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Asset pricing in production economies with extrapolative expectations

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ABSTRACT

Introducing extrapolative bias into a standard production-based model with recursive preferences reconciles salient stylized facts about business cycles (low consumption volatility, high investment volatility relative to output) and financial markets (high equity premium, volatile stock returns, low and smooth risk-free rate) with plausible levels of risk aversion and intertemporal elasticity of substitution. Furthermore, the model captures return predictability based upon dividend yield, *Q*, and investment. Intuitively, extrapolative bias increases the variation in the wealth–consumption ratio, which is heavily priced under recursive preferences; adjustment costs decrease the covariance between marginal utility and asset returns. Empirical support for key implications of the model is also provided.

1. Introduction

During the millennial high tech boom, the U.S. economy grew rapidly, and expectations among many investors about future growth were higher than subsequent realizations. In contrast, after the credit crisis of 2008, growth has been low and pessimistic expectations for future growth have been prevalent. This raises the questions of whether there is a general tendency for individuals to overextrapolate recent economic growth, and if so what effect this has on consumption and asset pricing.

Evidence from both psychology and finance indicates that extrapolative bias is pervasive in human judgement and decisions (see, e.g., Gilovich et al., 1985; Hirshleifer, 2001; Barberis and Thaler, 2003; Fuster et al., 2010). In laboratory experiments, Tversky and Kahneman (1974) provide evidence consistent with individuals following the representativeness heuristic, wherein observations are perceived as being more indicative (representative) of population distributions than they really are. This results in the so-called 'law of small numbers', a belief updating process whereby individuals overweight small numbers of observations. In an investment setting, this would imply that when investors see a firm realizing high earnings growth, for example, they may classify it as a growth firm and discount inadequately for the regression phenomenon.

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Empirically, several field and experimental studies find that the trading of individual and professional investors seems to reflect extrapolation of past performance.[1] Theoretical models and discussions have also emphasized how extrapolation can affect capital market behavior. In the model of Barberis et al. (1998), the representativeness heuristic causes overreaction anomalies in the stock market. Fuster et al. (2010) suggest that extrapolation is important for understanding macroeconomic fluctuations. Indeed, Barberis (2011) proposes that overextrapolation may explain the 2008 credit crisis.

As is well known, production-based asset pricing models face an even greater challenge than endowment-based models in explaining consumption and asset return behavior, as such models allow greater scope for endogenous consumption and dividend smoothing (see, e.g., Rouwenhorst, 1995; Jermann, 1998; Lettau and Uhlig, 2000; Boldrin et al., 2001).[2] It has been recognized for some time that relaxing the assumption of perfect rationality might help explain macroeconomic and financial empirical puzzles.[3] Recently, Fuster et al. (2010, 2011) argue that quasi-rational models deserve greater attention.

In this spirit, our paper introduces extrapolative expectations into a standard dynamic stochastic general equilibrium (DSGE) model featuring recursive preferences to study the implications of the model for asset prices, consumption, investment, and output. Specifically, the model assumes that the true average productivity growth is unobservable, and that the representative individual has to estimate it from historical data. The individual uses a smoothed average of past realized technology growth to estimate future technology growth, with greater weight on the most recent growth realization.

Introducing extrapolative expectations greatly improves upon traditional rational models in matching key stylized facts about both asset prices and macroeconomic quantities. Specifically, our model produces large and volatile excess stock market returns and low and smooth risk-free rates, with a relative risk aversion (RRA) of four and a preference intertemporal elasticity of substitution (IES) of two. Moreover, the model can replicate the predictability of excess market returns by the price–dividend ratio, Tobin's Q, and investment rates, consistent with known evidence. Importantly, extrapolative expectations also improves the model's ability to match the relative volatilities of investment growth.

The intuition for the high equity premium in our model is straightforward. First, extrapolation of past growth trends cause excess volatility in productivity growth expectations. This increases the volatility of investment, and since variations in investment are driven by excessive volatility in perceived productivity, this results in excessive perceived volatility of the consumption growth rate. Second, owing to recursive preferences, this variation in the perceived consumption growth rate (i.e., the long-run risk) is heavily priced.

Finally, with reasonably high adjustment costs, the limited flexibility of investment tends to direct the payoff from high realized productivity growth more towards dividends and consumption rather than investment. This reduces marginal utility of consumption in the event of a favorable productivity shock. Since the positive productivity shock is associated with a high asset return, higher adjustment costs tend to decrease the covariance between return and marginal utility of consumption, thereby increasing the equity risk premium. Thus, taken together, the combination of extrapolation bias, adjustment costs, and recursive preferences can generate a large equity premium.

All three of adjustment costs, extrapolation and recursive preferences contribute to this outcome. Without capital adjustment costs, consumption is excessively smoothed, because after high growth realizations, firms would heavily reinvest cash flows, reducing the payouts and consumption or making them countercyclical; this would reduce equity risk. Without extrapolation bias, the perceived volatility of investment and consumption growth would be too small to explain a high equity premium. Finally, without recursive preferences, the time-variation in perceived consumption growth is not priced, and thus the equity premium is small.

The intuition for conditional return predictability derives from extrapolative expectations and overreaction. By a standard argument, overreaction results in higher volatility in the stock market and the wealth–consumption ratio, and predictability of stock returns by valuation ratios and investment rates.

As observed by Barlevy (2004) and Lansing (2012), a rational model with capital adjustment costs faces difficulty in generating sufficient investment volatility. Owing to capital adjustment costs, investment growth in the rational model exhibits about the same volatility as output growth, whereas investment growth in the data is about two times more volatile than output growth. In our model, the excess volatility of perceived productivity growth causes excess volatility of investment, thereby improving the fit with investment volatility in the data.

Lustig et al. (2013) document that both the wealth–consumption ratio and the return on the consumption claim are volatile, a challenge for traditional leading asset-pricing models.[4] Our paper shows that extrapolation can help produce high

[1] See, e.g., Smith et al. (1988), Benartzi (2001), Haruvy et al. (2007), Greenwood and Nagel (2009), and Choi et al. (2010). In survey evidence, Case and Shiller (1988) report higher expectations of future housing price growth in cities that have experienced past recent price growth. Using both survey and experimental data, de Bondt (1993) finds that the forecasts of individual investors satisfy a simple trend-following mechanism. Vissing-Jorgensen (2003) provides survey evidence that investors who have experienced high portfolio returns in the past expect higher returns in the future. Furthermore, Ederington and Golubeva (2010) find that mutual fund investors reallocate toward stock funds after stock price increases, and into bond funds after bond price increases.

[2] Several endowment-based asset-pricing models can successfully match the first two moments of the excess stock market return and the risk-free rate (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006). However, the reconciliation of asset markets with aggregate quantities has proved to be a challenge for DSGE models.

[3] Early notable studies include de Long et al. (1990a,b) and Barsky and de Long (1993).

[4] In a recent paper, Ai (2010) proposes a learning model in production economy which can account for the dynamics of the wealth consumption ratio. However, Ai (2010) does not address the conditional moments of the stock returns or the quantities.

volatility for both the wealth consumption ratio and the return on the aggregate wealth, again owing to excessive variation in expectations about technological growth.

Two key refutable implications of the model are also investigated. First, owing to overextrapolation, investor expectations of future productivity growth is predicted to forecast future returns negatively. Second, investment rates are predicted to be positively contemporaneously correlated with perceived productivity growth. Using several proxies for perceived productivity growth, we provide empirical evidence supporting these implications.

Several previous studies examine the effects of extrapolative expectations. In a partial equilibrium model, Barsky and de Long (1993) show that extrapolation in estimating expected dividend growth contributes to volatility in price–dividend ratios. Cecchetti et al. (2000), Choi (2006), Lansing (2006), and Bansal and Shaliastovich (2010) study the effects of recency bias in an exchange economy.[5] They show that extrapolative bias can help explain a high equity premium and high stock market volatility. Fuster et al. (2010) study the implications for macroeconomic fluctuations of natural expectations (a weighted average of rational and extrapolative expectations) in an endowment economy with constant relative risk aversion (CRRA) preferences. Barberis et al. (2015) study a consumption-based asset pricing model and find that the stock price extrapolative bias captures many features of actual prices and returns, as well as the survey evidence on investor expectations. By embedding the "this time is different" learning bias (a bias akin to extrapolation) and overlapping generations into otherwise standard macro-finance models, Collin-Dufresne et al. (2015b) show that this bias can be a key determinant of the joint dynamics of macro-aggregates and asset prices. Empirically, de Bondt and Thaler (1985), Poterba and Summers (1988), Lakonishok et al. (1994), and La Porta et al. (1997) provide evidence of overreactions, which suggests that extrapolation can help explain stylized facts about predictability of aggregate market returns and the cross-section of stock returns (but see also Daniel and Titman, 2006).

Our paper differs from all of these studies in that our paper studies the effects of extrapolation on both quantities and prices simultaneously. Our production-based model has implications for both price-related variables such as the equity premium and interest rates, and for quantity variables such as consumption growth, output growth, and investments. The model can therefore be subject to stringent calibration to see whether it can accurately reflect the real-world data on both macroeconomic quantities and capital market prices simultaneously. As mentioned earlier, it is more challenging to produce a large equity premium when consumption can be endogenously smoothed by firms' investment decisions as in our model.

Our approach builds on a growing literature on long-run risk, especially as applied to production economies. Bansal and Yaron (2004) demonstrate that in an endowment economy with long-run risk in consumption and recursive preferences, consumption and asset-price properties can be reconciled with moderate risk aversion and an IES greater than one. Our paper differs in examining a production economy, so that aggregate consumption is endogenous, and in using a lower risk aversion coefficient (which is arguably more realistic) in our calibration. Tallarini (2000) works with a representative agent in a production economy with recursive preferences, but his model focuses on the case of a fixed IES and no capital adjustment costs. He shows that even with high risk aversion, his model has implications for macroeconomic quantities comparable to those obtained by Kydland and Prescott (1982). The production economy of Tallarini (2000) can generate a high Sharpe ratio with an high risk aversion. His model, however, generates a low equity premium.

The most closely related paper to this one is Kaltenbrunner and Lochstoer (2010) (KL (2010) hereafter), who show that long-run consumption risk can be endogenously generated even if the technology is i.i.d.. Our model extends that of KL (2010) by introducing extrapolation; their model is the special case of ours in which there is no extrapolative bias. KL (2010) features an IES larger than one and can produce a high Sharpe ratio with relatively low risk aversion. However, the volatility of equity returns is still very low, and hence this approach does not replicate the high equity premium found in the data. Another related study is Croce (2014), who studies a model featuring long-run productivity risk directly. As compared with Croce (2014), extrapolation bias in our model reduces the predictability in realized consumption growth to a more realistic level, produces a larger equity premium with lower risk aversion, and generates asset return predictability as well.

An earlier literature studies asset prices in a production economy with habit preferences, including influential papers by Jermann (1998), Lettau and Uhlig (2000), and Boldrin et al. (2001). Methodologically, our paper is closely related to Jermann (1998), who finds that the combination of capital adjustment costs and habit preferences can generate a low risk-free rate, a high equity premium, high volatility of excess returns, and high relative investment and low consumption volatility. More recently, Campanale et al. (2010) show that a production economy with convex capital adjustment costs and disappointment aversion can produce a high equity premium as well.

These models typically feature a very low IES, and hence imply excessively high volatility for the risk-free rate. This tends to result in an abnormally large term premium. Our model produces a low volatility for the risk-free rate and high volatility for the equity returns simultaneously.[6]

[5] Recency bias in Bansal and Shaliastovich (2010) endowment-based model is somewhat akin to extrapolation. In their setting, investors receive exogenous signals about consumption growth rather than simply observing past realizations, and overweight recent signals in forming their forecasts. In contrast, in our model investors update beliefs after observing past productivity realizations. Their quantitative analysis is based upon a very strong recency bias (50% weight on the latest signal), whereas in our analysis the extrapolation bias puts 2–5% weight on the latest observation. Also, their focus is on confidence risk (second moments) rather than the effects of learning per se.

[6] There is also a large literature examining the role of Bayesian learning in asset markets (e.g., Timmerman, 1993, 1996; Veronesi, 1999; Brennan and Xia, 2001; Brandt et al., 2004).

In sum, our paper shows that incorporating extrapolative bias into a standard production-based equilibrium asset pricing model substantially improves the model's ability to match both macroeconomic quantities and asset prices.

2. A production-based model with extrapolation

This section presents a production-based equilibrium model with extrapolative expectations to examine the joint dynamics of consumption, investment, output, and asset prices. For simplicity, the model assumes a representative agent economy. In the special case where information is complete and there is no extrapolative bias, our model is the same as the permanent shock model of KL (2010). We thus follow their notations in setting up the model.

2.1. Household's preferences

The terms investor, individual, and household are interchangeably used to refer to the representative household. Following the long-run risk literature, the representative household's preferences over the uncertain consumption stream C_t are described by the Epstein–Zin–Weil recursive utility function (e.g., Epstein and Zin, 1989; Weil, 1989)

$$V_t = \left[\left(1-\beta\right)C_t^{(1-\gamma)/\theta} + \beta\left(\hat{\mathbf{E}}_t V_{t+1}^{1-\gamma}\right)^{1/\theta}\right]^{\theta/(1-\gamma)}, \tag{1}$$

where $\hat{\mathbf{E}}_t(\cdot)$ is the expectation under the individual's subjective belief conditional on information available up to time t, the parameter $0 < \beta < 1$ is the time discount factor, $\gamma \geq 0$ is the risk-aversion parameter, $\psi \geq 0$ is the intertemporal elasticity of substitution (IES) preference parameter, and

$$\theta = \frac{1-\gamma}{1-\dfrac{1}{\psi}}.$$

The sign of θ is determined by the values of risk aversion and IES. When the risk aversion parameter exceeds the reciprocal of IES, the individual prefers early resolution of the uncertainty of consumption path. Hence, these preferences allow for a preference over the timing of the resolution of uncertainty.

The Euler equation describing the representative individual's optimization holds under the individual's belief, which, owing to extrapolative bias, generically does not match the true probability distribution. Thus, the pricing kernel is (e.g., Bansal and Yaron, 2004)

$$m_{t+1} \equiv \log(M_{t+1}) = \theta \log\beta - \left(\frac{\theta}{\psi}\right)g_{t+1} + (\theta-1)r_{a,t+1}, \tag{2}$$

where $r_{a,t+1}$ is the logarithm of the gross return on an asset that delivers aggregate consumption as its dividends each period. For any continuous return $r_{t+1} = \log(R_{t+1})$, including the one on the consumption claim

$$\hat{\mathbf{E}}_t[\exp(m_{t+1} + r_{t+1})] = 1. \tag{3}$$

The expectation operator $\hat{\mathbf{E}}_t(\cdot)$ applies to the individual's biased subjective belief; this is the key difference from a rational expectations model.

2.2. Productivity, capital accumulation, and belief update

There is a representative firm owned by the representative household, and the output, Y_t, is produced by a constant return-to-scale neoclassical production function

$$Y_t = \left(A_t L_t\right)^{1-\alpha} K_t^{\alpha}, \tag{4}$$

where $L_t \equiv 1$ is the normalized labor supply,[7] A_t is the production-enhancing technology, and the capital level, K_t, evolves as

$$K_{t+1} = (1-\delta_K)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t, \tag{5}$$

where I_t is the investment in period t, δ_K is the rate of depreciation of the capital, and $\phi(\cdot)$ is a concave function from Jermann (1998) that allows for convex capital adjustment costs

[7] In other words, the model assumes an exogenous wage process such that it is optimal for the firm to always hire at full capacity ($L_t = 1$). In this case, one can show that the operating profit function of the representative firm is linearly homogenous in capital (see KL, 2010).

$$\phi\left(\frac{I_t}{K_t}\right) = a_1 + \frac{a_2}{1-\frac{1}{\xi}}\left(\frac{I_t}{K_t}\right)^{1-(1/\xi)}, \quad \xi > 0. \tag{6}$$

The adjustment cost is parameterized inversely by ξ. The constants a_1 and a_2 are set such that there are no adjustment costs in the nonstochastic steady state (see, also Boldrin et al., 2001; KL, 2010). The adjustment cost allows the shadow price of installed capital to diverge from the price of an additional unit of capital, and hence it permits variation in Tobin's Q. The aggregate resource constraint is

$$Y_t = C_t + I_t,$$

where C_t is the aggregate consumption. Labor is paid at its marginal product. Thus, wages, ω_t, and firm dividend payouts, D_t, satisfy $\omega_t = (1-\alpha)Y_t$, and $D_t = \alpha Y_t - I_t$, respectively. Letting the total factor productivity (TFP) growth rate be denoted by

$$g_{A,t} = \log\left(\frac{A_t}{A_{t-1}}\right),$$

the dynamics of the data-generating process for the productivity growth is assumed to satisfy

$$g_{A,t+1} = \mu_A + \sigma_A \epsilon_{A,t+1}, \tag{7}$$

where $\epsilon_{A,t+1}$ is i.i.d. standard normal. It is assumed that σ_A is known to the representative individual, but that the true growth rate in productivity μ_A is not observable. In practice, it is much easier to estimate the variance than the mean (see, e.g., Merton, 1980).

The individual is subject to extrapolative bias and updates his perceived growth rate at time t for period $t+1$, $\hat{\mu}_t$, as

$$\hat{\mu}_t = (1-\rho-\tilde{\rho})\bar{\mu} + \rho\hat{\mu}_{t-1} + \tilde{\rho} g_{A,t}, \tag{8}$$

where $\tilde{\rho}$ reflects the degree of overextrapolation of the most recent growth shock, ρ reflects the persistence of extrapolations made from past growth shocks, and $\bar{\mu}$ is the long-run mean in the individual's belief. In our calibration, $\bar{\mu}$ is equal μ_A, the true expected rate of productivity growth. So by Eq. (7), the individual believes that productivity growth follows

$$g_{A,t+1} = \hat{\mu}_t + \sigma_A \hat{\epsilon}_{A,t+1}, \tag{9}$$

where the individual perceives $\hat{\epsilon}_{A,t+1}$ to be i.i.d. standard normal.

By Eq. (8), the individual takes an average of recent past productivity growth with geometrically declining weights and projects that growth rate forward to forecast the future. The steepness of the decline (measured inversely by ρ) can be viewed as the degree of myopia in updating. If ρ is small and $\tilde{\rho}$ is large, the individual extrapolates placing a heavy weight on recent realizations of technological growth rates. On the other hand, when ρ is close to one, the individual places heavy weight on distant past growth rates. In a sense, when ρ is small and $\tilde{\rho}$ is large, the individual is both extrapolative and myopic. Finally, in the special case where $\tilde{\rho} = 1-\rho$, the above setting is similar to that of Barsky and de Long (1993). Moreover, in this case, the extrapolative learning matches the 'constant-gain' learning rule, which is popular in adaptive learning literature (see Sargent, 1993).

The learning scheme in (8) is quite intuitive as a way of capturing overextrapolation. A possible motivation derives from structural breaks in productivity growth. If individuals believe that a structural break might have occurred, they put less weights on distant past observations and more weights on recent observations. Another possible motivation for the updating rule in (8) is time-varying expected productivity growth. Indeed, Barsky and de Long (1993) present an example in which the true long-run growth rate is a random walk, and the optimal estimation of the expected growth rate exactly follows Eq. (8) with $\tilde{\rho} = 1-\rho$. They use this specification to study excess stock market volatility in a partial equilibrium framework. In our general equilibrium framework, to ensure stationarity and the finiteness of asset prices, $\rho + \tilde{\rho}$ is always fixed at $0.9999 < 1$ in our calibration.[8] Therefore, in the calibrations with overextrapolation, the degree of overextrapolation is measured inversely by a single parameter, ρ.

Lastly, in principle, based on productivity realizations investors could eventually learn to correct their mistaken extrapolative belief as given in Eq. (8). However, as is standard in behavioral models, investors are assumed to be subject to psychological bias, and are not able to completely 'learn their way out' of their bias. Psychological literature suggests that such imperfect learning can be a consequence of inherent cognitive constraints, or of overconfidence. Also, it could be argued that investors are even less rational than assumed in our model, and in particular do not fully understand the structure of the economy in which they participate. This is probably the case, but for reasons of parsimony we believe it is useful, at least as a first step, to try to understand the consequences of a single deviation from rationality before studying more complicated combinations of psychological effects.

[8] If $\rho + \tilde{\rho} = 1$, the perceived growth rate is a random walk as shown in (10). Under some preferences, it is possible that the value function is infinite. To rule out this possibility, $\rho + \tilde{\rho}$ is set to be less than one.

2.3. Model solution

Solving the model numerically is straightforward. Since the quantities in the economy are cointegrated with the aggregate productivity and the problem is homogeneous in A_t, variables are first scaled by the aggregate productivity. Then the value function is solved with the usual value function iteration. Readers can refer to KL (2010) for details on the numerical solution. The only difference from the standard rational model is that, under the perception of the individual, by (9) the dynamics of the state variable, $\hat{\mu}_t$, are

$$\hat{\mu}_{t+1} = (1-\rho-\tilde{\rho})\mu_A + \rho\hat{\mu}_t + \tilde{\rho}g_{A,t+1} = (1-\rho-\tilde{\rho})\mu_A + (\rho+\tilde{\rho})\hat{\mu}_t + \tilde{\rho}\sigma_A\hat{\epsilon}_{A,t+1}. \tag{10}$$

Thus, $\rho+\tilde{\rho}$ determines the persistence of the perceived technological growth rate under the individual's own belief. Thus, the perceived growth is quite persistent. This property is not unique to our extrapolative learning scheme. In an i.i.d growth economy with Bayesian learning on the true mean growth rate, Collin-Dufresne et al. (2015a) show that in the agent's filtration, the mean expected consumption growth rate is time-varying with a unit root.

Once the value function is solved numerically, variables of interest can be obtained. For example, from Epstein and Zin (1989), the log wealth–consumption ratio is

$$wc_t \equiv \log\left(\frac{W_t}{C_t}\right) = \log\left(\frac{1}{1-\beta}\right) + \left(1-\frac{1}{\psi}\right)\log\left(\frac{V_t}{C_t}\right). \tag{11}$$

Following a standard argument of Cochrane (1991), the return on investment is

$$R_{I,t+1} = \phi'(I_t/K_t)\left[\alpha\left(\frac{A_{t+1}}{K_{t+1}}\right)^{1-\alpha} + \frac{1-\delta_K+\phi(I_{t+1}/K_{t+1})}{\phi'(I_{t+1}/K_{t+1})} - \frac{I_{t+1}}{K_{t+1}}\right]. \tag{12}$$

The log return on investment is therefore $r_{I,t} = \log(R_{I,t})$. Notice that the return on investment is the same as the return on the equity claim.[9] Finally, it follows from Epstein and Zin (1989), the risk-free rate can be calculated numerically as

$$r_{f,t} = -\log\left[\hat{\mathbf{E}}_t\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(\frac{V_{t+1}(K_{t+1},\hat{\mu}_{t+1},A_{t+1})}{\hat{\mathbf{E}}_t\left(V_{t+1}^{1-\gamma}(K_{t+1},\hat{\mu}_{t+1},A_{t+1})\right)^{1/(1-\gamma)}}\right)^{(1/\psi)-\gamma}\right)\right]. \tag{13}$$

In calibration, results on levered equity market returns, $r_{E,t}$, are reported. Following Boldrin et al. (1995) and Croce (2014), our model introduces constant financial leverage, and the levered excess return is defined as $r_{E,t+1} - r_{f,t} \equiv (r_{I,t+1} - r_{f,t})(1+B/E)$, where B/E is the average debt–equity ratio. B/E is set to 2/3 since the actual debt to equity ratio is around 2/3 (see, e.g., Benninga and Protopapadakis, 1990). Alternative ways to introduce leverage are also discussed in Section 5.

2.4. The basic idea

The ability of the model to reconcile a high equity premium, a low risk-free rate, and a smooth consumption growth with low risk aversion comes from the way that extrapolative bias interacts with recursive preferences. Extrapolation bias causes excessive variation in perceived productivity growth, which in turn induces volatile fluctuations in investors' expectations of consumption growth. With recursive preferences, fluctuations in expected consumption growth are priced.

Following Epstein and Zin (1989), the pricing kernel can be rewritten as

$$m_t \approx \hat{\mathbf{E}}_{t-1}(m_t) - \gamma\hat{\epsilon}_{c,t} - \frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\hat{\epsilon}_{wc,t}, \tag{14}$$

where $\hat{\epsilon}_{c,t}$ is the short-run shock in consumption growth, and $\hat{\epsilon}_{wc,t}$ is the shock in the log wealth–consumption ratio. These shocks are derived under the subjective belief since it is the investor perception that determines asset prices. Under log-linear approximation in the spirit of Campbell et al. (2003) and Bansal and Yaron (2004), shocks to the wealth–consumption ratio can be approximated by

$$\hat{\epsilon}_{wc,t} \approx (E_t - E_{t-1})\sum_{j=1}^{\infty}\kappa_1^j(1-1/\psi)\Delta c_{t+j}, \tag{15}$$

where $\kappa_1 = \frac{W/C-1}{W/C}$ and W/C is the unconditional mean of the wealth–consumption ratio. Thus, long-run risk comes from shocks to the wealth–consumption ratio, or shocks to expected future consumption growth. To generate a highly volatile

[9] As in KL (2010), one can show that the conditions in Restoy and Rockinger (1994) are satisfied, and thus the investment return and the stock return are the same.

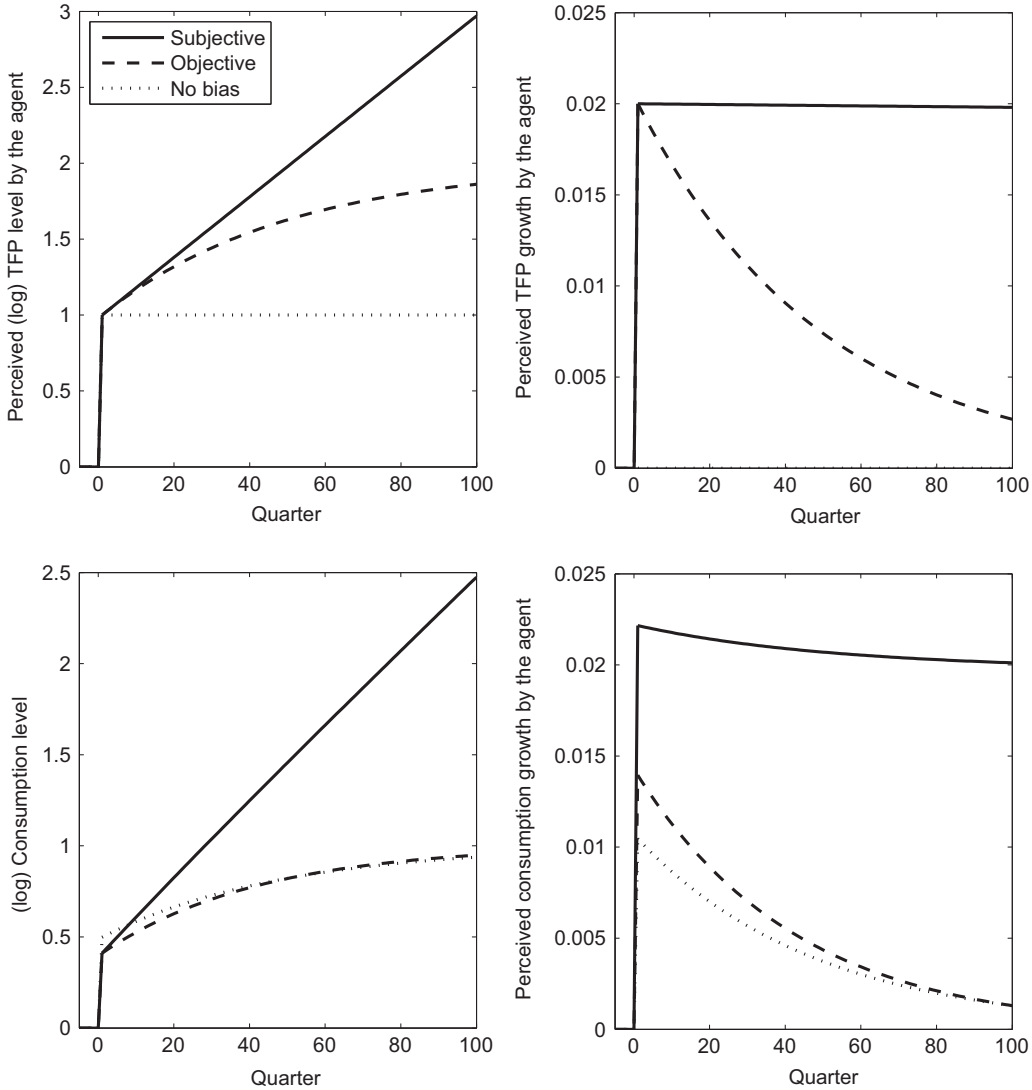

Fig. 1. Impulse response functions. This figure plots the impulse response functions for the perceived (log) TFP level and perceived TFP growth by the agent, (log) consumption level, and perceived consumption growth by the agent to a positive one-standard-deviation TFP growth shock from the model under the subjective and objective measures with and without extrapolative bias. The impulse response functions are estimated from the log-linearization as in KL (2010) under the benchmark parameterization. The process of each variable is expressed using lag operators, and the coefficient of the order T in the Taylor expansion around zero represents the impulse response in quarter T. The top left panel plots the response function for the agent's perception of the (log) TFP level. The top right panel plots the response function for the perceived TFP growth by the agent. The bottom left panel presents the response function for the (log) consumption level, and the bottom right panel presents the response function for the perceived consumption growth by the agent. The solid and dashed lines are for result under the subjective and objective measures, respectively, for the specification where there is extrapolative bias. The dotted line is for the specification where there is no extrapolative bias.

pricing kernel, which is a prerequisite for matching evidence of a high equity premium, our model needs a persistent and volatile expected consumption growth under the subjective belief.

In the supplementary material,[10] we follow the approximation argument in KL (2010) and provide an approximate analytical solution for the model. The key findings from the log-linearization approximation are summarized as follows. In contrast with an $ARMA(1, 1)$ process for consumption growth and an $AR(1)$ process for the expected consumption growth in KL (2010), when there is extrapolative bias the consumption growth is approximately an $ARMA(2, 2)$ and the perceived expected consumption growth follows an $ARMA(2, 1)$ process under both the subjective and the objective measures. In particular, it is shown that both endogenous capital accumulation (just as in KL, 2010) and investor's misperception on the persistence of the expected TFP growth (i.e., Eq. (10)) contribute to the perceived persistence of consumption growth. Although the productivity shocks are i.i.d. in our model, a persistent and predictable component in consumption growth

[10] The supplementary material is available at http://www.users.cla.umn.edu/~jianfeng/Extrapolative_IA.pdf.

(i.e., long-run risk) is endogenously generated as a consequence of consumption smoothing, the same as in KL (2010). More importantly, owing to extrapolation-induced overreaction, the volatility of the perceived expected consumption growth tends to increase as the extrapolation bias becomes more severe (i.e., as $\tilde{\rho}$ increases).

To illustrate this mechanism more clearly, the impulse response functions for (log) perceived TFP level, perceived TFP growth, (log) consumption, and perceived consumption growth are plotted under both subjective and objective measures with and without extrapolative bias in Fig. 1. Fig. 1 displays the impulse response functions to a positive one-standard-deviation shock to TFP growth. First consider the impulse response functions when the representative agent is subject to extrapolative bias. Because $\rho + \tilde{\rho}$ is close to one in our benchmark calibration, the expected TFP growth is very persistent under the subjective belief (the top two panels). This is in contrast with a less persistent TFP growth rate under the objective measure, whose dynamics are the weighted average of the perceived and realized TFP growth (i.i.d.). The endogenous consumption responses are plotted in the bottom two panels. Due to the difference in persistence and volatility in the perceived TFP growth under these two measures, the expected consumption growth is also more volatile and more persistent under the subjective measure than under the objective measure. Since asset prices are determined by the dynamics under the subjective measure, our model is able to produce a volatile pricing kernel.

Our next step is to compare the cases with and without extrapolative bias. Because the underlying process for the TFP is random walk, there is no predictable component in TFP when the individual is rational. A positive realized TFP shock increases the TFP level permanently, but does not change the expected TFP growth. More interestingly, when there is extrapolation bias, the consumption response to TFP shocks is weaker than in the case of no extrapolation bias as in KL (2010). This is because higher perceived TFP growth induces more investment now and less current consumption. However, the perceived future consumption growth by the agent would be higher with extrapolation bias than without extrapolation bias. Therefore, the introduction of extrapolative bias substantially increases the amount of long-run consumption risks under the subjective measure.

Again, since asset prices are determined by the dynamics under the subjective measure, the pricing kernel in our extrapolation model is more volatile than that in a standard long-run risk model (KL, 2010). At the same time, the smaller consumption response with extrapolative bias allows us to increase the capital adjustment cost, which helps raise the volatility of stock returns or the quantity of risk in the economy. In addition, by comparing the impulse responses of consumption under the objective measure with and without the extrapolative bias, it is seen that the consumption dynamics under these two situations are similar and the expected consumption growth under those two cases is less volatile and less persistent than that under the subjective measure with extrapolation bias. Since the realized consumption is generated by the dynamics under the objective measure, it is smooth relative to the output. In sum, these features combined explain why our model generates a large and volatile equity premium while keeping a smooth consumption process relative to the output.

The supplementary material provides more detailed analysis regarding the dynamics of consumption growth and returns under both the subjective and objective beliefs using the log-linear approximation. It also provides a more detailed approximation analysis of how extrapolation bias raises the volatility of the pricing kernel. Here we now proceed to numerical analysis of model calibration.

3. Calibration

This section examines different versions of the model to explore the importance of the different model assumptions for explaining stylized facts about asset pricing while replicating salient business cycle evidence about output, consumption and investment volatility. In addition, this section also examines the conditional performance of the model such as the

Table 1
Parameter choices for benchmark calibration. This table reports the parameter choices for the benchmark calibration. The model is simulated in quarterly frequency. These parameters are in quarterly frequency as well. The first five parameters are fixed for all of our calibration, whereas the last five parameters vary across calibrations.

Statistics	Variable	B.I
Fixed parameter:		
Mean technology growth (%)	μ_A	0.4
Volatility of the innovation in technology growth (%)	σ_A	4.11
Share of capital	α	0.36
Depreciation (%)	δ_K	0.021
Leverage	B/E	2/3
Varying parameter:		
Extrapolation parameter	ρ	0.98
Risk aversion	γ	4.00
IES	ψ	2.00
Capital adjustment cost	ξ	1.50
Subjective discount factor	β	0.991

return predictability by price–dividend ratio, investment, and aggregate Q. As is standard in the real business cycle literature, the model is calibrated at a quarterly frequency. Since the model is in real and per capita form, all calibration is done with real, per capita empirical counterparts.

3.1. Parameter choices

Table 1 reports the parameter values used for our benchmark calibration. Most of the parameter values are borrowed from the real business cycle literature. Following Boldrin et al. (2001), the capital share (α) is set to a value of 0.36, the quarterly depreciation rate (δ_K) is set at 0.021, and the quarterly average log productivity growth rate (μ_A) is fixed at 0.4%. This set of parameters is chosen to match the long-run growth rate of the economy; these parameters do not substantially affect model dynamics. Finally, the volatility of productivity growth is fixed at $\sigma_A = 0.041$, the same value as in KL (2010), to match observed output volatility since 1929. As our model is an exogenous growth model, all endogenous variables in the long run grow at the same rate as productivity.

Mehra and Prescott (1985) suggest that the conventional range of risk aversion should be less than 10. Our benchmark calibration therefore chooses $\gamma = 4$. This risk aversion coefficient is lower than that typically employed in the existing asset pricing literature. Following Ai (2010) and Croce (2014), the IES (ψ) is fixed at 2, which is consistent with estimates of Attanasio and Vissing-Jorgensen (2003), Bansal et al. (2007), Bansal et al. (2007), and van Binsbergen et al. (2012). For example, the estimated IES ranges from 1.73 to 2.09 in van Binsbergen et al. (2012).[11]

Empirical studies do not offer precise guidance for calibrating the pure time discount factor (β), capital adjustment costs (ξ), and extrapolative bias (ρ). Given the central role played by these parameters for business cycles and asset returns, reasonable ranges for them are examined to verify whether the model can match the empirical moments.

For example, the time discount parameter β is chosen to keep the level of interest rates low. For the capital adjustment cost parameter ξ , KL (2010) choose $\xi = 18$ and 0.7 for two baseline models, while Jermann (1998) and Boldrin et al. (2001) choose $\xi = 0.23$. As will be shown below, reducing the value of ξ (i.e., raising the adjustment costs) tends to improve the performance of our model. An intermediate value of 1.5 is chosen for our benchmark calibration. In addition to the benchmark calibration, a detailed sensitivity analysis is performed to provide us with insights about the model mechanisms at work.

Since the more innovative feature of the model, overextrapolation, is reflected inversely in the parameter ρ, results for different values of the extrapolation parameter are reported with benchmark value of $\rho = 0.98$. First, bearing in mind that extrapolative learning here is basically the same as the constant-gain learning in macro literature, a value of $\rho = 0.98$ is consistent with existing studies. Orphanides and Williams (2005), for example, choose $\rho = 0.98$ to match the inflation forecasts from the Survey of Professional Forecasters (SPF). Milani (2007) estimates a DSGE model on several macroeconomic time series, and find that $\rho = 0.9817$ fits the data the best. More recently, using micro-level forecast data, Malmendier and Nagel (2015) also confirm that $\rho \approx 0.98$ is a good approximation for aggregate-level extrapolation parameter.

An alternative way to gauge the extrapolation parameter is to use survey data on productivity or GDP growth. The volatility of productivity growth is 0.22% based on the SPF 10-year productivity median forecast. However, this long-term survey starts with 1992, a well-known stable period (great moderation) compared to the full sample, despite the recent financial crisis. For example, the volatility of realized GDP growth from 1992 to 2010 is only about 37% of the volatility in the full sample from 1929 to 2010. Taking this effect into account, the volatility of perceived productivity growth in the full sample should be around 0.22%/0.37 = 0.60%. With $\rho = 0.98$, the implied volatility of the perceived productivity growth is 0.41%, which is smaller than 0.60%.

Another important caveat is that survey information may be unreliable since the respondents do not have a strong stake in understanding the survey questions. Indeed, Abel et al. (2008) find that inflation expectations of market participants are about two times more volatile than those of professional forecasters. Thus, even if the perceived long-run growth rate is smooth based on survey evidence, the true investor perception could be quite volatile, implying a larger true extrapolation bias. Given these concerns, ρ is set to 0.95 in an alternative calibration, which implies volatility of 0.66% for perceived productivity growth, slightly higher than the 0.60% benchmark.

To further discipline our parameter choice, we also verify that the model-implied volatility of perceived consumption growth is comparable to that in a standard Bansal and Yaron (2004) setting. As will be shown later, even with $\rho = 0.95$ the model-implied volatility of the perceived consumption growth rate is indeed similar in magnitude to the volatility of expected growth in a standard long-run risk model.

Although the values for μ_A, σ_A, α, and δ_K are fixed, ρ, ψ, γ, β, and ξ are allowed to vary across different calibrations to match the key moments in the data. As mentioned earlier, $\rho + \tilde{\rho}$ is always fixed at 0.9999 except the cases in which there is no extrapolative bias ($\rho = 1$ and $\tilde{\rho} = 0$).

[11] In contrast, using aggregate data, early studies, such as Hall (1988) and Campbell et al. (1989), typically found the IES to be much less than 1. However, Vissing-Jorgensen (2002) and Guvenen (2006) point out that there is a downward bias in the IES estimation using aggregate data. Once heterogeneity – especially limited asset market participation – is taken into account, the estimated IES is much larger.

Table 2
Moments for quantities and prices. This table reports the results for six calibrations. Calibrations B.I-V are based on post-1929 sample, where the volatility of the innovation in technology growth σA=4.11. Calibration B.I is the benchmark calibration. Calibrations II considers a higher extrapolative bias (i.e., lower ρ). Calibrations III and IV consider a similar specification as the permanent TFP shocks case in Kaltenbrunner and Lochstoer (2010) with low and high capital adjustment cost (i.e., higher and lower ξ), respectively. Based on the benchmark specification, Specification V considers a lower adjustment cost (i.e., higher ξ). Calibration VI is based on the post-War II sample, where σ_A is set to 1.93%. All the numbers are annualized. The mean and the volatility are reported in percentages. Δc_t denotes consumption growth; Δy_t denotes output growth; Δi_t denotes investment growth; IK denotes investment over capital; x_t denotes the perceived consumption growth by the agent in the model; $R_{f,t}$ denotes the risk-free rate; $R_{E,t} - R_{f,t}$ denotes the levered equity excess returns; $w - c$ denotes the log wealth–consumption ratio; r_{wc} denotes the log return on the aggregate consumption claim.

Parameter	Data	B.I	II	III	IV	V	VI
Risk aversion (γ)	NA	4.00	2.00	4.00	4.00	4.00	4.00
IES (ψ)	NA	2.00	2.00	2.00	2.00	2.00	2.00
Time discount (β)	NA	0.991	0.991	0.9945	0.9945	0.991	0.993
Adjustment cost (ξ)	NA	1.50	1.50	10.00	1.50	15.00	0.85
Extrapolation (ρ)	NA	0.98	0.95	1.00	1.00	0.98	0.95
TFP volatility (σ_A) (%)	NA	4.11	4.11	4.11	4.11	4.11	1.93
$\sigma(\Delta c_t)$	2.93	3.80	2.43	2.42	4.57	1.43	1.46
$\sigma(\Delta c_t)/\sigma(\Delta y_t)$	0.52	0.72	0.46	0.46	0.87	0.27	0.59
$\sigma(\Delta i_t)/\sigma(\Delta y_t)$	3.32	1.72	2.41	2.18	1.29	2.79	2.04
Adj. cost/output (%)	NA	0.37	0.48	0.08	0.30	0.07	0.14
$E(R_{f,t})$	0.86	1.29	0.87	2.22	1.89	2.26	1.17
$\sigma(R_{f,t})$	0.97	0.28	0.31	0.36	0.16	0.41	0.16
$E(R_{E,t} - R_{f,t})$	6.33	5.75	6.12	0.60	2.20	0.94	5.30
$\sigma(R_{E,t} - R_{f,t})$	19.42	10.42	14.55	2.15	7.81	1.89	10.14
Sharpe ratio	0.33	0.55	0.42	0.28	0.28	0.50	0.52
E (10-year term premium)	1.79	−0.70	−0.92	−0.37	−0.17	−0.96	−0.59
σ (10-year term premium)	NA	0.07	0.10	0.19	0.06	0.15	0.06
$\sigma(w - c)$	22.21–34.48	12.40	19.94	4.57	3.19	13.33	10.62
$\sigma(r_{wc})$	9.88-12.44	9.58	15.25	4.88	6.03	7.84	8.40
$E(w - c)$	4.47–5.02	4.50	4.55	5.39	5.39	4.54	4.71
$E(r_{wc})$	1.72–2.16	4.79	5.06	1.23	1.56	3.65	4.05
$\sigma(x_t)$	NA	0.79	1.11	0.71	0.32	1.04	0.52
$\sigma(x_t)/\sigma(\Delta c_t)$	NA	0.21	0.46	0.29	0.07	0.73	0.35
$AC(x_t)$	NA	0.87	0.79	0.83	0.88	0.82	0.79

3.2. Unconditional moments

The model is simulated for 400,000 quarters of artificial data to estimate 'population' values for a variety of statistics. Small sample properties of the model are also examined by simulating 400 quarters of artificial data each time and repeating the procedure 1000 times. The main results are found in Table 2, which includes the summary statistics of both quantities and asset prices from the six different parameterizations. In general, consumption growth is smoother than output growth, while investment growth is more volatile than output growth, consistent with the data. The moments for consumption, GDP, investment, risk-free rate and stock market returns in the data column are taken directly from Bansal and Yaron (2004) and KL (2010), both of which are based on a data sample from 1929 to 1998.

3.2.1. The benchmark calibration

In our benchmark calibration (i.e. Model I in Table 2), the output volatility and the mean growth rates of the economy are pinned down by the technology parameters and are chosen to match the data. We therefore omit them and only report the volatility of the variables of interest. With an adjustment cost of $\xi = 1.5$ and an extrapolation parameter of $\rho = 0.98$, the model generates smooth consumption and volatile investment relative to output that is in a magnitude similar to, but still smaller than, that in the data as reported in KL (2010). Moreover, in the second calibration with stronger extrapolation bias, the quantities are even closer to those in the data.

The benchmark model I matches the moments of asset prices well. With a risk aversion coefficient of 4 and an IES of 2, the model produces a sizable equity premium of 5.75%, compared with 6.33% in the historical data. The volatility of the excess return is 10.42%, a reasonable number given the relatively small risk aversion coefficient. Since the model roughly matches the macroeconomic quantities, this is a significant success for a production-based model. For example, KL (2010) produce a volatility of 0.66% for the excess return on the (unlevered) equity claim in one of their benchmark calibrations. In addition, this volatility can be further increased if one chooses $\rho = 0.95$ as in our model II. Intuitively, investor over-extrapolation causes a perception of high volatility of expected consumption growth; together with recursive preference this can result in high risk premia without high consumption volatility.

The benchmark calibration also produces smooth interest rates, even smoother than those in the data. This is a standard feature in models with Epstein–Zin preferences and IES greater than one. On the other hand, standard habit models in

production economies tend to produce an excessively volatile interest rate. A usual side effect of highly volatile interest rates is an excessively large term premium (see, e.g., Jermann, 1998; Abel, 1999). Large bond term premia are related to overly volatile interest rates, as term premia are compensation for real interest rate risk. In our model, owing to the high IES, the interest rate is very smooth despite the extrapolative expectations, and hence the term premium (defined as the difference between the 10-year bond yield and 3-month interest rate) is also small, consistent with the data.[12] For the benchmark model, it produces a downward-sloping real yield curve. The short-term real rate is already low at 1.29%, while the long real rates are an additional 0.70 percentage point lower. This pattern does not match the positive real term premium of 1.79% for the U.S. TIPS data between January 1998 and October 2009, but in term of magnitude, our term premium is only slightly negative.[13]

The bottom of Table 2 reports summary statistics for the perceived expected consumption growth by the individual, and the aggregate wealth portfolio. The volatility of perceived expected growth rate in the model is 0.79%. The ratio of the volatility of the perceived expected growth to the volatility of realized growth rate is slightly less than 21%. The perceived expected growth rate is not directly observable, but its magnitude can be compared with existing studies. In the one channel model of Bansal and Yaron (2004), for example, the volatility ratio of expected growth and realized growth is 34.4% for quarterly frequency calibration. Thus, the volatility of the perceived growth rate in our calibration does not appear too large.

Owing to the high variation in the perceived expected growth rate, our model produces an annual volatility of the wealth–consumption ratio of about 12.40%, which is about half of that in the data as reported in Lustig et al. (2013). This is a success compared to the standard long-run risk models; Ai (2010), for example, shows that the standard long-run risk model only produces a volatility of 4.70% (see Table I in Ai, 2010). From Eq. (14), this high volatility in the wealth–consumption ratio is the main driving force underlying the high and volatile equity returns. Our model also generates a high volatility of the return on aggregate wealth, again consistent with the data.

Finally, when ρ is reduced to 0.95 (as in model II), the model matches the moments in the data even more closely. For example, consumption growth is smoother, and investment growth and asset returns are more volatile. With larger extrapolation bias, the perceived productivity growth becomes more volatile, and hence the volatility of investment and perceived expected consumption growth is also larger. Relative investment volatility increases from 1.72 to 2.41, which is very close to that in the data. Moreover, due to high long-run consumption risk, the equity premium is still very large despite a lower risk aversion in model II, and equity returns are highly volatile. The risk-free rate is also slightly more volatile owing to greater volatility of the perceived expected consumption growth rate.

However, in both calibrations, the mean return on the wealth portfolio is larger than that in the data, as calculated by Lustig et al. (2013). In the model, the consumption claim is riskier than the firm's dividend payout claim since the payout is less procyclical than consumption. Thus, the return on the consumption claim is too high relative to the data. Potential resolutions for this issue are discussed in Section 5.

3.2.2. The mechanism of the model

To identify the key mechanism behind the empirical success of the model, below we follow Jermann (1998) by calibrating the model at different parameter combinations. This way helps identify which ingredients are key to replicating different aspects of the data.

We start with a calibration (model III) with a very low capital adjustment cost and no extrapolation ($\rho = 1$, $\tilde{\rho} = 0$, and $\hat{\mu}_t = \mu_A$), and then sequentially add the adjustment cost and extrapolation into the model. The outcome of model III is consistent with the standard RBC models and KL (2010). Consumption is smooth, and investment is more volatile than output. These patterns are consistent with the quantity data. However, the equity premium is very low (less than 1%), as is the volatility of the stock return (less than 3%). Intuitively, owing to low adjustment costs, the individual can easily smooth consumption by adjusting the amount of investment. This reduces consumption risk and therefore equity risk premium.

Model IV increases the capital adjustment cost. Not surprisingly, this greatly reduces investment volatility. Consistent with the intuition above, this in turn increases the volatility of consumption growth. So the model sacrifices good matching of this quantity moment, but does have the benefit of raising the equity premium slightly, and generating greater returns volatility.

This type of finding is well known for standard DSGE models (e.g., Jermann, 1998). To produce a high equity premium, the adjustment cost cannot be too small; otherwise, consumption is too smooth, and hence the equity premium is small. However, if the capital adjustment cost is high, the investment volatility is too low compared with the data. Thus, it is hard to match both the quantities and asset prices simultaneously. In two important papers, Jermann (1998) and Boldrin et al. (2001) show that introducing habit preference can help to match these stylized facts.

[12] Campbell et al. (2003) argue that extrapolative expectations in general equilibrium tend to lead to volatile interest rates. In our benchmark models, owing to high adjustment costs, the perceived expected consumption growth is not very volatile. This fact, together with a high IES value, results in a smooth risk-free rate.

[13] The data on the US real term structure is downloaded from Professor J. Huston McCulloch's Web site (http://www.econ.ohio-state.edu/jhm/ts/ts.html). For the international evidence, Piazzesi and Schneider (2006) find a negative real term premium (the difference in real yield between 10-year bond and 2.5-year bond) of −0.18% in the sample of December 1995–March 2006 in UK. However, in the longer sample of January 1985–June 2015, the real term premium in UK is slightly positive at 0.33%.

Compared with Model IV, the benchmark calibration I introduces a relatively small extrapolative bias. Owing to bias in expectations, the perceived expected growth rate of productivity varies over time, leading to greater fluctuation in (perceived) optimal investment. For example, after a few positive productivity shocks, the individual perceives that the future growth rate is likely to be very high. Thus, compared with the rational case, the individual tends to invest more heavily to exploit this productivity. Similarly, after negative shocks, the individual tends to underinvest relative to the rational case. Therefore, even with a relatively high adjustment cost ($\xi = 1.5$), the model can produce high volatility of investment growth. With a slight extrapolative bias in the benchmark model I, the investment volatility relative to output increases to 1.72 from 1.29 in model IV.

With extrapolation, benchmark calibration I also generates smoother consumption than in model IV, which is more consistent with the data. Moreover, despite the smoothness of consumption in benchmark model I, the equity return is more volatile, and the equity premium larger. Despite the higher return volatility, the Sharpe ratio of the market increases (consistent with the equity premium puzzle), because the rise in the equity premium is proportionately even larger.

These differences from model IV reflect a key mechanism in our model. Although the volatility of realized consumption growth is smaller in benchmark calibration I, the *perceived* expected consumption growth rate is *more* volatile. The long-run risk literature shows that the variation of the perceived expected growth rate commands an especially high price of risk. Owing to the persistence in perceived expected growth rates, news regarding future expected growth rates results in large reactions in the price–dividend ratio and the stock return. Since these reactions are negatively associated with the marginal rate of substitution of the representative agent, this effect increases the equity risk premium.

Furthermore, owing to capital adjustment costs, the firm cannot easily alter its investment. For example, after a favorable productivity realization, realized asset returns increase. With higher adjustment costs, the firm pays out more dividends. Hence, compared to the case of low capital adjustment costs, the level of consumption is higher, and thus the marginal utility of consumption is reduced. Thus, high capital adjustment costs decrease the covariation between asset returns and the marginal utility from consumption, which in turn increases equity risk. Together, these effects result in a high equity risk premium.

The reason for the higher volatility of the perceived expected consumption growth rate is similar to the reason for higher investment volatility. After favorable productivity shocks, the individual invests more than in the rational case and consumes less in the current period. As a result, he perceives high future consumption growth resulting from high perceived future productivity and the current high investment. Thus, extrapolative bias amplifies the volatility of both investment and *perceived* expected consumption growth. On the other hand, the extrapolative bias smooths *actual* consumption growth: with extrapolative expectations the representative investor has more incentive to make more investments after good productivity shocks, and less investments after bad shocks; investments tend to absorb more of the payoff variation resulting from productivity shocks, leading to smoother actual consumption growth.

Model V maintains extrapolation bias, but reduces the adjustment cost by setting ξ to 15. With an extremely low capital adjustment cost, the individual can now easily adjust his investment according to perceptions of growth opportunities, making investment very volatile. This ease in shifting investment in response to opportunities allows the perceived expected consumption growth to become much more volatile.[14] In the face of a positive shock, the individual consumes less and invests more, and hence perceived expected consumption growth is high.

Despite the high volatility of the perceived expected consumption growth, and hence high long-run risk, the equity premium is still very small in model V. Furthermore, the volatility of the levered equity is just 1.89%, compared with the 10.42% in benchmark calibration I. This is because with low capital adjustment costs, the firm can easily invest more in the face of a good productivity shock, and thus the firm's payout and consumption are *less procyclical or even countercyclical*. This effect tends to increase the covariation between asset returns and the marginal utility of consumption, and hence leads to a very low risk premium for the equity claim.[15]

Lastly, model VI provides a calibration to match the post-WWII quantity data. During the post-WWII period, both consumption and GDP growth volatility is lower. The volatilities for consumption growth and GDP growth are 1.51% and 2.46% respectively, and the volatility ratio of investment to GDP and the volatility ratio of consumption to GDP is 2.60 and 0.61, respectively. Thus, here the model is calibrated to produce lower volatility for consumption and GDP growth. With extrapolation parameter $\rho = 0.95$, the model-implied volatility of the perceived consumption growth rate is similar in magnitude to the volatility of expected growth in a standard long-run risk model (i.e., $\sigma(x_t)/\sigma(\Delta c_t) = 35\%$). With lower consumption volatility, the equity premium and stock market volatility is also lowered in this calibration.

Nonetheless, the model still produces a sizeable equity premium 5.30% and stock market volatility of 10.14% relative to the case of no extrapolative expectations. In addition, the volatility ratio of investment to GDP and the volatility ratio of consumption to GDP are 2.04 and 0.59, respectively. Even for the case of a smaller extrapolation bias ($\rho = 0.98$ untabulated analysis), the equity premium and stock return volatility are 3.57% and 8.77%, respectively.

In sum, to generate a high equity premium, it is necessary to have sufficiently high capital adjustment costs as well as extrapolative bias. Extrapolative bias generates high volatility of perceived consumption growth and hence of the pricing kernel. Capital adjustment costs prevent firms from investing so much that dividend payouts become minimally procyclical

[14] However, the volatility of actual consumption remain relatively small, since individuals are also in part adjusting investment to smooth consumption.

[15] Our findings with low adjustment costs are consistent with Carceles-Poveda and Giannitsarou (2008), who study the effects of constant gain learning on asset prices in a production economy. With CRRA preferences and no adjustment costs, they find that the effects on volatility and the equity premium are quite small.

Table 3
Long horizon stock return predictability by DP, IK, and Tobin's Q. Cumulative 1-, 3-, and 5-year excess stock market returns are regressed onto the dividend–price ratio, investment over capital, and Tobin's Q. This table reports results on both coefficients and R^2s. The model is simulated for 400 quarters and simulations are repeated 1000 times to obtain small sample values. The median (50%) values are reported. The data column reports the corresponding results from real data. Due to data availability constraints, the sample periods for DP, IK, and Q regressions are 1927Q1–2009Q4, 1929–2009, and 1962Q1–2009Q4, respectively.

Statistics	Horizon	DP		IK		Q	
		50%	data	50%	data	50%	data
Panel A: benchmark calibration I: $\rho = 0.98$							
Coef	1	5.95	13.06	−2.63	−4.12	−0.10	−0.04
	3	16.35	32.67	−7.22	−7.99	−0.27	−0.11
	5	25.33	50.28	−11.11	−8.06	−0.42	−0.15
R^2	1	3.99	5.48	3.77	1.33	3.74	2.58
	3	11.21	13.51	10.22	3.66	10.35	9.68
	5	17.64	23.49	16.21	2.20	16.32	13.68
Panel B: calibration II: $\rho = 0{:}\ 95$							
Coef	1	7.49	13.06	−4.59	−4.12	−0.18	−0.04
	3	18.42	32.67	−11.30	−7.99	−0.43	−0.11
	5	25.79	50.28	−15.80	−8.06	−0.61	−0.15
R^2	1	8.28	5.48	8.32	1.33	8.39	2.58
	3	20.13	13.51	20.54	3.66	20.58	9.68
	5	28.35	23.49	28.98	2.20	29.05	13.68
Panel C: calibration III: $\rho = 1$							
Coef	1	0.35	13.06	−0.18	−4.12	−0.04	−0.04
	3	0.99	32.67	−0.49	−7.99	−0.13	−0.11
	5	1.58	50.28	−0.79	−8.06	−0.20	−0.15
R^2	1	0.62	5.48	0.62	1.33	0.67	2.58
	3	1.93	13.51	1.92	3.66	1.95	9.68
	5	3.07	23.49	3.09	2.20	3.07	13.68

(or even countercyclical), which would reduce the riskiness of equity. With both model ingredients, there is a high market price of risk, and equity is perceived as very risky, resulting in a high equity premium.

3.3. Return predictability

In the data, excess returns are predictable by the dividend–price ratio (e.g., Campbell and Shiller, 1988; Fama and French, 1988), aggregate Q (e.g., Kothari and Shanken, 1997; Pontiff and Schall, 1998), and the investment rate (IK) (e.g., Cochrane, 1991). Despite a debate over robustness (e.g., Goyal and Welch, 2008), the trend of the literature favors such variables having power to predict returns (e.g., Ang and Bekaert, 2007; Campbell and Thompson, 2008; Cochrane, 2008; Rapach et al., 2010).

The objective of this section is to show that with extrapolation bias, our model can help reproduce these empirical patterns. To do so, the standard regression of future return on the dividend–price ratio is performed. When the perceived technology expected growth rate, $\hat{\mu}_t$, is high, the firm invests more and the dividend payout is small. Thus, the dividend–price ratio is small. In this circumstance, an econometrician would forecast a lower expected return, in anticipation of lower productivity growth, than do extrapolative individuals. Thus, the dividend–price ratio positively predicts future returns in the model.

To quantify this, the 1-, 3-, and 5-year stock market excess returns are regressed onto the lagged dividend–price ratio. Table 3 reports both regression coefficients and R^2 statistics. The results are based on 1000 simulations, each with 400 quarters of simulated data. Both the median values and the 95% confidence intervals are reported.

It is well known that in the data, the coefficient for DP is positive and the coefficients for IK and Q are negative. In addition, the R^2s increase with time horizon. Our model replicates this feature for both coefficients and R^2 statistics. Moveover, as extrapolative bias increases (a smaller ρ), the predictive power of the dividend–price ratio is stronger. In addition, our untabulated analysis finds that the return predictability results based on a long sample with 400,000 observations remain similar to those based on small samples.

Intuitively, if extrapolative bias is strong, then after a few good shocks to productivity growth, the individual is more over-optimistic about the future productivity growth. Hence, the firm invests more and the dividend is especially low. This reduces the dividend-price ratio, and makes the future return reversal especially strong since on average the future realized productivity growth is much lower than the individual's expectation. At the opposite extreme, if there is no extrapolation bias, there is no return predictability, as shown in Panel C. These findings are consistent with those of Ai (2010). With rational learning in a production-based long-run risk model, Ai (2010) finds that the price–dividend ratio predicts excess returns in the wrong direction.

Analogously, the investment rate (IK) and Tobin's Q should also predict future equity returns. After a few favorable shocks to productivity growth, the individual is overoptimistic about future productivity growth, and hence the firm tends to invest more to exploit technology growth. Thus, IK is large, and owing to adjustment costs, Tobin's Q is also large. However, on

average the future realized productivity growth is lower than the individual's expectation. Thus, the value of the firm is expected to decline from the viewpoint of the econometrician who analyzes the historical data.

In Table 3, excess returns are also regressed onto lagged investment rates and aggregate Q. Consistent with the data, both investment rates and aggregate Q negatively predict future returns. When extrapolation bias is eliminated ($\rho = 1$), there is no return predictability by the investment rate or aggregate Q. On the other hand, as extrapolative bias increases (i.e., a lower value of ρ), the predictive abilities of IK and Q become stronger. The intuition is similar as before. These results highlight the key role of extrapolation in explaining the conditional moments of returns in a production economy.

In sum, extrapolative bias not only helps match the first two unconditional moments of the data, it also helps generate predictive patterns in returns as observed in the data.

3.4. Implied consumption dynamics

It is important to verify whether the implied consumption dynamics in the model resemble those in the data. In the data, the predictable component in consumption growth is small. So we next verify that the high equity premium is not due to an excessively predictable component in consumption growth. Table 4 presents summary statistics for the consumption growth from the simulated data.

The general pattern is that the autocorrelation of consumption growth is very small, especially for a high value of extrapolative bias ρ. For our benchmark case I, the first-order autocorrelation is only 4%. Even for the case of $\rho = 0.95$, the autocorrelation of consumption growth is still only 18%. Thus, the model does not produce an excessively predictable component in consumption. Moreover, this table also presents results for different values of the extrapolative bias parameter and adjustment costs.

In general, increasing extrapolative bias tends to raise the predictable component of consumption growth and the volatility of consumption growth. Intuitively, when the individual is more extrapolative, his perceived technology growth rate varies more, resulting in greater variation in consumption growth. In addition, as shown in calibrations III and IV, increasing the adjustment costs tend to reduce consumption predictability since it is difficult to smooth consumption with high adjustment costs. In particular, consumption is less predictable in our benchmark model I with extrapolation than in model III without extrapolation but with smaller adjustment costs. In summary, consumption growth in our benchmark model is not excessively predictable compared with the data.

In unreported analysis, consumption dynamics are further studied by changing the IES. In general, a high IES leads to more predictable and volatile consumption growth. If the IES is larger, the individual can easily substitute consumption intertemporally. Hence, in the face of a positive (negative) technology shock, the individual can invest more (less) now and consume less (more) in the future, raising the importance of the predictable component in the growth rate.

4. Further implications of the model

This section examines two additional implications of the model. First, owing to extrapolation, the model implies that the perceived technological growth rate, $\hat{\mu}_t$, will negatively predict returns. Second, the model implies that the aggregate investment rate is positively related to perceived technological growth.

Although the state variable is unobservable (just as is the surplus ratio in the habit-formation model and the expected growth rate in the long-run risk model), one can construct a proxy for our state variable from the historical data. As observable proxies for $\hat{\mu}_t$, an exponentially weighted moving average of realized GDP growth from Saint Louis FED and of realized productivity growth from Bureau of Labor Statistics (BLS) is used. The exponential decay parameter $\tilde{\rho}$ is set to be 0.95. Our results remain similar if this value is 0.98. The first proxy based on GDP is indirect; the second proxy based on productivity is more directly linked to the state variable in our model. However, the second proxy is more subject to misspecification and measurement errors. Thus, the first proxy is used for our main analysis and the second is used for robustness checks.

Table 4
Consumption dynamics. This table reports the 1st, 4th, 8th, 12th, 16th, and 20th order autocorrelations of consumption growth under different calibrations as in Table 2. Calibration I corresponds to the case when $\rho = 0.98$. Calibration II corresponds to the case when $\rho = 0.95$. Calibration III corresponds to the case when $\rho = 1$ and $\xi = 10$. Calibration IV corresponds to the case when $\rho = 1$ and $\xi = 1.5$. The model is simulated for 400 quarters and simulations are repeated 1000 times to obtain small sample values. The median values under each calibration are reported. Quarterly seasonally adjusted consumption data are obtained from Bureau of Economic Analysis, 1947Q1–2009Q4.

Horizon	Data	Calibration			
		I	II	III	IV
1	0.26	0.04	0.18	0.14	0.02
4	0.15	0.03	0.16	0.13	0.01
8	−0.03	0.03	0.13	0.11	0.01
12	0.03	0.02	0.10	0.09	0.01
16	0.07	0.02	0.08	0.08	0.01
20	−0.04	0.02	0.06	0.06	0.01

Table 5
Predictability of return, GDP growth, and TFP growth. This table reports the long-horizon predictive regressions of market return, GDP growth, and TFP growth onto perceived growth $\hat{\mu}$. $\hat{\mu}$ is calculated from an exponentially weighted moving average of realized GDP growth rates or TFP growth rates from BLS with exponential decay parameter $\rho = 0.95$. t-statistics calculated from Newey and West (1987) standard errors (t_{NW}) and Hodrick (1992) standard errors (t_{HD}) are reported. The data sample for all raw series is 1947Q2–2009Q4. The resulting data sample for $\hat{\mu}$ is 1957Q1–2009Q4.

Panel A: return predictability by $\hat{\mu}$ calculated from GDP growth and TFP growth

Horizon	$\hat{\mu}(GDP)$	t_{NW}	t_{HD}	R^2	$\hat{\mu}(TFP)$	t_{NW}	t_{HD}	R^2
1-year	−23.08	−1.71	−1.64	0.04	−13.84	−1.38	−1.23	0.02
2-year	−38.45	−1.91	−1.42	0.06	−18.69	−1.26	−0.89	0.02
3-year	−67.70	−3.35	−1.83	0.16	−31.77	−1.91	−1.04	0.05
4-year	−92.32	−4.46	−1.98	0.26	−46.13	−2.21	−1.15	0.08
5-year	−111.68	−4.52	−1.98	0.31	−61.07	−2.39	−1.20	0.12

Panel B: GDP growth and TFP growth predictability by calculated from GDP growth $\hat{\mu}$

Horizon	GDP growth				TFP growth			
	$\hat{\mu}(GDP)$	t_{NW}	t_{HD}	R^2	$\hat{\mu}(GDP)$	t_{NW}	t_{HD}	R^2
1-year	−0.61	−0.25	−0.36	0.00	−4.09	−2.35	−2.37	0.09
2-year	−4.18	−1.02	−1.31	0.03	−5.94	−2.29	−1.90	0.10
3-year	−6.77	−1.39	−1.60	0.06	−6.83	−2.55	−1.59	0.09
4-year	−8.80	−1.69	−1.72	0.08	−6.75	−2.30	−1.29	0.07
5-year	−9.17	−1.90	−1.55	0.08	−6.02	−2.00	−0.97	0.05

Our analysis first considers the implication for return predictability. Table 5 shows that the proxy for the perceived growth rate can indeed significantly negatively predicts future excess market returns in the data. t-statistics calculated from Newey and West (1987) standard errors and Hodrick (1992) standard errors are reported. Ang and Bekaert (2007) show that Newey-West standard errors tend to overreject the null, whereas Hodrick's standard errors tend to underreject the null.

The analysis shows that the predictive power of the perceived expected growth is statistically significant. In terms of economic magnitude, a one-standard-deviation-increase in the perceived expected growth leads to about a 3.65% decrease in subsequent annual returns. The same pattern, although slightly weaker, remains if the alternative proxy for $\hat{\mu}_t$, the smoothed average of realized productivity growth from BLS, is used. In the model, the predictive power of the perceived growth $\hat{\mu}_t$ is very similar to that of the dividend–price ratio, IK, and Q as shown in Table 3 since the dividend–price ratio, IK, and Q are all highly correlated to the state variable $\hat{\mu}_t$. Thus, comparing Tables 3 and 5, the predictive power of $\hat{\mu}_t$ in the model is similar to that in the data.

In contrast, if the negative output gap (NGAP) of Cooper and Priestley (2009) is used as a proxy for 'true expected growth' in the data-generating process,[16] the opposite results follows, as shown in Cooper and Priestley (2009). That is, true expected growth (NGAP) positively forecasts future returns. This distinct predictive power of perceived expected growth and 'true' growth highlights the key role of investor perception for asset prices. In addition, NGAP is indeed significantly associated with future GDP growth and productivity growth in the data.[17]

In contrast, the predictive ability of $\hat{\mu}_t$ for GDP and productivity growth is weaker and is opposite to that of NGAP as shown in Panel B of Table 5. This negative predictive power of $\hat{\mu}_t$ may be due to the initial inefficient overinvestment and the subsequent decrease in productivity after a positive shock to the perceived expected growth. Of course, our simple model cannot generate this predictable component in productivity growth, as, for parsimony, it is assumed that the realized productivity growth is i.i.d. Lastly, the above estimation of $\hat{\mu}_t$ can easily be related to business cycles, and thus its predictive ability for returns and GDP growth can be potentially consistent with a risk-based model. These findings therefore provide further collaborating evidence using survey data.

In particular, using Livingston Survey data, Campbell and Diebold (2009) and Goetzmann et al. (2010) find that the forecasted growth rate is also negatively related to future aggregate returns, which is consistent with our model. In addition, in cross-sectional analysis, Imrohoroglu and Tuzel (2014) show that firm-level productivity is positively related to contemporaneous stock returns and negatively related to future excess returns, consistent with our model as well.

Nonetheless, a possible concern is that both the forecasted growth rate based on survey data and the perceived growth $\hat{\mu}_t$ based on lagged growth include a genuine expected growth component, and thus their contrarian predictive power for stock returns can be easily consistent with risk-based stories as well. To address this issue, realized GDP growth is first regressed onto lagged GDP growth, the DP ratio, and the term premium to estimate objective expected GDP growth.[18] Then, the difference between forecasted GDP growth and the estimated expected growth (i.e., the misperception on GDP growth) and the objective expected growth are used to predict aggregate excess returns.

[16] The output gap measures the difference between industrial production and its long-run deterministic trend. As a result, the negative output gap (NGAP) should positively predict future output growth, and thus NGAP is a natural proxy for true expected output growth.

[17] These results are available upon request.

[18] It is well known that the stock market is a leading indicator, and the term premium is probably the most famous predictor of GDP growth. Thus, our regressions use these two variables along with lagged GDP growth to estimate objective expected GDP growth.

Table 6
Return predictability by perceived growth. Panel A of this table reports the long-horizon predictive regressions of market excess returns onto misperception in GDP growth calculated as the difference between forecasted growth based on Livingston Survey and the estimated objective growth based on realized GDP growth rates. In particular, we run the following regression:

$$\sum_{j=1}^{H} r_{t+j} = \alpha + \beta_1 \cdot (g_{forecasted} - g_{expected}) + \beta_2 \cdot g_{expected} + error_{t,t+H}$$

, where $g_{expected}$ are estimated based on a regression of realized GDP growth on the lagged GDP growth rate, the lagged DP ratio, and the lagged term premium with lag = 2. The data sample for all raw series is 1952Q4–2009Q4. Panel B reports the result from the same analysis using simulated data. The model is simulated for 400 quarters and simulations are repeated 1000 times to obtain small sample values. The median values under each calibration are reported. The parameter ρ is set at 0.95. t-statistics calculated from Newey and West (1987) standard errors (t_{NW}) and Hodrick (1992) standard errors (t_{HD}) are reported.

Horizon	β_1	t_{NW}	t_{HD}	β_2	t_{NW}	t_{HD}	R^2
Panel A: empirical data							
1-year	−6.97	−2.84	−2.44	−1.56	−0.50	−0.26	0.07
2-year	−12.52	−2.69	−2.64	−6.83	−1.26	−0.73	0.12
3-year	−14.30	−2.62	−2.23	−6.94	−0.98	−0.61	0.13
4-year	−12.59	−2.74	−1.63	−4.62	−0.92	−0.35	0.09
5-year	−13.16	−2.27	−1.55	−5.94	−1.02	−0.41	0.08
Panel B: simulated data							
1-year	−4.38	−3.81	−2.86	−2.31	−1.38	−1.13	0.09
2-year	−7.68	−4.65	−2.67	−5.08	−2.20	−1.44	0.16
3-year	−10.32	−5.46	−2.52	−7.42	−2.81	−1.51	0.22
4-year	−12.43	−6.15	−2.39	−9.31	−3.29	−1.52	0.26
5-year	−14.10	−6.71	−2.27	−10.93	−3.73	−1.51	0.30

Panel A of Table 6 shows that only the misperception component has predictive power. Objective GDP growth has little power in predicting aggregate stock returns. In addition, the same analysis is performed with simulated data from the model. In particular, we replace survey forecasts with perceived GDP growth and estimate true expected GDP growth in the same fashion as in Panel A. The result in Panel B indicates that the model produces regression coefficients similar to those in the data. However, this misperception component in the data is still weakly correlated to one-year ahead TFP growth, albeit at an insignificant level of −16.64% (t-stat = 1.30). This correlation is smaller than the correlation between perceived growth $\hat{\mu}_t$ and one-year ahead TFP growth as reported in Table 5, which is −29.78% (t-stat = 2.37). Although a potential risk-based explanation cannot be fully ruled out for our empirical findings, it is reassuring that the evidence based on survey data is consistent with our model implications as well.

The surplus ratio in the habit-formation model could also be approximated by smoothed average of past growth rates, a variable similar to our proxy for the perceived expected growth rate. Thus, the habit-formation model would have exactly the same prediction as ours. However, after favorable shocks, the individual in the habit model expects a lower future return, whereas the individual in our model does not anticipate a lower subsequent return.

Using survey data on investor expectations, Durell (1999), Vissing-Jorgensen (2003) show that in good times, when stock prices are high, investors do not expect lower returns. If anything, the opposite holds. However, investors are supposed to perceive and accept such lower returns in habit models owing to high risk tolerance. In contrast, our model is consistent with this survey evidence as the individual in our model does not perceive a lower future return after good shocks.

Although our model is for the aggregate market, La Porta (1996) focuses on analyst expectations in the cross-section and finds that stocks with the highest growth forecasts earn much lower subsequent returns than stocks with the lowest growth forecasts, potentially consistent with the cross-sectional extension of our current model.

We now turn to the implication of the model for the determinants of the aggregate investment rate.[19] The model implies that the perceived expected growth rate should be positively correlated with the aggregate investment rate. The top panel of Fig. 2 plots the proxy for our state variable and the investment rate, both calculated from the data. The model predicts a strong comovement between this two variables, and this figure confirms this prediction. Owing to the high autocorrelation of the series in levels, the bottom panel in Fig. 2 plots the four-quarter differences in investment rates, a measure used by Hassett and Hubbard (1997) and Philippon (2009), and the four-quarter differences in $\hat{\mu}_t$. It confirms the strong comovement as well.

Following the existing empirical literature, a predictive regression for investment rates is performed under the interpretation that expectations in our model cause investment flows. Table 7 reports the results from a formal regression analysis by regressing investment rates (in difference) onto lagged perceived expected growth rates and lagged cash flow. It is well-known that the stock market based measure of Q performs poorly in predicting investment rates, while the cash flow can forecast investment rates better. Our results confirm the predictive ability of cash flow. However, the perceived expected growth is a much stronger predictor of the investment rates. Moreover, after controlling for the perceived growth, cash flow is no longer significant. Finally, as a comparison, the explanatory power of our proxy is similar to the bond's Q in Philippon (2009).

[19] It is well-known that Q does not have much power in determining the investment rate. The existing literature typically attributes this failure to measurement errors in Q.

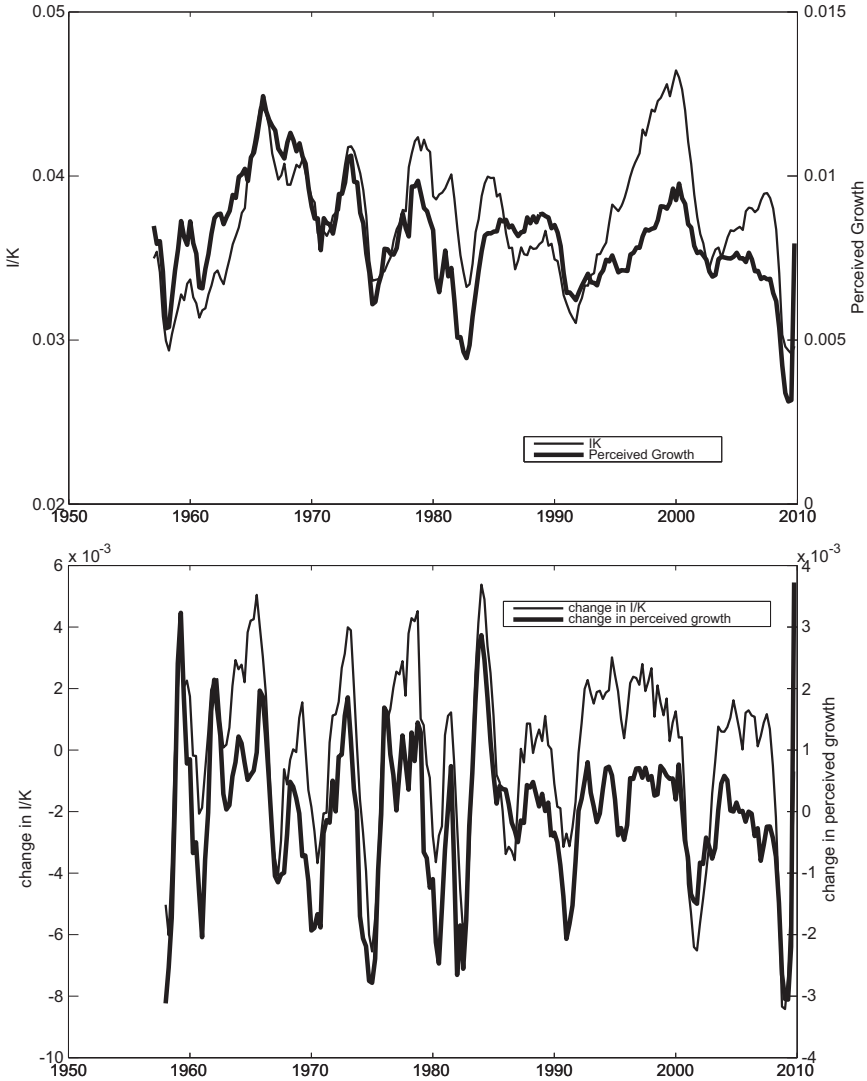

Fig. 2. Evolution of I/K and the perceived growth rate. The top figure plots the evolution of I/K and the perceived growth rate. The bottom figure plots the evolution of change in I/K from $t-4$ to t and the change in the perceived growth rate from $t-4$ to t. I/K is nonresidential fixed investment divided by the replacement cost. The perceived growth rate is calculated as the exponentially weighted moving average of realized GDP growth rates with exponential decay parameter $\rho = 0.95$. All data are from NIPA. The data sample for all raw series is 1947Q2–2009Q4. The resulting data sample for $\hat{\mu}_A$ is 1957Q1–2009Q4.

Table 7
Predictability of investment rates by perceived growth and cash flow. This table reports the predictive regressions of the investment rate, I/K, onto cash flow (corporate profits to capital ratio) and perceived growth calculated from an exponentially weighted moving average of realized GDP and TFP growth rates with exponential decay parameter $\rho = 0.95$. The data sample for all raw series is 1947Q2–2009Q4. The resulting data sample for $\hat{\mu}_t$ is 1957Q1–2009Q4. Newey and West (1987) standard errors with lag of 4 are used to calculate the t-statistics, which are reported in parenthesis.

	I	II	III	IV
$\hat{\mu}_A$ from GDP		1.81	1.86	
		(9.65)	(10.34)	
$\hat{\mu}_A$ from TFP				1.48
				(5.00)
Cash flow	0.17	0.01		
	(6.43)	(0.50)		
R-sqr	0.30	0.66	0.66	0.29

In sum, with different proxies for our state variable, there is preliminary support for the two direct testable implications of the model.

5. Conclusion

Considerable evidence from the psychology of judgment and from investor behavior suggests that individuals tend to overextrapolate small samples of past performance in forecasting future performance. Standard attempts to resolve the equity premium puzzle based on habit formation or long term risk in endowment economies have not been immediately successful in production economies owing to endogenous consumption and dividend payout smoothing. Existing models typically imply interest rates that are too volatile or excess stock returns that are too smooth. By introducing extrapolation into an otherwise standard model of production-based asset pricing in a dynamic equilibrium setting with capital adjustment costs, the model reproduces the major stylized facts about both macroeconomic quantities and capital market prices as observed in the data. The key to this is the interaction of recursive preferences, adjustment costs, and extrapolation bias.

Finally, we acknowledge a limitation of the model and suggest possible extensions. Since all variables are driven by one shock, it is hard to simultaneously match the moments for both the firms' payout claim and the aggregate consumption claim. The consumption claim is riskier than the firm's (unlevered) dividend payout claim, since the payout is less procyclical than consumption. In the data, aggregate stock market dividends are highly procyclical; dividend payout in the model is not as procyclical. A possible way to address this issue would be to introduce an additional productive sector into the model. With this additional sector, it would be possible to delink consumption and dividends, and hence potentially match both the consumption and the dividend claim jointly.

Another possible approach to improving model performance would be to introduce sticky wages as in Danthine and Donaldson (2002). With rigid wage processes, dividends are effectively levered up and delinked from consumption and output, thus matching the joint dynamics of consumption and payout better. A third possible approach would be to follow previous studies by exogenously defining equity market dividends as a levered claim to the consumption stream (among others, see Abel, 1999; Bansal and Yaron, 2004). This way, the dividend claim can be riskier than the consumption claim.

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Appendix A. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2015.08.006.

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