Index Investing and Asset Pricing under Information Asymmetry and Ambiguity Aversion

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April 7, 2019

Abstract

In a setting with information asymmetry and a tradable value-weighted market index, ambiguity averse investors hold undiversified portfolios, and assets have non-zero alphas. But when a passive fund offers the risk-adjusted market portfolio (*RAMP*) whose weights depend on information precisions as well as market values, all investors hold the same portfolios as in the economy without model uncertainty and thus engage in index investing. So *RAMP* improves participation and risk sharing. Asset alphas are zero with *RAMP* as pricing portfolio. *RAMP* can be implemented by a fund of funds even if no single manager has sufficient knowledge to do so.

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1 Introduction

Index investing has long been recommended by legendary practitioners (such as Jack Bogle and Warren Buffett) and leading scholars (such as William Sharpe and John Cochrane) as means by which retail investors can attain attractive combinations of risk and return.[1] Market indexes are also important in asset pricing: they are used as pricing portfolios for determining assets' risk premia.

These two roles of market indexes, facilitating index investment and pricing assets, are highlighted in the Capital Asset Pricing Model (Sharpe 1964; Lintner 1965). In a perfect financial market with rational investors and homogeneous beliefs, the *Value-Weighted Market Portfolio* (*VWMP*) is, in equilibrium, the portfolio that all investors should hold. Also, in equilibrium, *VWMP* is the pricing portfolio, so that correctly priced assets have zero alphas with respect to it.

In practice, however, investors have heterogeneous beliefs and are imperfectly rational, violating the CAPM assumptions. With regard to beliefs, some investors are better informed than others about asset payoffs, and even investors with equally good information receive heterogeneous signals. So investors should not hold the same portfolio.

Second, evidence from the laboratory and the field indicates that people are ambiguity averse. This can discourage investors from participating in the market and thus affect asset risk premia.[2] In particular, if all investors have common knowledge of the structure of the capital market, they will have the courage to invest in all assets, because their risk will be reduced by the information they glean from asset prices (i.e., price signals), which partially aggregate other investors' information.[3] In contrast, if some investors face model uncertainty (also known as ambiguity) about the financial market, they may be unable to extract information from the price and may take zero positions in assets that they are ambiguous about. The non-participation caused by ambiguity aversion may obstruct the information aggregation of asset prices, and undermine the roles of *VWMP* as a common holding of investors and as a pricing portfolio.

[1] In recent years investors have increasingly followed this recommendation. For example, according to Morningstar *Direct Asset Flows Commentary*, February 20, 2018, net flows into passive U.S. equity funds rose by $233.5 billion for the 12 months ended January 2018.

[2] Dimmock, Kouwenberg, and Wakker (2016), Dimmock et al. (2016), and Bianchi and Tallon (2018) measure ambiguity attitudes using artificial events based on Ellsberg urn experiments. Anantanasuwong et al. (2019) directly elicit ambiguity attitudes using an incentivized survey. These studies provide evidence suggesting that ambiguity aversion reduces market participation.

[3] Investor learning from asset prices has been extensively studied in rational expectations equilibrium models, such as Grossman and Stiglitz (1980), Hellwig (1980), and Admati (1985).

These points raise doubt about whether, in a financial market with information asymmetry and ambiguity aversion, the *VWMP* is dysfunctional in facilitating index investment and pricing assets. If, in equilibrium, investors are not willing to hold a *VWMP* fund as a component of their optimal portfolios, this raises the question of whether a different index can be designed that investors will use and benefit from. If so, is there an asset pricing model based on such a new index?

In this paper, we address these questions in a noisy rational expectations equilibrium setting with asymmetric information about asset payoffs, ambiguity averse investors who do not know some parameters of the financial market, and a passive fund that offers an index portfolio (not necessarily the *VWMP*). In particular, in addition to standard problems of information asymmetry, the ambiguity averse investors do not know the precisions of some assets' random supplies and optimize under worst-case assumptions about supply variance (that is, the max-min expected utility proposed by Gilboa and Schmeidler (1989)).[4]

We first provide a positive analysis of index investing and asset pricing when a passive fund offers investors the standard market index, *VWMP*. We show that the *VWMP* is dysfunctional under information asymmetry and ambiguity aversion. In equilibrium, investors who believe that some assets' random supplies may be extremely volatile hold zero position of the passive fund. That is, *VWMP* does not help ambiguity averse investors participate fully in the market via index investing. The problem is that these investors believe that the price signals are extremely noisy in the worst-case scenario, so that *VWMP* is too subjectively risky for them to hold.

Second, using *VWMP* as the pricing portfolio, individual assets have non-zero alphas in equilibrium. The non-zero alphas arise from both the information asymmetry and the ambiguity aversion. On the one hand, the non-zero alphas relative to *VWMP* are equivalent to the assertion that *VWMP* is not mean-variance efficient conditional only on asset prices. With information asymmetry, the efficient portfolio conditional on asset prices must reflect the average precision of investor private signals and the precision of random supply, because these precisions determine the distribution of price signals. The market capitalization weights in *VMWP* do not depend upon these two precisions; hence, assets have non-zero alphas relative to *VWMP*. On the other hand, compared

[4]While the assumptions that investors are subject to model uncertainty about the precisions of asset random supplies and that they have max-min utilities are sufficient for our main results, our results, especially those in the economy with the newly designed index, are robust to the parameters that the investors are ambiguous about and to the utility representations of the ambiguity aversion preference.

with the equilibrium asset holdings where investors commonly know the financial market, some investors hold less positions of the assets that they are ambiguous about, so those assets will have lower equilibrium prices and higher risk premia. We name the extra risk premia the "ambiguity premia," which also contribute to the non-zero alphas.

Given the failures of *VWMP*, we investigate whether a new index can be designed to facilitate index participation and serve as the pricing portfolio. The key is to have asset weights depend appropriately upon the precisions of investor private signals and the precisions of random supply, as well as assets' market values. Since these precisions measure the amount of risk an investor can reduce by trading based on the price signal, the weights are, in this sense, risk-adjusted. We, therefore, call the newly designed index the *Risk-Adjusted Market Portfolio* (*RAMP*). Specifically, relative to *VWMP*, *RAMP* has lower investment in more volatile assets (conditional on asset prices), i.e., it is a defensive (low volatility) investing strategy.

Importantly, while investors commonly know how *RAMP* is constructed (as a function of financial market parameters *including supply shock precisions*), investors who are ambiguous about the financial market do not know the exact composition of *RAMP* (since they do not know the values of the supply precisions). *RAMP* does not rely on private information about asset payoffs, so *RAMP* is in a sense of a passive index. Given these points, *RAMP* can be viewed as a "smart beta" investing strategy, a general approach that recently has gained popularity in investment practice.[5]

With the passive fund offering *RAMP*, we show that there is an equilibrium in which investors' asset holdings are exactly the same as those in the economy without model uncertainty. In particular, all investors, including ambiguity averse investors, hold exactly one share of *RAMP* (by delegating the passive component of their portfolios to the index fund) and additionally hold positions based upon their private information signals. Hence, investors do employ the index investment strategy in equilibrium. In other words, delegation of investment to the index fund solves the problem of ambiguity aversion and nonparticipation. Also, using *RAMP* as the pricing portfolio, in equilibrium, assets' alphas are zero, implying a new version of CAPM security market line under information asymmetry and ambiguity aversion.

The key intuition for why investors hold *RAMP* as the passive component of their portfolios derives from a new separation theorem, which applies in the setting with no

[5]Smart beta strategies seek to passively follow indexes that use alternative weighting schemes such as volatility rather than weighting solely by traditional market capitalization.

model uncertainty. In this setting, there is a rational expectations equilibrium in which any investor's equilibrium risky asset holding can be decomposed into two components. The first is just *RAMP*, which is a common deterministic component in all investors' equilibrium holdings. The second is the investor's *information-based portfolio*, which includes a non-zero position in an asset if and only if the investor receives a private signal about the asset.

This new separation theorem differs from the separation theorem derived in the literature in that an investor's optimal portfolio is separated by her conditionally independent signals of asset payoffs.[6] We, therefore, call it the *Information Separation Theorem*. Specifically, *RAMP* is an investor's equilibrium holding, when she trades based only on price signal. In contrast, an investor's information-based portfolio is her optimal portfolio based on her private signals and conditional on asset prices (as exogenous parameters). Therefore, any investor's information-based portfolio is independent of price signals.

The Information Separation Theorem provides new insight into why the investors' equilibrium asset holdings in the setting where ambiguity averse investors are subject to model uncertainty are the same as in the economy without model uncertainty. To deepen the intuition, we consider the following proposed strategy profile: each investor holds exactly one share of the fund, and additionally holds her information-based portfolio (which could be a nullity). Given that all other investors behave as prescribed, no investor has an incentive to deviate.

The key insight is that the fund provides investors with a channel to share risks. Consider, for example, an investor and a vector of precisions of assets' supply shocks that is possible according to her subjective belief. Given the value of this vector, the investor would be in a possible world without model uncertainty. In such a possible world, since all other investors are holding effectively one share of the fund and their own information-based portfolios, they are holding the same portfolios as they would in the rational expectations equilibrium in this world. Hence, the market clearing condition implies that the pricing function is the same as the one in the rational expectations equilibrium. Therefore, if the investor knew the parameter values that characterize this possible world, her optimal portfolio choice would consist of *RAMP* and her own

[6]This separation derives from the model assumption that any investor is a price taker and so her private signal and the price signal are conditionally independent. Hence, it follows from the Bayes' rule that investors can optimally construct a portfolio based on each of her signals and then sum all the constructed portfolios together to get the equilibrium asset holdings.

information-based portfolio.

Because the fund offers *RAMP* in any possible world, without the knowledge of the exact world, the investor's optimal portfolio choice is to hold one share of the fund together with her own information-based portfolio. Investors may disagree with each other about financial market parameters (owing to having different prior supports on their distributions) and thus the fund's composition and risks. Nevertheless, by holding one share of the fund along with their own information-based portfolios, each investor knows that her position is, in the proposed equilibrium, optimal in each possible world.

The fact that the investment strategy of holding one unit of the fund and her own information-based portfolio is optimal to an investor (given other investors' strategies) for any possible world in her belief support implies that such an investment strategy maximizes the investor's max-min utility. Put differently, a strong min-max property holds in the equilibrium. Therefore, ambiguity averse investors hold a diversified portfolio, *RAMP*, by engaging in index investing.

The above argument makes clear that an investor's willingness to hold the fund is an equilibrium outcome; the reasoning relies on her conjecture about the willingness of all other investors to hold the fund and their own information-based portfolios. So *RAMP* is not addressing ambiguity aversion (solely) by diversification, a principle which can benefit investors even in a partial equilibrium setting. The willingness to hold *RAMP* derives from an understanding that other investors will, in equilibrium, also be willing to hold *RAMP*. Indeed, even though investors know the fund's investment strategy (the function to construct *RAMP*), off the equilibirum path, an investor will not in general find it optimal to hold the fund if she knew that other investors are choosing off-equilibrium asset holdings. Hence, it is only in equilibrium, and by virtue of our new separation theorem, that we can conclude that ambiguity averse investors optimally hold the fund offering *RAMP*.

Interestingly, although investors agree to hold the fund in equilibrium, they disagree on the fund's composition, because it depends on financial market parameters about which investors have heterogeneous uncertainties. So our assumption that there is a passive fund that observes supply volatilities and offers *RAMP* accordingly does not assume away model uncertainty. Model uncertainty in equilibrium remains; it is just that investors find delegation to be an optimal way of addressing it.[7]

[7]Mele and Sangiorgi (2015) show that in a model without a passive fund offering *RAMP*, ambiguity averse investors acquire costly information about model parameters. In comparison with their finding,

The Information Separation Theorem also shows why all assets have zero alphas relative to *RAMP*. The fact that an investor with no private information optimally holds *RAMP* (through the fund) implies that *RAMP* is mean-variance efficient, so that the CAPM security market line holds with *RAMP* as the pricing portfolio. Because the pricing portfolio does not depend on the realization of the random supply shock, and the weight of each asset in the pricing portfolio are conditional on asset prices, which are publicly observable, the portfolio is potentially observed by an econometrician. This makes the model empirically testable.

Given the potential benefits of *RAMP* to investors, it is important to consider whether the portfolio is implementable in practice as an index fund. First, although we assume that a single passive fund knows all parameters of the financial market, enabling it to offer *RAMP*. However, it turns out that this assumption is not necessary. Suppose instead that no single agent knows all parameters of the financial market. Nevertheless, as long as each parameter is known by some agents, knowledgeable agents can start competitive specialized funds (e.g., geography-specific funds or industry-specific funds). This enables investors at large to hold *RAMP* by holding all specialized funds. To put this another way, it is possible to offer *RAMP* by creating as a fund of funds.

Second, if private agents do not have the knowledge about financial markets needed to offer a *RAMP* fund, government could potentially collect the needed information and make it public. This can enable the formation of *RAMP* funds and improve market participation and risk sharing. For either government or private parties, the cost of collecting relevant information about the financial market have been declining as big data technology develops.

A possible practical problem with implementation of *RAMP* is helping naïve investors understand its benefits. This suggests that improvements in financial literacy may be a further ingredient needed to have more investors obtain the benefits of market participation and risk sharing via *RAMP*.

Our findings contribute to the growing literature on the economic consequences of index investment. Chabakauri and Rytchkov (2016) find that the introduction of index trading increases volatilities and correlation of stock returns and that such an effect arises from the improved risk sharing. Bond and García (2018) show that as indexing becomes cheaper, indexing increases while individual asset trading decreases, and ag-

our results indicate that delegation can serve as a substitute for information acquisition as a means of addressing ambiguity aversion.

gregate price efficiency decreases while relative price efficiency increases. Baruch and Zhang (2018) prove that it is optimal for nonindex investors to index and establish a version of CAMP in a very different economic setting. All these papers assume that some investors are constrained to make index investing. By contrast, our paper assumes that all investors can freely employ index investing strategy and analyzes the different effects of different market indexes (*VWMP* versus *RAMP*) on portfolio choice and asset pricing.

Past literature has analyzed extensively the extent to which ambiguity aversion hinders market participation. This research mainly considers investor uncertainty about the first moment of asset payoffs and study market participation decisions in partial equilibrium frameworks (e.g., Bossaerts et al. (2010), Cao, Wang, and Zhang (2005), Dow and Werlang (1992), Easley and O'Hara (2009), Easley and O'Hara (2010), Epstein and Schneider (2010), and Cao, Han, Hirshleifer, and Zhang (2011)).[8] Our paper differs from this literature in considering investor model uncertainty about the second moment of asset payoffs in a rational expectations equilibrium setting, and in studying index investing as a way to address ambiguity aversion.

A very different version of the CAPM has been derived in somewhat similar model setups where all investors are perfectly rational (see, for example, Easley and O'Hara (2004), Biais, Bossaerts, and Spatt (2010), and the online appendix of Van Nieuwerburgh and Veldkamp (2010)). In these models, the market portfolio for CAPM pricing is the ex-post total supply of the risky assets, the sum of the endowed risky assets and the random supply of risky assets. This market portfolio is mean-variance efficient conditional on the average investor's information set, and so the CAPM security market line holds from the perspective of the average investor. The version of the CAPM security market line we derive differs in that the pricing portfolio is *RAMP*, which is determined ex ante (prior to the realization of the random supply shocks), and in that risk premia are conditional only upon the public information (market prices). This makes the market portfolio more directly observable to an econometrician.

Our setting endogenizes investor trust in fund managers (Gennaioli, Shleifer, and Vishny 2015). An insight that our approach reveals is that inducing investors to make index investing requires more than investor trust in the honesty and superior knowledge of fund managers about the financial market. It is crucial that investors foresee

[8] An exception is Watanabe (2016), who assumes that investors are ambiguous about the mean of the asset's random supply shock. The focus of Watanabe's paper is on market fragility.

an equilibrium in which other investors also trust the fund managers and trade accordingly. Off the equilibrium, an investor would not be willing to hold the fund, even if she trusts the fund manager. In a new working paper, Li and Wang (2018), a representative investor faces model uncertainty about the financial market and so is uncertain about the composition of the efficient portfolio offered by a fund based on the fund's knowledge of the financial market. They call such an uncertainty the "delegation uncertainty." In their partial equilibrium setting where there is no risk sharing, the delegation uncertainty causes the investor to partially delegate, leading to higher CAPM alphas. By contrast, in our model with *RAMP*, delegation uncertainty is endogenously eliminated by risk sharing among investors: holding one share of the fund and the information-based portfolio is optimal to an investor in every possible world in her subjective belief support, when other investors are employing the same investment strategy. So an investor is not concerned about the exact asset holdings through the fund, which vary across possible worlds in her belief support.

2 The Financial Market

A continuum of investors with measure one, who are indexed by i and uniformly distributed over $[0, 1]$ trade assets at date 0 and consume at date 1.

Assets. At date 0, each investor i can invest in a riskfree asset and $N \geq 2$ independent risky assets.[9] Let $\mathcal{Q}$ be the set of all risky assets. At date 1, the riskfree asset pays r units, and risky asset n pays f_n units of the single consumption good. In addition to trading directly, investors can also hold individual risky assets through a passive fund that commits to offering a portfolio X, which is an N-dimension column vector with the nth element being the shares of the nth risky asset in X.

Letting D_i be the vector of shares of the risky assets held by investor i, and d_i (a scalar) the shares of the fund held by investor i, investor i's effective risky asset holding is $d_i X + D_i$.

Return Information. Let $F = (f_1, f_2, \ldots, f_N)'$ be the vector of risky assets' returns.

[9] If assets are correlated with full rank prior variance-covariance matrix, asset payoffs can be decomposed into orthogonal factors, and investors' private signals are about these factors. Van Nieuwerburgh and Veldkamp (2009), Van Nieuwerburgh and Veldkamp (2010) and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) show that such a model has the same solution as in the setting with independent assets. In such a setting, investors hold factors instead of individual assets.

We assume that all investors share a common uniform improper prior of F, and so no investor has prior information about any risky asset's return.[10] Each investor i receives a vector of private signals S_i about asset returns, $S_i = F + \epsilon_i$, where F and ϵ_i are independent; and ϵ_i and ϵ_j are also independent. Each ϵ_i is normally distributed, with mean zero and precision matrix $\mathbf{\Omega}_i$. We assume that $\mathbf{\Omega}_i$ is diagonal for all $i \in [0,1]$, so investor i's private signal about asset n's payoff is uninformative about that of asset k.

Investor i is an informed investor of asset n if and only if the nth diagonal entry of $\mathbf{\Omega}_i$ is strictly positive. Let λ_n be the measure of informed investors of asset n; we assume that $\lambda_n \in (0,1)$, for all $n \in \mathcal{Q}$. Let $\text{diag}(\lambda)$ be the $N \times N$ diagonal matrix with the nth diagonal entry being λ_n.

For simplicity, we assume that the private signals of all informed investors of asset n have the same precision $\kappa_n > 0$. Let $\mathbf{\Omega}$ be the $N \times N$ diagonal matrix with the nth diagonal entry being κ_n. Letting $\mathbf{\Sigma}$ be the matrix of the average precision of private signals, we have

$$\mathbf{\Sigma} = \int_0^1 \mathbf{\Omega}_i \mathrm{d}i = \mathbf{\Omega}\text{diag}(\lambda) \tag{1}$$

As is standard, the independence of the errors implies that in the economy as a whole signal errors average to zero, so that the equilibrium pricing function does not depend on the error realizations (though it does depend on their distributions).

Random Supply. Let Z denote the random vector of supplies of all risky assets. We assume that Z is independent of F and of ϵ_i (for all $i \in [0,1]$). We further assume that Z is normally distributed with mean 0 and the precision matrix $\boldsymbol{U}$. By independence of assets, $\boldsymbol{U}$ is diagonal and positive definite, with the nth diagonal entry being u_n.

All parameters are common knowledge to investors except $\boldsymbol{U}$. We assume that for each asset n, a subset of uninformed investors do not know u_n. We say that such a group of investors are subject to *model uncertainty* (or are *ambiguous*) about asset n. Any investor i who is ambiguous about asset n will have her own subjective prior belief about u_n with the support $\left(0, \bar{u}_n^i\right)$, where $\bar{u}_n^i > 0$. So we allow different investors who

[10] The uniform improper prior assumption is for simplicity. Our model can be extended to an economy with a normal prior. Specifically, as we shall show, the investors' equilibrium asset holdings and the asset pricing implications are qualitatively same when the passive fund is offering the newly designed *RAMP*. By contrast, in the case where the passive fund offers *VWMP*, the investors may hold non-zero positions of risky assets based on the prior information; however, the investors' equilibrium asset holdings still differ from those in the economy without model uncertainty. We discuss in more detail an ambiguity averse investor's asset holdings in the economy with normal prior distributions of asset payoffs and a fund offering *VWMP* in the Online Appendix.

are ambiguous about a particular asset n to have different supports of their beliefs about u_n. Let $\mathcal{U}_i$ be the set of all possible subjective beliefs of investor i about $\boldsymbol{U}$, let $\boldsymbol{U}_i$ be a typical element in $\mathcal{U}_i$, and let $\underline{\boldsymbol{U}}_i$ be the lower bound of $\mathcal{U}_i$.

Let the measure of the group of investors who are ambiguous about n be $1 - q_n \in (0, 1 - \lambda_n)$. Let $\boldsymbol{Q}$ be the $N \times N$ diagonal matrix with the nth diagonal entry being q_n. For simplicity, we assume that an investor who is ambiguous about asset n is also uninformed about asset n. However, an investor who is uninformed about asset n may know u_n and so is not ambiguous about asset n.

Ambiguity Aversion. Taking the riskfree asset to be the numeraire, let P be the price vector of the risky assets. Also, let $W_i = (w_{i1}, w_{i2}, \ldots, w_{iN})'$ be the endowed shareholdings of investor i (we assume that the aggregate endowments of shares of each stock are strictly positive; that is, $W = \int_0^1 W_i \mathrm{d}i \gg 0$.) Then investor i's final wealth at date 1 is

$$\Pi_i = r\left[W_i' - \left(d_i X' + D_i'\right)\right] P + \left(d_i X' + D_i'\right) F. \tag{2}$$

The first term is the return of investor i's investment in the riskfree asset, and the second term is the total return from her investments in risky assets.

Since investor i is risk averse, if she knows all model parameters, at date 0 she maximizes a CARA expected utility function,

$$\mathbb{E}_i u(\Pi_i) = \mathbb{E}_i\left[-\exp\left(-\frac{\Pi_i}{\rho}\right)\right]. \tag{3}$$

The expectation in equation (3) is taken based on investor i's information about asset returns. Since the common prior about asset returns is uninformative, any investor i's information consists of the equilibrium price vector and the realization of a private signal S_i only.

However, investor i may be subject to model uncertainty about the precisions of some assets' random supplies. If so, owing to ambiguity aversion, she chooses an investment strategy (d_i, D_i) to maximize the infimum of her CARA utility. Formally, each investor i's decision problem is[11]

$$\max_{d_i, D_i} \inf_{\boldsymbol{U}_i \in \mathcal{U}_i} \mathbb{E}_i\left[-\exp\left(-\frac{\Pi_i}{\rho}\right)\right]. \tag{4}$$

[11] An investor's utility in this paper differs slightly from that defined in Gilboa and Schmeidler (1989). Since any investor's subjective prior about the precision of the random supply shock has a non-compact support, the investor maximizes the infimum rather than the minimum of her CARA utility among all possible precisions.

Equilibrium. We are interested in an equilibrium defined as follows.

Definition 1 *A pricing vector P^* and a profile of all investors' risky asset holdings $\{d_i^*, D_i^*\}_{i\in[0,1]}$ constitute an equilibrium, if:*

1. *Given P^*, (d_i^*, D_i^*) solves investor i's maximization problem in equation* (4)*, for all $i \in [0,1]$; and*

2. *P^* clears the market,*

$$\int_0^1 (d_i^* X + D_i^*)\,\mathrm{d}i = W + Z, \quad \textit{for any realizations of } F \textit{ and } Z. \tag{5}$$

As in the literature on rational expectations models, equilibrium prices play two roles: clearing the market and partially aggregating private information. Hence, a rational expectations equilibrium differs from a Walrasian equilibrium mainly in that the asset prices convey information about the asset payoffs to investors. This is especially important in our setting with model uncertainty. Since investors hold neither informative priors nor private signals about the payoffs of those assets they are ambiguous about, observation of asset prices is what allows them to update their beliefs to have finite conditional variances, making them willing to participate in the markets for those risky assets. Their ambiguity about random supply precisions, however, will hinder their learning from asset prices, discouraging them from holding non-zero positions of the assets they are ambiguous about.

3 The Value-Weighted Market Portfolio

We next discuss the investors' equilibrium asset holdings and the asset pricing implications when the fund is offering the investors with the value-weighted market portfolio (*VWMP*); formally, in this section, we assume that $X = W$. We show that ambiguity averse investors are not willing to hold a fund offering *VWMP* as the passive (noninformational) index component of their portfolios, and that *VWMP* is not the appropriate benchmark portfolio for pricing assets.

3.1 Equilibrium Asset Holdings

Since the composition of *VWMP* is commonly known, even if no fund is available that offers *VWMP*, an investor can attain the same effective risky asset holdings by directly

trading individual assets only. So an investor's optimal risky asset holdings are the same regardless of whether this passive fund is available. As a result, in order to analyze investors' equilibrium asset holdings when *VWMP* is publicly offered, we can first consider an economy without *VWMP*.

Investor i is risk averse, so she only holds a non-zero position of asset n, if her subjective belief of asset n's payoff has a finite variance, conditional on her information. When investor i is uninformed about asset n, however, she has neither prior information nor private information about the payoff of asset n. Hence, she estimates the payoff based on only the price, which partially aggregates informed investors' private information. Since the precision of asset n's random supply, u_n, is strictly positive (no matter how small it is), if investor i knows u_n, her belief of asset n's payoff has a finite variance and, therefore, she will hold a non-zero position of asset n.

On the other hand, if investor i is subject to model uncertainty about asset n, she does not know the precision of asset n's random supply. By assumption, investor i's subjective prior about u_n has the support $\left(0, \bar{u}_n^i\right)$. Since all random variables in the model are normally distributed, observing the asset price does not change the support of investor i's belief about u_n, although investor i may extract some information about u_n from asset n's price. Hence, the worst-case scenario is independent of asset n's price. Specifically, when the precision of the random supply is arbitrarily close to zero, price becomes almost uninformative. So as investor i considers the worst-case scenario in making the investment decision, she focuses on the possibility that the true u_n is very close to 0. For any non-zero position of asset n, as the price becomes almost uninformative, the payoff variance conditional upon investor i's information diverges to infinity. So holding a non-zero position is extremely risky in the worst-case scenario. To avoid this risk, investor i optimally chooses a zero position. Lemma 1 below summarizes the argument above.

Lemma 1 *An investor i who is ambiguous about asset n optimally holds a zero position in it.*

We now analyze the investors' equilibrium asset holdings. The model is similar to the rational expectations equilibrium model with multiple risky assets (Admati 1985). The key difference is that for each asset n, there are $1 - q_n$ measure investors who will hold a zero position (by Lemma 1). Proposition 1 below characterizes a linear rational expectations equilibrium. Recall that since the composition of *VWMP* is commonly known, whether or not such a portfolio is offered by the passive fund, investors will

have the same effective risky asset holdings.

Proposition 1 *In the model where the passive fund offers VWMP (formally, $X = W$), there exists a linear equilibrium with the pricing function*

$$P = \boldsymbol{B}_V^{-1}\left[F - A_V - \boldsymbol{C}_V Z\right], \tag{6}$$

where

$$A_V = \frac{1}{\rho}\left[\rho^2(\boldsymbol{\Sigma}\boldsymbol{Q}\boldsymbol{U}\boldsymbol{\Sigma}) + \boldsymbol{\Sigma}\right]^{-1} W \tag{7}$$

$$\boldsymbol{B}_V = r\boldsymbol{I} \tag{8}$$

$$\boldsymbol{C}_V = \frac{1}{\rho}\boldsymbol{\Sigma}^{-1}. \tag{9}$$

Any investor i's effective risky asset holding is

$$d_i X + D_i = \lim_{\boldsymbol{U}_i \to \underline{\boldsymbol{U}}_i}\left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma}\boldsymbol{U}_i\right)^{-1}\right]^{-1} W + \rho\boldsymbol{\Omega}_i\left(S_i - rP\right) \tag{10}$$

Equations (6) and (7) show that for each risky asset n, the measure of investors who know the precision of its random supply affects its equilibrium price. In particular, $\boldsymbol{QU}$ is the matrix of the average precisions of asset random supplies in the investors' subjective "worst-case scenarios," which positively affect the asset prices. Hence, *ceteris paribus*, if $q_k > q_n$, the equilibrium price of asset k is greater than that of asset n. Intuitively, when $q_k > q_n$, on average the subjective worst case for k is not as bad as for n, so the demand function for asset k is higher than the demand function for asset n. So when both assets have the same supply, asset k's price is higher than that of asset n.

Equation (10) characterizes investor i's effective risky asset holdings in equilibrium. For each asset n, if investor i knows the precision of its random supply, she will hold a position based on the equilibrium price. Formally, in such a case, the nth diagonal entry of $\underline{\boldsymbol{U}}_i$ is $u_n > 0$; hence, the first term in equation (10) is positive. Furthermore, if such an investor receives a private signal about asset n's payoff, the nth diagonal entry of $\boldsymbol{\Omega}_i$ is $\kappa_n > 0$, and so the second term is also positive.

At the other extreme, if investor i is ambiguous about asset n, both the nth diagonal entry of $\underline{\boldsymbol{U}}_i$ and the nth diagonal entry of $\boldsymbol{\Omega}_i$ are zero, implying that investor i holds a zero position of asset n. Therefore, even with a passive fund offering *VWMP*, ambiguity averse investors will not participate in some assets' markets. As a result, *VWMP* is not effective in encouraging ambiguity averse investors to hold better-diversified portfolios.

3.2 Asset Pricing with *VWMP*

We next analyze asset risk premia to see if *VWMP* is, as in the CAPM, the relevant pricing portfolio. Given any realized equilibrium price P, the volatility of asset payoffs derives from the supply shock only. Let $\text{diag}(P)$ be an $N \times N$ diagonal matrix, whose n^{th} diagonal element is the nth element of the vector P. Generically, as no asset has a zero price, $\text{diag}(P)$ is invertible. Then, by the definition of $\text{diag}(P)$,

$$\text{diag}(P)^{-1}P = \mathbb{1}, \tag{11}$$

where $\mathbb{1} = (1, 1, \ldots, 1)'$. From equation (6), we can calculate the difference between individual assets' expected rates of return and the riskfree rate as

$$\text{diag}(P)^{-1}\mathbb{E}(F) - r\mathbb{1} = \text{diag}(P)^{-1}A_V, \tag{12}$$

where A_V is characterized in equation (7), and the expectation is taken conditional on the asset prices.

Since the pricing portfolio is *VWMP*, we calculate the market capitalization weights for individual assets as

$$\omega_V = \frac{\text{diag}(P)W}{P'W}. \tag{13}$$

Then, the variance of *VWMP* is

$$Var(R_V) = \left(\frac{1}{P'W}\right)^2 W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W, \tag{14}$$

and the covariances between individual assets and *VWMP* are

$$Cov(R, R_V) = \frac{1}{P'W}\text{diag}(P)^{-1}\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V. \tag{15}$$

Here, $\boldsymbol{C}_V$ is characterized in equation (9).

Therefore, when the pricing portfolio is *VWMP*, the assets' betas are

$$\beta_V = \frac{Cov(R, R_V)}{Var(R_V)} = P'W\frac{\text{diag}(P)^{-1}\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}, \tag{16}$$

and their alphas are

$$\alpha_V = \text{diag}(P)^{-1}A_V - \beta_V\left[\mathbb{E}(R_V) - r\right] = \text{diag}(P)^{-1}\left[\boldsymbol{I} - \frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}\right]A_V. \tag{17}$$

Simple algebra verifies that *VWMP* does not successfully price assets in the capital market.

Proposition 2 *With VWMP being the pricing portfolio, the assets' alphas are not equal to zero.*

The non-zero alphas show that in contrast with the CAPM, in our model *VWMP* does not price assets correctly in the cross section. These alphas derive from both information asymmetry and ambiguity aversion. First, the traditional CAPM is based on homogeneous beliefs which are fully impounded in the market capitalization weights in *VWMP*. Hence, conditional on the asset prices, *VWMP* is mean-variance efficient. In contrast, in our setting, owing to information asymmetry, the average precision of private signals ($\boldsymbol{\Sigma}$) and the precision of random supply ($\boldsymbol{U}$) will determine the price signal distribution. This is directly implied by the equilibrium pricing function (equation (6)). Therefore, the weights in a portfolio that can price assets correctly must be functions of these two precisions. *VWMP*, however, has value weights, which are not functions of these two precisions, so it cannot be efficient conditional on asset prices in the financial market with information asymmetry. Hence, using *VWMP* as the pricing portfolio, the assets should have non-zero alphas.

Second, ambiguity aversion also affects appropriate index portfolio weights for asset pricing, which further contributes to nonzero alphas relative to *VWMP*. Intuitively, when more investors are ambiguous about an asset, its demand curve shifts leftward, leading to a lower price and a higher risk premium. We then refer to the increment in the asset's risk premium due to ambiguity aversion as the asset's *ambiguity premium*.

4 The Risk-Adjusted Market Portfolio

Proposition 1 and Proposition 2 show that as an index fund, the conventional stock market index, *VWMP*, fails to encourage better diversification and risk sharing by ambiguity averse investors, and fails as a benchmark for equilibrium asset pricing. The dysfunctions of *VWMP* derive from important limitations: assets are weighted in *VWMP* by their market capitalization only, so these weights depend upon neither the average precision of private signals nor the precision of random supply, which determine the distribution of price signal.

This raises the question of whether there is an alternative index which, if offered by a fund, ambiguous investors would be willing to hold. If so, this could improve diversification and risk-sharing between investors who are ambiguous about any given asset and those who are not. We will show that a new index design achieves these goals.

The index is formed by adjusting the market capitalization weights in *VWMP* by the

average precision of investor private signals and the precision of random supply. Since these precisions determine the amount of risks an investor can reduce by trading based on the price signal, we name the new index the *Risk-Adjusted Market Portfolio* (*RAMP*). We analyze portfolio choices and asset pricing, when there is a passive fund offering this index portfolio.

Formally, *RAMP* is defined as

$$X = \left[I + \frac{1}{\rho^2} \left(\boldsymbol{\Sigma} \boldsymbol{U} \right)^{-1} \right]^{-1} W. \tag{18}$$

Ambiguity averse investors do not know $\boldsymbol{U}$ and hence do not know the exact composition of X. However, the functional relationship between X and $\boldsymbol{U}$ that is specified in equation (18) is common knowledge to investors. *RAMP* does not depend on any of the private signals about asset payoffs, and so *RAMP* is a passive asset management product. Because of these features (i.e., passive but weighting assets not only by market capitalization), *RAMP* can be viewed as a type of *smart beta* strategy, an investing approach which has been growing in popularity in investment practice.[12]

More specifically, *RAMP* an be viewed as a defensive investment strategy in the sense that it underweights high volatility stocks. Specifically, *RAMP* differs from *VWMP* in that it contains a component $(\boldsymbol{\Sigma} \boldsymbol{U})^{-1}$. It then follows from equation (18) that *RAMP* includes fewer shares of more volatile assets. Therefore, holding *RAMP* will be a defensive investment strategy that can largely reduce the risk of loss.

4.1 Equilibrium Index Investing

The key result in this subsection is that with the passive fund offering *RAMP*, all investors employ the index investment strategy in equilibrium by holding one share of the fund, and their effective asset holdings are exactly the same as in the economy without model uncertainty. Proposition 3 formalizes this result.

Proposition 3 *In the model with a passive fund that commits to offering the portfolio X specified in equation* (18)*, there is an equilibrium in which*

1. *All investors will buy one share of the passive fund, and so* $d_i^* = 1$ *for all* $i \in [0, 1]$*;*

2. *Any investor i will hold an extra portfolio* $\rho \boldsymbol{\Omega}_i \left(S_i - rP \right)$*; and*

[12] By the end of December 2017, smart beta funds have surpassed $1 trillion in assets under management.

3. *For any given F and Z, the equilibrium price is*

$$P = \boldsymbol{B}^{-1}\left[F - A - \boldsymbol{C}Z\right], \tag{19}$$

where

$$A = \frac{1}{\rho}\left[\rho^2(\boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{\Sigma}) + \boldsymbol{\Sigma}\right]^{-1} W \tag{20}$$

$$\boldsymbol{B} = r\boldsymbol{I} \tag{21}$$

$$\boldsymbol{C} = \frac{1}{\rho}\boldsymbol{\Sigma}^{-1}. \tag{22}$$

The intuition of Proposition 3 builds upon a new separation theorem that applies in the setting without model uncertainty. We develop this intuition in detail in Section 4.2. In brief, *RAMP* provides investors with a channel to share risks, so that when other investors behave as prescribed, an ambiguity averse investor will find that holding *RAMP* and her information-based portfolio is optimal in each possible world in her subjective belief support. In the rest of this subsection, we discuss some properties of the equilibrium characterized in Proposition 3.

First, investors all hold exactly one share of the fund, even though they have heterogeneous priors about $\boldsymbol{U}$ and thus different beliefs about the fund's composition. Take two investors, Lucy and Martin, for an example, who do not receive private information about asset payoffs. Lucy believes that for all n, u_n could be arbitrarily close to 0; Martin knows the true precisions of all supply shocks. Hence, Lucy and Martin have very different beliefs about the composition of the fund. By Proposition 3, however, both Lucy and Martin will hold one share of the passive fund, and neither holds any extra active positions because they don't have any private information about assets' payoffs. Hence, Lucy and Martin hold the same overall portfolio. So, if a *RAMP* index fund is available, differences in investors' holdings arise solely from differences in their private signals about asset payoffs, not from differences in their model uncertainties.

Second, Proposition 3 shows that the willingness of investors to buy an index is based on an understanding of equilibrium risk-sharing, rather than just a partial equilibrium understanding that there can be risk-reduction benefits to the investor in isolation to diversifying her portfolio. Specifically, consider an investor who faces model uncertainty about a subset of traded assets, and views the return distributions as exogenous. Even if she can indirectly trade those assets through a passive fund, it may not be optimal for her to do so, because she cannot calculate the fund's expected return and risk. Therefore,

arguments based on the incentive of individuals to diversify do not, under radical ignorance, justify holding of the fund. In contrast, in our equilibrium setting, an investor optimally holds the fund, given her belief that other investors will also do so (together with their direct portfolios). Hence, she is willing to hold the fund too, which achieves the benefit of optimally sharing risks with other investors.[13]

Proposition 3 more broadly suggests that the reason why actual investors often fail to diversify goes beyond investor ambiguity aversion. In particular, for an investor to hold the fund, all other investors need to behave according to the prescribed equilibrium strategy profile. If imperfectly rational investors reason about possible portfolios based solely on partial equilibrium risk and return arguments, portfolios containing assets that investors are ambiguous about might seem extremely risky (or in the limiting case, infinitely risky). Proposition 3 shows that, owing to equilibrium considerations, even ambiguity averse investors, if otherwise rational, will hold such assets. But actual investors may not understand the equilibrium reasoning which underlies this result.

Third, and finally, comparing the equilibrium characterized in Proposition 3 with that in Proposition 1, we can see the different effects of *RAMP* and *VWMP* on the asset prices and investors' equilibrium asset holdings. Specifically, whether the passive fund offers *RAMP* or *VWMP*, conditional on the asset payoffs, the price volatility is same. This follows from the fact that $\boldsymbol{B}$ and $\boldsymbol{C}$ in the equilibrium pricing function with *RAMP* are equal to $\boldsymbol{B}_V$ and $\boldsymbol{C}_V$ in the equilibrium pricing function with *VWMP*, respectively. For any give asset payoffs (F) and asset random supplies (Z), however, the asset prices are higher in the economy with *RAMP* being the index, because $A < A_V$. This then directly implies that an informed investor's asset holdings based on her private signals are higher in the economy with *VWMP*. On the other hand, while an informed investor's asset holdings based on the price signals are the same in both the economies with *RAMP*

[13]The fact that equilibrium rather than just diversification considerations are crucial for the index investment result can be seen more concretely by considering the off-equilibrium possibility that other investors trade in a fashion that causes asset prices to be almost uninformative. In such a scenario, an ambiguity averse investor would not hold the passive fund, because *RAMP* would be perceived as extremely risky. Specifically, suppose that the off-equilibrium trading strategy profile of other investors leads asset price informativeness to converge to zero as the random supply shock precisions go to zero. This convergence could be even faster than the convergence of the asset positions in *RAMP* to zero. Hence, taking any nonzero position of the passive fund will give the investor infinite risks in the worst-case scenario, since she believes that random supply shock precisions could be extremely close to zero. Therefore, the investor will not hold the fund. In contrast, in such a case, an uninformed investor who knows the supply shock precisions may still hold asset positions that are bounded away from zero, because the investor can extract asset payoff information from asset prices, resulting in finite risk.

and with *VWMP*, an ambiguity averse investor's asset holdings are higher with *RAMP*. The market clearing condition then implies that when the passive fund offers *VWMP*, the informed investors trade more aggressively based on their private signals to absorb the random supplies, and hence they should receive higher premia, which can be called ambiguity premia. This in turn suggests that when the passive fund offers *RAMP*, the assets' ambiguity premia should disappear, which we verify in Subsection 4.3.

4.2 The Information Separation Theorem

Proposition 3 is surprising, since the fund is at an informational disadvantage relative to informed investors. There is a clear potential benefit to ambiguous investors to holding the fund to take advantage of the fund's knowledge of the precisions of the supply shocks. But the informational disadvantage of the fund relative to some investors also might seem to be a potential danger for ambiguous investors. Indeed, off equilibrium, if other investors do not trade as prescribed in Proposition 3, an ambiguity averse investor in general may not hold the fund.

So a valid intuition for Proposition 3 must go beyond the passive fund's superior knowledge about the parameters of the financial markets. To build insight into this result, we now derive a new separation theorem for financial markets with asymmetric information, but without model uncertainty or funds.

To do so, we simplify the model described in Section 2 by assuming that $\boldsymbol{U}$ is common knowledge among all investors and that there is no passive fund.[14] Then the model is a traditional rational expectations equilibrium model with multiple risky assets, analyzed by Admati (1985). Proposition 4 characterizes a linear rational expectations equilibrium and shows investors' optimal risky assets holding when all parameters are common knowledge.

Proposition 4 *In the model whose parameters are all common knowledge among investors, there exists an equilibrium with the pricing function*

$$P = \boldsymbol{B}^{-1}\left[F - A - \boldsymbol{C}Z\right], \tag{23}$$

[14]The theorem we are about to state does not require the assumption of an uninformative prior. Hence, we prove a general version of Proposition 4 in the appendix for the case of independent assets with normal priors. Since both the equilibrium pricing function and investors' equilibrium holdings are continuous in the prior precisions of assets' payoffs, substituting zero prior precisions will lead to exact Proposition 4.

where

$$A = \frac{1}{\rho}\left[\rho^2(\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma}\right]^{-1} W \tag{24}$$

$$\boldsymbol{B} = r\boldsymbol{I} \tag{25}$$

$$\boldsymbol{C} = \frac{1}{\rho}\boldsymbol{\Sigma}^{-1}. \tag{26}$$

Any investor i's risky asset holding is

$$D_i = \left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma U}\right)^{-1}\right]^{-1} W + \rho\boldsymbol{\Omega}_i\left(S_i - rP\right). \tag{27}$$

Owing to supply shocks, asset prices are not fully revealing, so information asymmetry persists in equilibrium and different investors have different asset holdings. An investor's asset holding is the sum of two components. The first term in equation (27),

$$\left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma U}\right)^{-1}\right]^{-1} W$$

is just *RAMP*, which is deterministic.

The second component of any investor's risky asset holding, the second term in (27), is what we call information-based portfolio. This position, $\rho\boldsymbol{\Omega}_i\left(S_i - rP\right)$, consists of extra holdings in the securities about which the investor has information. Investor i holds such an extra position of an asset n if and only if the nth diagonal entry of $\boldsymbol{\Omega}_i$ is $\kappa_n > 0$. This suggests that any investor i holds direct positions of a risky asset because possessing an informative signal about such an asset reduces its conditional volatility (independent of the signal realization). Investor i's direct positions of a risky asset also come from her speculation, which is taken to exploit superior information. Different investors, even if they are informed about asset n, hold different speculative portfolios, because they receive heterogeneous private signals.

Crucially, in each investor's equilibrium asset holdings in equation (27), the two components depend upon investors' information sets in different ways. The first component, *RAMP*, is formed based only on the information that the investor gleans from asset prices; it is independent of the investor's private information. In contrast, the second component, the information-based portfolio, can be formed based only on the investor's own private information; it is independent of the information content of the market price. Since the supply shock precisions do not affect the distributions of the

private signals, it follows that the information-based portfolio is *independent of the supply shock precisions*. The reason for this independence is that each individual investor is "small" and thus her trading cannot affect the asset prices and thus the price informativeness. [15] This independence implies a new separation theorem under asymmetric information.

Theorem 1 (The Information Separation Theorem) *When the characteristics of all assets are common knowledge, equilibrium portfolios have three components: a deterministic risk-adjusted market portfolio (RAMP); an information-based portfolio based upon private information and equilibrium prices but no extraction of information from prices; and the riskfree asset.*

Theorem 1 indicates that any investor can form an optimal portfolio in separate steps: (1) buy one share of *RAMP*; (2) buy the information-based portfolio using only private information, not the information extracted from price; and (3) put any left-over funds into the riskfree asset. This separation theorem derives from market equilibrium as well as optimization considerations. This differs from those (non-informational) separation theorems in the literature that are based solely on individual optimization arguments.[16]

In our model, the fund can provide *RAMP* because it knows all the model parameters, and *RAMP* does not include any investor's private information. Meanwhile, the information-based portfolio is exactly the same as the direct holdings of the risky assets in Proposition 3. To form the information-based portfolio, an investor does not need to extract information from the equilibrium price: she can treat the equilibrium prices as given parameters, and solve for the information-based portfolio from her CARA utility maximization problem as in a partial equilibrium model.

The Information Separation Theorem provides the intuition for investors' equilibrium investment strategies in the setting with model uncertainty. Consider the model in which investors are uncertain about the precisions of some assets' supply shocks. The easiest way to build the intuition is to first suppose that investors have min-max

[15] Vives (2008) derives investors' equilibrium asset holdings in a single-asset environment with a normal prior and zero aggregate endowment. Therefore, his result cannot be directly used in our analysis when investors are ambiguity averse about some assets.

[16] It may seem puzzling that none of the three portfolio components depend on the information that an investor extracts from price. How then does this information enter into the investor's portfolio decision? The answer is that *RAMP* is optimal precisely because of the ability of investors to extract information from price. As mentioned before, *RAMP* is deterministic; it does not depend on the private signals. But the fact that *RAMP* is an optimal choice is true only because investors update their beliefs based on price. So the optimal portfolio choice is indeed influenced by such information extraction.

preferences, and then show that our conclusions also apply to max-min preferences as well. With min-max preferences, we argue that given other investors' strategies of holding *RAMP* and their own information-based portfolios, an investor's optimal trading strategy is constant across all possible worlds in her subjective belief support. Hence, the investor's optimal trading strategy with max-min utility is the same as that with min-max utility.

Specifically, when investor i has a min-max utility, for each possible world $\boldsymbol{U}_i \in \mathcal{U}_i$, she can solve her optimal risky asset holdings, assuming that the equilibrium pricing function is the one in equation (19) with $\boldsymbol{U}$ being $\boldsymbol{U}_i$. Importantly, because all other investors are holding one share of the fund along with their own direct information-based portfolios, they are effectively holding the risky assets as in the world with $\boldsymbol{U}_i$ being common knowledge. Therefore, in the possible world $\boldsymbol{U}_i$, the market clearing condition implies that the pricing function is the one specified in equation (19) with $\boldsymbol{U}$ being $\boldsymbol{U}_i$. That is, investor i's belief about the pricing function is correct. So, she would like to hold the risky assets as in the world $\boldsymbol{U}_i$. Such risky asset holdings can be implemented by holding one share of the passive fund and her information-based portfolio, so investor i would like to use the investment strategy in Proposition 3. Furthermore, investor i is still uncertain about $\boldsymbol{U}$, so holding the risk-adjusted market portfolio through holding one share of the fund is strictly preferred.

In the above, for any given possible world, holding one share of the fund and her own information-based portfolio maximizes investor i's expected CARA utility (given that all other investors trade according to the prescribed strategy profile). Since such a trading strategy is optimal across all possible worlds, it maximizes investor i's max-min utility. That is, a strong max-min property holds in the equilibrium, and hence, in our model with investors having max-min utilities, the investment strategy of holding one share of the fund and the information-based portfolio is also optimal to investors.

So the Information Separation Theorem helps explain why ambiguity averse investors are willing to hold the fund that offers *RAMP* in an equilibrium. Indeed, the same argument can also be applied when investors have heterogeneous risk tolerances. In the Online Appendix, we extend our model to allow for heterogeneous risk tolerances. We find that in such an extension, investors are still willing to employ the index investment strategy, but their equilibrium holdings of the passive fund depend on their risk tolerances.

4.3 CAPM Pricing with *RAMP*

Propositions 3 and 4 indicate that the model with ambiguity aversion and a fund that offers *RAMP* has an equilibrium in which investors' effective risky assets holdings are exactly the same as in the rational expectations equilibrium without model uncertainty. This suggests that asset risk premia should not have any ambiguity premia. In addition, as shown in the Information Separation Theorem, *RAMP* is the efficient portfolio conditional only on the price signals. This suggests that a version of CAPM security market line will hold under information asymmetry and ambiguity aversion, with *RAMP* as the benchmark pricing portfolio.

To formally analyze asset alphas relative to *RAMP* as an asset pricing benchmark, we first return to the special case of no model uncertainty. From equation (23), the equilibrium pricing function in this case is

$$P = \frac{1}{r}\left[F - A - \frac{1}{\rho}\boldsymbol{\Sigma}^{-1}Z\right], \tag{28}$$

where $A = \frac{1}{\rho}[\rho^2(\boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{\Sigma}) + \boldsymbol{\Sigma}]^{-1}W$. We then have

$$\mathrm{diag}(P)^{-1}\mathbb{E}(F) - r\mathbb{1} = \mathrm{diag}(P)^{-1}A. \tag{29}$$

Here, $\mathbb{E}(F)$ is the expected payoff conditional on the equilibrium price. The LHS of equation (29) is just the vector of the risky assets' equilibrium risk premia.

Given a realized equilibrium price, *RAMP* has the value $P'X$. Then the vector of the weights of risky assets in *RAMP* is

$$\omega = \frac{1}{P'X}\mathrm{diag}(P)X.$$

Hence, conditional on the price P, the difference between the expected return of *RAMP* and the riskfree rate is

$$\begin{aligned}\mathbb{E}(R_X) - r &= \omega'\mathrm{diag}(P)^{-1}\mathbb{E}(F) - r \\ &= \frac{1}{P'X}X'\mathrm{diag}(P)\mathrm{diag}(P)^{-1}(A + rP) - r \\ &= \frac{1}{P'X}X'A, \end{aligned} \tag{30}$$

where the expectations are all conditional on the equilibrium price.

The variance of *RAMP* is

$$\mathbb{V}(R_X) = \mathbb{E}\left[\left(\omega'\mathrm{diag}(P)^{-1}\boldsymbol{C}Z\right)\left(\omega'\mathrm{diag}(P)^{-1}\boldsymbol{C}Z\right)'\right] = \left(\frac{1}{P'X}\right)^2 X'\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}X, \tag{31}$$

and the covariance between all risky assets and *RAMP* is

$$Cov(R, R_X) = \frac{1}{P'X}\text{diag}(P)^{-1}\boldsymbol{CU}^{-1}\boldsymbol{C}X. \tag{32}$$

Let α_X be the CAPM alpha with *RAMP* being the pricing portfolio. From equations (29)-(32), and since $X = \rho(\boldsymbol{CU}^{-1}\boldsymbol{C})^{-1}A$, we have the following proposition.

Proposition 5 (Risk premia with Supply Shocks) *In the model with all parameters being common knowledge, asset risk premia satisfy the CAPM security market line where the relevant market portfolio for pricing is RAMP.*

This result may seem surprising, since investors have heterogeneous asset holdings, and since the portfolios held by informed investors are not mean-variance efficient with respect to the public information set. Nevertheless, in equilibrium, there are no extra risk premia incremental to those predicted by the CAPM using *RAMP*.

The CAPM pricing relation using *RAMP* is equivalent to the assertion that *RAMP* is mean-variance efficient conditional only on asset prices. This efficiency can be seen from the utility maximization problem of an investor who is uninformed about all assets. Such an investor balances the expected returns and the risks of her holdings, and her information consists of the equilibrium price only. In equilibrium, such an investor holds *RAMP*, implying that *RAMP* is mean-variance efficient conditional only on equilibrium prices.

Privately informed investors also hold *RAMP* as the passive component of their portfolios; this is the piece that does not depend upon their private signals (except to the extent that their signals are incorporated into the publicly observable market price). In addition, they have other asset holdings to take advantage of the greater safety of assets they have more information about, and for speculative reasons based upon their private information. *RAMP* is not mean-variance efficient with respect to their private information sets, but it is efficient with respect to the information set that contains only publicly available information.

In the special case of asymmetric information but no model uncertainty, *RAMP* is a natural candidate for the CAPM pricing portfolio, because it is the common component in all investors' risky asset holdings. We show that *RAMP* is mean-variance efficient unconditional on any investor's private information. Therefore, the CAPM security market line relation holds without conditioning on private information, with respect to *RAMP*.

What is perhaps more surprising is that when there is a passive fund that offers *RAMP*, even with model uncertainty, *RAMP* is the appropriate CAPM pricing portfolio. To see how the presence of a fund offering *RAMP* affects asset risk premia when there is model uncertainty, consider equilibrium asset holdings as in Proposition 3. This shows that even when investors are uncertain about the precisions of asset supply shocks, they all hold one share of the passive fund, eliminating ambiguity premia. So asset risk premia satisfy the CAPM.

Corollary 1 presents this even more surprising result.

Corollary 1 *In the model where investors are uncertain about the precisions of some assets' supply shocks, and a passive fund is offering RAMP that is specified in equation* (18)*, asset risk premia satisfy the CAPM with RAMP being the pricing portfolio.*

5 Centralized versus Distributed Implementation of the *RAMP* Fund

We have shown that our new index, *RAMP*, can encourage all investors, including ambiguity averse ones, to employ index investment strategy, and thus in equilibrium investor risky asset holdings are exactly same as those in the economy without model uncertainty.

In order to offer *RAMP*, the passive fund needs to have full knowledge about the relevant parameter values that characterize the capital market. Improving information processing technologies and "big data" may have improved the feasibility of this over time. It is also possible that the regulatory powers of government may give it advantages for collecting information relevant for estimating the relevant parameter values. If so, in principle the government itself could offer *RAMP*, or could publicly disclose relevant information that helps others estimate relevant parameters. So potentially either government or private funds can contribute to the offering of *RAMP*, promoting investor participation and risk sharing. There can also be agency problems associated with either public or private fund providers, a topic that we do not focus on in this paper.

By whatever means, if a single agents has access to all relevant parameter values, a centralized approach to offering *RAMP* is straightforward. The agent calculates *RAMP* based on equation (18) and publicly announces the formula for calculating portfolio

weights. If there are multiple agents with the requisite information, competition between them can drive fees to a very low level.

What is less obvious is that even if no single agent knows all the relevant parameters, a decentralized approach can also implement *RAMP*. Suppose that each parameter is known by a nontrivial set of investors. Specifically, suppose that the set of all traded assets can be partitioned into M subsets. In the partition j, there are $m_j \geq 1$ assets. We assume that there is a positive measure of investors, who know all parameters about assets in partition j but do not have any private signals about the payoffs of such assets. We call these investors "Group j uninformed investors." (In the extreme case, $M = N$, and so, in each partition, there is only one asset. Then, we are in the setting described in Section 2.)

We consider the following equilibrium. Each of the Group j uninformed investors commits to offering a portfolio Y_j. Here, Y_j can be seen as a "local" fund, which includes only assets in partition j. For each asset n included in asset partition j, Y_j includes exactly the same position as in the portfolio X.

First, the fund fee will be zero in an equilibrium. Since there are infinitely many funds who are committing to offering Y_j, the jth local fund industry is perfectly competitive. Hence, the fund fee should be the same as the marginal cost of offering Y_j, which is zero.

Second, and more importantly, as required in the existing index fund industry, all local funds are required to disclose their asset holdings at the end of the period. Then, if a local fund of Group j that deviates from Y_j, its portfolio holding will differ from other Group j local funds' portfolio holdings. Hence, such a deviation is observable ex post and verifiable. Ex post, once a fund's deviation is detected, we assume that the fund will be heavily punished or incur a large reputation cost. It follows that no local fund is willing to deviate from its commitment to invest in Y_j. This indeed follows the idea of "Nash implementation" in the mechanism design literature.

Finally, any investor will first buy one share of the Group j local fund, for each j. By doing so, any investor will form an asset holding $(Y_1', Y_2', \ldots, Y_M')' = X'$, which is exactly *RAMP* specified in equation (18). Then, investors will hold their own information-based portfolios. Obviously, investors are effectively holding one share of the passive fund and their own information-based portfolios, which are their optimal investment strategies in the equilibrium described in Proposition 3.

6 Concluding Remarks

We analyze here two major roles played by market indexes, facilitating diversified investing, and providing an appropriate asset pricing benchmark, in a financial market with information asymmetry, model uncertainty, and ambiguity aversion. We show that with the Value-Weighted Market Portfolio (*VWMP*) as index, investors' equilibrium asset holdings differ from those in the economy without model uncertainty, because ambiguity averse investors take zero positions of the assets whose parameters they are sufficiently uncertain about. So ambiguity averse investors do not employ an index investment strategy, which hinders diversification and risk sharing. This also implies that in comparison to a market without model uncertainty, informed investors need to hold extra positions to absorb a greater proportion of outstanding shares, including random supplies, and thus they will require ambiguity premia in expected returns. So information asymmetry and ambiguity aversion lead to non-zero alphas of assets relative to *VWMP* as the pricing portfolio.

We derive a new market index that adjusts market value weights to take into account the average precision of investor private signals and the precision of random supply of different assets, i.e., the amount of risk reduction investors obtain by conditioning on price as a signal. We call this index design the *Risk-Adjusted Market Portfolio* (*RAMP*). *RAMP* is a defensive strategy in the sense that, relative to the value-weighted market, it underweights assets that are more volatile.

The ability of investors to invest in a *RAMP* index fund has major implications for equilibrium trading and asset pricing. In equilibrium, regardless of investors' heterogeneity in their ambiguity aversion, all investors hold exactly one share of the index as the passive (non-information-based) component of their portfolios. That is, *RAMP* induces investors to diversify by employing an index investment strategy. This improves the sharing of risk between investors who face model uncertainty about an asset and those who do not. In equilibrium, all investors' asset holdings are exactly the same as those in the economy without model uncertainty. Finally, the CAPM pricing relationship holds with respect to *RAMP* as the benchmark pricing portfolio, even though investors have asymmetric information and face model uncertainty.

These properties of *RAMP* derive from a new information separation theorem as shown in the special case of no model uncertainty even if no index fund is available. The information separation theorem says that to attain her optimal asset holdings, an

investor first constructs an optimal portfolio based on each of her signals (i.e., price signal and private signal) and then sums all these optimal portfolios together. Then, in the setting with model uncertainty, when other investors are holding a passive fund offering *RAMP* and their information-based portfolios, in any possible financial market in her subjective belief support, an investor's optimal investment strategy is also to hold the fund and her own information-based portfolio. Therefore, providing *RAMP* to all investors facilitates their asset market participation and risk sharing.

The design of *RAMP* has important empirical and policy implications. First, because it underweights high volatility stocks, *RAMP* is a defensive investing strategy. The investment strategy of following *RAMP* can also be viewed as a smart beta strategy, which is gaining increasing popularities. In order to implement *RAMP*, the government may employ big data analysis to gather all the information about capital market parameters and to offer *RAMP*. At the same time, the government should educate investors about the market equilibrium and risk sharing, and may consider recommending *RAMP* to investors through their retiring investments.

A Omitted Proofs

Proof of Lemma 1:

Because investor i is ambiguous about asset n, by assumption, she does not have private signal about asset n's payoff; that is, $\kappa_i = 0$. Hence, investor i's only information about the distribution of asset n's payoff is its price, which may partially aggregate informed investors' private signals. Suppose the uninformed investors' aggregate demand for asset n is $(1-\lambda_n)D(p_n)$. Since uninformed investors do not observe u_n, $D(p_n)$ is not a function of u_n.

Given any P and any $u_n \in \left(0, \bar{u}_n^i\right)$, we derive investor i's expected utility conditional on P_n as follows. Suppose asset n's pricing function in a linear equilibrium is

$$f_n = a + bp_n + cz_n,$$

where a, b, and c are undetermined parameters. Since informed investors know u_n, they can extract information from the price without any ambiguity. Therefore, any informed investor j's demand is

$$D_j = \rho\left[\kappa_n s_j + \frac{u_n}{c^2}a + \frac{u_n}{c^2}(b-r)p_n - r\kappa_n p_n\right].$$

Then, the informed investors' aggregate demand will be

$$\lambda_n \rho\left[\kappa_n f_n + \frac{u_n}{c^2}a + \frac{u_n}{c^2}(b-r)p_n - r\kappa_n p_n\right].$$

Then, the market clearing condition implies that

$$\lambda_n \rho\left[\kappa_n f_n + \frac{u_n}{c^2}a + \frac{u_n}{c^2}(b-r)p_n - r\kappa_n p_n\right] + (1-\lambda_n)D(P_n) = w_n + z_n.$$

Matching the coefficient of the market clearing condition and the pricing function, we have

$$\begin{aligned}
a &= \frac{w_n}{\lambda_n \kappa_n \rho} - \frac{u_n}{c^2 \kappa_n}a \\
bp_n &= -\frac{(1-\lambda_n)D(p_n)}{\lambda_n \kappa_n \rho} - \frac{u_n}{c^2 \kappa_n}(b-r)p_n + rp_n \\
c &= \frac{1}{\lambda_n \kappa_n \rho}
\end{aligned}$$

Therefore, for any given $u_n \in \left(0, \bar{u}_n^i\right)$, conditional on the price P_n, $|\mathbb{E}(f_n - rp_n|p_n)| < +\infty$. On the other hand, the variance of asset n's payoff conditional on p_n is

$$\mathbb{V}\left(f_n|p_n\right) = c^2 u_n^{-1},$$

which diverges to $+\infty$ as u_n goes to 0. Hence, any non-zero position D_i of asset n brings investor i a utility

$$-\exp\left(-\frac{1}{\rho}w_i r p_n\right)\exp\left[-\frac{1}{\rho}D_i\mathbb{E}\left(f_n - rp_n|p_n\right) + \frac{D_i^2}{2\rho^2}\mathbb{V}\left(f_n|p_n\right)\right], \tag{33}$$

which goes to $-\infty$ as u_n goes to 0. Therefore, if investor i is ambiguous about asset n, investor i will hold a zero position of asset n.

Q.E.D.

Proof of Proposition 1:

We assume that the pricing function can be written as

$$F = A_V + B_V P + C_V Z,$$

where B_V is nonsingular. Then, conditional on the asset prices P, an investor i's updated belief about asset payoffs is

$$F|P \sim \mathcal{N}\left(A_V + \boldsymbol{B}_V P, \boldsymbol{C}_V' \boldsymbol{U}_i^{-1} \boldsymbol{C}_V\right),$$

where $\boldsymbol{U}_i \in \mathcal{U}_i$

Then, each investor i's optimal asset holdings are

$$d_i X + D_i = \rho\left\{(\boldsymbol{C}_V \boldsymbol{U}_i^{-1}\boldsymbol{C}_V')^{-1}(\boldsymbol{B}_V - r\boldsymbol{I}) - r\boldsymbol{\Omega}_i\right\}P + \rho\boldsymbol{\Omega}_i S_i + \rho[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}A_V]. \tag{34}$$

It follows from Lemma 1 that if investor i is ambiguous about asset n, her holding of asset n is zero. Then, aggregating all investors' effective asset holdings and applying the market clearing condition yield

$$\begin{aligned} D &= \rho\left[\left(\boldsymbol{C}_V'\right)^{-1}\boldsymbol{Q}\boldsymbol{U}\left(\boldsymbol{C}_V\right)^{-1}A_V\right] + \rho\boldsymbol{\Sigma}F + \rho\left[\left(\boldsymbol{C}_V'\right)^{-1}\boldsymbol{Q}\boldsymbol{U}\left(\boldsymbol{C}_V\right)^{-1}\left(\boldsymbol{B}_V - r\boldsymbol{I} - r\boldsymbol{\Sigma}\right)\right]P \\ &= W + Z. \end{aligned}$$

Therefore, by matching coefficients, we have

$$\begin{aligned} \boldsymbol{C}_V &= \frac{1}{\rho}\boldsymbol{\Sigma}^{-1} \\ \boldsymbol{B}_V &= r\boldsymbol{I} \\ A_V &= \frac{1}{\rho}\left[\rho^2\boldsymbol{\Sigma}\boldsymbol{Q}\boldsymbol{U}\boldsymbol{\Sigma}+\boldsymbol{\Sigma}\right]^{-1} W. \end{aligned}$$

Substituting these parameters into the pricing function and the individual investor's asset holding function, we get Proposition 1.

Q.E.D.

Proof of Proposition 2:

From equation (17), individual assets' alphas are

$$\alpha_V = \mathrm{diag}(P)^{-1}\left[\boldsymbol{I} - \frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}\right] A_V.$$

Because $\frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W} \neq \boldsymbol{I}$, $\alpha_V = 0$ if and only if A_V is an eigenvector of $\frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}$ with the associated eigenvalue 1. Therefore, A_V should not be a function of $\boldsymbol{Q}$, since $\boldsymbol{I} - \frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}$ is not a function of $\boldsymbol{Q}$. However, it follows from Proposition 1 that $A_V = \frac{1}{\rho}\left[\rho^2\boldsymbol{\Sigma}\boldsymbol{Q}\boldsymbol{U}\boldsymbol{\Sigma}+\boldsymbol{\Sigma}\right]^{-1} W$, and so A_V does depend on $\boldsymbol{Q}$. Therefore, A_V is not an eigenvector of $\frac{\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V WW'}{W'\boldsymbol{C}_V\boldsymbol{U}^{-1}\boldsymbol{C}_V W}$ with the associated eigenvalue 1. Hence, $\alpha_V \neq 0$.

Q.E.D.

Proof of Proposition 3:

We first verify that the market clearing condition holds. Each investor i's effective risky assets holding is

$$d_i^* X + D_i^* = \left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma}\boldsymbol{U}\right)^{-1}\right]^{-1} W + \rho\boldsymbol{\Omega}_i\left(S_i - rP\right).$$

Then, using the pricing function (equation (19)), the aggregate demand can be calculated as

$$\begin{aligned}
& \int_0^1 \left(d_i^* X + D_i^*\right) \mathrm{d}i \\
= & \left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma}\boldsymbol{U}\right)^{-1}\right]^{-1} W + \rho\boldsymbol{\Sigma}\left(F - rP\right) \\
= & \left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma}\boldsymbol{U}\right)^{-1}\right]^{-1} W + \rho\boldsymbol{\Sigma}\left(\frac{1}{\rho}\left(\boldsymbol{\Sigma} + \rho^2\boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{\Sigma}\right)^{-1} W + \frac{1}{\rho}\boldsymbol{\Sigma}^{-1} Z\right) \\
= & \left[I + \frac{1}{\rho^2}\left(\boldsymbol{\Sigma}\boldsymbol{U}\right)^{-1}\right]^{-1} W + \left[\boldsymbol{I} + \rho^2\boldsymbol{\Sigma}\boldsymbol{U}\right]^{-1} W + Z \\
= & \rho^2\boldsymbol{\Sigma}\boldsymbol{U}\left[\boldsymbol{I} + \rho^2\boldsymbol{\Sigma}\boldsymbol{U}\right]^{-1} W + \left[\boldsymbol{I} + \rho^2\boldsymbol{\Sigma}\boldsymbol{U}\right]^{-1} W + Z \\
= & W + Z.
\end{aligned}$$

Therefore, the market clears.

Now, for any investor i, we consider a general investment strategy $d_i X + D_i$. Denote by D_{in} investor i's direct holding of asset n. Suppose that investor i is informed about asset n. Then, the pricing function (19) implies that investor i's optimal holding of asset n is

$$\left[1 + \frac{1}{\rho^2}\left(\lambda_n \kappa_n u_n\right)^{-1}\right]^{-1} w_n + \rho\kappa_n\left(s_{in} - rp_n\right) = x_n + \rho\kappa_n\left(s_{in} - rp_n\right).$$

Therefore, any combination of d_i and D_{in} such that

$$d_i x_n + D_{in} = x_n + \rho\kappa_n\left(s_{in} - rp_n\right)$$

can lead to the optimal holding of asset n for investor i.

Now, consider an asset n that investor i is uninformed about. For any given d_i and D_{in}, investor i is effectively holding a position $d_i x_n + D_{in}$ of asset n. Then, for any given u_n Such a holding will bring invest i a utility

$$-\exp\left(-\frac{1}{\rho} w_{in} r p_n\right) \exp\left[-\frac{1}{\rho}\left(d_i x_n + D_{in}\right)\mathbb{E}\left(f_n - rp_n | p_n\right) + \frac{\left(d_i x_n + D_{in}\right)^2}{2\rho^2}\mathbb{V}\left(f_n | p_n\right)\right]. \tag{35}$$

There are two cases. In the first case where $\underline{u}_n^i = 0$, similarly to Lemma 1, if $D_{in} \neq 0$, the infimum of such a utility is $-\infty$, since $\mathbb{V}(f_n|P_n) \to +\infty$ as $u_n \to 0$. Therefore, $D_{in}^* = 0$. Next, substituting X_n into equation (35), the investor's utility given u is

$$-\exp\left(-\frac{1}{\rho} w_{in} r p_n\right) \exp\left[-\left(d_i - \frac{1}{2} d_i^2\right)\frac{\rho u_n \lambda_n^2 \kappa_n^2 w_n^2}{\left[\lambda_n \kappa_n + \rho^2 u_n \lambda_n^2 \kappa_n^2\right]^2}\right]. \tag{36}$$

It follows from equation (36) that for any d_i, the infimum of the investor's utility is at most $-\exp\left(-\frac{1}{\rho}w_{in}rP_n\right)$. Since the investor can get the utility at least $-\exp\left(-\frac{1}{\rho}w_{in}rP_n\right)$ by employing the investment strategy $d_i^* = 1$, there is no profitable deviation.

In the second case, $\underline{u}_n^i > 0$. We first assume that any investor i has min-max utility, and then finally show that her max-min utility is the same as her min-max utility, which implies a strong min-max property. Then, investor i's optimal investment strategy with a max-min utility is the same as the optimal investment strategy with a min-max utility. Since investor i does not know u_n, d_i and D_{in} are not functions of u_n. For any given u_n, we can solve d_i^* and D_{in}^* by the first order condition of the following maximization problem:

$$\max_{d_i, D_{in}} (d_i x_n + D_{in}) \frac{w_n}{\rho\left[\lambda_n \kappa_n + \rho^2 u_n \lambda_n^2 \kappa_n^2\right]} - \frac{(d_i x_n + D_{in})^2}{2\rho} \frac{1}{\rho^2 \lambda_n^2 \kappa_n^2 u_n}. \tag{37}$$

The second order condition of such a maximization problem holds, because the utility function in equation (37) is strictly concave.

Differentiating the utility function in equation (37) with respect to d_i, we get one of the first-order conditions:

$$x_n \frac{w_n}{\rho\left[\lambda_n \kappa_n + \rho^2 u_n \lambda_n^2 \kappa_n^2\right]} - \frac{(d_i x_n + D_{in})\, x_n}{\rho} \frac{1}{\rho^2 \lambda_n^2 \kappa_n^2 u_n} = 0.$$

So,

$$d_i x_n + D_{in} = d_i \frac{\rho^2 u_n \lambda_n^2 \kappa_n^2}{\lambda_n \kappa_n + \rho^2 u_n \lambda_n^2 \kappa_n^2} w_n + D_{in} = \frac{\rho^2 u_n \lambda_n^2 \kappa_n^2}{\lambda_n \kappa_n + \rho^2 u_n \lambda_n^2 \kappa_n^2} w_n + D_{in}.$$

Then, $d_i^* = 1$ and $D_i^* = 0$, because they are not functions of u_n. Therefore, with a min-max utility, if an investor i is uninformed about asset n, she will hold exactly one share of the passive fund and a zero position of asset n.

Because investor i's optimal investment strategy $(d_i^*, D_{in}^*) = (1, 0)$ is constant across all possible u_n, we have

$$\min_{u_n} \max_{d_i, D_{in}} u((d_i, D_{in}), u_n) = \min_{u_n} u((1,0), u_n) \leq \max_{d_i, D_{in}} \min_{u_n} u((d_i, D_{in}), u_n).$$

Generally, by the min-max utility, we have

$$\min_{u_n} \max_{d_i, D_{in}} u((d_i, D_{in}), u_n) \geq \max_{d_i, D_{in}} \min_{u_n} u((d_i, D_{in}), u_n).$$

Then, we have

$$\min_{u_n} \max_{d_i, D_{in}} u((d_i, D_{in}), u_n) = \max_{d_i, D_{in}} \min_{u_n} u((d_i, D_{in}), u_n).$$

This implies a strong min-max property, and hence, $(d_i^*, D_{in}^*) = (1, 0)$ is also the optimal investment strategy of investor i, when she has a max-min utility.

In sum, given the pricing function specified in equation (19), it is optimal for any investor i to choose the investment strategy $d_i^* = 1$ and

$$D_{in}^* = \begin{cases} 0, & \text{if she is uninformed about asset } n; \\ \rho\kappa_n \left(S_{in} - rP_n\right), & \text{if she is informed about asset } n. \end{cases}$$

Q.E.D.

Proof of Proposition 4:

Let's first prove a more general version of Proposition 4, when investors hold a common prior belief about F, $F \sim \mathcal{N}\left(\overline{F}, \boldsymbol{V}\right)$. As is standard in the literature of rational expectations equilibrium, we consider the linear pricing function

$$F = A + \boldsymbol{B}P + \boldsymbol{C}Z, \quad \text{with } C \text{ nonsingular.} \tag{38}$$

If and only if B is nonsingular, equation (38) can be rearranged to

$$P = -\boldsymbol{B}^{-1}A + \boldsymbol{B}^{-1}F - \boldsymbol{B}^{-1}\boldsymbol{C}Z, \tag{39}$$

which solves for prices. Recall that $S_i = F + \epsilon_i$, so conditional on F, P and S_i are independent. Therefore, we can write down assets' payoffs' posterior means and posterior variances conditional on all information that are available to investor i as follows.

First consider investor i's belief about F conditional on P. Conditional on P, F is normally distributed with mean $A + \boldsymbol{B}P$ and precision $[\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}']^{-1}$. On the other hand, conditional on S_i, investor i's belief about F is also normally distributed, with mean S_i and precision $\boldsymbol{\Omega}_i$. Therefore, investor i's belief about F conditional on what the investor observes, P and S_i, is also normally distributed. The mean of the conditional distribution of F is the weighted average of the expectation conditional on the price P, the expectation conditional on investor i's private signal S_i, and the prior mean $\overline{F}$. Therefore, the conditional mean of F is

$$\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1} + \boldsymbol{\Omega}_i + \boldsymbol{V}^{-1}\right]^{-1} \left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}\left(A + \boldsymbol{B}P\right) + \boldsymbol{\Omega}_i S_i + \boldsymbol{V}^{-1}\overline{F}\right]. \tag{40}$$

The precision of the conditional distribution of F is

$$(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1} + \boldsymbol{\Omega}_i + \boldsymbol{V}^{-1}. \tag{41}$$

Then, from any investor i's first order condition, investor i's demand is

$$\begin{aligned}
D_i &= \rho\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}+\boldsymbol{\Omega}_i+\boldsymbol{V}^{-1}\right] \\
&\quad \left\{\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}+\boldsymbol{\Omega}_i+\boldsymbol{V}^{-1}\right]^{-1}\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}(A+\boldsymbol{B}P)+\boldsymbol{\Omega}_i S_i+\boldsymbol{V}^{-1}\overline{F}\right]-rP\right\} \\
&= \rho\left\{\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}(A+\boldsymbol{B}P)+\boldsymbol{\Omega}_i S_i+\boldsymbol{V}^{-1}\overline{F}\right]-\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}+\boldsymbol{\Omega}_i+\boldsymbol{V}^{-1}\right]rP\right\} \\
&= \rho\left\{(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}(\boldsymbol{B}-r\boldsymbol{I})-r\boldsymbol{\Omega}_i-r\boldsymbol{V}^{-1}\right\}P \\
&\qquad +\rho\boldsymbol{\Omega}_i S_i+\rho[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}A+\boldsymbol{V}^{-1}\overline{F}]. \qquad (42)
\end{aligned}$$

Integrating across all investors' demands gives the aggregated demand as

$$\begin{aligned}
\int_0^1 D_i \mathrm{d}i =& \rho\left\{(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}(\boldsymbol{B}-r\boldsymbol{I})-r\left(\int_0^1 \boldsymbol{\Omega}_i \mathrm{d}i\right)-r\boldsymbol{V}^{-1}\right\}P \\
&+\rho\left(\int_0^1 \boldsymbol{\Omega}_i S_i \mathrm{d}i\right)+\rho[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}A+\boldsymbol{V}^{-1}\overline{F}]. \qquad (43)
\end{aligned}$$

By equation (1), we have $\int_0^1 \boldsymbol{\Omega}_i \mathrm{d}i = \boldsymbol{\Sigma}$. Also, note that

$$\int_0^1 \boldsymbol{\Omega}_i S_i \mathrm{d}i = \boldsymbol{\Sigma}F.$$

Therefore, from the market clearing condition, we have

$$\int_0^1 D_i \mathrm{d}i = Z + W. \qquad (44)$$

In an equilibrium, both equation (38) and equation (44) hold simultaneously for any realized F and Z, therefore, by matching coefficients in these two equations, we have

$$\begin{aligned}
\rho\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}A+\boldsymbol{V}^{-1}\overline{F}\right]-W &= -\boldsymbol{C}^{-1}A \qquad (45) \\
\rho\left[(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}')^{-1}(\boldsymbol{B}-r\boldsymbol{I})-r\boldsymbol{\Sigma}-r\boldsymbol{V}^{-1}\right] &= -\boldsymbol{C}^{-1}\boldsymbol{B} \qquad (46) \\
\rho\boldsymbol{\Sigma} &= \boldsymbol{C}^{-1} \qquad (47)
\end{aligned}$$

Therefore, from equation (47), we have

$$\boldsymbol{C} = \frac{1}{\rho}\boldsymbol{\Sigma}^{-1}$$

Obviously, $\boldsymbol{C}$ is positive definite and symmetric. Then from equation (45), we have

$$[\rho^2(\boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{\Sigma})+\boldsymbol{\Sigma}]A = \frac{1}{\rho}W - \boldsymbol{V}^{-1}\overline{F}.$$

Because both $(\boldsymbol{\Sigma U \Sigma})$ and $\boldsymbol{\Sigma}$ are both positive definite, we have

$$A = [\rho^2(\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma}]^{-1}\left(\frac{1}{\rho}W - \boldsymbol{V}^{-1}\overline{F}\right).$$

From equation (46), we have

$$[\rho^2(\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma}](\boldsymbol{B} - r\boldsymbol{I}) = r\boldsymbol{V}^{-1}.$$

Again, because $[\rho^2(\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma}]$ is positive definite, we have

$$\boldsymbol{B} = r\boldsymbol{I} + r[\rho^2(\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma}]^{-1}\boldsymbol{V}^{-1}.$$

Obviously, $\boldsymbol{B}$ is invertible. By substituting A, $\boldsymbol{B}$, and $\boldsymbol{C}$ into equation (39), we solve the equilibrium pricing function.

Now, let's look at any investor i's holding. Substituting the coefficients into investor i's holding function (42), we have

$$D_i = \left(\boldsymbol{I} + \frac{1}{\rho^2}(\boldsymbol{\Sigma U})^{-1}\right)^{-1} W + \rho\left[I + \rho^2\boldsymbol{\Sigma U}\right]^{-1}\boldsymbol{V}^{-1}(\overline{F} - rP) + \rho\boldsymbol{\Omega}_i(S_i - rP).$$

Finally, because the pricing function P and any investor i's demand function D_i are continuous in $\boldsymbol{V}^{-1}$, we can substitute $\boldsymbol{V}^{-1} = 0$ to get Proposition 4.

Q.E.D.

Proof of Proposition 5:

By equations (30), (31), and (32), we have

$$\begin{aligned} & \frac{\frac{1}{P'X}\mathrm{diag}(P)^{-1}\boldsymbol{CU}^{-1}\boldsymbol{C}X}{\left(\frac{1}{P'X}\right)^2 X'\boldsymbol{CU}^{-1}\boldsymbol{C}X}\frac{X'A}{P'X} \\ = \quad & \frac{\mathrm{diag}(P)^{-1}\boldsymbol{CU}^{-1}\boldsymbol{C}X}{X'\boldsymbol{CU}^{-1}\boldsymbol{C}X}X'A. \end{aligned}$$

This is the RHS of the Security Market Line relation. We want to show that this equals the difference between the risky assets' rates of return and the riskfree asset's rate of return, which is shown to be $\mathrm{diag}(P)^{-1}A$ from equation (29).

Then, we have

$$
\begin{aligned}
& \frac{\mathrm{diag}(P)^{-1}\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}X}{X'\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}X}X'A = \mathrm{diag}(P)^{-1}A \\
\Leftrightarrow \quad & \mathrm{diag}(P)^{-1}\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}XX'A = \mathrm{diag}(P)^{-1}AX'\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}X \\
\Leftrightarrow \quad & \boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}XX'A = AX'\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C}X.
\end{aligned}
$$

The last equation holds because $X = \rho(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C})^{-1}A$ and $(\boldsymbol{C}\boldsymbol{U}^{-1}\boldsymbol{C})^{-1}$ is a symmetric matrix.

Q.E.D.

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B Online Appendix (Not For Publication)

B.1 Normal Prior in Economy with *VWMP*

In this appendix, we consider ambiguity averse investors' asset holdings when they have normal priors about assets' payoffs. Because all assets are assumed to be independent, we focus on one risky asset here, and so we omit notations' subscripts in this section.

Equation (33) in the paper implies that if an investor is ambiguous about the asset, she wants to chooses D, her effective holding of the asset, such that

$$\inf_{u\in(0,\bar{u})} D\mathbb{E}\left(f - rp|p\right) - \frac{D^2}{2\rho}Var(f|p)$$

is maximized. Denote by ν and $\bar{f}$ the precision and the mean of the prior distribution of the asset's payoff, respectively. Suppose that the pricing function is $f = a + bp + cz$, the investor will then choose her effective asset holding d to maximize

$$\inf_{u\in(0,\bar{u})} D\left[\frac{u(a+bp)+\nu\bar{f}}{u+\nu} - rp\right] - \frac{D^2}{2\rho}\frac{1}{u+\nu}. \tag{48}$$

Because of the max-min utility, we fix any D and solve for the u that will solve the infimum problem. Differentiating

$$D\left[\frac{u(a+bp)+\nu\bar{f}}{u+\nu} - rp\right] - \frac{D^2}{2\rho}\frac{1}{u+\nu}$$

with respect to u yields

$$\frac{1}{(u+\nu)^2}\left[\nu(a+bp)D - \nu\bar{f}D + \frac{D^2}{2\rho}\right].$$

Hence, the sign of the derivative is either globally positive or globally negative, implying that the ambiguity averse investor will evaluate her asset holding based on either $u \to 0$ or $u \to \bar{u}$.

While whether the investor will evaluate her asset holding based on either $u \to 0$ or $u \to \bar{u}$ is indeterminate,[17] we conclude here that the ambiguity averse investor's asset holding surely differs from that in the economy without model uncertainty. And this is enough for us to establish the benchmark, since with *RAMP* as the index, investors' equilibrium asset holdings are exactly same as those in the economy without model uncertainty.

[17] This depends on the realized asset price, which is endogenously determined in equilibrium. Therefore, completely solving the model with normal prior distributions when *VWMP* is publicly offered is rather intractable.

B.2 Heterogeneous Risk Aversions

To evaluate the robustness of our conclusions, we now analyze an extension in which investors have heterogeneous risk tolerances. This also suggests further empirical implications.

In the model described in Section 2, investors share a same risk aversion coefficient ρ. Such an assumption leads to investors' homogeneous holdings of the passive fund. Indeed, in the equilibrium characterized in Proposition 3, all investors hold one share of the passive fund. However, it is conceivably that differences in risk tolerances, and investor unawareness of other investors' risk tolerances, could resurrect investors' heterogeneous holdings of the passive fund. We extend the model in Section 2 by assuming that any investor i ($i \in [0,1]$) has the risk aversion coefficient ρ_i. Here, ρ_i is a continuous function of i. Let

$$\overline{\rho} = \int_0^1 \rho_i \mathrm{d}i \quad \text{and} \quad \overline{\Sigma} = \int_0^1 \rho_i \Omega_i \mathrm{d}i.$$

Here, $\overline{\rho}$ is the average risk tolerance, and $\overline{\Sigma}$ is the average precision of investors' private information that is weighted by their risk tolerances. We assume that any investor i knows ρ_i, but she does not know the distribution of ρ_j and thus the average risk tolerance $\overline{\rho}$. The passive fund cannot evaluate each individual investor's risk tolerance, but it has accurate information about the distribution of investors' risk tolerances; hence, it knows $\overline{\rho}$ and $\overline{\Sigma}$. Then, the passive fund offers the portfolio

$$\overline{X} = \left[\overline{\rho} + \left(\overline{\Sigma} U\right)^{-1}\right]^{-1} W. \tag{49}$$

to all investors. Proposition 6 shows that investors with different risk tolerances hold different numbers of shares of the passive fund.

Proposition 6 *In the model with investors' heterogeneous risk tolerances, there exists an equilibrium in which any investor i with the risk tolerance ρ_i holds ρ_i shares of the passive fund and her own information-based portfolio $\rho_i \Omega_i (S_i - rP)$.*

Proof of Proposition 6:

We first analyze the model in which investors have heterogeneous risk tolerances and all parameters are common knowledge. We again consider the linear pricing function as in equation (38),

$$F = A + BP + CZ, \quad \text{with } C \text{ nonsingular.}$$

Therefore, conditional on the price, assets' payoffs have the conditional distribution is

$$F|P \sim \mathcal{N}\left(A + BP, CU^{-1}C'\right).$$

An investor i gleans such information from the price. Therefore, an investor i's demand is

$$D_i = \rho_i \left[(CU^{-1}C')^{-1}(B - rI) - r\Omega_i\right] P + \rho_i \Omega_i S_i + \rho_i (CU^{-1}C')A. \tag{50}$$

Then, by integrating all investors' demands and equalizing the aggregate demand and the total supply (the aggregate endowments and the supply shocks), we can derive the pricing function

$$P = B^{-1}\left[F - A - CZ\right], \tag{51}$$

where

$$\begin{aligned} A &= \left[\overline{\Sigma} + \overline{\rho}\left(\overline{\Sigma}U\overline{\Sigma}\right)^{-1}\right]^{-1} W & (52)\\ B &= rI & (53)\\ C &= \overline{\Sigma}^{-1}. & (54) \end{aligned}$$

Any investor i's risky asset holding is

$$D_i = \rho_i\left[\overline{\rho} + \left(\overline{\Sigma}U\right)^{-1}\right]^{-1} W + \rho_i\Omega_i\left(S_i - rP\right). \tag{55}$$

Because the passive fund provides the portfolio $\overline{X}$ specified in equation (49), Equation (55) can be rewritten as

$$D_i = \rho_i\overline{X} + \rho_i\Omega_i\left(S_i - rP\right). \tag{56}$$

Then, when investors are uncertain about some parameters and thus are subject to ambiguity aversions, they still want to hold the passive fund. In particular, investor i first buys ρ_i shares of a passive fund and then use her own private information to form the information-based portfolio $\rho_i\Omega_i^{-1}\left(S_i - rP\right)$. Finally, investor i invests the rest of her endowments in the riskfree asset.

Q.E.D.

B.3 Less Trusting Investors

In the model described in Section 2, investors are perfectly rational except that they are ambiguity averse. As we argue in the paper, when the passive fund offers *RAMP*, it is optimal for an ambiguity averse investor to hold the *RAMP* fund (together with her own information-based portfolio), if and only if she believes that all other investors are also holding the *RAMP* fund and their information-based portfolios. In this appendix, however, we show that such an equilibrium requirement can be relaxed.

We consider a setting where there are some investor with the measure h (here, $h > 0$ but very small) do not trust the fund. The lack of trust may arise from the potential agency problem or from the fact that these investors do not think the fund manager has the information about the financial market. Therefore, even though the fund publicly commits to an investment strategy, these "less trusting" investors still think the fund is very risky.

For simplicity, we assume that the less trusting investors do not have private information and face model uncertainty about all risky assets. Therefore, even if the fund

is still publicly available, the less trusting investors will not participate in the capital market at all.

By the same arguments as in the paper, if the passive fund offers an adjusted *RAMP*, which is defined as

$$X_h = \left[hI + \frac{1}{\rho^2} \left(\boldsymbol{\Sigma U} \right)^{-1} \right]^{-1} W, \tag{57}$$

we show that the following proposition hold.

Proposition 7 *With the passive fund offering X_h, there is an equilibrium in which*

1. *Each trusting investor i's asset holding is $d_i^* = 1$ and $D_i^* = \rho \boldsymbol{\Omega}_i \left(S_i - rP \right)$;*
2. *The pricing function is*

$$P = \boldsymbol{B}_h^{-1} \left[F - A_h - \boldsymbol{C}_h Z \right], \tag{58}$$

where

$$A_h = \frac{1}{\rho} \left[h\rho^2 (\boldsymbol{\Sigma U \Sigma}) + \boldsymbol{\Sigma} \right]^{-1} W \tag{59}$$

$$\boldsymbol{B}_h = r\boldsymbol{I} \tag{60}$$

$$\boldsymbol{C}_h = \frac{1}{\rho} \boldsymbol{\Sigma}^{-1}; \tag{61}$$

3. *and, using X_h as the pricing portfolio, a version of CAPM security market line holds.*