over the signal values $\mathbf{s}$ gives

$$E[(B - P_g) - (B - P_m)|\mathbf{s}] = \mu(B - \bar{\theta}_m) < 0. \tag{35}$$

It follows that on average growth firms (firms with new investments to undertake) have more negative book-to-price differences than do mature firms. In other words, consistent with intuition, firms with new investments to undertake tend to have low book-to-price ratios.

This analysis indicates that the growth firm has more negative $B - P$ on average, because it is an upward rescaling. But there is another possible reason for a firm to have a big negative $B - P$, which is that private signals drive up P. So $B - P$ is a proxy for two different things: overoptimism about the firm based on overreaction to private signals; or availability of a risky growth opportunity. Thus, even in a setting in which there is no market misvaluation of growth per se, growth and misvaluation are bound together in the characteristic and in characteristics-based factor loadings.[15]

[15] More generally, growth could itself be misvalued, rather than just being correlated with proxies for misvaluation. For example, in the model of Barberis, Shleifer, and Vishny (1998), investors overextrapolate earnings growth.

From (8), we observe that the bigger the loading on the systematic factors and the more positive the signal, the more negative is the loading of the first sector on the HML portfolio. Thus, stocks with high book-to-market ratios tend to have high systematic factor risk, and a negative loading on the HML portfolio. In conjunction with (35), this observation indicates that in our model, firms with growth opportunities tend to have low book-to-market but high market betas. This contrasts with explanations for the book-to-market effect based on low book-to-market firms having low systematic risk (Berk, Green, and Naik (1999)). Despite their high betas, owing to investor overconfidence, firms with low book-to-market tend to be overpriced, and therefore subsequently earn low returns. Thus, our analysis reconciles simply the evidence that low book-to-market firms have high risk (market betas) with their low expected returns.

7 Summary

This paper evaluates the meaning of factor pricing models such as that of Fama and French (1993) in which price-related 'characteristics' are used to form factor-mimicking portfolios. Empirically, the loadings with respect to these portfolios ('covariances') have been related to fundamental macroeconomic variables such as GDP and inflation, and have been found to predict future returns. Such tests have been interpreted in the finance field as providing an indication of whether stock prices are determined by rational risk premia, or reflect irrational investor misvaluation.

We consider a simple model of the cross section of security returns in which investors are risk neutral and overconfident. Overconfidence is a pervasive bias that has been documented extensively in the literature on cognitive psychology, and which has been the basis for some previous models of securities market under- and over-reactions. In our model, even though risk is not priced, the implications for the pricing of characteristics-

based portfolios and their loadings are qualitatively similar in some important ways to the empirical implications of rational asset pricing models.

A basic finding is that a characteristics-based portfolio such as HML formed in a manner akin to that of Fama and French (1993) yields positive expected returns. Intuitively, such a portfolio loads positively on factors (or residual components) with negative signals and negatively on factors with positive signals. Since underpriced stocks in the long run earn high returns and overpriced stocks earn low returns, holding a characteristics-based portfolio generates a positive expected return.

The ability of loadings on characteristics-based factor portfolios to predict returns occurs for similar reasons in the rational and behavioral theories. In both cases, low price relative to future dividends is associated with factor comovement. The characteristics-based portfolio is constructed to identify the comovement in high return stocks. The loadings on this portfolio therefore identify whether some other stock shares in the comovement that has caused low price. comovement, and loadings

A fully rational explanation for the empirical finding that expected returns are related to book-to-market, and therefore that an HML portfolio earns high returns, is related to growth options. In the model of Berk, Green, and Naik (1999), low book-to-market firms earn low returns because such firms have growth options, and under certain circumstances such firms can have low systematic risk. However, this perspective does not explain the empirical evidence that firms with low book-to-market tend to have high market betas. In our model (which allows for the prospect of investment and growth but not real options), firms with strong growth prospects tend to have low book-to-market and *high* market betas, consistent with this evidence. Despite their high betas, owing to investor overconfidence, firms with low book-to-market tend to be overpriced, and therefore subsequently earn low returns. Thus, our analysis reconciles simply this

evidence.

We further find that the characteristics-based portfolio loads on fundamental macroeconomic variables (for the reason discussed above); that the loading of a security with respect to the characteristics-based portfolio is a positive cross-sectional forecaster of future returns; that a security's covariance with the characteristics-based portfolio in general has incremental power to predict returns after controlling for the characteristic; that loadings can be a stronger predictor of return than characteristics when factor mispricing is stronger than idiosyncratic mispricing; and that the characteristic completely dominates the covariance in predicting returns if the factor and idiosyncratic mispricing are equally strong.

These findings suggest that care is needed in interpreting empirical tests of characteristics-based factor pricing models such as those of Fama and French (1993), because the overlap in predictions between a rational factor pricing model and a behavioral factor pricing model are stronger than has previously been recognized. Not only do both approaches imply that loadings can predict returns, but even if covariances are stronger return predictors than characteristics, neither the rational approach nor the psychology-based approach is disconfirmed.

This overlap in predictions arises from the nature of the Fama and French (1993) test approach, not because of any inherently excessive flexibility of either the rational or behavioral models. Rational asset pricing models provide elegantly simple but powerful predictions about the relation of risk premia and interest rates to consumption covariances; methods other than the Fama-French characteristics-based portfolio technique exist to test whether the sign and magnitude risk premia do indeed match these covariances. Similarly, psychology-based models offer a rich set of testable implications that are distinct from those offered by a purely rational approach (see, e.g., Barberis,

Shleifer, and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998, 2001), and Barberis, Huang, and Santos (2001)).

Even within the realm of characteristics-based factor pricing tests as introduced by Fama and French, there are still some differences in the predictions of purely rational asset pricing theory and our overconfidence model. First, in the purely rational model characteristics should have no power to predict returns, whereas in the behavioral model characteristics will have at least some predictive power (except in a relatively extreme special case, no idiosyncratic mispricing). This distinction provides a motivation for the characteristics versus covariances tests of authors such as Daniel and Titman (1997), Davis, Fama, and French (2000), and Daniel, Titman, and Wei (2001). A related implication that distinguishes the behavioral approach from rational factor pricing is that (so long as there is any idiosyncratic mispricing) the coefficient in a regression of returns on loadings exceeds the coefficient on loadings in a regression on both loadings and on characteristics.[16]

Second, depending on one's priors, there is still some expectation in our overconfidence setting that in characteristics versus covariance tests, characteristics will be stronger predictors than covariances (though not necessarily completely dominant). This is because characteristics capture both factor an idiosyncratic mispricing, whereas loadings capture only factor mispricing. So although incremental predictive power of covariances does not oppose the behavioral model, if the predictive power of covariances is far greater than that of characteristics, acceptance of the behavioral approach would require that there be little idiosyncratic mispricing. If markets are highly liquid and a large set of arbitrageurs trade actively in many securities, the ability to diversify idiosyncratic

[16] As a practical matter, if there is measurement error in the factor loadings, even under the purely rational approach, adding characteristics to a loadings regression can reduce the coefficient on loadings. Such a mismeasurement perspective makes the rational factor pricing approach more flexible in accommodating different empirical outcomes, which can be viewed as either a vice or a virtue.

risk can mute idiosyncratic mispricing more than factor mispricing. However, it is not clear that the premises needed for this conclusion are realistic, and there is evidence that mispricing is present both in common factors and in idiosyncratic components of returns.[17]

When neither approach can be rejected using characteristics-based factors and a characteristics-versus-covariances methodology, some alternative directions for asset pricing tests may be fruitful. A more direct approach to testing rational asset pricing, using the covariances of securities returns with proxies for aggregate consumption (rather than portfolios formed from price-based characteristics) to predict returns, is not subject to the critique we offer here. The characteristics-based factors can also potentially be developed to distinguishes alternatives more sharply. This would involve calibration using such variables as mean returns, the security covariance structure, and independent estimates of the degree of risk aversion or of psychological biases such as overconfidence.

Our analysis implies that the many tests that use the Fama-French factors as risk-controls to demonstrate anomalous return patterns need to be interpreted carefully. Since, as we demonstrate, a result that covariances predict returns does not distinguish between rational factor pricing and a standard behavioral approach, the use of such controls can mask possible psychological effects on asset prices. Specifically, a test variable in an event or asset pricing study could be a proxy for irrational beliefs and market inefficiency, but a conventional, naive interpretation of factor pricing controls would lead the researcher to conclude that the phenomenon derives from a rational premium for covariance risk.

[17] E.g., Cohen and Polk (1995), Moskowitz and Grinblatt (1999), Grundy and Martin (2001), Hirshleifer and Jiang (2005), and Hou, Peng, and Xiong (2005).

Appendix

Proof of Proposition 3: To compute the conditional expectations in Proposition 3, we use the following well-known result. Let X_1 and X_2 be random variables with means μ_1 and μ_2. Further, let $\Sigma_{YX} = [\mathrm{cov}(Y, X_1), \mathrm{cov}(\mathrm{Y}, \mathrm{X}_2)]$ denote the covariance vector between a variable Y and the vector $[X_1, X_2]$. Finally, let Σ_{XX} denote the variance-covariance matrix of X_1 and X_2. Then, the OLS predictor of Y based on X_1 and X_2, denoted by b_{OLS}, is given by

$$b_{OLS} = \Sigma_{YX}\Sigma_{XX}^{-1}. \tag{36}$$

In our case, returns for the selected security (i.e., security 1) are

$$Y = \beta_1 f(1-k) - k\beta_1 e + \eta(1-k_1) - k_1\nu,$$

loadings are

$$X_1 = a_1(f+e),$$

and characteristics are

$$X_2 = b_1(f+e) + b_2(\eta+\nu),$$

where $k \equiv v_f/(v_f+v_e)$, $k_1 = v_\eta/(v_\eta+v_c')$,

$$a_1 = -\frac{\beta_1 v_f^2 v_e W(f+e)}{(v_f+v_c)(v_f+v_e)},$$

$b_1 = -\beta_1 v_f/(v_f+v_c)$, and $b_2 = -v_\eta/(v_\eta+v_c')$. The variance-covariance matrix of X_1 and X_2 is given by

$$\begin{bmatrix} a_1^2(v_f+v_\epsilon) & a_1 b_1(v_f+v_e) \\ a_1 b_1(v_f+v_e) & b_1^2(v_f+v_e)+b_2^2(v_\eta+v_\nu) \end{bmatrix}.$$

Similarly, the covariances of Y with X_1 and X_2 conditional on a specific security 1, are respectively given by

$$\beta_1 v_f(1-k)a_1 - k\beta_1 a_1 v_e,$$

and

$$\beta_1 v_f(1-k)b_1 - k\beta_1 b_1 v_e + (1-k)b_2 v_\eta - k_1 b_2 v_\eta.$$

The proposition thus follows by first applying the law of iterated expectations to first calculate the relevant the unconditional covariance and variance (i.e., the covariance and variance without conditioning on a specific security) in Σ_{YX} and Σ_{XX} above, and then by an application of Equation (36) . ‖

Proof of Proposition 5: Adopting the notation in the proof of Proposition 3, we need to calculate the OLS predictor of Y based on X_1 and X_2, where the variables Y, X_1 and X_2 (i.e., the return, HML loading, and characteristic, respectively) now become:

$$Y = \beta_1 f_1(1-k) - k\beta_1 e_1 + \gamma_1 f_2(1-k) - k\gamma_1 e_2 + \eta(1-k_1) - k_1\nu,$$

$$X_1 = a_1(f_1 + e_1) + a_1'(f_2 + e_2),$$

and

$$X_2 = b_1(f_1 + e_1) + b_2(f_2 + e_2) + b_3(\eta + \nu),$$

where k and k_1 are unchanged from those in the proof of Proposition 3,

$$a_1 = -\frac{\beta_1 W v_f^2 v_e}{(v_f + v_e)(v_f + v_c)},$$

$$a_1' = -\frac{\gamma_1 W' v_f^2 v_e}{(v_f + v_e)(v_f + v_c)},$$

$b_1 = -\beta_1 v_f/(v_f + v_c)$, $b_2 = -\gamma_1 v_f/(v_f + v_c)$, and $b_3 = -v_\eta/(v_\eta + v_c')$.

The variance-covariance matrix of X_1 and X_2 is given by

$$\begin{bmatrix} (a_1^2 + a_1{}'2)(v_f + v_e) & (a_1 b_1 + a_1' b_2)(v_f + v_e) \\ (a_1 b_1 + a_1' b_2)(v_f + v_e) & (b_1^2 + b_2^2)(v_f + v_e) + b_3^2(v_\eta + v_\nu) \end{bmatrix}.$$

Similarly, the covariances of Y with X_1 and X_2 conditional on a specific security 1, are respectively given by

$$\beta_1 v_f(1-k)a_1 + \gamma_1 v_f(1-k)a_1' - k\beta_1 a_1 v_e - k\gamma_1 a_1' v_e,$$

and

$$\beta_1 v_f(1-k)b_1 - k\beta_1 b_1 v_e + \gamma_1 v_f(1-k)b_2 - k\gamma_1 b_2 v_e + (1-k)b_3 v_\eta - k_1 b_3 v_\eta.$$

The proposition then follows upon an application of Equation (36) and the law of iterated expectations. ‖

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