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Taking the road less traveled by: Does conversation eradicate pernicious cascades? ☆

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Abstract

We offer a model in which sequences of individuals often converge upon poor decisions and are prone to fads, despite communication of the payoff outcomes from past choices. This reflects both direct and indirect action-based information externalities. In contrast with previous cascades literature, cascades here are spontaneously dislodged and in general have a probability less than one of lasting forever. Furthermore, the ability of individuals to communicate can reduce average decision accuracy and welfare.
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1. Introduction

Social observers have often commented on the 'folly,' 'fickleness,' or 'madness' of mass behavior. This may implicitly reflect the fact that a high benchmark is appropriate for judging the *potential* accuracy of mass decisions. If large numbers of rational individuals were to communicate their imperfectly correlated information signals, we would expect convergence toward nearly perfect decision making.

A popular explanation for the failure to meet this benchmark is that people are imperfectly rational. An alternative explanation that has been explored in the literature on information cascades or herding is that the rational choices of individuals cause information to be aggregated poorly (see, e.g., Banerjee [2], Bikhchandani, Hirshleifer and Welch [4]).

In this approach, an individual's use of information gleaned from observation of predecessors makes his own action less informative to later observers. Such an individual will sometimes find it optimal to choose an action consistent with the choices or experience of predecessors regardless of his own (possibly opposing) private signal. In such a situation he is in an *information cascade*, and his action is uninformative to later individuals. Under appropriate conditions, a cascade eventually forms with probability one. Furthermore, if individuals are ex ante identical, and the only sources of information available to an individual is his private signal and past actions, then all subsequent individuals are also in a cascade, and information aggregation ceases. Thus, the system reaches an equilibrium in which the great mass of individuals make inefficient decisions. As a result, decisions are fragile: individuals are prone to shifting their behavior in the event of even a modest external shock.

The analysis of cascades has been generalized in various ways and applied in many fields, such as economics, politics, finance, law, corporate strategy, zoology, sociology, demography, organizational behavior.[2] However, in the basic cascades approach, individuals can observe the actions of predecessors but do *not* observe either past private information signals, or payoffs from past actions. Thus, models of cascades rule out by assumption some potentially profitable communication activity.

Such an assumption is reasonable in some contexts, such as when the relevant individuals are not personally acquainted, when decision outcomes resolve with long delay, or when conflicts of interest interfere with communication. However, in many situations verbal communication of the information or experiences of previous decision makers is important.[3] Conversation also influences individuals' financial investment decisions.[4]

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[2] Information cascades or related forms of herd behavior have been derived in models where the timing of action is endogenous, the ability to observe past actions is limited, or players have heterogeneous private values or reputational concerns (e.g., Chamley and Gale [15], Gale [20], Ottaviani and Sorensen [29,30], Celen and Kariv [10], Chari and Kehoe [11], Chamley [13], Guarino, Harmgart, and Huck [23], Cipriani and Guarino [12], Dasgupta and Prat [17]). Reviews of information cascades or herding theory, social learning, and applications include Bikhchandani, Hirshleifer and Welch [5], Brunnermeier [7], and Chamley [14].

[3] Such circumstances include both everyday choices people make among movies (see Shiller [31, Chapter 8]) or auto mechanics to the adoption of new technologies. Several empirical studies are discussed in Herr, Kardes and Kim [24].

[4] A survey by Shiller and Pound [32] asked individual investors what first drew their attention to the company whose stock they had purchased most recently. Almost all investors named sources which involved direct personal interaction; direct personal interaction was also important for institutional investors. Kelly and O'Grada [27] and Hong, Kubik and Stein [25] provide evidence that social interactions between individuals affect decisions about equity participation and other financial decisions.

On the other hand, if conversation perfectly conveyed *all* information (and individuals were rational and ex ante identical), everyone would become equally informed, and everyone would make equally good and highly accurate decisions. The unrealism of this implication suggests that assuming *perfect* communication is too extreme.

In this paper, we consider some plausible barriers to communication, and explore whether they clog the information pipeline so tightly that most individuals make inefficient decisions. We extend the cascades model of Bikhchandani, Hirshleifer and Welch [4] by allowing conversation to convey information about past action choices or payoffs, but not about the private reasons for decisions (i.e., private information signals). Payoff consequences of past actions are objective and measurable, whereas reasons for past actions are internal and subjective. Subjective information tends to be less credible because it is hard to verify directly. Furthermore, reasons are often less salient than payoffs, and are often hard to describe succinctly. Even someone who is inarticulate about reasons for choices can say whether the choice was a success. Thus, in our model, individuals observe past actions and receive private information about each project alternative. Simultaneously, there is a continual flow of public payoff information whose arrival depends on the project choice.

Allowing for some communication opens the possibility that individuals will eventually converge upon correct outcomes. It seems plausible that the continual arrival of payoff information (not allowed for in the standard cascades model) might eventually provide enough of a nudge to dislodge any temporary fixation on a mistaken outcome, thereby causing actions to become informative and induce further experimentation. With sufficient experimentation, the ability to observe past payoffs could potentially resolve all uncertainty about which action is superior and thereby bring about the correct action choice in the long run.

Our analysis shows that this intuition is not in general correct. Under a fairly broad set of circumstances, moderate limits to observability and communication cause individuals to fall into mistaken cascades. Even individuals very late in the decision queue make inefficient decisions frequently. The reason that efficiency is not achieved is that information aggregation is limited by two types of information externalities.

The first type, *direct action-based information externalities*, is also present in the basic cascades model: individuals fail to take into account the benefit that adjusting their actions in response to private signals confers upon later decision makers. In addition, in our model there are also *indirect action-based information externalities*, in which an individual's action choice affects the information of later individuals indirectly by generating informative payoff outcomes.[5] As William Blake puts it, "If others had not been foolish, we should be so" (*The Marriage of Heaven and Hell*, 1794).

In contrast with the basic cascades model, we find that cascades have a positive probability of being spontaneously dislodged—endogenously, not owing to an external shock. Thus, cascades in general have a probability between zero and one of lasting forever (i.e., of individuals continuing to act independently of their signals and following their immediate predecessors). There is a non-zero probability that individuals choose one of the possible actions only a finite number of times. If this happens, observation of payoffs eliminates uncertainty about the quality of the project chosen infinitely often, but uncertainty about the alternative project persists forever. So

[5] The information about payoff may be acquired by direct observation, or by conversation; our model does not distinguish these alternatives. Previous models in which there are information externalities associated with observation of past payoffs include Vives [33], Caplin and Leahy [8], and Bolton and Harris [6]. However, these papers do not explore how indirect action-based information externalities influence the formation of cascades.

individuals in the model will always have reason to wonder wistfully whether the other project, 'the road less traveled by,' was in fact the better one.

Finally, we explore whether the ability to observe past payoffs in addition to actions improves welfare and decision accuracy.[6] One might hope that even if a structure in which payoffs are observable does not maximize individuals' average expected payoffs, observing past payoffs would at least increase ex ante average welfare relative to not observing payoffs. However, we show this is not always the case. Observation or communication of past payoffs can trigger mistaken cascades earlier, by rendering the decisions of early individuals less informative (both directly and indirectly) to subsequent individuals. In consequence, delay in communication of payoffs can improve average decision quality and welfare.

Fixation upon a mistaken decision has also been previously analyzed in 'bandit' models of experimentation without private information. This literature has focused on the problem of insufficient *generation* of new public information (payoffs on the less-favored project). As in the bandit models, in our model individuals learn only about the payoffs of alternatives that are actually adopted. In contrast with traditional bandit models, but as in the multi-person bandit model of Bolton and Harris [6], we examine how the actions of an individual affect others.[7]

Our contribution relative to either the basic cascades approach or the multi-person bandit model is to see whether inefficiency can persist when there are much richer opportunities to learn from predecessors. In contrast with the basic cascades model, in our setting there is continual arrival of new payoff information, which in the long run resolves the true value of any project that is taken often enough. This raises the possibility that whenever people are stuck on the wrong choice, newly arriving signals can jostle them into further experimentation. In both the cascades and the bandit models, society can settle into the wrong choice, and informational shocks are not enough to dislodge it. We show that society still tends to settle upon mistaken actions despite fairly extensive information arrival and sharing. So the problem of information blockage identified in the theory of information cascades requires surprisingly mild limits to observation or communication.

Another strand of research that studies word-of-mouth learning (Ellison and Fudenberg [18,19]; Banerjee and Fudenberg [3]) has allowed for observation of payoff outcomes, but in settings that differ from ours in important respects (such as a continuum of agents, imperfect rationality, or small observation samples).[8] This literature describes conditions under which information aggregation is efficient. Given the several major modeling differences, it is not obvious that the greater communication of payoffs in the word-of-mouth models is the key difference as compared with the basic cascades model that promotes efficiency. Our analysis clarifies this issue by examining whether allowing observability of payoffs allows rational individuals move rapidly toward accurate decisions in the cascades model.

[6] Our focus is on the possibility that individuals are able to obtain *more* information (through conversation or observation) than in the basic cascades model. Some papers have analyzed information cascades or herd behavior when individuals observe less information about past actions than in the basic cascades model (Celen and Kariv [9], Guarino, Harmgart, and Huck [23]).

[7] In Bolton and Harris [6], there is a positive externality in generating new payoff information; trying a less-favored action alternative generates information that is useful for the other player. This externality results in too little experimentation. As a result, individuals can converge upon the wrong action.

[8] In the word-of-mouth learning papers with a continuum of decision makers there is no problem of insufficient experimentation. In these models every project is taken by an infinite number of individuals in the initial round. If the information generated thereby were aggregated efficiently, correct actions would result. Thus, the potential problem in the word-of-mouth learning literature is imperfect information aggregation rather than insufficient experimentation.

An experimental literature has verified that information cascades are common in the laboratory (see, e.g., Anderson and Holt [1], Hung and Plott [26], Celen and Kariv [9]). Studies also find biases in different directions in how individuals weight the actions of predecessors relative to personal private signals.[9] One reason it remains useful to analyze the fully rational case is that it provides a benchmark against which to measure the possible effects on social outcomes of irrationality. Furthermore, in some realistic settings participants have extensive experience and receive extensive feedback on the results of their actions, promoting rational behavior.

2. Conversation about payoffs and cascades: Definitions and an illustrative example

We assume that there is an exogenous sequence of individuals, each of whom decides which among a set of projects to adopt. Each individual learns (either through direct observation, or through conversation) the decisions of all those ahead of him. Each individual privately observes an identically distributed and conditionally independent signal about *expected* profitability of different action choices (projects). In addition, the individual may observe signals about the payoff outcomes resulting from the action choices of previous individuals.

Since our setting allows for greater observation and/or communication than in the original cascades models, we need to generalize slightly the definition of a cascade.

Definition. An **information cascade** occurs if, given observation of and communication with others, an individual follows the same action as an immediate predecessor independent of the individual's private signal.

Denote individual i by I^i. Since I^1 has no immediate predecessor, I^1 cannot be in a cascade. We also define the notion of a cascade continuing to multiple individuals, or lasting forever.

Definition. An information cascade *continues* if a sequence of consecutive individuals are in a cascade. An information cascade *lasts forever* if, starting with some individual I^i, all individuals I^j, where $j \geqslant i$, are in a cascade.

The first definition generalizes the notion of an information cascade to apply in a setting where individuals can observe more than just past actions, such as the payoff outcomes resulting from past action choices. In the special case of our setting where there is no acquisition of a signal about past payoff outcomes, Bikhchandani, Hirshleifer and Welch [4] show that if signals are discrete and likelihood ratios are bounded, then as the number of individuals becomes sufficiently large, the probability that an information cascade eventually starts approaches one. (Related results are provided by Banerjee [2], Welch [34] and Lee [28].) In the traditional models of information cascades with ex ante identical individuals, a cascade, once started, lasts forever in the absence of external shock. Our definition allows for the possibility that a cascade starts, but does not continue or last forever. Letting W^i denote the payoff of I^i and δ be the discount factor over time, we define the average realized payoff as

$$W(\delta) \equiv (1-\delta)\sum_{i=1}^{\infty} \delta^i W^i. \tag{1}$$

[9] Goeree et al. [21] find that people rely on their personal private signals too much relative to predecessors' actions, which tends to dislodge mistaken cascades. In contrast, in a different setting Goeree and Yariv [22] find that individuals rely too heavily on predecessors' actions relative to their own private signals.

Lemma 1. $\lim_{\delta \to 1} E[W(\delta)] = \lim_{i \to \infty} E[W^i]$.

Definition. An observation/communication structure is **long-run inefficient** if the choices made achieve lower $\lim_{\delta \to 1} E[W(\delta)]$ than when all information is aggregated optimally.

This definition therefore involves both expectations and averaging over individuals. If cascades lead to wrong decisions being made with non-negligible probability even in the long run, then decisions are long-run inefficient, or in the terminology of Bikhchandani, Hirshleifer and Welch [4], idiosyncratic. They also show that cascades are fragile with respect to public shocks in the sense that an exogenous additional public disclosure with modest precision will with substantial probability shift the behavior of the next individual.

If all information were aggregated efficiently, then almost surely the correct decision would be made by all but a finite number of individuals. More generally, we call a situation where all but a finite number of individuals make the correct choice 'long-run efficient.' We illustrate with an example that cascades can form in a regime with perfect observation/communication of past payoffs, even if these payoffs are perfect indicators of the future values. Suppose that there are two projects Y and Z. The payoff of project Y is either $v_Y = 0$ or 2, with probabilities μ and $1 - \mu$; the payoff of project Z is known to all, 1. Successive individuals receive a series of direct private signals about the value of project Y; conditional on the value of the project these value signals are i.i.d. Each signal has two possible realizations H and L, with $Pr(H|v_Y = 2) = p$, $Pr(H|v_Y = 0) = 1 - p$, $p > 1/2$.

Let $v_Y(\emptyset)$ be the prior mean payoff on Y, and let $v_Y(S)$ denote the posterior expected payoff on Y when a sequence of signals S is observed. We assume that $1/2 < \mu < p$, which implies that $v_Y(\emptyset) < 1 < v_Y(H)$ and $v_Y(L) < 1$.

If the first individual (I^1) obtains an L signal, which suggests that project Y is inferior to project Z, his action, Z, reveals his signal. Clearly, if the second individual (I^2) receives an L signal, he will also choose Z. I^2 will also choose Z if he receives an H signal, because the posterior mean payoff of Y after observing an LH sequence, $v_Y(LH)$, is the same as the prior mean, $v_Y(\emptyset) < 1$, the payoff of Z. So if I^1 chooses Z, a cascade on Z forms; I^2 chooses Z no matter what signal he receives. The resulting cascade is inefficient. The true payoff of Y could be higher than Z; if the information of many individuals were aggregated optimally, they would converge upon the correct decision with probability 1.

Learning about past payoffs of the chosen actions does nothing to break the cascade on project Z, because observation of its constant payoff tells successors nothing about the profitability of project Y (which is not chosen, hence there its payoffs are not observed). Thus, in the absence of interfering shocks, cascades on Z persist forever. If, on the other hand, I^1 selects Y, then all uncertainty about Y is resolved and the ensuing cascade is always correct.

Furthermore, the mistaken cascade is fragile with respect to exogenous shocks. If we introduce an exogenous release of a public signal of modest precision at a given date, then this news event can break the cascade and affect the behavior of many later individuals. Consider, for example, a single public signal with the same precision as the private signals that individuals receive. The arrival of a public signal realization of H on Y can shatter a cascade on Z. If the next individual also observes a private H signal, he chooses Y. If $v_Y = 2$, then thereafter all individuals choose Y instead of Z.

In this simple example, there can only be a mistaken cascade upon one of the projects, Z. Once such a cascade starts, it lasts forever (in the absence of exogenous shocks) and ends all experimentation. More generally, in the model of Section 3, there is a positive probability of an incorrect cascade on either project. Furthermore, there exist sample paths on which both projects are tried any number of times, yet mistaken cascades occur; and cascades have only a positive probability (less than one) of lasting forever.

3. Transient payoff shocks and noisy transmission of payoffs information

In the example in Section 2, all individuals who adopt the same project receive the same payoff. We now offer a general model that allows for transient payoff shocks, i.e., payoffs that are stochastic conditional on the project states. The project states are unknown to individuals but are constant over time. The true expected gain from each project is the same for all individuals, but two different individuals may gain different amounts from the same project.

We show that under mild distributional assumptions, even though payoffs are observable, eventually all individuals ignore their private information and choose identical actions. Mistaken cascades occur with positive probability and can last forever, leading to inefficient information aggregation. In our model, a given cascade endogenously does not necessarily last forever (without the intervention of exogenous shocks or shifts/heterogeneity in the characteristics of the decision makers). This finding differs from that of the standard cascades model.

We first consider the case in which communication of past payoffs is perfect. We then show that this analysis generalizes immediately to the case in which signals about past payoffs are noisy.

3.1. Transient payoff shocks

3.1.1. The economic setting

Let there be a sequence of individuals $i = 1, 2, 3, \dots$, with individual i denoted as I^i. Each individual must choose between two project alternatives. The state of project $\theta = Y, Z$ is u_θ or d_θ (up or down). The underlying state is unobservable to individuals and does not change over time. The project has two possible payoffs, $v_{\theta l} < v_{\theta h}$. The project payoff $\tilde{v}_\theta$ is stochastic conditional on the project's state, and its distribution depends on the state but not on the individual. The payoffs $\tilde{v}_\theta$ and the project states s_θ are independent across projects.[10] Throughout the paper, we assume that the likelihood ratio of payoffs from the two projects are not identical, and that the likelihood ratio of signals across each project's states are not identical.

Individuals are risk neutral and their decision rule depends only on the expected payoff of project. Let π_{s_θ} denote the prior probability that the state of project θ is s_θ. Conditional on a state s_θ, the probability that the payoff of project θ takes on value $v_{\theta k}$, $k = l, h$, is denoted $Pr(v_{\theta k}|s_\theta)$. Assume $Pr(v_{\theta k}|s_\theta) > 0$ for all θ and k. The expected payoff of project θ conditional on state s_θ is

$$\bar{v}_\theta(s_\theta) = Pr(v_{\theta h}|s_\theta)v_{\theta h} + Pr(v_{\theta l}|s_\theta)v_{\theta l}.$$

We impose a no long-run ties assumption that if individuals learn enough about value, then they are not indifferent between the two project alternatives.

[10] It is straightforward to extend the model, including Propositions 1 and 2 to the case in which the number of projects, the number of possible values from each project and the number of signal values are more than two; we have verified this explicitly (proofs available upon request). However, for notational simplicity, we present here the binary case.

Assumption 1 *(No long-run ties).* If $\theta \neq \theta'$, then $\bar{v}_\theta(s_\theta) \neq \bar{v}_{\theta'}(s'_{\theta'})$, where $\bar{v}_\theta(s_\theta)$ denotes the expected payoff of project θ given that the state of project θ is s_θ.

Individuals observe predecessors' actions and learn the payoff experiences for all previously-selected projects. I^i observes a vector of direct private signals $\tilde{\sigma}^i = (\tilde{\sigma}^i_Y, \tilde{\sigma}^i_Z)$, where $\tilde{\sigma}^i_\theta$ is a signal about the state of project θ, with possible values L_θ and H_θ, where the θ subscript is omitted when the project being referred to is clear. These signals are independent across projects, and (conditional on state) are independent and identically distributed across individuals.

To solve the game, we apply perfect Bayesian equilibrium. Let a^i be I^i's action and let $A^i = (a^1, \ldots, a^i)$ represent the history of actions taken by individuals $1, 2, \ldots, i$. Let b^i be the *payoff* from I^i's action and let $B^i = (b^1, \ldots, b^i)$ represent the history of payoffs from the action of individuals $1, 2, \ldots, i$. I^i's conditional expected payoff from adopting project θ given his own signal realization $\sigma^i = \sigma = (\sigma_Y, \sigma_Z)$ and the history A^{i-1}, B^{i-1} is

$$V^i_\theta\left(A^{i-1}, B^{i-1}, \sigma\right) \equiv E\left[v_\theta \middle| \sigma^i = \sigma, A^{i-1}, B^{i-1}\right].$$

$V^i_\theta(A^{i-1}, B^{i-1}, \sigma)$ can be computed as a weighted average of $\bar{v}_\theta(s_\theta)$ over all states s_θ of project θ, where the weights are I^i's posterior probabilities of the states conditional on his information set $(A^{i-1}, B^{i-1}, \sigma)$. I^i adopts project θ if the conditional expectation of project θ is larger than that of the other project. When there is a tie, we follow the convention that I^i chooses project Y. (Similar results would obtain under randomization among tied projects.)

3.1.2. Implications

In this setting, mistaken and long-run inefficient cascades occur.

Proposition 1. *If Assumption* 1 *holds, and individuals can observe both actions and payoffs of adopted actions, then as the number of individuals increases*:

(i) *with probability one a cascade starts that lasts forever*;
(ii) *mistaken cascades occur with positive probability*;
(iii) *there is a positive probability that a mistaken cascade, having occurred, lasts forever*; *and*
(iv) *if payoffs are subject to nondegenerate transient shocks, then the probability that a given cascade lasts forever is less than one.*

Intuitively, for Part (i), as the number of times a project is chosen becomes larger and thereby there are more and more observations of its payoffs, the public information pool becomes increasingly informative about the true state of the project. Suppose it were the case that no cascade lasted forever. Then eventually both projects would be taken an infinite number of times, and repeated observation of payoffs would perfectly and publicly resolve their states. Of course, in any finite amount of time there is always a positive probability that payoff outcomes zigzag back and forth without ever resolving clearly the states, or that the payoff outcomes are misleading; but as the number of observations becomes large, in the limit the probability of such events is zero. With states resolved, since there are no long-run ties, everyone would cascade on the better project forever almost surely, which contradicts the assumption that no cascade lasts forever.

For Part (ii), an incorrect cascade is possible because at the point at which an individual optimally conforms with the evidence provided by the actions and payoffs of predecessors despite an opposing private signal, he is still uncertain about which alternative is better. For example,

Table 1
Payoff and signal probabilities.

	$Pr(v_\theta = 1 \mid u_\theta)$	$Pr(v_\theta = 1 \mid d_\theta)$	$Pr(H_\theta \mid u_\theta)$	$Pr(H_\theta \mid d_\theta)$
$\theta = Y$	0.8	0.2	0.8	0.2
$\theta = Z$	0.7	0.3	0.6	0.4

early payoffs may indicate a high expected value for project Z relative to project Y when in fact the true expected payoff of Y is higher.

Part (iii) indicates that with positive probability a mistaken cascade continues forever. For example, suppose the true mean value of project Y (given the actual state) is higher, but if individuals cascade upon project Z, they do not generate new information about Y. This information blockage can prevent people from ever discovering the superiority of Y. At any point in time, if Z is actually in its good state (which occurs with positive probability), individuals will tend to continue receiving good news about Z, in ignorance of the even higher true mean payoff of project Y. Thus, there is a positive conditional probability that no further switch ever occurs once a cascade starts.

Part (iv) indicates that the cascade can be broken endogenously, because project payoff provides a noisy indication of the underlying state. With a long enough sequence of low payoff outcomes for the adopted project, a cascade, whether correct or not, will be broken.

3.1.3. Numerical example of mistaken and long-run inefficient cascades with transient payoff shocks

The following example illustrates more concretely how mistaken cascades can occur, and how there is a probability between zero and one of a mistaken cascade lasting forever.

Projects Y and Z each have possible payoffs of 0 or 1. Each project can be in two states, which are equally likely. Conditional on the state, the payoff distribution and conditional expected payoffs are described in Table 1.

Table 1 indicates that $Pr(v_Y = 1|u_Y) = 0.8$, $Pr(v_Y = 1|d_Y) = 0.2$, $Pr(v_Z = 1|u_Z) = 0.7$, $Pr(v_Z = 1|d_Z) = 0.3$. So for a given state, the conditional expected payoff from the projects are equal to these probabilities, $\bar{v}_Y(u_Y) = 0.8$, $\bar{v}_Y(d_Y) = 0.2$, $\bar{v}_Z(u_Z) = 0.7$, $\bar{v}_Z(d_Z) = 0.3$. Thus the no tie condition is satisfied. Project Y's ex ante expected payoff is more sensitive to state than Z's.

Prior to his project choice, each individual receives private signals regarding the state of the two projects, $Pr(H_Y|u_Y) = 0.8$, $Pr(H_Y|d_Y) = 0.2$, $Pr(H_Z|u_Z) = 0.6$, $Pr(H_Z|d_Z) = 0.4$. The first individual I^1 has 4 possible information outcomes based on high versus low signals about each project. Since the signal on project Y is more precise than that of project Z, and project Y has higher payoff sensitivity, I^1 chooses Y if and only if his Y-signal is H. I^2 learns the payoff received from the project that I^1 adopted, as well as his own private signals about Y and about Z. Therefore, conditional on I^1's project choice, I^2 has 2^3 possible observation/communication outcomes.

Now suppose that the true states are u_Y and u_Z, so that Y is superior. If I^1 receives an L signal about Y, he chooses Z. Suppose he obtains a payoff of 1 from Z. I^2 infers from I^1's action that the first signal on project Y was L. So even if the I^2's private signal on Y is H, his conditional expectation of Y is 0.5. Thus, I^2 chooses Z even if he receives an L signal about it; the favorable payoff of I^1 is such a favorable indicator (precision 0.7) that it outweighs his own private signal (precision 0.6). Therefore I^2 is in an incorrect cascade—his action is incorrect (relative to the true state), it is independent of his private signal, and his action matches that of his immediate predecessor.

There is a positive probability that the cascade will be overturned as later individuals learn about earlier payoffs from adopted projects. If a sufficiently long sequence of 0 payoffs from Z occurs, the cascade will be overturned, since the posterior expectation of project value will approach 0.3. Nevertheless, since the true mean of Z is 0.7, the probability that the posterior mean of Z will stay above 0.5 forever is positive. With positive probability, the mistaken cascade lasts forever, so information aggregation is long-run inefficient.

3.2. No transient shocks

We have shown that mistaken cascades can occur and can last forever when individuals can observe actions and payoffs, and that these cascades have a positive probability of being broken. The possibility of cascades being dislodged spontaneously contrasts with the result of Banerjee [2] and Bikhchandani, Hirshleifer and Welch [4]. In their settings, if all individuals are identical and there are no interfering shocks, a cascade once started lasts forever. A similar outcome applies in our model in the special case in which payoffs depend on the state non-stochastically. In this special case, once a cascade starts (which by definition implies that at least one predecessor has already adopted the project), observing further identical payoffs does not reveal any new information. With no further information arriving, all succeeding individuals join the cascade. This proves the following corollary.

Corollary 1. *If payoffs are non-stochastic functions of state and individuals observe all past actions and project payoffs, then a cascade once started lasts forever.*

When payoffs are non-stochastic functions of state, mistaken cascades are highly fragile in the sense that a small shock (such as a new public information arrival) can easily shift the long run outcome. If the shock persuades just one individual to try the alternative project, then everyone thereafter shifts to the better project. An individual can be sure that he is in a correct cascade only if both projects have been tried previously or if the payoff of current project is higher than the highest possible value from the untaken project.

3.3. Noisy communication of payoffs

So far we have assumed that payoffs are perfectly observable. Proposition 1 and the corollary generalize immediately to the case in which individuals observe noisy signals about past project payoffs.

Suppose that each individual obtains (through conversation or direct observation) a set of noisy signals about the realized payoffs of previously-chosen projects. Suppose further for each project, the distribution of the payoff signals is not identical across states. Thus, although the signals are noisy, they are useful for learning about the project states and hence the true mean payoffs of the projects. Individuals do not know the true mean payoffs of the projects for two reasons. First, the realized payoffs are noisy indicators of the true mean because of the transient payoff shocks. Second, the signals that the individuals obtain by conversation are noisy indicators of the realized past payoffs. For any two-stage garbling of the project states resulting from stochastic payoffs and signal noise in payoffs, there is an isomorphic model in the setting of perfect communication of payoffs where the probability distribution of payoffs creates an equivalent garbling. Therefore, the same arguments in proving Proposition 1 can be applied to establish the following:

Proposition 2. *Suppose that Assumption* 1 *holds, individuals can observe past actions, decision payoffs are stochastically transient and that the observation or communication of payoffs is noisy. Suppose further that the distribution of the payoff signals is not identical across states. Then as the number of individuals increases*:

(i) *the probability that a cascade eventually starts and lasts forever approaches one*;
(ii) *incorrect cascades occur with positive probability*; *and*
(iii) *there is a positive probability less than one that a given cascade lasts forever.*

4. Observation/communication of payoffs can reduce average welfare and decision accuracy

Long-run efficient decision making is a stringent benchmark. It requires that almost everyone does the right thing. One might hope that even if a structure in which payoffs are observable is not long-run efficient, observing payoffs at least increases ex ante average welfare relative to not observing payoffs. The example below shows that this is not always the case.

Consider two projects Y and Z. The payoff of Y is 0 and $2+\epsilon$ with equal probability, and the payoff of Z is $1+\epsilon$, $1-\epsilon$ with equal probability, where $\epsilon > 0$ is a sufficiently small quantity. Each individual observes a signal about Y. Each signal has two possible realizations H and L, with $Pr(H|v_Y = 2+\epsilon) = p$, and $Pr(H|v_Y = 0) = 1-p$, where $p > 1/2$. There is no signal about Z.[11] We compare the ex ante expected welfare outcomes of two scenarios. In the first, individuals do not observe any payoffs from the adopted projects. In the second scenario, individuals can observe payoffs from the Z project if it is adopted but individuals cannot observe payoffs from project Y.[12]

Assumption 2. ϵ is sufficiently close to zero,

$$\epsilon < \frac{2p-1}{2-p}.$$

Assumption 2 ensures that if all individuals had to make decisions independently, they would adopt project Y without any signals or with an H signal, even if they observe the payoff of Z is high, and adopt Z if an L signal about Y is received, even if they observe the payoff of Z is low.

In Appendix A, we prove the following result:

Result 1. In the example, allowing individuals to observe payoffs of adopted projects on average makes individuals worse off. Outcomes with observable payoffs are long-run inefficient.

Allowing each individual to observe the payoff history of predecessors has two potentially opposing effects. On the one hand, it provides an individual with valuable information which can

[11] We can allow individuals to observe a very noisy signal of Z without affecting any of the results.

[12] Similarly, we can allow individuals to observe a very noisy signal of the predecessor's realized payoff from adopting project Y without affecting any of our results here.

be used to improve decisions. For example, the additional payoff information may show that a particular action history is not as compelling as the actions alone indicate. This can help increase expected efficiency by breaking an existing cascade. On the other hand, when payoff information reinforces the action history, it can cause individuals to settle into cascades even earlier than they would have otherwise. Such cascades tend to reduce the information content of the earlier individuals' actions, reducing expected efficiency. Whether learning about payoffs makes later individuals better or worse off thus depends on the tradeoff between the information conveyed directly by payoffs and the possible reduction of information resulting from the alteration in previous actions.

Interestingly, if observation/communication of payoffs is delayed, later individuals may sometimes obtain the benefit of observing payoffs without the cost of reduced informativeness of predecessors' actions and payoffs, making the average individual better off. We verify this is the case when we modify the example above by assuming that the payoffs of project Z can be observed with one period delay.

Result 2. In the example, delaying the observation/communication of payoffs improve the average welfare.

According to Alexander Pope[13]

A little learning is a dang'rous thing;
Drink deep, or taste not the Pierian spring:
There shallow draughts intoxicate the brain,
And drinking largely sobers us again.

Taking observation of past actions as a starting point, 'drinking largely' in our setting would be observation of all past private signals, which leads to correct decisions in the long run. A shallow draught of learning, observing past payoffs (but not past signals), can be harmful. Observing payoffs only with delay is an even shallower draught of learning. In our setting shallower can be better than shallow.

The rise of mass media and of interactive communication technology (printing, telegraph, telephone, television, email, the web) have made it easier to observe the payoff outcomes of others (or summary statistics of past payoffs). The analysis here suggests that the resulting improvements in decisions may not be as great as might have been expected. The recipient of extra information (beyond what he would have obtained by observing past actions) gains directly from the ability to make a more accurate decision. However, this benefit may be opposed by the fact that the actions of the recipient may be less informative to later decision makers.

5. Conclusion

We have examined how efficiently society aggregates information when individuals make decisions in sequence, observe past actions, and observe or communicate the payoffs resulting from past action choices. In previous models of cascades or herding, society is trapped in mistaken

[13] *Essay on Criticism*, 1711, Part 2, Lines 215–218, from *Representative Poetry Online*, http://rpo.library.utoronto.ca/display/index.cfm.

choices until an external shock arrives to dislodge the cascade. Here, individuals can *endogenously* talk their way out of mistaken cascades by learning about the past payoff outcomes.

If mistaken cascades are with probability one eventually dislodged, then in the long run individuals make accurate choices. However, our analysis indicates that even when project payoffs are communicated, there is still a positive chance that a mistaken cascade lasts forever, because of the combination of a direct action-based information externality (each individual ignores the information benefit to others of basing his action on his private signal), and an indirect action-based externality (each individual ignores the fact that taking a less-chosen action provides useful payoff-outcome information to others). Society is harmed by a lack of experimentation on 'the road less traveled by.'

Furthermore, we provide examples which show that the ability to observe/communicate past payoffs can, ex ante, reduce expected average welfare. Similarly, delay in observation or communication of payoffs can increase ex ante expected welfare. Delay or non-communication can help because they can encourage individuals to aggregate more information before cascades clog the flow of information. Taken together, the findings of this paper indicate that the problem of information blockage through information cascades requires surprisingly little in the way of limits to observation or communication.

Appendix A

Proof of Lemma 1. Let $W_0 \equiv \lim_{i\to\infty} E[W^i]$. Since $E[W^i] \to W_0$, for all $\epsilon > 0$, there exists an integer N such that $|E[W^i] - W_0| < \epsilon$ for all $i \geqslant N$. Thus,

$$\begin{aligned}\left|E\left[W(\delta) - W_0\right]\right| &= \left|(1-\delta)\sum_{i=1}^{\infty} \delta^{i-1}\left(E\left[W^i\right] - W_0\right)\right| \\ &\leqslant \left|(1-\delta)\sum_{i=1}^{N-1} \delta^{i-1}\left(E\left[W^i\right] - W_0\right)\right| + \delta^{N-1}\epsilon.\end{aligned}$$

Let $\bar{M} \equiv \sum_{i=1}^{N-1} \delta^{i-1}|E[W^i] - W_0|$, and let $\delta' \equiv 1 - (\epsilon/\bar{M})$. Then for all $1 > \delta > \delta'$, $|E[W(\delta) - W_0]| < 2\epsilon$. Since ϵ is arbitrary, we have $\lim_{\delta\to 1} E[W(\delta)] = W_0$. □

Proof of Proposition 1. We consider the case of two projects; the same conclusions hold for any number of projects, by similar reasoning. For clarity, we review our notation here. Individuals $i = 1, 2, 3, \ldots$ each choose between projects $\theta = Y, Z$. Project θ has state u_θ or d_θ (up or down), which is unobservable and fixed. The two possible project payoffs are $v_{\theta h} < v_{\theta l}$. The project payoff $\tilde{v}_\theta^i$ is stochastic conditional on the project's state; its distribution depends on the state but not on the individual. Let π_{s_θ} denote the prior probability that the state of project θ is s_θ. Conditional on a state s_θ, the probability that the payoff of project θ takes on value $v_{\theta k}$, $k = l, h$, is denoted $Pr(v_{\theta k}|s_\theta)$. The expected payoff of project θ conditional on state s_θ is

$$\bar{v}_\theta(s_\theta) = Pr(v_{\theta h}|s_\theta)v_{\theta h} + Pr(v_{\theta l}|s_\theta)v_{\theta l}.$$

(i): Assertion (i) in Proposition 1 states that a cascade eventually forms with probability one. Without loss of generality, suppose that the true state is (u_Y, u_Z) and the expected Y payoff in state u_Y exceeds the expected Z payoff in state u_Z, i.e., $\bar{v}_Y(u_Y) > \bar{v}_Z(u_Z)$. Let F_i denote the public information set available to I^i, i.e., all past project choices and payoffs. l^i_θ denote the likelihood ratio conditional on F^i for the two states of project θ just before I^i's action,

$$l^i_\theta = \frac{Pr(d_\theta | F^i)}{Pr(u_\theta | F^i)}. \tag{2}$$

Given this public likelihood ratio, the probability that project θ is in the up state is $1/(1 + l^i_\theta)$.

Using public information, I^i with likelihood ratios l^i_Y, l^i_Z will be indifferent between the two projects Y, Z if

$$f\left(l^i_Y, l^i_Z\right) \equiv \frac{\bar{v}_Y(u_Y)}{1 + l^i_Y} + \frac{l^i_Y \bar{v}_Y(d_Y)}{1 + l^i_Y} - \left(\frac{\bar{v}_Z(u_Z)}{1 + l^i_Z} + \frac{l^i_Z \bar{v}_Z(d_Z)}{1 + l^i_Z}\right) \tag{3}$$

is zero. The first two terms in f represent the conditional expected payoff from project Y based on public information and the next two terms in f represent the conditional expected payoff from project Z based on public information.

Since l^i_Y, l^i_Z is a martingale given state u_Y, u_Z, by the martingale convergence theorem (see Chung [16]), there exists a random vector (l^∞_Y, l^∞_Z) such that (l^i_Y, l^i_Z) converge to (l^∞_Y, l^∞_Z) with probability one as i goes to ∞.

At least one project will be taken infinitely often. Suppose that Y is taken infinitely often. Let ω denote a sample path such that (l^i_Y, l^i_Z) converges. Then along the path ω, for any $\epsilon > 0$, there exists an integer i' such that $|l^i_Y - l^\infty_Y| < \epsilon$ for all $i > i'$. Since by assumption $l^\infty_Y \neq \infty$, we will show that $l^\infty_Y = 0$ (i.e., d_Y is ruled out). To see this, suppose that $l^\infty_Y > 0$, then after the observation of an informative payoff from project Y, l^i_Y would drop by some non-negligible amount. It follows that for sufficiently small ϵ, the arrival of a new payoff observation will cause l^i_Y to fall outside the ϵ neighborhood of l^∞_Y for all i sufficiently large. This contradicts the fact that l^i_y converges along the sample path. Similarly if Z is taken infinitely often, then $l^\infty_Z = 0$ (i.e., d_Z is ruled out).

Next we show that for all but a set of sample paths with measure zero, only one of projects is taken infinitely often. The reason is that for sample paths on which (l^i_Y, l^i_Z) converges and both projects are taken infinitely often, $(l^i_Y, l^i_Z) \to (0, 0)$. But for all i sufficiently large, I^i will adopt project Y, since $f(0, 0) = \bar{v}_Y(u_Y) - \bar{v}_Z(u_Z) > 0$. This contradicts the premise that both projects Y and Z are taken infinitely often. Thus on sample paths in which both projects are taken infinitely often, (l^i_Y, l^i_Z) will not converge. By the martingale convergence theorem, such sample paths have a measure of zero. Thus with probability one all individuals eventually cascade on the same decision regardless of their private information.

(ii): Assertion (ii) states that an incorrect cascade can occur with positive probability. From Assertion (i), we know that with probability one, individuals will start to cascade on either project Y or Z. Without loss of generality, we assume project Y has the highest possible of the state-conditional expected payoffs, i.e., $\bar{v}_Y(u_Y) > \bar{v}_Z(u_Z) > \bar{v}_Y(d_Y)$. Suppose that individuals cascade on project Y given a series of signals and project payoffs. The same set of signals and payoffs has a positive probability of occurring in state (d_Y, u_Z), hence the correct choice should be Z. Similarly, suppose individuals cascade on project Z given a series of signals and project payoffs. The same combination of signals and payoffs has a positive probability of being observed in state (u_Y, d_Z), hence the correct choice should be Y. Consequently, an incorrect cascade occurs with positive probability.

(iii): In this part, we prove that mistaken cascades can start and last forever even when all payoffs resulting from adopted projects are publicly observable. From Parts (i) and (ii), we know that with positive probability a cascade starts. Without loss of generality, suppose that the true state is (u_Y, u_Z), with $\bar{v}_Y(u_Y) > \bar{v}_Z(u_Z)$, but a cascade has formed on project Z. We prove that such an incorrect cascade can last forever, despite the observations of payoffs from the chosen project Z.

Suppose an incorrect cascade on project Z starts with I^i. It must be true that $\mu_Z^i > \mu$ where μ_Z^i is the conditional expected payoff of project Z chosen by individual i and μ is the expected value of project Y given the most favorable possible signal values.

$$\mu_Z^i = \pi_Z^i(u_Z)\bar{v}_Z(u_Z) + \pi_Z^i(d_Z)\bar{v}_Z(d_Z), \tag{4}$$

where $\pi_Z^i(u_Z)$, $\pi_Z^i(d_Z)$ are the conditional probabilities that project Z is in the high or low state. These conditional probabilities are related to l_Z^i, the likelihood ratio of the down state versus the up state of project Z (see (2)):

$$\pi_Z^i(u_Z) = \frac{1}{1+l_Z^i}, \qquad \pi_Z^i(d_Z) = \frac{l_Z^i}{1+l_Z^i}.$$

Thus, we can rewrite Eq. (4) as

$$\begin{aligned}\mu_Z^i &= \left(\frac{1}{1+l_Z^i}\right)\bar{v}_Z(u_Z) + \left(\frac{l_Z^i}{1+l_Z^i}\right)\bar{v}_Z(d_Z) \\ &= \bar{v}_Z(d_Z) + \left(\frac{1}{1+l_Z^i}\right)\big[\bar{v}_Z(u_Z) - \bar{v}_Z(d_Z)\big].\end{aligned} \tag{5}$$

The expected payoff μ_Z^i is a decreasing function of the likelihood ratio l_Z^i. The likelihood ratio evolves with the observed payoffs from the adopted project Z. If the payoff from project Z adopted by I^i is $\tilde{v}_Z^i = v_{Zk}$, $k = h$ or l, then

$$l_Z^{i+1} = l_Z^i\left(\frac{Pr(v_{Zk}|d_Z)}{Pr(v_{Zk}|u_Z)}\right),$$

which implies that

$$\log\big(l_Z^{i+1}\big) = \log\big(l_Z^i\big) + \log\left(\frac{Pr(v_{Zk}|d_Z)}{Pr(v_{Zk}|u_Z)}\right).$$

Letting $x_Z^{i+1} \equiv \log(l_Z^{i+1}/l_Z^i)$ denote the change in the log-likelihood ratio,

$$\begin{aligned}E\big[x_Z^{i+1}|u_Z\big] &= E\left[\log\left(\frac{Pr(v_{Zk}|d_Z)}{Pr(v_{Zk}|u_Z)}\right)\Big|u_Z\right] \\ &< \log\left(E\left[\frac{Pr(v_{Zk}|d_Z)}{Pr(v_{Zk}|u_Z)}\Big|u_Z\right]\right) \\ &= \log\left(\left[Pr(v_{Zh}|u_Z)\frac{Pr(v_{Zh}|d_Z)}{Pr(v_{Zh}|u_Z)} + Pr(v_{Zl}|u_Z)\frac{Pr(v_{Zl}|d_Z)}{Pr(v_{Zl}|u_Z)}\right]\right) \\ &= \log\big[Pr(v_{Zh}|d_Z) + Pr(v_{Zl}|d_Z)\big] \\ &= \log[1] \\ &= 0,\end{aligned} \tag{6}$$

where the inequality follows from Jensen's inequality.

Note that the same argument can be used to show that $E[\log(l_Z^{j+1})] < E[\log(l_Z^j)]$ for all $j \geqslant i$. Thus, the log-likelihood ratio $\log(l_Z^j)$ follows a generalized random walk with a downward drift. This implies (see Chung [16, p. 263]) that with positive probability $l_Z^j < l_Z^i$, for all $j > i$. By Eq. (5), $\mu_Z^j > \mu_Z^i > \mu$ for all $j > i$, as the expected payoff is a decreasing function of the likelihood ratio. Thus, with positive probability the expected payoff of project Z is always larger than the expected value of project Y under the most favorable possible signal values for Y and an incorrect cascade can last forever.

(iv): Suppose that the conditional expected payoff from the down state of the adopted project θ^* is strictly below the maximum expected payoff of all other projects conditional, for each of those projects, on the observation by the next individual of one more signals that take on their best possible values. Then a long series of adverse payoffs can cause individuals to believe that project θ^* is in the down state. Since there is a positive probability that the most favorable signal for any other given project is received, a cascade can break. Thus, the probability a cascade lasts forever is less than one. □

Proof of Result 1. No Communication/Observation of Payoffs. Suppose individuals cannot observe or communicate past payoffs. First consider the case that project Y's payoff is $2 + \epsilon$. If I^1 receives an H signal which occurs with probability p, he chooses project Y. I^2 chooses project Y even if he receives an L signal because $E[v_Y|HL] = 0.5(2 + \epsilon)$ which is higher than the prior mean of project Z. There is a Y cascade immediately. If the signal sequence is LL, then I^1 and I^2 both adopt Z. I^3 chooses project Z independently of his signal and thus a Z cascade forms. Finally, if the sequence of the signals is LH, then I^1 chooses Z and I^2 chooses Y. I^3 now faces the same situation as I^1. Starting after the first two individuals, the probability of a project Y cascade is p, the probability of a Z cascade is $(1 - p)^2$, and the probability of no cascade is $p - p^2$. Consequently, the ratio of the likelihoods of a project Y cascade and a project Z cascade is $p/(1 - p)^2$. Since a cascade will form with probability one, the long-run probability of a project Y cascade is

$$\frac{p}{1 - p + p^2},$$

and the long-run probability of a project Z cascade is

$$\frac{(1 - p)^2}{1 - p + p^2}.$$

The same reasoning applies when the payoff of project Y is zero. Individuals' actions are the same as above for a given sequence of signals. A Y cascade occurs after I^1 receives an H signal, and a Z cascade forms after two L signals. Starting after the first two individuals, the probability of a project Y cascade is $1 - p$, the probability of a Z cascade is p^2, and the probability of no cascade is $p - p^2$. The long-run probability of a Y cascade is

$$\frac{1 - p}{1 - p + p^2},$$

and the long-run probability of a project Z cascade is

$$\frac{p^2}{1 - p + p^2}.$$

Averaging over the two cases, the expected payoff of I^i as $i \to \infty$ converges to

$$W_0 = 0.5\left[\left(\frac{p}{1-p+p^2}\right)(2+\epsilon) + \frac{(1-p)^2}{1-p+p^2}\right] + 0.5\left(\frac{p^2}{1-p+p^2}\right).$$

Communication/Observation of Payoffs. In contrast, if individuals can learn the payoffs of project Z, a cascade forms immediately after one round if the payoff of project Z is $1+\epsilon$. This is because if the signal is H, I^1 chooses Y and a cascade on Y occurs. There is no observation of project Z thereafter. If the signal is L, Z will be adopted. Even if I^2 observes H signal, he still chooses project Z since $E[v_Y|LH] = 0.5(2+\epsilon)$ is less than the observed payoff $1+\epsilon$ of Z.

If the payoff of project Z is $1-\epsilon$, it is straightforward to show that under the parameter restriction Assumption 2, the analysis is the same as the case above without payoff observation. Consequently, as $i \to \infty$, the expected payoff of I^i converges to

$$E[W_p] = 0.25\big[p(2+\epsilon) + (1-p)(1+\epsilon)\big] + 0.25p(1+\epsilon)$$
$$+ 0.25\left[\left(\frac{p}{1-p+p^2}\right)(1+2\epsilon) + 1 - \epsilon\right] + 0.25\left[\left(\frac{p^2}{1-p+p^2}\right)(1-\epsilon)\right].$$

We therefore have

$$W_0 - E[W_p] = 0.25\left[\frac{p(1-p)^2}{1-p+p^2}\right]\left(\frac{p}{1-p} - 1 - \epsilon\right) > 0.$$

The last inequality follows by the assumed parameter restriction. □

Proof of Result 2. The assumption here is that the payoff of project Z is observed with one period delay. Let (v_Y, v_Z) denote the payoffs of project Y and Z. We calculate the conditional expected payoffs given the states and then determine the unconditional expectations. There are four possible payoff combinations:

Case I. $(2+\epsilon, 1+\epsilon)$. If the first signal is H, a Y cascade immediately since the payoffs of Y are not observable. The probability of such a cascade is p.

If the first signal is L, I^1 adopts project Z. However, unlike the case when payoff of project Z is observed without delay, now a cascade does not always form on project Z. Instead, a Y cascade occurs with positive probability.

There are two subcases to consider. If I^2 receives another L signal, he adopts project Z. A Z cascade forms since I^3 further observes the past payoffs of project Z and adopts project Z regardless of his signal.

If the signal sequence is LH, then I^2 adopts project Y because he does not observe the payoff of project Z, and $E[v_Y|LH]$ is higher than the prior mean of project Z. I^3 infers the signal sequence from previous individuals' actions and he also observes that the payoff of project Z is $1+\epsilon$. If I^3 receives an H signal, he chooses project Y, and a Y cascade occurs if I^4 observes another H signal. If I^3 receives an L signal, he chooses project Z and so will I^4 regardless of his signals. Then a cascade on Z occurs. The ratio of the likelihoods of a project Z cascade and a project Y cascade is $(1-p)/p^2$. Therefore, conditional on signal sequence LH, the probability of a Z cascade is $(1-p)/(1-p+p^2)$ and the probability of a Y cascade is $p^2/(1-p+p^2)$.

The total probability of a Y cascade is

$$p_Y = p + p(1-p)\frac{p^2}{1-p+p^2} = \frac{p - p^2(1-p)^2}{1-p+p^2}.$$

Thus, the payoff of an individual sufficiently far along in the sequence is, in the limit,

$$E[W_I] = \frac{p - p^2(1-p)^2}{1-p+p^2} + 1 + \epsilon.$$

The expected payoffs of individuals far down the line for the other three cases can be derived similarly.

Case II. $(0, 1+\epsilon)$.

$$E[W_{II}] = \frac{1 - p - p^2(1-p)^2}{1-p+p^2}(-1-\epsilon) + 1 + \epsilon.$$

Case III. $(2+\epsilon, 1-\epsilon)$.

$$E[W_{III}] = \frac{p}{1-p+p^2}(1+2\epsilon) + 1 - \epsilon.$$

Case IV. $(0, 1-\epsilon)$.

$$E[W_{IV}] = \frac{p^2}{1-p+p^2}(1-\epsilon).$$

Finally, the expected payoff of an individual who is late in the sequence in the limit is

$$\begin{aligned} E[W_d] &= 0.25 E[W_I + W_{II} + W_{III} + W_{IV}] \\ &= 0.25\left\{\left[\frac{p - p^2(1-p)^2}{1-p+p^2} + 1 + \epsilon\right]\right. \\ &\quad + \left[\left(\frac{1 - p - p^2(1-p)^2}{1-p+p^2}\right)(-1-\epsilon) + 1 + \epsilon\right] \\ &\quad \left. + \left[\left(\frac{p}{1-p+p^2}\right)(1+2\epsilon) + 1 - \epsilon\right] + \left[\left(\frac{p^2}{1-p+p^2}\right)(1-\epsilon)\right]\right\}. \end{aligned}$$

It is straightforward to show that

$$E[W_d] - W_0 = 0.25\left[\frac{p^2(1-p)^2}{1-p+p^2}\right]\epsilon > 0.$$

Therefore,

$$E[W_d] > W_0 > E[W_p]. \qquad \square$$

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