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Security Analysis and Trading Patterns When Some Investors Receive Information Before Others

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ABSTRACT

In existing models of information acquisition, all informed investors receive their information at the same time. This article analyzes trading behavior and equilibrium information acquisition when some investors receive common private information before others. The model implies that, under some conditions, investors will focus only on a subset of securities ("herding"), while neglecting other securities with identical exogenous characteristics. In addition, the model is consistent with empirical correlations that are suggestive of oft-cited trading strategies such as profit taking (short-term position reversal) and following the leader (mimicking earlier trades).

IN EXISTING MODELS OF information acquisition, all informed investors receive their information at the same time. While these models provide many important insights, in reality some investors, either fortuitously or owing to superior skill, acquire pertinent information before others. By being first, an investor can exploit this information to great advantage. Information that is uncovered only slightly later is less valuable even if it has not yet been publicly revealed.

We show in this article that the sequential nature of information arrival has a significant effect on both the trading decisions and the types of information collected by investors. The model we develop describes the investment choices of a set of competitive, risk-averse investors who investigate the long-term prospects of firms. Lucky or high-ability investors uncover the payoff-relevant information early, while unlucky or low-ability investors uncover the information later.

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Earlier work by Copeland (1976), and later by Jennings, Starks, and Fellingham (1981) and Jennings and Barry (1983), also examines models in which information is obtained by different informed traders at different times. However, in this literature, uninformed investors do not condition their trades on public observables such as prices and order flows, which are potentially important sources of information about a security. More recent work has analyzed dynamic trading behavior where the uninformed investors condition their trades on all public information (see, for example, Brown and Jennings (1989), Grundy and McNichols (1989), Kim and Verrecchia (1991), and Wang (1993)). These recent models provide important insights in settings where all potentially informed investors receive information (public or private) simultaneously.

The model we consider consists of three types of traders: (i) a continuum of competitive, risk-averse investors with finite aggregate mass, some subset of which (perhaps randomly) discovers a private information signal about a security's terminal value before the other subset; (ii) liquidity traders whose share demands are unmodeled; and (iii) risk-neutral market makers who absorb the net demands of the other traders at competitive prices. As we are able to solve for the linear equilibria of our dynamic model in closed-form, the analysis represents methodological interest independent of the issues we address.

We find that in a partially revealing rational expectations equilibrium, investors who discover information early trade aggressively in the initial period and then partially reverse their trades in the next trading round, when the trades of the investors who become informed at this later date cause the price to more fully reflect the investors' information. This reversal occurs because the risk-averse, early-informed investors wish to reduce the long-term risk associated with price movements that arise from future events they cannot predict. The trades of the informed investors are consistent with certain oft-cited institutional strategies.[1] Specifically, the early informed appear to be short-term "profit takers" because of their position reversal. On the other hand, the late informed appear to "follow the leader" as their trades are positively correlated with those of the early informed.

Unlike most other dynamic models of informed trading, we analyze ex ante information acquisition. This allows us to address the tendency of investors to investigate and trade the same group of stocks, leaving others somewhat neglected. Since a greater mass of late-informed traders allows the early informed to unwind their trades at more attractive prices, both the aggressiveness and the expected utility of the early informed increases with the mass of late-informed traders. As a result, it turns out that prior to an investor's knowing whether he will be informed early or late, his ex ante utility can be increasing in the total mass of investors who collect the

[1]See Kraus and Stoll (1970), Friend, Blume, and Crockett (1970), Lakonishok, Shleifer, and Vishny (1992), Peles (1992), and Grinblatt, Titman, and Wermers (1993) for empirical analyses of institutional trading strategies.

information. In other words, under some conditions, an investor will find it more attractive to collect information about stocks that are followed by many investors than to collect information about otherwise identical stocks that are being ignored. This is because if an investor happens to obtain information early and expects late-informed investors to move prices in the direction of his private information later, he can reverse his position early and thereby reduce his risk exposure. This benefit does not obtain if he is the only investor following a stock. From this, it follows that multiple "herding" equilibria can exist with the property that some stocks receive substantial attention while other stocks with very similar characteristics are ignored.[2]

Recent literature, e.g., Brennan (1990), Shleifer, and Vishny (1990), and Froot, Scharfstein, and Stein (1992) (FSS), examines theoretical explanations for the herding behavior of institutional investors and security analysts.[3] Our analysis is most closely related to that of FSS, in that both articles explicitly model how information is conveyed by prices in securities markets. Specifically, in FSS informed investors can have their trades executed either early or late, which is similar to our assumption that private information is received either early or late. However, the informed investors have exogenous short horizons in the FSS model and are constrained to reverse their trades after they have initially been executed. While their initial trades affect the price, they are able to reverse their trades without moving the price.[4]

[2]Recent empirical studies appear to be consistent with the idea that analysts and investors tend to herd. For example, Bhushan (1989) reports that in a sample of 1409 observations, although the average number of analysts following a firm was 13.94, a number of firms were followed by only one analyst, while the maximum number of analysts was 77. Evidence of herding is found in the recent analyses of mutual fund behavior by Grinblatt, Titman and Wermers (1993) and of financial institutions by Peles (1992). Discussion in the popular press also suggests that some firms are virtually ignored while others are followed by the herd. For example, an article entitled "A Shifting Tide on Neglected Stock Research," *New York Times* (July 10, 1992) states that 23 percent of NYSE stocks and 61 percent of NASDAQ stocks had two or fewer analysts following them in early 1992.

[3]A different group of recent papers (Scharfstein and Stein (1990), Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Trueman (1994)) show that herd behavior in general can occur when individuals make use of private information at the time that they decide whether or not to conform or deviate. Our focus here is on herding in the decision of whether to *acquire* private information on the same security. Further, unlike the above articles, we also focus on equilibrium trading behavior after receipt of information.

[4]An implication of this time-varying market liquidity, pointed out by the authors, is that investors can sometimes profit by herding on imagined information or "noise." Trading on noise in the FSS model is similar to market manipulation. A group of investors tacitly coordinate their trades based on some noise variable by purchasing stock when the market is illiquid, thereby pushing up prices, and then selling the stock at this higher price when the market is highly liquid. Kyle (1985), Black (1992), and Jarrow (1992) argue that market prices cannot be profitably manipulated if market depth is relatively constant since the manipulators will have the same price impact when they try to sell the stock as they did when they originally purchased it. Similar reasoning indicates that the herding result in FSS also requires levels of market depth that change significantly over time. In our model, intertemporal depth variations are not a consideration, because investors behave competitively.

The most important distinction between the herding equilibria in the FSS model and those in ours is that all prices and informed investment choices are determined within our model's equilibrium, while in the FSS model, some informed trading strategies are determined exogenously. Specifically, we demonstrate that herding equilibria can obtain without exogenously specified short horizons. Our analysis instead demonstrates that risk-sharing considerations alone can lead to inefficient outcomes in information acquisition.[5]

Although our explanation for herding applies to rational investors, we also examine the effects of irrationality and incentive problems on the tendency to herd. Since the expected payoff associated with investigating a stock that is followed by others increases with the probability assessed by an investor that he will uncover the information early, confident investors who believe they will receive the information signal early will find it more attractive to investigate the stocks examined by other investors. It follows that the tendency to investigate the same stocks as those investigated by others is stronger if investors are either afflicted with hubris (overconfidence), or if, as money managers, they wish to signal their confidence to future clients.

The structure of the article is as follows. Section I describes the economic setting. Section II characterizes the equilibrium. Section III discusses ex ante information acquisition strategies. Section IV concludes. All proofs, unless otherwise stated, appear in the Appendix.

I. The Economic Setting

Consider a risky security that is exchanged for a riskless security at two trading dates, 1 and 2, with its liquidation value subsequent to date 2 given by

$$F = \bar{F} + \theta + \varepsilon. \tag{1}$$

$\bar{F}$ is public knowledge and is the unconditional mean, while θ and ε are independent normal random variables with mean zero.

The informed traders in our model can be interpreted as private investors or investment managers. We will refer to them henceforth as "investors." We assume that there are two types of informed investors. The "early informed" learn precisely the realization of θ when the market opens at date 1, while the "late informed" do not receive any information at date 1, but learn the realization of θ when the market opens at date 2. The error term ε remains unknown at both trading dates.

For tractability, we assume that the identically informed traders behave competitively. Competitive rational expectations models with a finite number of informed agents are subject to the problem that these agents take the equilibrium price as given even though their trades affect the market price.

[5] Paul (1993) also provides a model in which risk aversion causes inefficient investigation decisions. He shows in a static setting that the proportion of investors who analyze various pieces of information will not, in general, maximize the informativeness of stock prices.

Admati (1985), Ausubel (1990), Admati and Pfleiderer (1991), and Boot and Thakor (1993) address this problem by assuming that informed traders are individually infinitesimal and fall on a continuum, so that no informed trader can affect the price.[6] Following these articles, we assume that each informed trader is a point on a continuum of investors whose set has a finite measure. The mass (or measure) of the early informed traders is denoted by M, while the *total* mass of early- and late-informed traders is denoted by N.[7] Both groups of traders have negative exponential utility over terminal wealth with a common risk-aversion coefficient R. Each informed trader has an endowment of B_0 units of the riskless bond (i.e., claims to B_0 units of terminal wealth). In the next section, we analyze the equilibrium assuming that the masses of early- and late-informed traders are determined exogenously and are common knowledge. In Section III, we analyze equilibrium information acquisition under the assumption that when investors choose to acquire their information, they do not know whether they will receive information early or late.

Noise is introduced in the form of liquidity demand shocks for the security of z_1 and z_2, which arrive at dates 1 and 2, respectively. These shocks are normally distributed with mean zero and variance σ_z^2, and are independent of each other and of θ and ε.

There is also a group of risk-neutral market makers, who possess no information about the fundamental value of the risky security. These agents represent a competitive fringe of risk-neutral traders (e.g., floor brokers, scalpers, or institutions who monitor trading floor activities) who are willing to absorb the net demands of the other traders at competitive prices.

Our equilibrium concept closely parallels that used in independent work by Vives (1994). We assume that at both dates 1 and 2, informed investors submit demand schedules ("limit orders") as a function of their information and the market prices. The risk-neutral market makers observe the combined demand schedules of the informed and liquidity traders and set competitive prices at each date.

Let ϕ_1 and ϕ_2 denote the information sets of the early-informed traders at dates 1 and 2. The date 2 information set of the late-informed traders is identical to ϕ_2 and their date 1 information set is the same as that of the risk-neutral market markers. Let W^E and W^L respectively denote the wealth levels of the early- and late-informed traders when the security is liquidated. Then, at date 2, the early-informed investors maximize $E[-\exp(-RW^E) \mid \phi_2]$, and the late-informed investors maximize $E[-\exp(-RW^L) \mid \phi_2]$. Fur-

[6] Wang (1993) also assumes competitive behavior on the part of identically informed traders that form a finite fraction of the investor population.

[7] More formally, if $f_E(g)$ and $f_L(g)$ represent the density functions of early- and late-informed investors (with g being an indexing variable), we assume that $\int_{-\infty}^{\infty} f_E(g)\, d_g = M$ and $\int_{-\infty}^{\infty} f_L(g)\, dg = N - M$, where N and M are positive numbers such that $N > M$. (For other models that use a continuum of traders with finite aggregate mass, see Boot and Thakor (1993) and Bhattacharya and Spiegel (1991).)

thermore, at date 1, the investors maximize their expected utility based on their date 1 information.

Given prices P_1 and P_2, let $x_1(\theta, P_1)$ and $x_2(\theta, P_2)$ denote the demands (holdings) of the early-informed investors and $y_1(P_1)$ and $y_2(\theta, P_2)$ denote the desired holdings of the late-informed investors at date 1 and date 2, respectively. The market makers observe the noisy aggregate demand schedules $D_1(\cdot) = Mx_1(\theta, \cdot) + (N - M)y_1(\cdot) + z_1$ at date 1, and $D_2(\cdot) = Mx_2(\theta, \cdot) + (N - M)y_2(\theta, \cdot) + z_1 + z_2$ at date 2.[8]

Because market makers are risk neutral and competitive, they set prices that are semi-strong form efficient. Thus, at each date prices equal the expectation of the final value of the security, conditional on the information set of the market makers, i.e., $P_1 = E[F \mid D_1(\cdot)]$ and $P_2 = E[F \mid D_1(\cdot), D_2(\cdot)]$. The Appendix (see the proof of Lemma 1 below) shows that, given linear pricing functions, the demand schedules can be written as $D_1(P_1) = \tau_1 + f(P_1)$ and $D_2(P_2) = \tau_2 + g(P_2)$, where $f(\cdot)$ and $g(\cdot)$ are linear functions, and τ_1 and τ_2 are linear combinations of the informational variable θ and the liquidity trades z_1 and z_2. (See Vives (1994), Glosten (1989), and Bhattacharya and Spiegel (1991) for similar demonstrations.) The informative parts of the linear demand schedules are the variables τ_1 and τ_2. In a linear equilibrium, we therefore have $P_1 = E[F \mid D_1(\cdot)] = E[F \mid \tau_1]$ and $P_2 = E[F \mid D_1(\cdot), D_2(\cdot)] = E[F \mid \tau_1, \tau_2]$.

As Vives (1994) points out, the market efficiency condition above can be justified by Bertrand competition among risk-neutral market makers who each observe the aggregate demand schedule (i.e., the limit order book) at each date. However, our model with risk-neutral uninformed traders may also be viewed as the limit of a Walrasian model in which the number of uninformed traders grows without bound, so that the risk-bearing capacity of the market becomes large. Kyle (1989, Theorem 7.4, p. 336), in the context of a strategic one-period model with risk-averse informed and uninformed traders, shows that in such a limit prices become unbiased, in the sense that the price approaches the expected value of the security conditional on the market price. In the linear equilibria we analyze, since prices are linear functions of the part of the aggregate demand schedules that are informative about final value, conditioning on the demand schedules $D_1(P_1)$ and $D_2(P_2)$ is equivalent to conditioning on P_1 and P_2.

II. Equilibrium Trades and Prices

A. The Optimal Demands of the Informed

We will consider the linear equilibria implied by the model. In a linear equilibrium, prices are linear functions of the net demand quantities at dates 1 and 2, which, in turn, are linear functions of the random variables θ, z_1,

[8] This is observationally identical to assuming that the market makers observe the net desired *trades* as functions of market prices at each date, i.e., $Mx_1(\theta, \cdot) + (N - M)y_1(\cdot) + z_1$ at date 1 and $M[x_2(\theta, \cdot) - x_1(\theta_1, \cdot) + (N - M)[y_2(\theta, \cdot) - y_1(\cdot)] + z_2$ at date 2.

and z_2. To derive the linear equilibria, we therefore begin by postulating that the prices are linear functions of the private information variable θ and the liquidity demand shocks to date, such that

$$P_2 = \overline{F} + a\theta + bz_1 + cz_2 \tag{2}$$

$$P_1 = \overline{F} + e\theta + fz_1. \tag{3}$$

In the ensuing analysis, we derive a linear equilibrium in which this conjecture is confirmed to be correct.

Terminal wealths are $W^E = x_2(\overline{F} + \theta + \varepsilon) - x_1 P_1 - (x_2 - x_1)P_2 + B_0$, and $W^L = y_2(\overline{F} + \theta + \varepsilon) - y_1 P_1 - (y_2 - y_1)P_2 + B_0$. Since the date 2 wealth is conditionally normally distributed, one can use the mean-variance framework and standard methodology to show that the optimal risky holdings of each early- and late-informed individual at the end of date 2 are identical and are given by

$$x_2(\theta, P_2) = y_2(\theta, P_2) = \frac{\overline{F} + \theta - P_2}{R\sigma_\epsilon^{\,2}}. \tag{4}$$

A more complex problem is the calculation of date 1 demands of the early-informed traders. Let $\overline{P}_2$ and $\sigma_{P_2}^2$ denote the mean and variance of P_2 *conditional on* ϕ_1. The Appendix shows that the optimal date 1 demand of an early-informed trader is

$$x_1(\theta, P_1) = \frac{\overline{P}_2 - P_1}{R}\left(\frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_\varepsilon^{\,2}}\right) + \frac{\overline{F} + \theta - \overline{P}_2}{R\sigma_\varepsilon^{\,2}}. \tag{5}$$

Note that the demand represented by equation (5) consists of two components, one to exploit the expected price appreciation across dates 1 and 2, and another to lock in at the current price the expected demand at date 2.[9] Comparing equations (4) and (5), one finds that the early-informed trader expects to reverse the component of the period 1 trade designed to exploit the expected price appreciation $\overline{P}_2 - P_1$. The early informed expect to reverse a relatively large fraction of their date 1 positions if the absolute difference between $\overline{P}_2$ and P_1 is large and/or the absolute difference between the informed expected terminal value $\overline{F} + \theta$ and the price $\overline{P}_2$ is small. These differences are influenced by, among other factors, the mass of the late-informed traders.

The Appendix also shows that $y_1(P_1)$, the date 1 demand of the late-informed investors, equals zero in equilibrium. Intuitively, this is so because of

[9] The derivation of equation (5) relies on $\sigma_{P_2}^2$ being strictly positive. See footnote 10.

two reasons: (i) the equilibrium date 1 price does not offer a risk premium because of the presence of risk-neutral market makers, and (ii) the late-informed investors cannot hedge their date 2 demand in advance, because conditional on their date 1 information set (which does not contain the informational variable θ), the expected date 2 price is unbiased, and, therefore, their expected date 2 trade is zero. Since, in equilibrium, the late informed trade zero shares at date 1, for the remainder of the article we suppress subscripts on the late-informed demand quantities and let y denote their holdings at date 2.

B. Equilibrium

We now characterize equilibrium prices and trading behavior in the model. Given the linear pricing functions equations (2) and (3), from equations (4) and (5), the demand quantities of the informed x_1, x_2, and y are linear functions of normal random variables and are therefore normally distributed. By assumption, the liquidity trades z_1 and z_2 are also normally distributed. Thus, the expectation of F conditional on the demand schedules is linear, and the prices P_1 and P_2 are indeed linear functions of the information variable θ and the liquidity shocks to date, supporting the initial conjectures in equations (2) and (3). Solving for the linear equilibrium entails solving for the coefficients a, b, c, e, and f in equations (2) and (3). The following lemma, which follows from some tedious calculations, presents closed-form solutions for these coefficients.

LEMMA 1: *In a linear equilibrium, the coefficients a, b, c, e, and f in equations* (2) *and* (3) *satisfy*

$$a = \frac{\sigma_\theta^2\left[(kN - R\sigma_\varepsilon^2)^2 + R^2\sigma_\varepsilon^4\right]}{D} \tag{6}$$

$$b = \frac{kR^2\sigma_\varepsilon^4\sigma_\theta^2}{D} \tag{7}$$

$$c = \frac{kR\sigma_\varepsilon^2\sigma_\theta^2(kN - R\sigma_\varepsilon^2)}{D} \tag{8}$$

$$e = \frac{\sigma_\theta^2}{\sigma_\theta^2 + k^2\sigma_z^2} \tag{9}$$

$$f = ke, \tag{10}$$

where

$$D \equiv \sigma_\theta^2(kN - R\sigma_\varepsilon^2)^2 + R^2\sigma_\varepsilon^4(\sigma_\theta^2 + k^2\sigma_z^2). \tag{11}$$

In the partially revealing equilibria with $P_1 \neq P_2$, k solves the quadratic equation[10]

$$k^2 M\left[N^2\sigma_\theta^2 + R^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\varepsilon^2 + \sigma_\theta^2)\right] - kR\sigma_\varepsilon^2\sigma_\theta^2(2MN + R^2\sigma_\varepsilon^2\sigma_z^2)$$
$$+ 2MR^2\sigma_\varepsilon^4\sigma_\theta^2 = 0. \tag{12}$$

Note that a real solution to equation (12) exists only if the discriminant of the quadratic equation is positive, i.e., if

$$\frac{4MR^2\sigma_\varepsilon^2\sigma_z^2\left[2M(\sigma_\varepsilon^2 + \sigma_\theta^2) - N\sigma_\theta^2\right] + 4M^2N^2\sigma_\theta^2}{R^4\sigma_\varepsilon^4\sigma_z^4\sigma_\theta^2} < 1. \tag{13}$$

We will call the equilibria obtained by solving the quadratic equation "T^+" and "T^-" equilibria depending on whether one places a positive or negative sign on the discriminant, respectively. It can be verified that there is a greater expected price move across dates 1 and 2 (conditional on θ) if one places a positive sign on the discriminant rather than a negative sign. (Note that, provided equation (13) holds, both roots of the quadratic equation are positive.)[11]

C. *Trading Behavior in Equilibrium*

This subsection describes the trading patterns of the various traders in the T^+ and T^- equilibria, and derives empirical implications. The following two propositions begin by describing some relevant relationships between the price moves, the trades of the informed traders, and the private information variable θ.

PROPOSITION 1: *The price moves at dates 1 and 2 are positively correlated with the information θ, i.e.,*

$$\operatorname{cov}\left(P_1 - \bar{F}, \theta\right) > 0 \tag{14}$$

and

$$\operatorname{cov}(P_2 - P_1, \theta) > 0. \tag{15}$$

[10] The Appendix shows that the model yields a fifth-order polynomial equation for k. One of the roots of this equation implies fully revealing prices, two others imply $P_1 = P_2$, while the two remaining ones are solutions to equation (12) and imply $P_1 \neq P_2$. The fully revealing equilibrium is trivial. In this equilibrium, since prices fully reflect the information of the informed traders, the informed traders' optimal positions are always zero regardless of their information at both dates. Since prices are unbiased, the speculators are indifferent and absorb the liquidity trades. Further, if $P_1 = P_2$, then the trader's wealth is independent of his date 1 trade, so the date 1 demands of the informed are not well defined. We analyze equilibria in which non-null trades and price moves occur, and therefore focus on the two roots that yield $P_1 \neq P_2$.

[11] Multiplicity of equilibria and the possibility of a nonexistence of equilibrium also obtain in Grundy and McNichols (1989)—see their discussion on pages 501 to 504.

PROPOSITION 2:

1. *The trades of the early-informed traders at date 1 are positively correlated with the information* θ, *and with the date 2 price move* $P_2 - P_1$, *i.e.*,

$$\operatorname{cov}(x_1, \theta) > 0 \tag{16}$$

and

$$\operatorname{cov}(x_1, P_2 - P_1) > 0. \tag{17}$$

2. *The trades of the early-informed traders are negatively correlated with the price move at date 2 and the information variable* θ, *i.e.*,

$$\operatorname{cov}(x_2 - x_1, P_2 - P_1) < 0 \tag{18}$$

and

$$\operatorname{cov}(x_2 - x_1, \theta) < 0. \tag{19}$$

3. *The trades of the late-informed traders are positively correlated with the date 1 trades of the early-informed traders, i.e.*,

$$\operatorname{cov}(y, x_1) > 0. \tag{20}$$

The signs of the covariances in equations (17) and (18) imply that the early-informed traders, on average, make a profit on positions taken at date 1 and, on average, reverse their positions at date 2. This profit-taking behavior in our model arises naturally as a consequence of risk sharing between the early- and late-informed traders.[12] The early informed will appear to be profit takers in a particularly marked fashion if the time interval between dates 1 and 2 is small. This behavior generally tends to become more pronounced as the post-date 2 risk (σ_ε^2) increases and the proportion of late-informed traders increases, as Figures 1 and 2, generated for the T^+ equilibria, illustrate.[13]

Based on the latter comparative static, an empirical implication of the model is that apparent profit-taking behavior by large institutions (who are more likely to receive the information early) should be more strongly evident in stocks that have a greater proportion of late-informed traders. For example, some firms may be studied by a large number of relatively small institutions which may be less likely to receive information early, and only a few large institutions. To an observer who does not know the timing of private information arrival across institutions, this short-term position rever-

[12] A stricter definition of profit taking would stipulate that on an upward price move at date 2, informed traders buy at date 1 and reverse their positions at date 2, for *all possible realizations* of the random variables. Profit taking does not obtain in our model in this stricter sense. However, the definition implied by equations (17) and (18) seems more relevant from an empirical perspective.

[13] The simulations are performed for unit realizations of the random variables θ, z_1, and z_2. Though we focus on the T^+ equilibria, qualitatively similar results are obtained for the T^- equilibria.

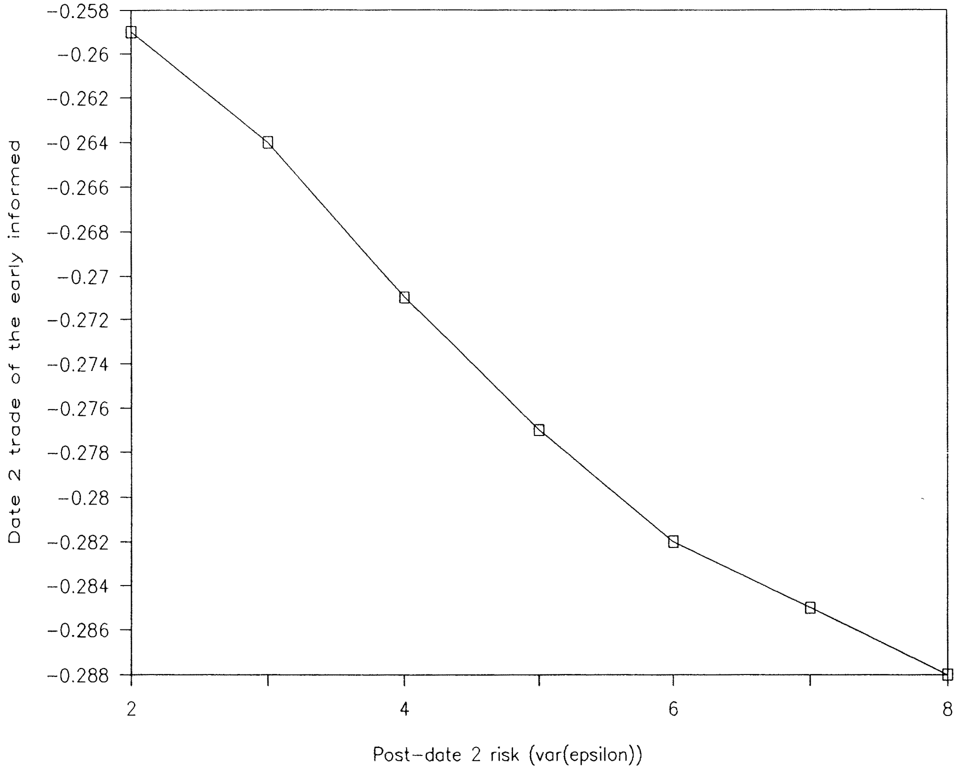

Figure 1. Date 2 trade of the early-informed traders as a function of the post-date 2 risk, σ_ε^2 (denoted by "var(epsilon)" in the figure). The total mass of informed traders is fixed at $N = 0.4$, and the mass of the early-informed traders is fixed at $M = 0.16$. The variance of the informational variable, ${\sigma_\theta}^2$, equals 1, the variance of liquidity trading in each period, ${\sigma_z}^2$, equals 0.4, and the risk-aversion coefficient of informed investors, R, equals 2.0.

sal will seem to occur without any triggering informational event (e.g., release of an accounting report).

In our analysis, early-informed traders tend to reverse their trades at date 2 even if there are no late-informed investors (i.e., if $M = N$); Proposition 2 imposes no restrictions on M and N. This implies that the early-informed, "on average," trade in a direction opposite to that implied by their information in round 2 even when *no* late-informed traders arrive to the market. This behavior is in contrast to Kyle's (1985) dynamic model in which the risk-neutral informed trader's orders are always positively correlated with the information variable.

This can be explained as follows. The date 2 price moves in the direction of the information on average (i.e., equation (15) holds) even if no late-informed investors enter in round 2, simply because there are two noisy signals conveying information about the final value at date 2 (the two order flows), as opposed to just one noisy signal at date 1. In response to this favorable price

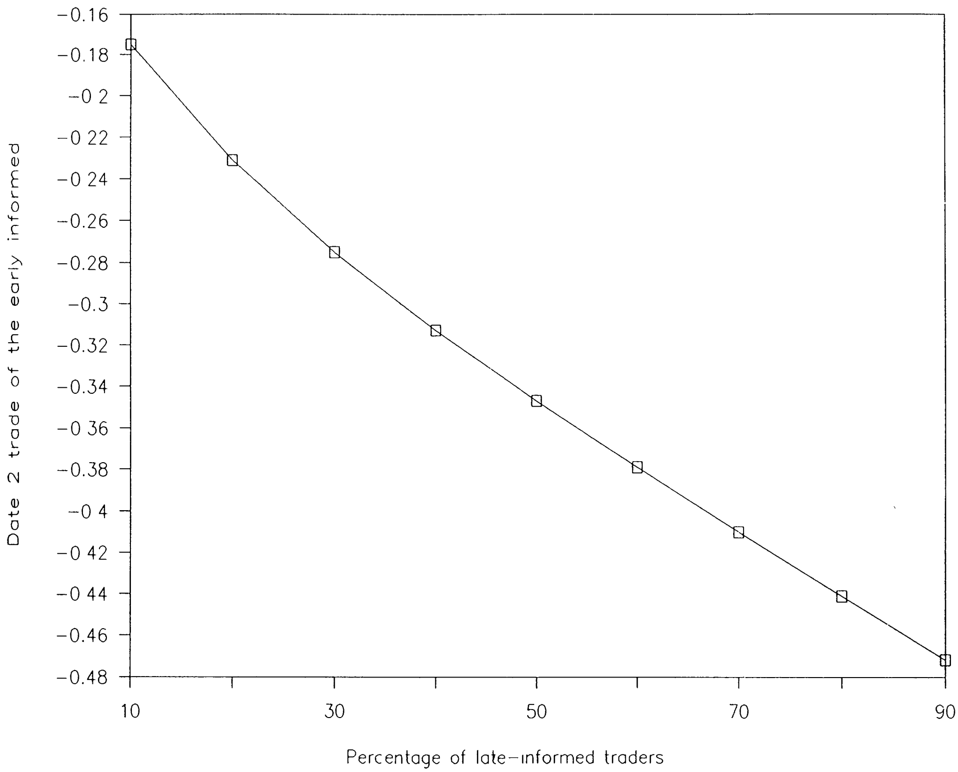

Figure 2. Date 2 trade of the early-informed traders as a function of the percentage of late-informed traders. The total mass of informed traders is fixed at $N = 0.4$. The variance of the informational variable, σ_θ^2, equals 1, the post-date 2 risk, σ_ε^2, equals 8.0, the variance of liquidity trading in each period, σ_z^2, equals 0.4, and the risk-aversion coefficient of informed investors, R, equals 2.0.

move, the informed traders partially reverse their position (on average) at date 2 to reduce its overall riskiness.

It follows that trading volume is stimulated by the nonsimultaneity of information arrival in the sense that different groups of the informed take both the buy and the sell side of the market at date 2, even though they observe a common informational signal. This is again in contrast to standard models with simultaneous information arrival in which all informed who observe strongly correlated signals trade in the same direction, and trading volume is limited by the amount of liquidity trading in the market.

Part 3 of Proposition 2 implies that the late-informed traders appear to "follow the leader." Relatively small institutions that have limited resources and contacts may be more likely to receive the information late.[14] Based on

[14] The cascades model of Bikhchandani, Hirshleifer, and Welch (1992) provides a possible alternative explanation for follow-the-leader behavior, if smaller institutions tend to have less precise information than larger ones.

this observation, our model predicts that the appearance of following the leader would be more prevalent in the trades of small institutions. Our analysis provides an important distinction that must be kept in mind in order to interpret empirical evidence on follow-the-leader type behavior correctly. The trades of apparent "followers" need not actually be *caused* by the observation of the trades of the "leaders" within our model. Instead, followers are responding at a later time to the same information observed earlier by leaders.

We next show that since price changes are uncorrelated (under risk neutrality of market makers, prices follow a martingale), there is no correlation between the trades of the late-informed traders and price movements.

PROPOSITION 3:

1. *Successive price changes are uncorrelated, i.e.,*

$$\operatorname{cov}\left(P_2 - P_1, P_1 - \bar{F}\right) = \operatorname{cov}(F - P_2, P_2 - P_1) = 0. \tag{21}$$

2. *The trades of the late-informed traders are uncorrelated with both the contemporaneous and the past price move, i.e.,*

$$\operatorname{cov}(y, P_2 - P_1) = \operatorname{cov}\left(y, P_1 - \bar{F}\right) = 0. \tag{22}$$

Part 1 of Proposition 3 implies that, in our model, it is not possible for a trader to profitably "trend chase" or be a "contrarian," i.e., to systematically earn profits by trading based on earlier price moves alone. This is simply because individuals are competitive and equilibrium prices are unbiased conditional on all available public information at each date.[15]

Part 2 of the proposition appears counterintuitive. Casual intuition may suggest that the late-informed traders would appear to follow a "relative strength" strategy on average, i.e., they would buy stocks on positive information subsequent to purchases by early-informed investors whose buying pressure initially pushed up prices. However, in our model, this intuition is incorrect. Depending on whether or not θ has an extreme realization, late-informed investors can trade either in the same or opposite direction of the date 1 price move. The late informed observe the price move and an information signal. Based on these, they judge whether the price move is justified based on the signal. The price can be either too high or too low owing to liquidity trades at date 1. If the date 1 price is too high, the late informed tend to sell; if the price is too low, they buy. (Actually, they decide based on the date 2 price, which also reflects date 2 liquidity trades, but this does not affect the argument.) On average, the price is set correctly at the unconditional expected value, since otherwise market makers would trade to exploit the difference. Thus, for any given date 1 price, the late informed are just as likely to buy as to sell, and there is no correlation between the date 1 price

[15] In a model with risk-averse uninformed traders, Wang (1993) shows that "trend chasing" arises because the uninformed respond to past price moves caused by informed trades.

change and the trades of the late informed. On average, as part 2 of Proposition 3 states, late-informed traders are neither contrarians nor trend chasers.

III. Ex Ante Information Acquisition

This section examines the ex ante gains from collecting information. We first show that our model implies strategic complementarities across informed investors in the following sense: the ex ante benefit to being informed about a given stock can *increase* in the total mass of individuals who are informed about the stock. This can lead to equilibria in which certain stocks receive disproportionate attention from investors, even though such stocks may be identical in terms of their exogenous characteristics (the joint cash flow and signal distribution) to other stocks.

The reason is as follows. Holding a position for long periods of time results in the bearing of stock performance risk that is unrelated to the trader's information (represented by the post-date 2 risk σ_ε^2). This means that if a large mass of informed individuals receive the information late, traders who receive information early can reduce the overall riskiness of their positions by taking a large position at date 1 and reversing it at date 2, when the late informed make the price more fully reflect the information. The risk-reduction effect implies that the benefit to early informed of having late informed trade can exceed the cost to the late informed of being preceded by the early informed. Provided that individuals do not know whether they will receive the information early or late, this can create a joint benefit to collecting information on the same security. Investors therefore may all prefer to investigate the same security rather than dispersing.

We argue in Subsections *C* and *D* that the tendency to herd on the same stock can be strengthened if investors are overconfident about their own ability or have an incentive to signal their ability.

A. The Ex Ante Benefit from Acquiring the Information

The calculation of the ex ante benefit from trading on the information is complicated. From the earlier analysis, one can write $x_1 = k_1\theta + k_2 z_1$, where k_1 and k_2 are defined as

$$k_1 \equiv \frac{(a-e)\sigma_\varepsilon^2 + c^2(1-e)\sigma_z^2}{Rc^2\sigma_\varepsilon^2\sigma_z^2} \tag{23}$$

and

$$k_2 \equiv \frac{(b-f)\sigma_\varepsilon^2 - c^2 f\sigma_z^2}{Rc^2\sigma_\varepsilon^2\sigma_z^2}. \tag{24}$$

The end-of-date 3 wealth of the early informed is

$$W^E = x_2(\bar{F} + \theta + \varepsilon) - x_1 P_1 - (x_2 - x_1)P_2 + B_0. \tag{25}$$

Substituting for x_1, x_2, P_1, and P_2, from equations (2), (3), (4), and (5), respectively, we have

$$W^E = J_1\theta^2 + J_2\theta\varepsilon + J_3\theta z_1 + J_4\theta z_2 + J_5\varepsilon z_1 + J_6\varepsilon z_2 + J_7 z_1^2 + J_8 z_1 z_2 + J_9 z_2^2 + B_0, \tag{26}$$

where the J_i's depend on a, b, c, and k in a complicated nonlinear way. Similarly the end-of-date 3 wealth of the late informed can be written as

$$W^L = y(\bar{F} + \theta + \varepsilon - P_2) + B_0. \tag{27}$$

Substituting for x_2, and P_2 from equations (4) and (2), we have

$$W^L = B_0 + I_1\theta^2 + I_2\theta\varepsilon + I_3\theta z_1 + I_4\theta z_2 + I_5\varepsilon z_1 + I_6\varepsilon z_2 + I_7 z_1^2 + I_8 z_1 z_2 + I_9 z_2^2 + B_0, \tag{28}$$

where the I_i's again depend on a, b, c, and k. (The actual expressions for the Is and Js are given in the Appendix.)

The ex ante expected utilities of the early- and late-informed individuals are $E[-\exp(-RW^E)]$ and $E[-\exp(-RW^L)]$, respectively. Construct the matrices

$$Y = R \times \begin{bmatrix} 2J_1 + R^{-1}\sigma_\theta^{-2} & J_2 & J_3 & J_4 \\ \cdot & R^{-1}\sigma_\varepsilon^{-2} & J_5 & J_6 \\ \cdot & \cdot & 2J_7 + R^{-1}\sigma_z^{-2} & J_8 \\ \cdot & \cdot & \cdot & 2J_9 + R^{-1}\sigma_z^{-2} \end{bmatrix} \tag{29}$$

and

$$Z = R \times \begin{bmatrix} 2I_1 + R^{-1}\sigma_\theta^{-2} & I_2 & I_3 & I_4 \\ \cdot & R^{-1}\sigma_\varepsilon^{-2} & I_5 & I_6 \\ \cdot & \cdot & 2I_7 + R^{-1}\sigma_z^{-2} & I_8 \\ \cdot & \cdot & \cdot & 2I_9 + R^{-1}\sigma_z^{-2} \end{bmatrix}. \tag{30}$$

The $\cdot$'s reduce notational clutter, as the matrices are symmetric. Define Π to be the variance-covariance matrix of the random vector $[\theta, \varepsilon, z_1, z_2]$. ($\Pi$ is a diagonal matrix.) Then, provided $Det(Y)$ and $Det(Z)$ are positive,[16] the Appendix shows that the expected utilities of each early- and late-informed individual are

$$-[Det(\Pi)Det(Y)]^{-\frac{1}{2}}\exp[-RB_0] \tag{31}$$

and

$$-[Det(\Pi)Det(Z)]^{-\frac{1}{2}}\exp[-RB_0], \tag{32}$$

[16] If the determinant of the matrix Y (Z) is not positive, the ex ante expected utility of an early- (late-) informed individual does not exist.

respectively. Defining these expressions divided by $\exp[-RB_0]$ to be EU^I and EU^{II}, before learning one's type, the benefit from trading on the information is

$$[\gamma EU^I + (1-\gamma)EU^{II}]\exp[-RB_0], \tag{33}$$

where γ denotes the ex ante probability of being an early-informed trader. For simplicity, we assume that the proportions of early- and late-informed individuals are exogenous and known to all individuals. Thus $M = \gamma N$. Let $E_N[U] \equiv \gamma E_N U^I + (1-\gamma)E_N U^{II}$, where the subscript N is added to emphasize the dependence of the expected utilities on the total mass of informed investors. The expression for $E_N[U]$ is complex and does not permit the derivation of formal comparative statics. We therefore focus on numerical simulations for the remainder of the article. The proposition below provides a numerical example in which $E_N[U]$ (and therefore the certainty equivalent of each potentially informed individual) increases in the total mass of individuals who collect the information. Following the numerical example, we provide more intuition and simulations on the conditions under which this complementarity obtains.

PROPOSITION 4: *There exist parameter values such that the ex ante expected utility from trading on the information increases in the total mass of individuals who trade on the information.*

Proof: Let $R = 2.5$, $\sigma_\varepsilon^2 = 8.0$, $\sigma_\theta^2 = 1.0$, $\sigma_z^2 = .4$, $B_0 = 0$, and $\gamma = 40\%$.[17,18] Then there exists a T^+ equilibrium under which the relationships between N and the expected utilities of each early- and late-informed individual and the ex ante expected utility $E_N[U]$ are as given in Table I.

The intuition behind the above proposition is as follows. Note from Table I that the expected utility of each late-informed individual decreases in the mass of informed individuals, while the opposite holds for the expected utility of an early-informed individual.[19] Adding more informed individuals to the market implies that both early- and late-informed individuals face greater competition. However, from the perspective of an early-informed trader, the

[17] The chosen parameter set under which the complementarity result obtains is not a set of measure zero. That is, our result continues to obtain for a range of values around the specified values for each parameters. See Subsection III.*B* later in the article.

[18] The value for the risk-aversion coefficient and the range of values of N considered are consistent with historical estimates of the market risk premium and other macroeconomic data (see the numerical simulations for representative agents in Sundaresan (1989) and Leland (1992)). The orders of magnitude of σ_ε^2 and σ_θ^2 are consistent with an annual return standard deviation of 20 percent (for realistic ranges of stock prices), as reported by Mehra and Prescott (1985) and several others. The values chosen for σ_z^2, B_0, and γ are not crucial for the illustration.

[19] In Table I, the ex ante utility for $N = 0$ represents the ex ante utility when only a single informed investor collects information, because each informed investor is assumed to be infinitesimally small. The expected utility for $N = 0$ is obtained by calculating the expected utility of an informed investor when $P_1 = P_2 = \bar{F}$. The prices each equal the unconditional expected value in this case because the demand schedule does not convey any information.

Table I

Expected Utility of an Informed Trader as a Function of the Total Mass of Informed Traders

This table provides the ex ante utility of an informed trader as a function of the total mass of informed traders. N denotes the total mass of informed agents. $E_N U^I$ and $E_N U^{II}$ denote the expected utility of an early- and a late-informed investor, respectively. $E_N[U]$ denotes the ex ante expected utility of an informed investor prior to knowing whether he will be an early-informed or a late-informed investor. The holdings of the riskless bond, B_0, equal zero, the post-date 2 risk, σ_ε^2, equals 8.0, the variance of liquidity trading in each period, σ_z^2, equals 0.4, the risk-aversion coefficient of informed investors, R, equals 2.5, and the probability of receiving information early, γ, equals 0.4.

N	0	0.2	0.5	0.6	0.8	1.0
$E_N U^I$	−0.94294	−0.94216	−0.94017	−0.93661	−0.93104	−0.92247
$E_N U^{II}$	−0.94294	−0.94307	−0.94387	−0.94525	−0.94737	−0.95046
$E_N[U]$	−0.94294	−0.94271	−0.94239	−0.94180	−0.94084	−0.93927

price at date 2 more closely reflects the information if more late-informed individuals trade at date 2. This implies that by partially reversing his position at date 1, the early-informed trader can reduce his risk exposure more than if there were fewer late-informed individuals. This is beneficial. For some parameter values, this benefit dominates the cost of greater competition.

Proposition 4 demonstrates the existence of equilibria in which disproportionate attention is paid to one security relative to another security with independent and identically distributed future payoffs. For example, consider two securities that have payoff distributions given by $\bar{F}_A + \theta_A + \varepsilon_A$ and $\bar{F}_B + \theta_B + \varepsilon_B$, respectively, where the $\bar{F}$s are nonstochastic and the θ's and ε's are mutually independent and normally distributed with means of zero. Further, suppose that $\sigma_{\theta_A}^2 = \sigma_{\theta_B}^2 = 1.0$ and $\sigma_{\varepsilon_A}^2 = \sigma_{\varepsilon_B}^2 = 8.0$, as in the numerical illustration above. In Table I, N may be viewed as the proportion of informed investors following a given stock (note that N ranges from 0 to 1.0 in the table), so that the maximum mass of potential informed investors is 1.0 (100 percent). Now suppose that each informed investor may choose to obtain information either on stock A or stock B. Then it is an equilibrium for the entire mass of investors to study stock A and no investor to study stock B (and vice versa).

To see that such herding is an equilibrium, note that if all investors follow one stock and none follow the other stock, so that each investor earns an $E_{N=1}[U]$ of -0.93927 (corresponding to a certainty equivalent of 0.025), it does not pay for an investor to deviate to the other stock, as, if he did so, he would earn $E_{N=0}[U]$ (i.e., that corresponding to $N = 0$), which is lower than $E_{N=1}[U]$. Further, since $E_N[U]$ increases in N, for all investor allocations across the two stocks except for the 50/50 one, the investors following the stock with a lower investor following each have an incentive to deviate to the stock with a higher investor following. Thus, the only other equilibrium is the "knife-edge" one in which the investors split 50/50 across the two stocks and are indifferent to following one stock over another. This equilibrium, however, is not robust, in the sense that small increases in the investor mass studying a given stock would create an incentive for investors to shift to this stock.[20]

The intuition for the herding equilibrium is that if an early-informed investor expects a large mass of investors to trade on the signal at date 2 and thereby cause the signal to be substantially reflected in the date 2 price, he is able to trade aggressively and reverse his position quickly without having to bear risk from news arrival that is unrelated to his information signal. Thus, prior to knowing whether an investor will be informed early or late, the gains to analyzing a stock that is being studied by a large mass of investors can be higher than the gains to studying a stock that is only being analyzed by a small mass. In the following subsection, we further investigate the conditions under which this comparative static obtains.

B. What Factors Promote Herding?

Whether the ex ante utility of an informed investor increases in the total mass of informed investors is parameter dependent in our model. The easiest way to see this is to examine intuitively the effect of increasing the total mass of investors on the expected utility of early- and late-informed investors. Increasing the total mass of investors implies increasing the masses of both early- and late-informed investors. Now, increasing the mass of late-informed investors should (i) decrease the expected utility of a late-informed investor (due to increased competition) but (ii) increase the expected utility of an early-informed investor, because the early informed are more effectively able to reverse their position in round 2. However, increasing the mass of early-informed investors should (iii) decrease the expected utility of both early- and late-informed investors (because of competition). From this it follows that whether the complementarity result obtains depends on whether effect (ii) is strong enough to dominate effects (i) and (iii).

To obtain more perspective on the above intuition, we performed additional numerical simulations. These simulations used the following parameter ranges (grid sizes for increments in each parameter are in parentheses): R ranged from 1.0 to 3.0 (0.1), σ_z^2 from 0.2 to 6.2 (1.0), σ_θ^2 from 0.8 to 1.5 (0.1), σ_ε^2 from 1.0 to 8.0 (1.0), and γ from 0.1 to 1.0 (0.1). (Not all parameter values yielded a partially revealing equilibrium, which exists only if equation (13) is

[20] Our assumption that the two stocks have i.i.d. payoffs abstracts from the rich set of issues that arise under correlated information structures, as analyzed in Admati (1985) and Admati and Pfleiderer (1987). The latter article investigates the viability of various types of information allocations under endogenous information acquisition. In particular, it investigates the conditions under which different pieces of correlated information signals all tend to be *concentrated* in the hands of a relatively small fraction of investors, and those under which the information signals are dispersed among many investors. In contrast, we explore the existence of equilibria in which investors all gather information about a given signal to the exclusion of other *independent* signals. Thus, our focus is mainly on the firm-specific component of security values.

satisfied.) The qualitative results from the simulations are presented below for specific parameter values.

A parameter that is crucial in determining whether the complementarities of the type described above obtain is σ_ε^2, which measures the post-date 2 risk. The tendency for herding equilibria to obtain generally increases in σ_ε^2, because a large σ_ε^2 implies that the benefit to the early-informed traders of reversing their positions at date 2 is large. Furthermore, the complementarity has a strong tendency to obtain if the date 2 competition is sufficiently intense that the date 2 price is close to being fully revealing. This implies that the perceived loss in expected utility because of the possibility of being a late-informed trader is low for an additional potentially informed individual. The simulation in Figure 3 illustrates the strengthening of the complementarity result as the variance of the post-date 2 risk (σ_ε^2) increases.

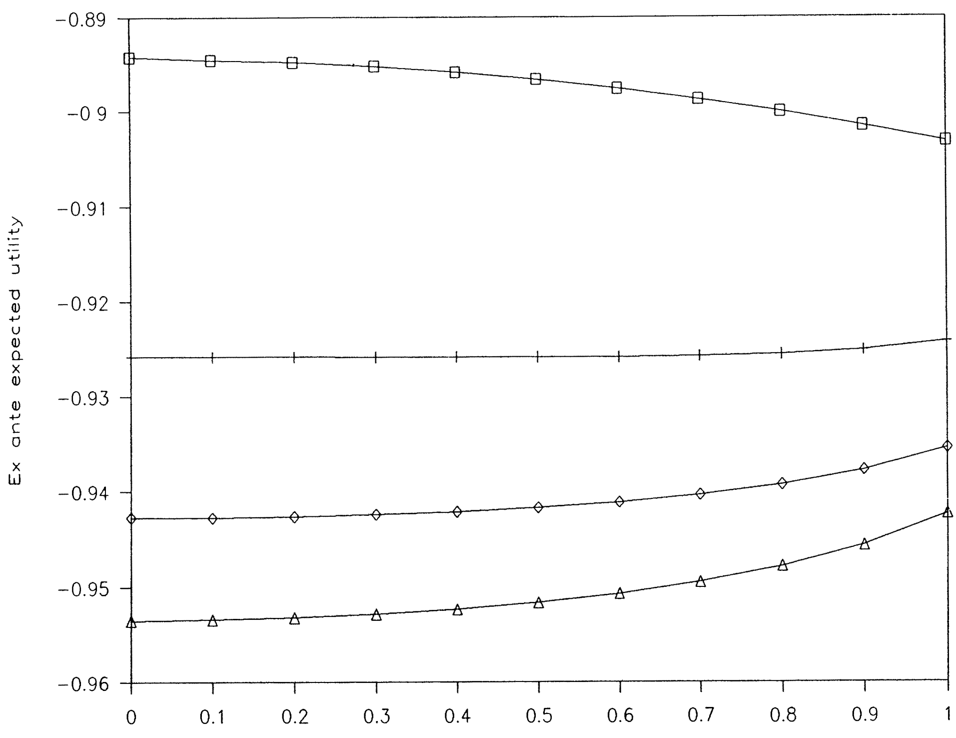

Figure 3. The expected utility of an informed trader as a function of the total mass of informed traders, for different levels of the post-date 2 risk, σ_ε^2 (denoted by "ve" in the figure). The holdings of the riskless bond, B_0, equal zero, the variance of the informational variable, σ_θ^2, equals 1, the variance of liquidity trading in each period, σ_z^2, equals 0.4, the risk-aversion coefficient of informed investors, R, equals 2.0, and the probability of receiving information early, γ, equals 0.4. The holdings of the riskless bond, B_0, are set to zero. $\square$, ve = 4; +, ve = 6; $\diamond$, ve = 8; and $\triangle$, ve = 10.

Further numerical analysis indicates that two other parameters that influence the tendency for herding equilibria to obtain are the ex ante probability of receiving information early, γ, and the variance of the information variable, σ_θ^2. In Figures 4 and 5, we use the base values of Proposition 4 to analyze how ex ante utility changes as the mass of informed investors increases, for different values of γ and σ_θ^2, respectively.

Figure 4 shows that for low values of γ, strategic complementarities do not obtain (there is very little chance that an investor will receive information early and capture the risk-sharing gains from herding), while for high γ the tendency to herd increases considerably. (Note, however, that the tendency cannot monotonically increase in γ.) Figure 5 indicates that the tendency to herd decreases in σ_θ^2. The intuition is that high values of σ_θ^2 increase the profit potential from obtaining information, which implies that, as the mass of informed investors increases, the effect of increased competition among informed investors dominates the benefit from the more effective position reversal at date 2. Our simulations suggest, however, that changing the

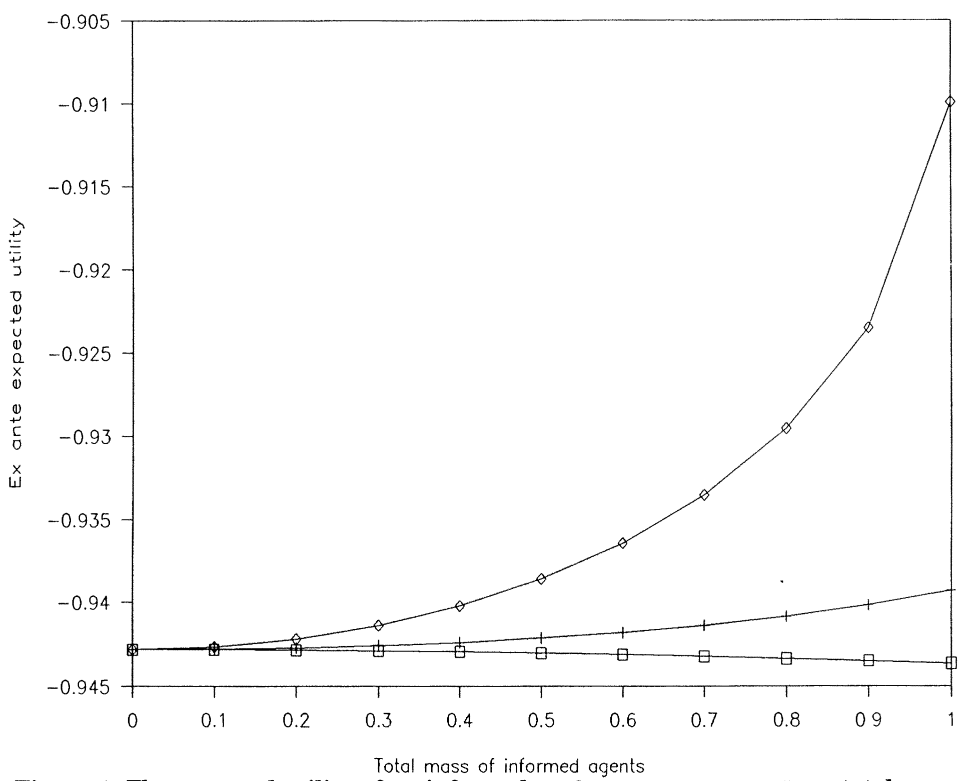

Figure 4. The expected utility of an informed trader as a function of the total mass of informed traders, for different levels of the probability of being early informed (denoted by "gamma" in the figure). The holdings of the riskless bond, B_0, equal zero, the variance of the informational variable, σ_θ^2, equals 1, the post-date 2 risk, σ_ε^2, equals 8.0, the variance of liquidity trading in each period, σ_z^2, equals 0.4, and the risk-aversion coefficient of informed investors, R, equals 2.5. □, gamma = 0.2; +, gamma = 0.4; and ◇, gamma = 0.6.

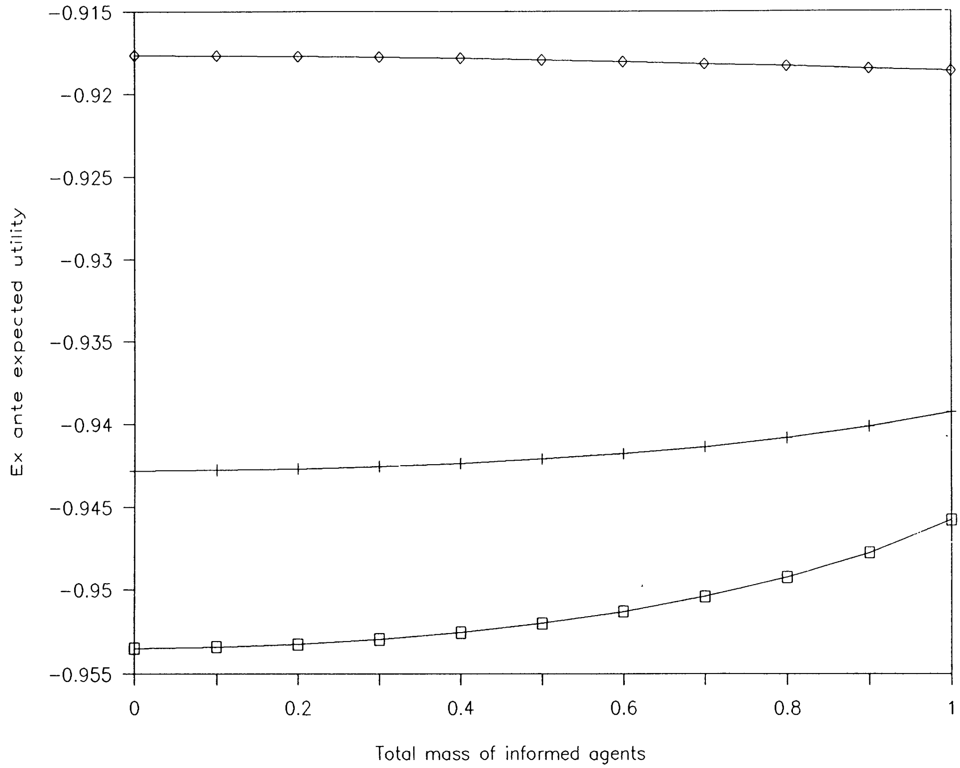

Figure 5. The expected utility of an informed trader as a function of the total mass of informed traders, for different levels of the informational variance σ_θ^2 (denoted by "vt" in the figure). The holdings of the riskless bond, B_0, equal zero, the post-date 2 risk, $\sigma_\varepsilon{}^2$, equals 8.0, the variance of liquidity trading in each period, $\sigma_z{}^2$, equals 0.4, the risk-aversion coefficient of informed investors, R, equals 2.5, and the probability of receiving information early, γ, equals 0.4. □, vt = 0.8; +, vt = 1.0; and ◇, vt = 1.5.

parameters R and $\sigma_z{}^2$ within reasonable ranges does not have a significant effect on the tendency for complementarities across informed traders to obtain.

In our model, both the early- and late-informed traders observe the same variable θ. In a more general setting, a parameter that should also be critical in determining whether herding equilibria obtain is the degree of correlation in the informed traders' signals. If the informed investors' signals are only weakly correlated, the tendency to herd should diminish considerably, as the early informed can less accurately forecast the price move at date 2, which reduces their ability to trade aggressively at date 1 and unwind a large portion of their trades at date 2. Unfortunately, equilibria in dynamic models with imperfectly correlated signals involve solutions of systems of nonlinear equations, which make it difficult to prove existence or uniqueness (or lack thereof) of equilibria. We performed a numerical analysis of a model in which the early informed observed θ with a noise term δ and the late informed observed θ perfectly. The results suggested that (for reasonable parameter values) increasing σ_δ^2 decreased the tendency of herding equilibria to obtain. The computational details of this model are available upon request from the authors.

C. Hubris among Investors

Proposition 4 shows that rational investors who expect their private information signals to arrive at different times will sometimes converge upon the same security in order to share risk. However, herding can also result from irrational behavior. We will now argue that because some investors receive information earlier than others, overconfidence by investors will promote herding.

DeLong *et al.* (1991) state that "one of the best documented biases (in experimental psychology is) the tendency to underestimate variances and to be overconfident" and that "experts and novices alike are too certain about their predictions given the true odds of being wrong" and provide several supporting citations from the psychology literature (e.g., Alpert and Raiffa (1982), Einhorn and Hogarth (1978), and Tversky and Kahneman (1974)).[21] In our model, if investors are overly confident about their own abilities to estimate value (afflicted with "hubris"), herding is more likely to obtain. To see this, note that in this model strategic complementarities in investigation across individuals occur because the benefit from being an early-informed individual in a stock dominates the utility loss from being a late-informed individual under some parameters. If an investor overestimates his probability of receiving information early, it is evident that the benefit he *perceives* from herding is strengthened.

More specifically, consider the numerical example in the proof of Proposition 4, and suppose that $\sigma_\varepsilon^2 = 5.0$ rather than 8.0. In this case, strategic complementarity does not obtain when all potentially informed investors rationally assess the probability of receiving information early to be 0.4. However, it can be shown that if each investor incorrectly assesses a probability of 0.7 or higher of receiving information early (even though he knows that there will be a 40/60 percent split between early and late informed), then strategic complementarity again obtains, and, consequently, herding is an equilibrium. This suggests that herding equilibria will obtain under a larger parameter range with hubris. The intuition, which is fairly general, is simply that in the calculation of the ex ante expected utility, the weight placed on the expected utility from receiving information early is higher under hubris, which increases the perceived benefit from analyzing the same stock as other investors.

D. Reputational Considerations

We have shown earlier that owing to risk-sharing considerations, when all investors assess the same probability of receiving information early, they will

[21] See, however, Gigerenzer (1993) for a skeptical viewpoint.

sometimes congregate upon one stock to the exclusion of another with identical characteristics. However, an investor (say, a money manager) may know whether he is more or less skillful or well positioned than other investors. In such a situation, a money manager's decision as to whether to herd (pool) or separate can potentially be used as a signal of his ability. In this section, we argue in the context of a numerical example that if money managers are concerned with their reputations, then their incentive to herd can be strengthened.

Suppose that there are two types of money managers: high-ability and low-ability. The total mass of all managers is 1.0, and there are equal masses of managers of each type. Furthermore, suppose that the high- and low-ability managers have probabilities of 0.85 and 0.15 of receiving the information early, respectively. We view the manager as an institution that investigates and trades on its own account, but can also benefit from having a favorable reputation. In contrast to the earlier sections, we will assume here that the manager receives a fixed payment that is proportional to his perceived ability as a manager, as well as obtaining direct trading profits. The fixed payment to the manager may be interpreted as the benefit associated with having a better reputation in the future.

Except for the different probabilities of being early or late informed, the following discussion assumes the parameter values of Proposition 4. As in the discussion following that proposition, we assume there are two stocks with i.i.d. payoffs and that each manager can follow either stock 1 or 2.

D.1. Managerial Abilities Known to All

We first consider the case where investor abilities are known to all. Suppose that both high- and low-ability managers follow the same stock. It can be shown that in this situation, the high-ability managers earn a certainty equivalent of 0.0453 while the low-ability managers earn a certainty equivalent of 0.0212. In this case, a defection from this pool by a high-ability manager reduces his certainty equivalent to 0.0236.[22] However, if a low-ability manager defects, his certainty equivalent *increases* to 0.0236, implying that in this case pooling (herding) is not an equilibrium.[23]

D.2. Privately Known Managerial Ability

Now suppose that abilities are known only to the managers. We assume that in addition to utility for trading the managers receive a wage represent-

[22] The certainty equivalent from being the lone defector to another stock is calculated as the expected utility of an informed investor when $N = 0$ (so that $P_1 = P_2 = \bar{F}$), as the defector is infinitesimally small.

[23] The discussion focuses upon a single set of parameters. However, the comparative statics regarding the respective certainty equivalents discussed in this paragraph, which are crucial to the analysis, obtain for a wide range of parameter values. For example, keeping the other parameters fixed at the levels in the proof of Proposition 4, the comparative statics obtain for σ_ε^2 ranging from 3.0 to 8.0 and the probability of receiving information early for a high- (a low-) ability manager ranging from 0.8 (0.2) to 1.0 (0.0). Similar wide ranges can be delineated for the other parameters.

ing future benefits of w_H if the manager's decision of what stock to analyze leads to the perception that he has high ability, and w_L if his choice leads to the perception that he has low ability. If the manager's type cannot be inferred from his investigation choice, he receives a "pooling" wage of w_P. These wages satisfy the conditions $w_H > w_P > w_L$.

The basic idea is that because bad managers have an incentive to conceal their lack of ability, they tend to herd with good managers when otherwise they would not do so. Let us propose a perfect Bayesian equilibrium in which both types of managers focus on one stock, with the out-of-equilibrium belief that any defector has low ability. It is evident that a high-ability manager does not want to defect from the pool because this reduces his gains from trade as well as his wage. Suppose further that the wage differential $w_P - w_L$ is greater than the expected gain to a low-ability manager from defecting (in our example, the latter quantity is the difference $0.0236 - 0.0212$). In this case, a low-ability type does not have an incentive to defect to the other stock either so a pooling equilibrium will exist.

The out-of-equilibrium belief that supports this equilibrium is in fact sensible, in the sense that the equilibrium survives the Cho and Kreps (1987) concept of equilibrium dominance (or the "intuitive criterion"). First, as already noted, a high-ability manager does not want to defect under the belief that a defection is by a low-ability manager as this would both lower his wage and his expected gains from trade. The question that remains is whether a high-ability manager could defect and convincingly argue that he was indeed a high-ability manager and not a low-ability manager as the out-of-equilibrium beliefs postulate. It is straightforward to show that a high-ability type cannot credibly communicate his type with such a defection. If investors believed that a defector had high ability, then such a defection would profitably be mimicked by low-ability types. Thus, the proposed pooling equilibrium survives the Cho-Kreps refinement. Intuitively, a high-ability manager cannot credibly separate himself by defecting, because the low-ability manager gains just as much reputational benefit as the high-ability manager does from being viewed as having high ability, and the low-ability manager gains *greater* utility for trading from defecting than does a high-ability manager.

IV. Conclusion

In existing models of information acquisition, all informed investors receive their information at the same time. As the mass of informed investors increases, prices become more informative, thereby decreasing the value of that information. The mass of investors collecting information thus increases in equilibrium until the point where the value of the information falls to the cost of producing it. Our analysis suggests that the exact timing of when investors uncover relevant information may be even more important than the accuracy of the information. Since investors who receive information early trade differently from investors who receive information late, the equilibrium in a securities market where investors receive their information before others can be fundamentally different from the equilibria in models of information acquisition in which investors receive their information simultaneously.

For example, owing to risk-sharing considerations, the value to an investor of investigating a particular stock can increase with the mass of other investigators also doing so. This results in equilibria where some stocks receive considerable attention from investors while others are ignored ("herding"). In contrast to other recent work on herding by Brennan (1990) and Froot, Scharfstein, and Stein (1992), in our model exogenous information revelation or short-horizons are not necessary for herding equilibria to obtain. Furthermore, in our analysis herding can only occur when a group of traders possess superior information; i.e., it does not pay to herd on noise.

While we demonstrate that inefficient herding equilibria can arise simply from the empirically reasonable assumption that investors observe an information signal at stochastically different times, we also show that the tendency to herd is strengthened if investors are overconfident (a documented cognitive bias) about whether they will receive information early. Furthermore, we demonstrate that herding among money managers can also obtain as a pooling equilibrium in a scenario where manager abilities vary and competent managers wish to signal their ability to acquire information early.

Explicit consideration of the stochastic timing of private information arrival has implications for the literature on trading volume and on institutional investor trading strategies. The differential timing of information arrival tends to stimulate volume by causing different groups of the informed to take both the buy and the sell side of the market in the same trading round. This is so even though they observe a *common* informational signal.

With respect to trading strategies, in our model certain investors make trades that are correlated with, but not caused by, other trades and price moves. Traders appear to follow the leader, but this is not a result of imitation. Rather, followers and leaders observe a common signal at different times. Position reversal (which gives the appearance of profit taking) occurs not as a result of favorable price moves or public revelation of private information, but because early-informed traders wish to reduce their positions over time to avoid risk. Position reversal is *correlated* with favorable price moves, since on average the early informed do profit on their early positions. Thus, our model highlights the crucial distinction between correlation and causation in the analysis of institutional trading strategies.

Appendix

Derivation of equation (5): Substituting for x_2 from equation (4) into the expression for W^E, we have

$$W^E \frac{(\bar{F} + \theta - P_2)}{R\sigma_\varepsilon^2}(\bar{F} + \theta + \varepsilon) - \frac{(\bar{F} + \theta - P_2)}{R\sigma_\varepsilon^2}P_2 - x_1(P_1 - P_2) + B_0$$

$$= \frac{(\bar{F} + \theta - P_2)^2}{R\sigma_\varepsilon^2} + \frac{(\bar{F} + \theta - P_2)\varepsilon}{R\sigma_\varepsilon^2} - x_1(P_1 - P_2) + B_0. \tag{34}$$

Now, from the formula for the characteristic function of a normal distribution, it follows that if $u \sim N(\mu, \sigma^2)$, then $E(\exp(vu)) = \exp(\mu v + (1/2)\sigma^2 v^2)$. In our case, setting $u = W^E$, $v = -R$, and using the fact that, from the perspective of the early informed, the only unknown at date 2 is the random variable ε, we have

$$E\left(-\exp(-RW^E) \mid \phi_2\right) = -\exp\left\{-R\left[B_0 - x_1P_1 + x_1P_2 + \left(\bar{F} + \theta - P_2\right)^2/(2R\sigma_\varepsilon^2)\right]\right\}. \quad (35)$$

It follows that at date 1, the early-informed traders maximize the derived expected utility of their date 2 wealth

$$E\left[\left[-\exp\left\{-R\left[B_0 - x_1P_1 + x_1P_2 + \left(\bar{F} + \theta - P_2\right)^2/(2R\sigma_\varepsilon^2)\right]\right\}\right]\middle|\phi_1\right]. \quad (36)$$

Now, equation (36) can be written as

$$-\left[2\pi\sigma_{P_2}^2\right]^{-\frac{1}{2}}\int_{-\infty}^{\infty}\exp\left\{-R\left[B_0 - x_1P_1 + x_1P_2 + \frac{\left(\bar{F} + \theta - P_2\right)^2}{(2R\sigma_\varepsilon^2)}\right] - \frac{1}{2}\frac{\left(P_2 - \bar{P}_2\right)^2}{\sigma_{P_2}^2}\right\} d\left(P_2 - \bar{P}_2\right). \quad (37)$$

Completing squares, the expression within the exponential above can be written as

$$-\left[\tfrac{1}{2}w^2 s + hw + l\right], \quad (38)$$

where

$$w = P_2 - \bar{P}_2$$

$$h = Rx_1 - \frac{\left(\bar{F} + \theta - \bar{P}_2\right)}{\sigma_\varepsilon^2}$$

$$s = \frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_\varepsilon^2}$$

$$l = Rx_1\left(\bar{P}_2 - P_1\right) + \frac{\left(\bar{F} + \theta - \bar{P}_2\right)^2}{2\sigma_\varepsilon^2} + RB_0,$$

Define $u \equiv \sqrt{s}\,w + h/\sqrt{s}$. Then, expression (38) becomes $-(1/2)u^2 + (1/2)h^2/s - l$. The Jacobian of the transformation from w to u is $s^{-\frac{1}{2}}$, and thus the integral (37) becomes

$$-\left[2\pi\sigma_{P_2}^2 s\right]^{-\frac{1}{2}}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{2}u^2+\frac{1}{2}\frac{h^2}{s}-l\right)du$$

$$=-\frac{1}{\left(\sigma_{P_2}^2 s\right)^{\frac{1}{2}}}\exp\left(\frac{1}{2}\frac{h^2}{s}-l\right). \qquad (39)$$

Solving for the optimal x_1 by maximizing the above objective, we obtain equation (5).

Proof that, in equilibrium, the late-informed agents will not trade in period 1: Let ϕ_l denote the information set of the late-informed traders *in period 1.* The terminal wealth of the late informed is

$$y_2(\bar{F}+\theta+\varepsilon)-y_1P_1-(y_2-y_1)P_2+B_0$$

or

$$y_2\left(\bar{F}+\theta+\varepsilon-P_2\right)-y_1P_1+y_1P_2.$$

At date 2, the only random variable is ε, so that conditional payoffs are normally distributed. Maximizing the mean-variance objective that results from an exponential-normal structure, the date 2 holdings of the late informed are

$$y_2=\frac{\bar{F}+\theta-P_2}{R\sigma_\varepsilon^2}. \qquad (40)$$

At date 1, the late-informed traders maximize the derived expected utility of their time 2 wealth, which is given by

$$E\left[\left[-\exp\left\{-R\left[B_0-y_1P_1+y_1P_2-\left(\bar{F}+\theta-P_2\right)^2/(2R\sigma_\varepsilon^2)\right]\right\}\right]\middle|\phi_l\right]. \qquad (41)$$

Let μ_{P_2} and μ_θ denote the means and Σ denote the covariance matrix of the random vector (P_2, θ) *conditional on* ϕ_l. Then, the expression within the exponential above can be written as

$$-\left[\tfrac{1}{2}x'Ax+h'x+l\right],$$

where

$$x=[\theta-\mu_\theta, P_2-\mu_{P_2}]$$

$$h=\left[-Ry_1+\frac{\left(\mu_{P_2}-\bar{F}-\mu_\theta\right)}{\sigma_\varepsilon^2}, \frac{\left(\bar{F}+\mu_\theta-\mu_{P_2}\right)}{\sigma_\varepsilon^2}\right]$$

$$A=\left[\Sigma^{-1}+\begin{bmatrix}\sigma_\varepsilon^{-2} & -\sigma_\varepsilon^{-2}\\ -\sigma_\varepsilon^{-2} & \sigma_\varepsilon^{-2}\end{bmatrix}\right]$$

$$l=Ry_1(P_1-\mu_{P_2})+g$$

and where g is an expression which does not involve y_1. Using standard results on multivariate normal distributions, equation (41) is given by

$$-\frac{1}{(Det(\Sigma))^{\frac{1}{2}}|Det(A)|^{\frac{1}{2}}}\exp\left(\frac{1}{2}h'A^{-1}h - l\right). \tag{42}$$

Thus, the optimal y_1 solves

$$\left[\frac{dh}{dy_1}\right]' A^{-1}\frac{dh}{dy_1} - \frac{dl}{dy_1} = 0. \tag{43}$$

Substituting, we have

$$y_1(P_1) = \frac{\mu_{P_2} - P_1}{RS_1} + \frac{\bar{F} + \mu_\theta - \mu_{P_2}}{R\sigma_\varepsilon^2}\frac{S_1 - S_2}{S_1}, \tag{44}$$

where S_1 and S_2 are the elements in the first row of the matrix A^{-1}. It is easy to verify that, in our equilibrium, S_1 is nonzero in general.

Let the superscript * denote equilibrium values. Since the late informed receive their information only at date 2, ϕ_l is identical to the information set of the market makers at date 1, which consists of the date 1 demand schedule $D_1(\cdot)$ (alternatively, the date 1 equilibrium price P_1^*). Thus,

$$\mu_{P_2^*} = \bar{F} + E[(E[\theta \mid D_1(\cdot), D_2(\cdot)] \mid D_1(\cdot))] = \bar{F} + E[\theta \mid D_1(\cdot)] = P_1^*. \tag{45}$$

Therefore, $\mu_{P_2^*} - P_1^*$ is zero. Further, $\bar{F} + \mu_\theta = \bar{F} + E[\theta \mid D_1(\cdot)] = P_1^* = \mu_{P_2^*}$. Thus, $\bar{F} + \mu_\theta - \mu_{P_2^*}$ is also zero. From equation (44), we then have $y_1(P_1^*) = 0$. Thus, in equilibrium, the late informed do not trade at period 1.

Proof of Lemma 1: Notice that the early-informed traders can invert the pricing function (3) to disentangle the noise term z_1, while the market makers cannot. That is $\phi_1 \equiv [\theta, P_1] \equiv [\theta, z_1]$. Thus, from equation (2), we have $\bar{P}_2 = E(P_2 \mid \theta, z_1) = \bar{F} + a\theta + bz_1$ and $\sigma_{P_2}^2 = \text{var}(P_2 \mid \theta, z_1) = c^2\sigma_z^2$. Substituting for $\bar{P}_2$ and $\sigma_{P_2}^2$ from above into equation (5), we have

$$x_1 = \theta\frac{a\sigma_\varepsilon^2 + c^2\sigma_z^2}{Rc^2\sigma_\varepsilon^2\sigma_z^2} + z_1\frac{b}{Rc^2\sigma_z^2} + \frac{(\bar{F} - P_1)(\sigma_\varepsilon^2 + c^2\sigma_z^2)}{Rc^2\sigma_\varepsilon^2\sigma_z^2}. \tag{46}$$

From equation (46) above, we can write the total period 1 demand of the informed and liquidity traders, denoted by $D_1(P_1) = Mx_1 + Ny_1 + z_1$, as

$$M\left[\theta\frac{a\sigma_\varepsilon^2 + c^2\sigma_z^2}{Rc^2\sigma_\varepsilon^2\sigma_z^2} + z_1\frac{b}{Rc^2\sigma_z^2} + \frac{(\bar{F} - P_1)(\sigma_\varepsilon^2 + c^2\sigma_z^2)}{Rc^2\sigma_\varepsilon^2\sigma_z^2}\right] + Ny_1(P_1) + z_1. \tag{47}$$

where $y_1(P_1)$ is given by equation (44). Thus, the period 1 net demand reveals a noisy transformation of the private information to the market makers.

Define

$$\tau_1 \equiv \frac{M(a\sigma_\varepsilon^2 + c^2\sigma_z^2)}{Rc^2\sigma_\varepsilon^2\sigma_z^2}\theta + \frac{Mb + Rc^2\sigma_z^2}{Rc^2\sigma_z^2}z_1. \tag{48}$$

Expression (47) can then be written as

$$\frac{M(\bar{F} - P_1)(\sigma_\varepsilon^2 + c^2\sigma_z^2)}{Rc^2\sigma_\varepsilon^2\sigma_z^2} + Ny_1(P_1) + \tau_1.$$

Note that at date 1, the late informed have the same information set as the market makers. Thus, $y_1(P_1)$ does not convey any information to the market makers. The informative part of the demand in equation (47) is therefore the variable τ_1. Thus, prices must satisfy $P_1 = E[\bar{F} + \theta + \varepsilon \mid \tau_1] = \bar{F} + E[\theta \mid \tau_1]$, which implies that P_1 takes the form $\bar{F} + \kappa\tau_1$, where κ is the coefficient in the regression of θ on τ_1, i.e., $\kappa = \text{cov}(\theta, \tau_1)/\text{var}(\tau_1)$. Defining

$$k \equiv \frac{Mb\sigma_\varepsilon^2 + Rc^2\sigma_\varepsilon^2\sigma_z^2}{M(a\sigma_\varepsilon^2 + c^2\sigma_z^2)}, \tag{49}$$

we have

$$P_1 = \bar{F} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + k^2\sigma_z^2}(\theta + kz_1). \tag{50}$$

Thus, equating coefficients in equations (3) and (50) above, we obtain equations (9) and (10) of the lemma. At the end of date 2, the total holdings of the informed and the total demand shocks are

$$D_2(P_2) = \frac{N}{R\sigma_\varepsilon^2}(\bar{F} + \theta - P_2) + z_1 + z_2 \tag{51}$$

Again, define

$$\tau_2 \equiv \frac{N}{R\sigma_\varepsilon^2}\theta + z_1 + z_2. \tag{52}$$

The demand in equation (51) can be written as

$$\frac{N}{R\sigma_\varepsilon^2}(\bar{F} - P_2) + \tau_2.$$

As the informative part of the demand is τ_2, the period 2 price satisfies

$$P_2 = E[\bar{F} + \theta + \varepsilon \mid \tau_1, \tau_2]. \tag{53}$$

To compute the conditional expectation in equation (53), we use the well-known result (see, e.g., Anderson (1984, Chapter 2)) that if there exist random vectors v_1 and v_2 such that

$$(v_1, v_2) \sim \mathbf{N}\left[(\mu_1, \mu_2), \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right],$$

then the conditional distribution of v_1 given $v_2 = \mathbf{X}_2$ is normal with a mean given by the vector

$$E(v_1 \mid v_2 = \mathbf{X}_2) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{X}_2 - \mu_2). \tag{54}$$

Letting $v_1 = \theta$ and $v_2 = \left[k\tau_1, \dfrac{R\sigma_\varepsilon^2}{N}\tau_2\right]$, (a vector observationally equivalent to $[\tau_1, \tau_2]$), we have $\Sigma_{11} = \sigma_\theta^2$, $\Sigma_{12} = [\sigma_\theta^2, \sigma_\theta^2]$, and

$$\Sigma_{22} = \begin{bmatrix} \sigma_\theta^2 + k^2\sigma_z^2 & \sigma_\theta^2 + \dfrac{kR\sigma_\varepsilon^2\sigma_z^2}{N} \\ \sigma_\theta^2 + \dfrac{kR\sigma_\varepsilon^2\sigma_z^2}{N} & \sigma_\theta^2 + \dfrac{2R^2\sigma_\varepsilon^4\sigma_z^2}{N^2} \end{bmatrix}. \tag{55}$$

Using equation (54), one obtains $P_2 = \bar{F} + E(\theta \mid \tau_1, \tau_2) = a'\theta + b'z_1 + c'z_2$, where a', b', and c' are given by the RHS of equations (6) to (8). Equating coefficients from equation (2), we obtain equations (6) to (8) in Lemma 1. Substituting for a, b, and c from equations (6) to (8) into equation (49) above, we obtain

$$\begin{aligned} k(kN - R\sigma_\varepsilon^2)^2\big[k^2M\{N^2\sigma_\theta^2 + R^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\varepsilon^2 + \sigma_\theta^2)\} \\ -kR\sigma_\varepsilon^2\sigma_\theta^2(2MN + R^2\sigma_\varepsilon^2\sigma_z^2) + 2MR^2\sigma_\varepsilon^4\sigma_\theta^2\big] = 0. \end{aligned} \tag{56}$$

This equation has four possible solutions. The first solution is $k = 0$ and implies that $a = e = 1$ and $b = c = e = 0$, i.e., fully revealing prices. The second solution is $k = R\sigma_\varepsilon^2/N$ and, upon inspection of equations (6) to (10), this solution necessitates $c = 0$ and $P_2 = P_1$. However, under these solutions, since $\sigma_{P_2}^2$ is necessarily zero (see equation (5)), the date 1 demands of the early-informed traders are not well defined. The other two solutions are obtained by setting the quadratic expression within square brackets to zero (i.e., by solving equation (12)), and imply $P_1 \neq P_2$.

Proof of Proposition 1: Equations (14) and (15) follow upon substituting for P_1 and P_2 from equations (2) and (3), and using the facts that $a < 1$ and $a > e$ (from equations (6) and (9)).

Proof of Proposition 2: Equation (16) follows from substituting for the pricing functions from equations (2) and (3) into equation (4) and again using the facts that $a < 1$ and $a > e$. To prove equation (17), note from equation (5) and the pricing functions (2) and (3) that

$$\operatorname{cov}(x_1, P_2 - P_1) = \sigma_\theta^2 \left[\frac{(a-e)^2}{R} \left(\frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_\varepsilon^2} \right) + \frac{(1-a)(a-e)}{R\sigma_\varepsilon^2} \right] + \sigma_z^2 \left[\frac{(b-f)^2}{R} \left(\frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_\varepsilon^2} \right) - \frac{b(b-f)}{R\sigma_\varepsilon^2} \right]. \quad (57)$$

However, from equations (6) to (12), it follows that $a < 1$, $a > e$, $b < f$, and $f > 0$ (note from equation (12) that both solutions for k are positive if they are real). Thus, the above covariance is positive. From equations (2), (3), (4), and (5), we have

$$\operatorname{cov}(x_2 - x_1, P_2 - P_1) = -c^2\sigma_z^2 - R^{-1}\left(\sigma_{P_2}^{-2} + \sigma_\varepsilon^{-2}\right)\left[(a-e)^2\sigma_\theta^2 + (b-f)^2\sigma_z^2\right] \quad (58)$$

and

$$\operatorname{cov}(x_2 - x_1, \theta) = -(a-e)\sigma_\theta^2 \frac{1}{R}\left(\frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_\varepsilon^2} \right), \quad (59)$$

both of which are necessarily negative (the latter because $a > e$ from equations (6) and (9)). Similarly, we have

$$\operatorname{cov}(y, x_1) = (1-a)^2\sigma_\theta^2 + b^2\sigma_z^2, \quad (60)$$

which is positive.

Proof of Proposition 3: Part 1 follows from substituting for the price coefficients from equations (6) to (10) into the pricing functions (2) and (3). To prove part 2, note that equation (4) implies that the date 2 order of the late-informed traders is proportional to $\theta - P_2$. However, recalling that $F = \overline{F} + \theta + \varepsilon$ and that ε is uncorrelated with P_1 and P_2, from part 1 it follows that the covariance between $\theta - P_2$ and $P_2 - P_1$ and that between $\theta - P_2$ and $P_1 - \overline{F}$ is necessarily zero.

Derivation of expressions (31) and (32): Now, $E(-\exp(-W^E))$ can be written as

$$-(2\pi)^{-2}(Det(\pi))^{-\frac{1}{2}} \int_{R^4} \exp\left(-W^E - \tfrac{1}{2}t\Sigma^{-1}t'\right) dt, \quad (61)$$

where t is the vector $[\theta, \varepsilon, z_1, z_2]$. Completing squares, the term within the exponential above can be written as $-\frac{1}{2}t'Yt$ where Y is defined in the text. Note that the integral in equation (61) is finite if and only if Y is positive definite. If this is the case (i.e., if $Det(Y)$ is positive), one can write $Y = \nu\nu'$. Defining $\mu \equiv \nu' t$, we have $-\frac{1}{2}t'Yt = \frac{1}{2}\mu'\mu$. The Jacobian of this transformation is $(Det(Y))^{-\frac{1}{2}}$, and therefore the integral in equation (61) becomes

$$\int_{R^4} \exp(-\tfrac{1}{2}t'Yt)\,dt = (Det(Y))^{-\frac{1}{2}} \int_{R^4} \exp(-\tfrac{1}{2}\mu'\mu)\,d\mu$$
$$= (2\pi)^2 (Det(Y))^{-\frac{1}{2}}, \qquad (62)$$

where the last equality obtains from using the fact that the normal density integrates to unity. Observing that $E(-\exp(-W^E))$ is the product of the terms outside the integral in equation (61) with the terms on the RHS of the last equality in equation (62) yields equation (31). The proof of equation (32) is identical to the one above, except that the matrix Y in the above proof is replaced by the matrix Z.

Expressions for the Elements of the Matrices Y and Z

The following are the expressions for the elements of the matrices Y and Z of Section III.

$$R\sigma_\varepsilon^2 J_1 = a^2 + a(k_1 R\sigma_\varepsilon^2 - 2) + 1 - ek_1 R\sigma_\varepsilon^2$$
$$R\sigma_\varepsilon^2 J_2 = 1 - a$$
$$R\sigma_\varepsilon^2 J_3 = a(2b + k_2 R\sigma_\varepsilon^2) + b(k_1 R\sigma_\varepsilon^2 - 2) - R\sigma_\varepsilon^2(ek_2 + fk_1)$$
$$R\sigma_\varepsilon^2 J_4 = c(2a + k_1 R\sigma_\varepsilon^2 - 2)$$
$$R\sigma_\varepsilon^2 J_5 = -b$$
$$R\sigma_\varepsilon^2 J_6 = -c$$
$$R\sigma_\varepsilon^2 J_7 = b^2 + (b - f)k_2 R\sigma_\varepsilon^2$$
$$R\sigma_\varepsilon^2 J_8 = c(2b + k_2 R\sigma_\varepsilon^2)$$
$$R\sigma_\varepsilon^2 J_9 = c^2$$
$$R\sigma_\varepsilon^2 I_1 = (a - 1)^2$$
$$R\sigma_\varepsilon^2 I_2 = (1 - a)$$
$$R\sigma_\varepsilon^2 I_3 = 2b(a - 1)$$
$$R\sigma_\varepsilon^2 I_4 = 2c(a - 1)$$
$$R\sigma_\varepsilon^2 I_5 = -b$$
$$R\sigma_\varepsilon^2 I_6 = -c$$
$$R\sigma_\varepsilon^2 I_7 = b^2$$
$$R\sigma_\varepsilon^2 I_8 = 2bc$$
$$R\sigma_\varepsilon^2 I_9 = c^2,$$

where k_1 and k_2 are defined by equations (23) and (24).

REFERENCES

Admati, A. R., 1985, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* 53, 629–657.

——— and P. Pfleiderer, 1987, Viable allocations of information in financial markets, *Journal of Economic Theory* 43, 76–115.

———, 1991, Sunshine trading and financial market equilibrium, *Review of Financial Studies* 3, 443–481.

Alpert, M., and H. Raiffa, 1982, A progress report on the training of probability assessors, in D. Kahneman, P. Slovic, and A. Tversky, Eds.: *Judgement Under Uncertainty: Heuristics and Biases* (Cambridge University Press, Cambridge, England).

Anderson, T. W., 1984, *An Introduction to Multivariate Statistical Analysis* (John Wiley & Sons, New York).

Ausubel, L., 1990, Insider trading in a rational expectations economy, *American Economic Review* 80, 1022–1041.

Banerjee, A., 1992. A simple model of herd behavior, *Quarterly Journal of Economics* 107, 797–817.

Bhattacharya, U., and M. Spiegel, 1991, Insiders, outsiders, and market breakdowns, *Review of Financial Studies* 4, 255–282.

Bhushan, R., 1989, Firm characteristics and analyst following, *Journal of Accounting and Economics* 11, 255–274.

Bikhchandani, S., D. Hirshleifer, and I. Welch, 1992, A theory of fads, fashions, customs, and cultural change as informational cascades, *Journal of Political Economy* 100, 992–1096.

Black, F., 1992, Equilibrium exchanges, Working paper, Goldman Sachs and Co.

Boot, A. W., and A. V. Thakor, 1993, Security design, *Journal of Finance* 48, 1349–1378.

Brennan, M. J., 1990, Latent assets, *Journal of Finance* 45, 709–730.

Brown, D. P., and R. Jennings, 1989, On technical analysis, *Review of Financial Studies* 2, 527–551.

Cho, I.-K., and D. M. Kreps, 1987, Signalling games and stable equilibria, *Quarterly Journal of Economics* 102, 179–221.

Copeland, T. E., 1976, A model of asset trading under the assumption of sequential information arrival, *Journal of Finance* 31, 1149–1168.

DeLong, B., A. Shleifer, L. Summers, and R. Waldmann, 1991, The survival of noise traders in financial markets, *Journal of Business* 64, 1–20.

Einhorn, H., and R. Hogarth, 1978, Confidence in judgement: Persistence of the illusion of validity, *Psychological Review* 85, 395–416.

Friend, I., M. Blume, and J. Crockett, 1970, *Mutual Funds and Other Institutional Investors: A New Perspective* (McGraw-Hill, New York).

Froot, K. A., D. S. Scharfstein, and J. C. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Gigerenzer, G., 1993, The bounded rationality of probabilistic mental models, in K. I. Manktelow and D. E. Over, Eds.: *Rationality: Psychological and Philosophical Perspectives* (Routledge, London).

Grinblatt, M., S. Titman, and R. Wermers, 1993, Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior, Working paper, University of California, Los Angeles.

Grundy, B., and M. McNichols, 1989, Trade and the revelation of prices through direct disclosure, *Review of Financial Studies* 2, 495–526.

Jarrow, R. A., 1992, Market manipulation, bubbles, corners, and short squeezes, *Journal of Financial and Quantitative Analysis* 27, 311–336.

Jennings, R. H., L. Starks, and J. Fellingham, 1981, An equilibrium model of asset trading with sequential information arrival, *Journal of Finance* 36, 143–162.

Jennings, R., and C. B. Barry, 1983, Information dissemination and portfolio choice, *Journal of Financial and Quantitative Analysis* 18, 1–19.

Kim, O., and R. Verrecchia, 1991, Market reactions to anticipated public announcements, *Journal of Financial Economics* 30, 273–310.

Kraus, A., and H. R. Stoll, 1970, Parallel trading by institutional investors, *Journal of Financial and Quantitative Analysis* 7, 2107–2138.

Kyle, A. S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

———, 1989, Informed speculation with imperfect competition, *Review of Economic Studies* 56, 317–356.

Lakonishok, J., A. Shleifer, and R. W. Vishny, 1992, The impact of institutional trading on stock prices, *Journal of Financial Economics* 20, 23–43.

Leland, H., 1992, Insider trading: Should it be prohibited? *Journal of Political Economy* 100, 859–887.

Mehra, R., and E. C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Paul, J., 1993, Crowding out and the informativeness of stock prices, *Journal of Finance* 48, 1475–1496.

Peles, N., 1992, Empirical evidence on institutional investor trading patterns, Working paper, Columbia University.

Scharfstein, D. S., and J. C. Stein, 1990, Herd behavior and investment, *American Economic Review* 80, 465–479.

Shleifer, A., and R. W. Vishny, 1990, Equilibrium short horizons of investors, *American Economic Review* 80, 148–153.

Sundaresan, S., 1989, Intertemporally dependent preferences and the volatility of consumption and wealth, *Review of Financial Studies* 2, 73–89.

Trueman, B., 1994, Analyst forecasts and herding behavior, *Review of Financial Studies* 7, 97–124.

Tversky, A., and D. Kahneman, 1974, Judgement under uncertainty: heuristics and biases, *Science* 185, 1124–1131.

Vives, X., 1994, Short-term investment and the informational efficiency of the market, forthcoming, *Review of Financial Studies*.

Wang, J., 1993, A model of intertemporal asset prices under asymmetric information, *Review of Economic Studies* 60, 249–282.