Chapter Outline

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In this chapter we will go into various commands that go beyond OLS. This chapter is a bit different from the others in that it covers a number of different concepts, some of which may be new to you. These extensions, beyond OLS, have much of the look and feel of OLS but will provide you with additional tools to work with linear models.

The topics will include robust regression methods, constrained linear regression, regression with censored and truncated data, regression with measurement error, and multiple equation models.

4.1 Robust Regression Methods

It seems to be a rare dataset that meets all of the assumptions underlying multiple regression. We know that failure to meet assumptions can lead to biased estimates of coefficients and especially biased estimates of the standard errors. This fact explains a lot of the activity in the development of robust regression methods.

The idea behind robust regression methods is to make adjustments in the estimates that take into account some of the flaws in the data itself. We are going to look at three approaches to robust regression: 1) regression with robust standard errors including the **cluster** option, 2) robust regression using iteratively reweighted least squares, and 3) quantile regression, more specifically, median regression.

Before we look at these approaches, let's look at a standard OLS regression using the elementary school academic performance index (elemapi2.dta) dataset.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/elemapi2
```

We will look at a model that predicts the api 2000 scores using the average class size in K through 3 (**acs_k3**), average class size 4 through 6 (**acs_46**), the percent of fully credentialed teachers (**full**), and the size of the school (**enroll**). First let's look at the descriptive statistics for these variables. Note the missing values for **acs_k3** and **acs_k6**.

```
summarize api00 acs_k3 acs_46 full enroll

Variable |     Obs        Mean   Std. Dev.       Min        Max
---------+--------------------------------------------------------
   api00 |     400    647.6225    142.249        369        940
  acs_k3 |     398     19.1608   1.368693         14         25
  acs_46 |     397    29.68514   3.840784         20         50
    full |     400       84.55   14.94979         37        100
  enroll |     400     483.465   226.4484        130       1570
```

Below we see the regression predicting **api00** from **acs_k3**, **acs_46 full** and **enroll**. We see that all of the variables are significant except for **acs_k3**.

```
regress api00 acs_k3 acs_46 full enroll

  Source |       SS       df       MS                  Number of obs =     395
---------+------------------------------                F(  4,   390) =   61.01
   Model |  3071909.06     4  767977.265                Prob > F      =  0.0000
Residual |  4909500.73   390  12588.4634                R-squared     =  0.3849
---------+------------------------------                Adj R-squared =  0.3786
   Total |  7981409.79   394  20257.3852                Root MSE      =  112.20

------------------------------------------------------------------------------
   api00 |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  acs_k3 |   6.954381   4.371097      1.591   0.112       -1.63948    15.54824
  acs_46 |   5.966015   1.531049      3.897   0.000       2.955873    8.976157
    full |   4.668221   .4142537     11.269   0.000       3.853771    5.482671
  enroll |  -.1059909   .0269539     -3.932   0.000      -.1589841   -.0529977
   _cons |  -5.200407   84.95492     -0.061   0.951      -172.2273    161.8265
------------------------------------------------------------------------------
```

We can use the **test** command to test both of the class size variables, and we find the overall test of these two variables is significant.

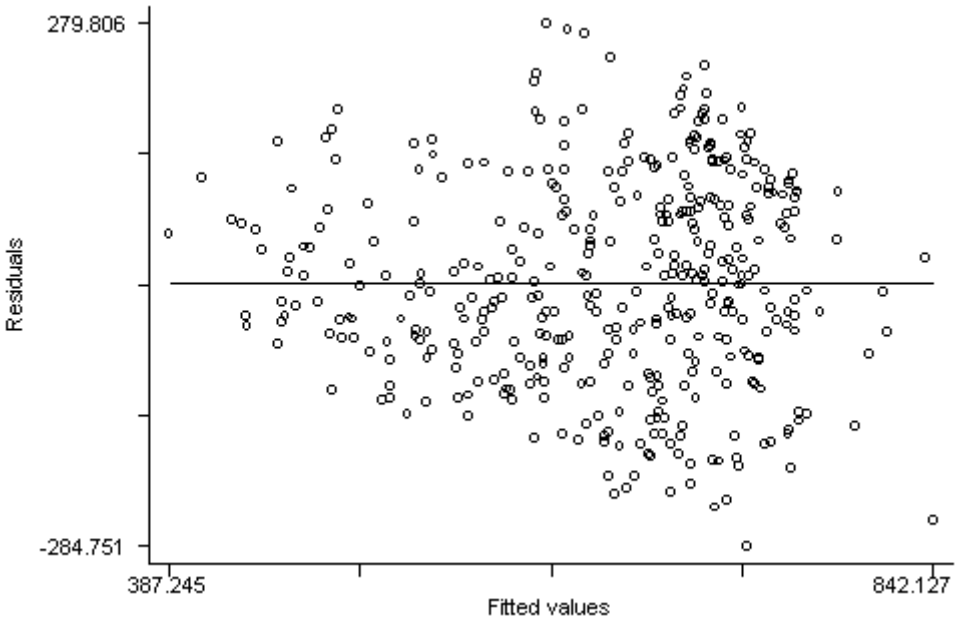
```
test acs_k3 acs_46
```

```
( 1)  acs_k3 = 0.0
( 2)  acs_46 = 0.0

      F(  2,   390) =   11.08
           Prob > F =    0.0000
```

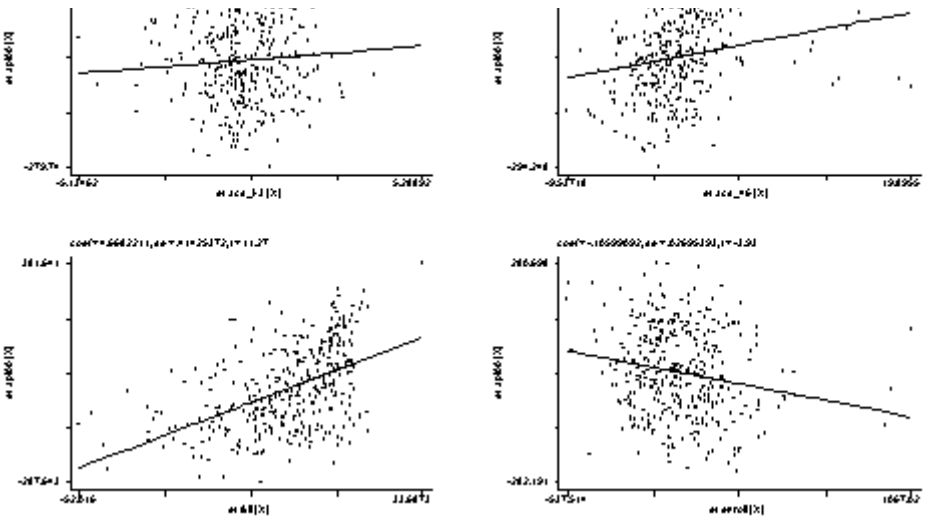
Here is the residual versus fitted plot for this regression. Notice that the pattern of the residuals is not exactly as we would hope. The spread of the residuals is somewhat wider toward the middle right of the graph than at the left, where the variability of the residuals is somewhat smaller, suggesting some heteroscedasticity.

rvfplot



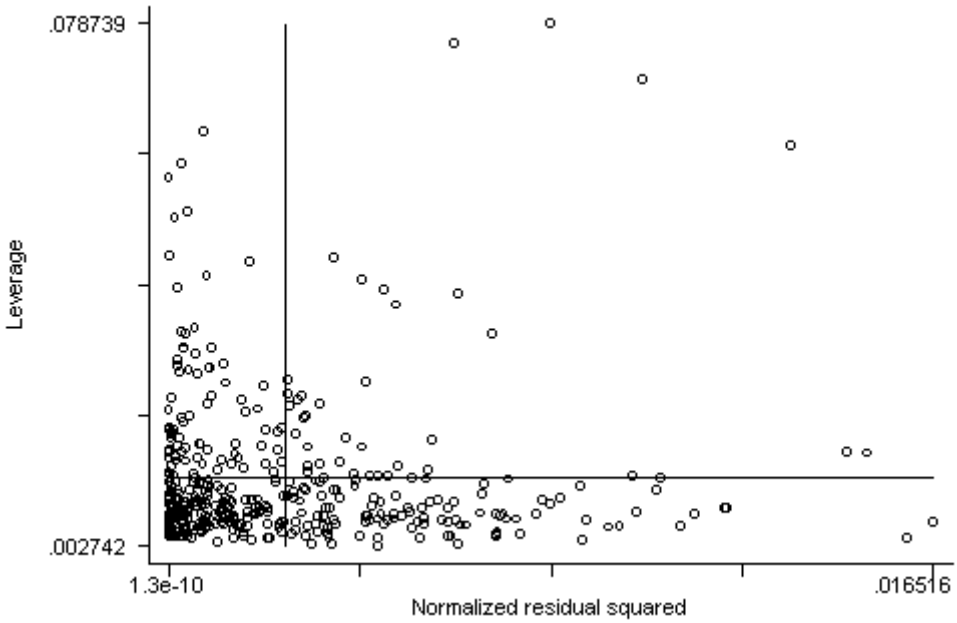
Below we show the **avplots**. Although the plots are small, you can see some points that are of concern. There is not a single extreme point (like we saw in chapter 2) but a handful of points that stick out. For example, in the top right graph you can see a handful of points that stick out from the rest. If this were just one or two points, we might look for mistakes or for outliers, but we would be more reluctant to consider such a large number of points as outliers.

avplots



Here is the **lvr2plot** for this regression. We see 4 points that are somewhat high in both their leverage and their residuals.

```
lvr2plot
```



None of these results are dramatic problems, but the **rvfplot** suggests that there might be some outliers and some possible heteroscedasticity; the **avplots** have some observations that look to have high leverage, and the **lvr2plot** shows some points in the upper right quadrant that could be influential. We might wish to use something other than OLS regression to estimate this model. In the next several

sections we will look at some robust regression methods.

4.1.1 Regression with Robust Standard Errors

The Stata **regress** command includes a **robust** option for estimating the standard errors using the Huber-White sandwich estimators. Such robust standard errors can deal with a collection of minor concerns about failure to meet assumptions, such as minor problems about normality, heteroscedasticity, or some observations that exhibit large residuals, leverage or influence. For such minor problems, the robust option may effectively deal with these concerns.

With the **robust** option, the point estimates of the coefficients are exactly the same as in ordinary OLS, but the standard errors take into account issues concerning heterogeneity and lack of normality. Here is the same regression as above using the **robust** option. Note the changes in the standard errors and t-tests (but no change in the coefficients). In this particular example, using robust standard errors did not change any of the conclusions from the original OLS regression.

```
regress api00 acs_k3 acs_46 full enroll, robust

Regression with robust standard errors                Number of obs =     395
                                                      F(  4,   390) =   84.67
                                                      Prob > F      =  0.0000
                                                      R-squared     =  0.3849
                                                      Root MSE      =  112.20

------------------------------------------------------------------------------
             |               Robust
       api00 |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
      acs_k3 |   6.954381   4.620599      1.505   0.133      -2.130019    16.03878
      acs_46 |   5.966015   1.573214      3.792   0.000       2.872973    9.059057
        full |   4.668221   .4146813     11.257   0.000       3.852931    5.483512
      enroll |  -.1059909   .0280154     -3.783   0.000      -.1610711   -.0509108
       _cons |  -5.200407   86.66308     -0.060   0.952      -175.5857    165.1849
------------------------------------------------------------------------------
```

4.1.2 Using the Cluster Option

As described in Chapter 2, OLS regression assumes that the residuals are independent. The **elemapi2**

dataset contains data on 400 schools that come from 37 school districts. It is very possible that the scores within each school district may not be independent, and this could lead to residuals that are not independent within districts. We can use the **cluster** option to indicate that the observations are clustered into districts (based on **dnum**) and that the observations may be correlated within districts, but would be independent between districts.

By the way, if we did not know the number of districts, we could quickly find out how many districts there are as shown below, by **quietly** tabulating **dnum** and then displaying the macro **r(r)** which gives the numbers of rows in the table, which is the number of school districts in our data.

```
quietly tabulate dnum
display r(r)
37
```

Now, we can run regress with the **cluster** option. We do not need to include the robust option since robust is implied with cluster. Note that the standard errors have changed substantially, much more so, than the change caused by the **robust** option by itself.

```
regress api00 acs_k3 acs_46 full enroll, cluster(dnum)

Regression with robust standard errors                Number of obs =     395
                                                      F(  4,    36) =   31.18
```

```
                                                      Prob > F      =  0.0000
                                                      R-squared     =  0.3849
Number of clusters (dnum) = 37                        Root MSE      =  112.20

------------------------------------------------------------------------------
             |               Robust
      api00 |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
      acs_k3 |   6.954381   6.901117      1.008   0.320      -7.041734     20.9505
      acs_46 |   5.966015   2.531075      2.357   0.024       .8327565    11.09927
        full |   4.668221   .7034641      6.636   0.000        3.24153    6.094913
      enroll |  -.1059909   .0429478     -2.468   0.018      -.1930931   -.0188888
       _cons |  -5.200407   121.7856     -0.043   0.966       -252.193    241.7922
------------------------------------------------------------------------------
```

As with the **robust** option, the estimate of the coefficients are the same as the OLS estimates, but the standard errors take into account that the observations within districts are non-independent. Even though the standard errors are larger in this analysis, the three variables that were significant in the OLS analysis are significant in this analysis as well. These standard errors are computed based on aggregate scores for the 37 districts, since these district level scores should be independent. If you have a very small number of clusters compared to your overall sample size it is possible that the standard errors could be quite larger than the OLS results. For example, if there were only 3 districts, the standard errors would be computed on the aggregate scores for just 3 districts.

4.1.3 Robust Regression

The Stata **rreg** command performs a robust regression using iteratively reweighted least squares, i.e., **rreg** assigns a weight to each observation with higher weights given to better behaved observations. In fact, extremely deviant cases, those with Cook's D greater than 1, can have their weights set to missing so that they are not included in the analysis at all.

We will use **rreg** with the generate option so that we can inspect the weights used to weight the observations. Note that in this analysis both the coefficients and the standard errors differ from the original OLS regression. Below we show the same analysis using robust regression using the **rreg** command.

```
rreg api00 acs_k3 acs_46 full enroll, gen(wt)

Robust regression estimates                           Number of obs =     395
                                                      F(  4,   390) =   56.51
                                                      Prob > F      =  0.0000

------------------------------------------------------------------------------
       api00 |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
      acs_k3 |   6.110881   4.658131      1.312   0.190      -3.047308    15.26907
      acs_46 |   6.254708   1.631587      3.834   0.000       3.046901    9.462516
        full |   4.796072   .4414563     10.864   0.000        3.92814    5.664004
      enroll |  -.1092586   .0287239     -3.804   0.000      -.1657316   -.0527855
       _cons |  -6.788183    90.5336     -0.075   0.940      -184.7832    171.2068
------------------------------------------------------------------------------
```

If you compare the robust regression results (directly above) with the OLS results previously presented, you can see that the coefficients and standard errors are quite similar, and the t values and p values are also quite similar. Despite the minor problems that we found in the data when we performed the OLS analysis, the robust regression analysis yielded quite similar results suggesting that indeed these were minor problems. Had the results been substantially different, we would have wanted to further investigate the reasons why the OLS and robust regression results were different, and among the two results the robust regression results would probably be the more trustworthy.

Let's calculate and look at the predicted (fitted) values (**p**), the residuals (**r**), and the leverage (hat) values (**h**). Note that we are including **if e(sample)** in the commands because **rreg** can generate weights of missing and you wouldn't want to have predicted values and residuals for those observations.

```
predict p if e(sample)
(option xb assumed; fitted values)
(5 missing values generated)
```

```
predict r if e(sample), resid
(5 missing values generated)

predict h if e(sample), hat
(5 missing values generated)
```

Now, let's check on the various predicted values and the weighting. First, we will sort by **wt** then we will look at the first 15 observations. Notice that the smallest weights are near one-half but quickly get into the .7 range.

```
sort wt
list snum api00 p r h wt in 1/15

          snum   api00          p          r          h          wt
  1.       637     447   733.1567  -286.1568   .0037645   .55612093
  2.      5387     892   611.5344   280.4655   .0023925   .57126927
  3.      2267     897   621.4881   275.5119    .010207   .58433963
  4.        65     903   631.2718   271.7282   .0105486   .59425026
  5.      3759     585   842.4838  -257.4838   .0414728   .63063771
  6.      5926     469   715.2266  -246.2266   .0058346   .65892631
  7.      1978     894   650.7816   243.2184   .0058116    .6665881
  8.      3696     483   721.3105  -238.3105   .0052619   .67834344
  9.      5222     940    707.648    232.352   .0041016   .69303069
 10.       690     424   654.5795  -230.5795   .0094319   .69701005
 11.      3785     459   687.3311  -228.3311   .0081474   .70245717
 12.      2910     831   604.4401     226.56   .0536809   .70650365
 13.       699     437   660.2588  -223.2588   .0059152   .71449402
 14.      3070     479   698.1256  -219.1256   .0043322   .72399766
 15.      1812     917   698.9828   218.0172   .0099871   .72670695
```

Now, let's look at the last 10 observations. The weights for observations 391 to 395 are all very close to one. The values for observations 396 to the end are missing due to the missing predictors. Note that the observations above that have the lowest weights are also those with the largest residuals (residuals over 200) and the observations below with the highest weights have very low residuals (all less than 3).

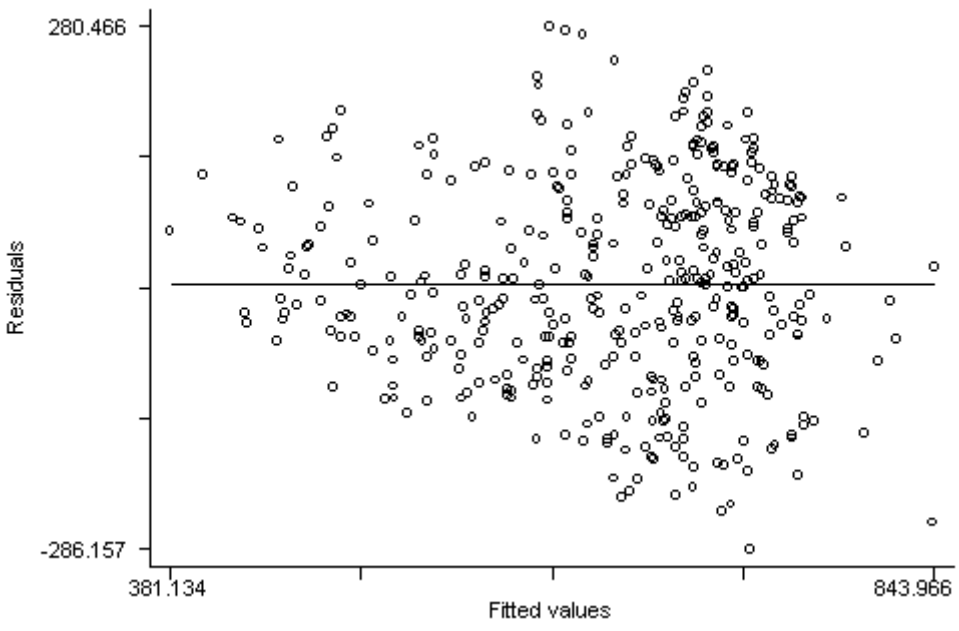
```
list snum api00 p r h wt in -10/l

          snum   api00          p          r          h          wt
391.      3024     727   729.0243  -2.024302   .0104834   .99997367
```

```
392.      3535     705    703.846   1.154008   .0048329   .99999207
393.      1885     605    605.427  -.4269809   .0144377   .99999843
394.      1678     497   496.8011   .1989256   .0243301   .99999956
395.      4486     706   705.8076    .192455   .0142448   .99999986
396.      4488     521          .          .          .          .
397.      3072     763          .          .          .          .
398.      3055     590          .          .          .          .
399.       116     513          .          .          .          .
400.      4534     445          .          .          .          .
```

After using **rreg**, it is possible to generate predicted values, residuals and leverage (hat), but most of the regression diagnostic commands are not available after **rreg**. We will have to create some of them for ourselves. Here, of course, is the graph of residuals versus fitted (predicted) with a line at zero. This plot looks much like the OLS plot, except that in the OLS all of the observations would be weighted equally, but as we saw above the observations with the greatest residuals are weighted less and hence have less influence on the results.

```
graph r p, yline(0)
```



To get an **lvr2plot** we are going to have to go through several steps in order to get the normalized squared residuals and the means of both the residuals and the leverage (hat) values.

First, we generate the residual squared (**r2**) and then divide it by the sum of the squared residuals. We then compute the mean of this value and save it as a local macro called **rm** (which we will use for

then compute the mean of this value and save it as a local macro called **rm** (which we will use for creating the leverage vs. residual plot).

```
generate r2=r^2
(5 missing values generated)

sum r2

Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+--------------------------------------------------------
      r2 |     395    12436.05    14677.98   .0370389    81885.7

replace r2 = r2/r(sum)
(395 real changes made)

summarize r2

Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+--------------------------------------------------------
      r2 |     395    .0025316     .002988   7.54e-09   .0166697

local rm = r(mean)
```

Next we compute the mean of the leverage and save it as a local macro called **hm**.

```
summarize h

Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+--------------------------------------------------------
       h |     395    .0126422    .0108228   .0023925   .0664077

local hm = r(mean)
```

Now, we can plot the leverage against the residual squared as shown below. Comparing the plot below with the plot from the OLS regression, this plot is much better behaved. There are no longer points in the upper right quadrant of the graph.

```
graph h r2, xline(`hm') xline(`rm')
```

```
graph h r2, yline( nm ) xline( rm )
```

Let's close out this analysis by deleting our temporary variables.

```
drop wt p r h r2
```

4.1.4 Quantile Regression

Quantile regression, in general, and median regression, in particular, might be considered as an alternative to **rreg**. The Stata command **qreg** does quantile regression. **qreg** without any options will actually do a median regression in which the coefficients will be estimated by minimizing the absolute deviations from the median. Of course, as an estimate of central tendency, the median is a resistant measure that is not as greatly affected by outliers as is the mean. It is not clear that median regression is a resistant estimation procedure, in fact, there is some evidence that it can be affected by high leverage values.

Here is what the quantile regression looks like using Stata's **qreg** command. The coefficient and standard error for **acs_k3** are considerably different when using **qreg** as compared to OLS using the **regress** command (the coefficients are 1.2 vs 6.9 and the standard errors are 6.4 vs 4.3). The coefficients and standard errors for the other variables are also different, but not as dramatically different.

Nevertheless, the **qreg** results indicate that, like the OLS results, all of the variables except **acs_k3** are significant.

```
qreg api00 acs_k3 acs_46 full enroll

Median regression                                   Number of obs =       395
  Raw sum of deviations    48534 (about 643)
  Min sum of deviations 36268.11                   Pseudo R2     =    0.2527

------------------------------------------------------------------------------
       api00 |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
      acs_k3 |   1.269065   6.470588      0.196   0.845      -11.45253    13.99066
      acs_46 |    7.22408   2.228949      3.241   0.001       2.841821    11.60634
        full |   5.323841   .6157333      8.646   0.000       4.113269    6.534413
      enroll |  -.1245734   .0397576     -3.133   0.002      -.2027395   -.0464073
       _cons |   17.15049   125.4396      0.137   0.891      -229.4719    263.7729
------------------------------------------------------------------------------
```

The **qreg** command has even fewer diagnostic options than **rreg** does. About the only values we can obtain are the predicted values and the residuals.

```
predict p if e(sample)
(option xb assumed; fitted values)
(5 missing values generated)

predict r if e(sample), r
(5 missing values generated)

graph r p, yline(0)
```

Stata has three additional commands that can do quantile regression.

iqreg estimates interquantile regressions, regressions of the difference in quantiles. The estimated variance-covariance matrix of the estimators is obtained via bootstrapping.

sqreg estimates simultaneous-quantile regression. It produces the same coefficients as qreg for each quantile. **sqreg** obtains a bootstrapped variance-covariance matrix of the estimators that includes

quantile. **sqreg** obtains a bootstrapped variance-covariance matrix of the estimators that includes between-quantiles blocks. Thus, one can test and construct confidence intervals comparing coefficients describing different quantiles.

bsqreg is the same as sqreg with one quantile. **sqreg** is, therefore, faster than bsqreg.

4.2 Constrained Linear Regression

Let's begin this section by looking at a regression model using the **hsb2** dataset. The **hsb2** file is a sample of 200 cases from the Highschool and Beyond Study (Rock, Hilton, Pollack, Ekstrom & Goertz, 1985). It includes the following variables: **id**, **female**, **race**, **ses**, **schtyp**, **program**, **read**, **write**, **math**, **science** and **socst**. The variables **read**, **write**, **math**, **science** and **socst** are the results of standardized tests on reading, writing, math, science and social studies (respectively), and the variable **female** is coded 1 if female, 0 if male.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/hsb2
```

Let's start by doing an OLS regression where we predict **socst** score from **read**, **write**, **math**, **science** and **female** (gender)

```
regress socst read write math science female

  Source |       SS       df       MS                  Number of obs =     200
---------+------------------------------                F(  5,   194) =   35.44
```

```
   Model |  10949.2575     5   2189.8515             Prob > F      =  0.0000
Residual |  11986.9375   194  61.7883375             R-squared     =  0.4774
---------+------------------------------             Adj R-squared =  0.4639
   Total |   22936.195   199  115.257261             Root MSE      =  7.8606

------------------------------------------------------------------------------
   socst |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    read |   .3784046   .0806267      4.693   0.000       .2193872     .537422
   write |   .3858743   .0889283      4.339   0.000       .2104839    .5612646
    math |   .1303258   .0893767      1.458   0.146       -.045949    .3066006
 science |  -.0333925   .0818741     -0.408   0.684      -.1948702    .1280852
  female |  -.3532648   1.245372     -0.284   0.777      -2.809471    2.102941
   _cons |   7.339342   3.650243      2.011   0.046       .1400864     14.5386
------------------------------------------------------------------------------
```

Notice that the coefficients for **read** and **write** are very similar, which makes sense since they are both measures of language ability. Also, the coefficients for **math** and **science** are similar (in that they are both not significantly different from 0). Suppose that we have a theory that suggests that **read** and **write** should have equal coefficients, and that **math** and **science** should have equal coefficients as well. We can test the equality of the coefficients using the **test** command.

```
test read=write

 ( 1)  read - write = 0.0

       F(  1,   194) =    0.00
            Prob > F =    0.9558
```

We can also do this with the **testparm** command, which is especially useful if you were testing whether 3 or more coefficients were equal.

```
testparm read write, equal
```

```
( 1) - read + write = 0.0

      F(  1,   194) =    0.00
           Prob > F =    0.9558
```

Both of these results indicate that there is no significant difference in the coefficients for the reading and writing scores. Since it appears that the coefficients for **math** and **science** are also equal, let's test the equality of those as well (using the **testparm** command).

```
testparm math science, equal

 ( 1) - math + science = 0.0

      F(  1,   194) =    1.45
           Prob > F =    0.2299
```

Let's now perform both of these tests together, simultaneously testing that the coefficient for **read** equals **write** and **math** equals **science**. We do this using two **test** commands, the second using the **accum** option to accumulate the first test with the second test to test both of these hypotheses together.

```
test read=write

 ( 1)  read - write = 0.0
```

```
       F(  1,   194) =    0.00
            Prob > F =    0.9558

test math=science, accum

 ( 1)  read - write = 0.0
 ( 2)  math - science = 0.0

       F(  2,   194) =    0.73
            Prob > F =    0.4852
```

Note this second test has 2 df, since it is testing both of the hypotheses listed, and this test is not significant, suggesting these pairs of coefficients are not significantly different from each other. We can estimate regression models where we constrain coefficients to be equal to each other. For example, let's begin on a limited scale and constrain **read** to equal **write**. First, we will define a constraint and then we will run the **cnsreg** command.

```
constraint define 1 read = write
. cnsreg socst read write math science female, constraint(1)

Constrained linear regression                   Number of obs =     200
```

```
                                                      F(  4,   195) =   44.53
                                                      Prob > F      =  0.0000
                                                      Root MSE      =  7.8404
 ( 1)  read - write = 0.0
------------------------------------------------------------------------------
   socst |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    read |   .3818488   .0513899      7.430   0.000       .2804975    .4832002
   write |   .3818488   .0513899      7.430   0.000       .2804975    .4832002
    math |   .1303036   .0891471      1.462   0.145      -.0455126    .3061197
 science |  -.0332762   .0816379     -0.408   0.684      -.1942827    .1277303
  female |  -.3296237   1.167364     -0.282   0.778      -2.631904    1.972657
   _cons |   7.354148   3.631175      2.025   0.044       .1927294    14.51557
------------------------------------------------------------------------------
```

Notice that the coefficients for **read** and **write** are identical, along with their standard errors, t-test, etc. Also note that the degrees of freedom for the F test is four, not five, as in the OLS model. This is because only one coefficient is estimated for **read** and **write**, estimated like a single variable equal to the sum of their values. Notice also that the Root MSE is slightly higher for the constrained model, but only slightly higher. This is because we have forced the model to estimate the coefficients for **read** and **write** that are not as good at minimizing the Sum of Squares Error (the coefficients that would minimize the SSE would be the coefficients from the unconstrained model).

Next, we will define a second constraint, setting **math** equal to **science**. We will also abbreviate the constraints option to **c**.

```
constraint define 2 math = science
. cnsreg socst read write math science female, c(1 2)

Constrained linear regression                       Number of obs =     200
```

```
                                                      F(  3,   196) =   58.75
                                                      Prob > F      =  0.0000
                                                      Root MSE      =  7.8496
 ( 1)  read - write = 0.0
 ( 2)  math - science = 0.0
------------------------------------------------------------------------------
       socst |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
        read |   .3860376   .0513322      7.520   0.000       .2848033    .4872719
       write |   .3860376   .0513322      7.520   0.000       .2848033    .4872719
        math |   .0428053   .0519238      0.824   0.411      -.0595958    .1452064
     science |   .0428053   .0519238      0.824   0.411      -.0595958    .1452064
      female |   -.200875   1.163831     -0.173   0.863      -2.496114    2.094364
       _cons |   7.505658   3.633225      2.066   0.040       .3404249    14.67089
------------------------------------------------------------------------------
```

Now the coefficients for **read** = **write** and **math** = **science** and the degrees of freedom for the model has dropped to three. Again, the Root MSE is slightly larger than in the prior model, but we should emphasize only very slightly larger. If indeed the population coefficients for **read** = **write** and **math** = **science**, then these combined (constrained) estimates may be more stable and generalize better to other samples. So although these estimates may lead to slightly higher standard error of prediction in this sample, they may generalize better to the population from which they came.

4.3 Regression with Censored or Truncated Data

Analyzing data that contain censored values or are truncated is common in many research disciplines. According to Hosmer and Lemeshow (1999), a censored value is one whose value is incomplete due to random factors for each subject. A truncated observation, on the other hand, is one which is incomplete due to a selection process in the design of the study.

We will begin by looking at analyzing data with censored values.

4.3.1 Regression with Censored Data

In this example we have a variable called **acadindx** which is a weighted combination of standardized test scores and academic grades. The maximum possible score on **acadindx** is 200 but it is clear that the 16 students who scored 200 are not exactly equal in their academic abilities. In other words, there is variability in academic ability that is not being accounted for when students score 200 on acadindx. The

variable **acadindx** is said to be censored, in particular, it is right censored.

Let's look at the example. We will begin by looking at a description of the data, some descriptive statistics, and correlations among the variables.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/acadindx
(max possible on acadindx is 200)

describe
```

```
Contains data from acadindx.dta
  obs:           200                          max possible on acadindx is 200
 vars:             5                          19 Jan 2001 20:14
 size:         4,800 (99.7% of memory free)
-------------------------------------------------------------------------------
   1. id        float  %9.0g
   2. female    float  %9.0g       fl
   3. reading   float  %9.0g
   4. writing   float  %9.0g
   5. acadindx  float  %9.0g                  academic index
-------------------------------------------------------------------------------
```

summarize

```
Variable |     Obs        Mean   Std. Dev.       Min        Max
---------+--------------------------------------------------------
      id |     200       100.5   57.87918          1        200
  female |     200        .545   .4992205          0          1
 reading |     200       52.23   10.25294         28         76
 writing |     200      52.775   9.478586         31         67
acadindx |     200     172.185    16.8174        138        200
```

count if acadindx==200

```
   16
```

corr acadindx female reading writing

```
(obs=200)

         | acadindx   female  reading  writing
---------+------------------------------------
acadindx |   1.0000
  female |  -0.0821   1.0000

 reading |   0.7131  -0.0531   1.0000
 writing |   0.6626   0.2565   0.5968   1.0000
```

Now, let's run a standard OLS regression on the data and generate predicted scores in **p1**.

```
regress acadindx female reading writing

  Source |       SS       df       MS                  Number of obs =     200
---------+------------------------------                F(  3,   196) =  107.40
   Model |   34994.282     3  11664.7607                Prob > F      =  0.0000
Residual |   21287.873   196  108.611597                R-squared     =  0.6218
---------+------------------------------                Adj R-squared =  0.6160
   Total |   56282.155   199  282.824899                Root MSE      =  10.422

------------------------------------------------------------------------------
acadindx |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  female |  -5.832498    1.58821     -3.672   0.000      -8.964671   -2.700324
 reading |   .7184174   .0931493      7.713   0.000       .5347138     .902121
 writing |   .7905706   .1040996      7.594   0.000       .5852715    .9958696
   _cons |   96.11841   4.489562     21.409   0.000       87.26436    104.9725
------------------------------------------------------------------------------

predict p1
(option xb assumed; fitted values)
```

The **tobit** command is one of the commands that can be used for regression with censored data. The syntax of the command is similar to regress with the addition of the **ul** option to indicate that the right censored value is 200. We will follow the **tobit** command by predicting **p2** containing the **tobit** predicted values.

```
tobit acadindx female reading writing, ul(200)

Tobit estimates                                   Number of obs   =        200
                                                  LR chi2(3)      =     190.39
```

```
                                                  Prob > chi2     =     0.0000
Log likelihood = -718.06362                       Pseudo R2       =     0.1171

------------------------------------------------------------------------------
acadindx |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  female |  -6.347316   1.692441     -3.750   0.000      -9.684943   -3.009688
 reading |   .7776857   .0996928      7.801   0.000       .5810837    .9742877
 writing |   .8111221    .110211      7.360   0.000       .5937773    1.028467
   _cons |   92.73782   4.803441     19.307   0.000       83.26506    102.2106
---------+--------------------------------------------------------------------
     _se |   10.98973   .5817477              (Ancillary parameter)
------------------------------------------------------------------------------

Obs. summary:       184 uncensored observations
                     16 right-censored observations at acadindx>=200

predict p2
(option xb assumed; fitted values)
```

Summarizing the **p1** and **p2** scores shows that the **tobit** predicted values have a larger standard deviation and a greater range of values.

```
summarize acadindx p1 p2

Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+--------------------------------------------------------
acadindx |     200     172.185     16.8174       138        200
      p1 |     200     172.185    13.26087   142.3821   201.5311
      p2 |     200     172.704    14.00292   141.2211   203.8541
```

When we look at a listing of **p1** and **p2** for all students who scored the maximum of 200 on acadindx, we see that in every case the **tobit** predicted value is greater than the OLS predicted value. These predictions represent an estimate of what the variability would be if the values of **acadindx** could exceed 200.

```
list p1 p2 if acadindx==200
```

```
            p1         p2
 32.   179.175     179.62
 57.  192.6806   194.3291
 68.  201.5311   203.8541
 80.  191.8309    193.577
 82.  188.1537   189.5627
 88.  186.5725   187.9405
 95.  195.9971   198.1762
100.  186.9333   188.1076
132.  197.5782   199.7984
136.  189.4592   191.1436
143.  191.1846   192.8327
157.  191.6145   193.4767
161.  180.2511   181.0082
169.   182.275   183.3667
174.  191.6145   193.4767
200.  187.6616   189.4211
```

Here is the syntax diagram for tobit:

```
tobit depvar [indepvars] [weight] [if exp] [in range], ll[(#)] ul[(#)]
        [ level(#) offset(varname) maximize_options ]
```

You can declare both lower and upper censored values. The censored values are fixed in that the same lower and upper values apply to all observations.

There are two other commands in Stata that allow you more flexibility in doing regression with censored data.

cnreg estimates a model in which the censored values may vary from observation to observation.

intreg estimates a model where the response variable for each observation is either point data, interval data, left-censored data, or right-censored data.

4.3.2 Regression with Truncated Data

Truncated data occurs when some observations are not included in the analysis because of the value of the variable. We will illustrate analysis with truncation using the dataset, **acadindx**, that was used in the previous section. If **acadindx** is no longer loaded in memory you can get it with the following use command.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/acadindx
(max possible on acadindx is 200)
```

Let's imagine that in order to get into a special honors program, students need to score at least 160 on **acadindx**. So we will drop all observations in which the value of **acadindx** is less than 160.

```
drop if acadindx <= 160
(56 observations deleted)
```

Now, let's estimate the same model that we used in the section on censored data, only this time we will pretend that a 200 for **acadindx** is not censored.

```
regress acadindx female reading writing

      Source |       SS       df       MS              Number of obs =     144
-------------+------------------------------           F(  3,   140) =   33.01
       Model |  8074.79638     3  2691.59879            Prob > F      =  0.0000
    Residual |  11416.3633   140  81.5454524            R-squared     =  0.4143
-------------+------------------------------           Adj R-squared =  0.4017
       Total |  19491.1597   143  136.301816            Root MSE      =  9.0303

------------------------------------------------------------------------------
    acadindx |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -5.238495   1.615632    -3.24   0.001    -8.432687   -2.044303
     reading |   .4411066   .0963504     4.58   0.000     .2506166    .6315965
     writing |   .5873287   .1150828     5.10   0.000     .3598037    .8148537
       _cons |   125.6355   5.891559    21.32   0.000     113.9875    137.2834
------------------------------------------------------------------------------
```

It is clear that the estimates of the coefficients are distorted due to the fact that 56 observations are no longer in the dataset. This amounts to restriction of range on both the response variable and the predictor variables. For example, the coefficient for writing dropped from .79 to .59. What this means is that if our goal is to find the relation between **adadindx** and the predictor variables in the population,

then the truncation of **acadindx** in our sample is going to lead to biased estimates. A better approach to analyzing these data is to use truncated regression. In Stata this can be accomplished using the **truncreg** command where the **ll** option is used to indicate the lower limit of **acadindx** scores used in the truncation.

```
truncreg acadindx female reading writing, ll(160)
(note: 0 obs. truncated)

Truncated regression
Limit:   lower =        160                       Number of obs =    144
         upper =       +inf                       Wald chi2(3)  =  77.87
Log likelihood = -510.00768                       Prob > chi2   = 0.0000

------------------------------------------------------------------------------
    acadindx |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
eq1          |
      female |  -6.099602   1.925245    -3.17   0.002    -9.873012   -2.326191
     reading |   .5181789   .1168288     4.44   0.000     .2891986    .7471592
     writing |   .7661636     .15262     5.02   0.000     .4670339    1.065293
       _cons |   110.2892   8.673849    12.72   0.000     93.28877    127.2896
-------------+----------------------------------------------------------------
sigma        |
       _cons |   9.803572    .721646    13.59   0.000     8.389172    11.21797
------------------------------------------------------------------------------
```

The coefficients from the **truncreg** command are closer to the OLS results, for example the coefficient for **writing** is .77 which is closer to the OLS results of .79. However, the results are still somewhat different on the other variables, for example the coefficient for reading is .52 in the **truncreg** as compared to .72 in the original OLS with the unrestricted data, and better than the OLS estimate of .47 with the restricted data. While **truncreg** may improve the estimates on a restricted data file as compared to OLS, it is certainly no substitute for analyzing the complete unrestricted data file.

4.4 Regression with Measurement Error

As you will most likely recall, one of the assumptions of regression is that the predictor variables are measured without error. The problem is that measurement error in predictor variables leads to under estimation of the regression coefficients. Stata's **eivreg** command takes measurement error into account when estimating the coefficients for the model.

Let's look at a regression using the hsb2 dataset.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/hsb2

regress write read female

  Source |       SS       df       MS                  Number of obs =     200
---------+------------------------------                F(  2,   197) =   77.21
   Model |  7856.32118     2  3928.16059                Prob > F      =  0.0000
Residual |  10022.5538   197  50.8759077                R-squared     =  0.4394
---------+------------------------------                Adj R-squared =  0.4337
   Total |   17878.875   199   89.843593                Root MSE      =  7.1327

------------------------------------------------------------------------------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    read |   .5658869   .0493849     11.459   0.000        .468496    .6632778
  female |   5.486894   1.014261      5.410   0.000        3.48669    7.487098
   _cons |   20.22837   2.713756      7.454   0.000       14.87663    25.58011
------------------------------------------------------------------------------
```

The predictor read is a standardized test score. Every test has measurement error. We don't know the exact reliability of **read**, but using .9 for the reliability would probably not be far off. We will now estimate the same regression model with the Stata **eivreg** command, which stands for errors-in-variables regression.

```
eivreg write read female, r(read .9)

               assumed                              errors-in-variables regression
variable     reliability
```

```
-----------------------                                     Number of obs =     200
    read       0.9000                                       F(  2,   197) =   83.41
       *       1.0000                                       Prob > F      =  0.0000
                                                            R-squared     =  0.4811
                                                            Root MSE      = 6.86268


------------------------------------------------------------------------------
       write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
        read |   .6289607   .0528111     11.910   0.000        .524813    .7331085
      female |   5.555659   .9761838      5.691   0.000       3.630548     7.48077
       _cons |   16.89655   2.880972      5.865   0.000       11.21504    22.57805
```

Note that the F-ratio and the R^2 increased along with the regression coefficient for **read**. Additionally, there is an increase in the standard error for read.

Now, let's try a model with **read**, **math** and **socst** as predictors. First, we will run a standard OLS regression.

```
regress write read math socst female

      Source |       SS       df       MS                    Number of obs =     200
-------------+------------------------------                  F(  4,   195) =   64.37
```

```
   Model |  10173.7036     4  2543.42591             Prob > F      =  0.0000
Residual |  7705.17137   195  39.5136993             R-squared     =  0.5690
---------+------------------------------             Adj R-squared =  0.5602
   Total |   17878.875   199   89.843593             Root MSE      =   6.286

------------------------------------------------------------------------------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    read |   .2065341   .0640006      3.227   0.001       .0803118    .3327563
    math |   .3322639   .0651838      5.097   0.000       .2037082    .4608195
   socst |   .2413236   .0547259      4.410   0.000        .133393    .3492542
  female |   5.006263   .8993625      5.566   0.000       3.232537     6.77999
   _cons |   9.120717   2.808367      3.248   0.001       3.582045    14.65939
------------------------------------------------------------------------------
```

Now, let's try to account for the measurement error by using the following reliabilities: **read** – .9, **math** – .9, **socst** – .8.

```
eivreg write read math socst female, r(read .9 math .9 socst .8)

               assumed                          errors-in-variables regression
variable     reliability
```

```
-----------------------                                     Number of obs =     200
    read       0.9000                                       F(  4,   195) =   70.17
    math       0.9000                                       Prob > F      =  0.0000
   socst       0.8000                                       R-squared     =  0.6047
       *       1.0000                                       Root MSE      = 6.02062

------------------------------------------------------------------------------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    read |   .1506668   .0936571      1.609   0.109      -.0340441    .3353776
    math |    .350551   .0850704      4.121   0.000       .1827747    .5183273
   socst |   .3327103   .0876869      3.794   0.000        .159774    .5056467
  female |   4.852501   .8730646      5.558   0.000        3.13064    6.574363
   _cons |    6.37062   2.868021      2.221   0.027       .7142973    12.02694
------------------------------------------------------------------------------
```

Note that the overall F and R^2 went up, but that the coefficient for read is no longer statistically significant.

4.5 Multiple Equation Regression Models

If a dataset has enough variables we may want to estimate more than one regression model. For example, we may want to predict y1 from x1 and also predict y2 from x2. Even though there are no variables in common these two models are not independent of one another because the data come from the same subjects. This is an example of one type of multiple equation regression known as seemingly unrelated regression. We can estimate the coefficients and obtain standard errors taking into account the correlated errors in the two models. An important feature of multiple equation models is that we can test predictors across equations.

Another example of multiple equation regression is if we wished to predict y1, y2 and y3 from x1 and x2. This is a three equation system, known as multivariate regression, with the same predictor variables for each model. Again, we have the capability of testing coefficients across the different equations.

Multiple equation models are a powerful extension to our data analysis tool kit.

4.5.1 Seemingly Unrelated Regression

Let's continue using the **hsb2** data file to illustrate the use of seemingly unrelated regression. You can

load it into memory again if it has been cleared out.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/hsb2
(highschool and beyond (200 cases))
```

This time let's look at two regression models.

```
   science = math female
   write   = read female
```

It is the case that the errors (residuals) from these two models would be correlated. This would be true even if the predictor female were not found in both models. The errors would be correlated because all of the values of the variables are collected on the same set of observations. This is a situation tailor made for seemingly unrelated regression using the **sureg** command. Here is our first model using OLS.

```
regress science math female

<some output omitted>
------------------------------------------------------------------------------
 science |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
    math |   .6631901   .0578724     11.460   0.000        .549061    .7773191
  female |  -2.168396   1.086043     -1.997   0.047      -4.310159    -.026633
   _cons |   18.11813   3.167133      5.721   0.000        11.8723    24.36397
------------------------------------------------------------------------------
```

And here is our second model using OLS.

```
regress write read female

<some output omitted>
------------------------------------------------------------------------------
```

```
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+------------------------------------------------------------------
    read |   .5658869   .0493849     11.459   0.000        .468496    .6632778
  female |   5.486894   1.014261      5.410   0.000        3.48669    7.487098
   _cons |   20.22837   2.713756      7.454   0.000       14.87663    25.58011
------------------------------------------------------------------------------
```

With the **sureg** command we can estimate both models simultaneously while accounting for the correlated errors at the same time, leading to efficient estimates of the coefficients and standard errors. By including the **corr** option with **sureg** we can also obtain an estimate of the correlation between the errors of the two models. Note that both the estimates of the coefficients and their standard errors are different from the OLS model estimates shown above. The bottom of the output provides a Breusch-Pagan test of whether the residuals from the two equations are independent (in this case, we would say the residuals were not independent, p=0.0407).

```
sureg (science math female) (write read female), corr

Seemingly unrelated regression
------------------------------------------------------------
```

```
Equation      Obs  Parms        RMSE    "R-sq"       Chi2        P
-------------------------------------------------------------------
science       200      2    7.595793    0.4085   125.4142   0.0000
write         200      2    7.085844    0.4383   144.2683   0.0000
------------------------------------------------------------------------------
             |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
science      |
        math |   .6251409   .0570948     10.949   0.000       .5132373    .7370446
      female |  -2.189344   1.077862     -2.031   0.042      -4.301914   -.0767744
       _cons |   20.13265   3.125775      6.441   0.000       14.00624    26.25905
-------------+----------------------------------------------------------------
write        |
        read |   .5354838   .0487212     10.991   0.000       .4399919    .6309757
      female |   5.453748   1.006609      5.418   0.000        3.48083    7.426665
       _cons |   21.83439    2.67851      8.152   0.000        16.5846    27.08417
------------------------------------------------------------------------------

Correlation matrix of residuals:

             science    write
science       1.0000
  write       0.1447   1.0000

Breusch-Pagan test of independence: chi2(1) =     4.188, Pr = 0.0407
```

Now that we have estimated our models let's test the predictor variables. The test for **female** combines information from both models. The tests for **math** and **read** are actually equivalent to the z-tests above except that the results are displayed as chi-square tests.

```
test female

 ( 1)  [science]female = 0.0
 ( 2)  [write]female = 0.0
```

```
          chi2(  2) =   37.45
        Prob > chi2 =    0.0000

test math

 ( 1)  [science]math = 0.0

          chi2(  1) =  119.88
        Prob > chi2 =    0.0000

test read

 ( 1)  [write]read = 0.0

          chi2(  1) =  120.80
        Prob > chi2 =    0.0000
```

Now, let's estimate 3 models where we use the same predictors in each model as shown below.

```
    read  = female prog1 prog3
    write = female prog1 prog3
    math  = female prog1 prog3
```

If you no longer have the dummy variables for **prog**, you can recreate them using the tabulate command.

```
tabulate prog, gen(prog)
```

Let's first estimate these three models using 3 OLS regressions.

```
regress read female prog1 prog3

<some output omitted>
------------------------------------------------------------------------------
```

```
    read |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+------------------------------------------------------------------
  female |  -1.208582   1.327672     -0.910   0.364      -3.826939    1.409774
   prog1 |   -6.42937   1.665893     -3.859   0.000      -9.714746   -3.143993
   prog3 |  -9.976868   1.606428     -6.211   0.000      -13.14497   -6.808765
   _cons |    56.8295   1.170562     48.549   0.000       54.52099    59.13802
------------------------------------------------------------------------------
```

regress write female prog1 prog3

```
<some output omitted>
------------------------------------------------------------------------------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+------------------------------------------------------------------
  female |   4.771211   1.181876      4.037   0.000       2.440385    7.102037
   prog1 |  -4.832929   1.482956     -3.259   0.001      -7.757528   -1.908331
   prog3 |  -9.438071   1.430021     -6.600   0.000      -12.25827   -6.617868
   _cons |   53.62162   1.042019     51.459   0.000       51.56661    55.67662
------------------------------------------------------------------------------
```

regress math female prog1 prog3

```
<some output omitted>
------------------------------------------------------------------------------
    math |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+------------------------------------------------------------------
  female |  -.6737673   1.176059     -0.573   0.567      -2.993122    1.645587
   prog1 |  -6.723945   1.475657     -4.557   0.000      -9.634149    -3.81374
   prog3 |  -10.32168   1.422983     -7.254   0.000        -13.128   -7.515352
   _cons |   57.10551    1.03689     55.074   0.000       55.06062     59.1504
------------------------------------------------------------------------------
```

These regressions provide fine estimates of the coefficients and standard errors but these results assume the residuals of each analysis are completely independent of the others. Also, if we wish to test **female**, we would have to do it three times and would not be able to combine the information from all three tests into a single overall test.

Now let's use **sureg** to estimate the same models. Since all 3 models have the same predictors, we can use the syntax as shown below which says that **read**, **write** and **math** will each be predicted by **female**, **prog1** and **prog3**. Note that the coefficients are identical in the OLS results above and the **sureg** results below, however the standard errors are different, only slightly, due to the correlation among the residuals in the multiple equations.

```
sureg (read write math = female prog1 prog3), corr

Seemingly unrelated regression
------------------------------------------------------------------------------
```

```
Equation      Obs  Parms        RMSE    "R-sq"       Chi2        P
-------------------------------------------------------------------
read          200      3    9.254765    0.1811   44.24114   0.0000
write         200      3    8.238468    0.2408   63.41908   0.0000
math          200      3    8.197921    0.2304   59.88479   0.0000
------------------------------------------------------------------------------
             |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
read         |
      female |  -1.208582   1.314328     -0.920   0.358      -3.784618    1.367454
       prog1 |   -6.42937    1.64915     -3.899   0.000      -9.661645   -3.197095
       prog3 |  -9.976868   1.590283     -6.274   0.000      -13.09377   -6.859971
       _cons |    56.8295   1.158797     49.042   0.000        54.5583     59.1007
-------------+----------------------------------------------------------------
write        |
      female |   4.771211   1.169997      4.078   0.000       2.478058    7.064363
       prog1 |  -4.832929   1.468051     -3.292   0.001      -7.710257   -1.955602
       prog3 |  -9.438071   1.415648     -6.667   0.000      -12.21269   -6.663451
       _cons |   53.62162   1.031546     51.982   0.000       51.59982    55.64341
-------------+----------------------------------------------------------------
math         |
      female |  -.6737673   1.164239     -0.579   0.563      -2.955634    1.608099
       prog1 |  -6.723945   1.460826     -4.603   0.000      -9.587111   -3.860778
       prog3 |  -10.32168   1.408681     -7.327   0.000      -13.08264   -7.560711
       _cons |   57.10551   1.026469     55.633   0.000       55.09367    59.11735
------------------------------------------------------------------------------

Correlation matrix of residuals:

             read   write    math
 read  1.0000
write  0.5519  1.0000
 math  0.5774  0.5577  1.0000


Breusch-Pagan test of independence: chi2(3) =   189.811, Pr = 0.0000
```

In addition to getting more appropriate standard errors, **sureg** allows us to test the effects of the predictors across the equations. We can test the hypothesis that the coefficient for **female** is 0 for all

predictors across the equations. We can test the hypothesis that the coefficient for **female** is 0 for all three outcome variables, as shown below.

```
test female

 ( 1)  [read]female = 0.0
 ( 2)  [write]female = 0.0
 ( 3)  [math]female = 0.0

           chi2(  3) =   35.59
         Prob > chi2 =    0.0000
```

We can also test the hypothesis that the coefficient for **female** is 0 for just **read** and **math**. Note that [**read**]**female** means the coefficient for **female** for the outcome variable **read**.

```
test [read]female [math]female

 ( 1)  [read]female = 0.0
 ( 2)  [math]female = 0.0

           chi2(  2) =    0.85
         Prob > chi2 =    0.6541
```

We can also test the hypothesis that the coefficients for **prog1** and **prog3** are 0 for all three outcome variables, as shown below.

```
test prog1 prog3

 ( 1)  [read]prog1 = 0.0
 ( 2)  [write]prog1 = 0.0
```

```
 ( 3)  [math]prog1 = 0.0
 ( 4)  [read]prog3 = 0.0
 ( 5)  [write]prog3 = 0.0
 ( 6)  [math]prog3 = 0.0

           chi2(  6) =   72.45
         Prob > chi2 =    0.0000
```

4.5.2 Multivariate Regression

Let's now use multivariate regression using the **mvreg** command to look at the same analysis that we saw in the **sureg** example above, estimating the following 3 models.

```
read  = female prog1 prog3
write = female prog1 prog3
math  = female prog1 prog3
```

If you don't have the **hsb2** data file in memory, you can use it below and then create the dummy variables for **prog1** – **prog3**.

```
use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/hsb2
tabulate prog, gen(prog)
<output omitted>
```

Below we use **mvreg** to predict **read**, **write** and **math** from **female**, **prog1** and **prog3**. Note that the top part of the output is similar to the **sureg** output in that it gives an overall summary of the model for each outcome variable, however the results are somewhat different and the **sureg** uses a **Chi-Square** test for the overall fit of the model, and **mvreg** uses an **F-test**. The lower part of the output appears similar to the **sureg** output; however, when you compare the standard errors you see that the results are not the same. These standard errors correspond to the OLS standard errors, so these results below do not take into account the correlations among the residuals (as do the **sureg** results).

```
mvreg read write math = female prog1 prog3

Equation      Obs  Parms        RMSE    "R-sq"          F        P
------------------------------------------------------------------
```

```
read          200      4    9.348725    0.1811   14.45211   0.0000
write         200      4     8.32211    0.2408    20.7169   0.0000
math          200      4    8.281151    0.2304   19.56237   0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
read         |
      female |  -1.208582   1.327672     -0.910   0.364      -3.826939    1.409774
       prog1 |   -6.42937   1.665893     -3.859   0.000      -9.714746   -3.143993
       prog3 |  -9.976868   1.606428     -6.211   0.000      -13.14497   -6.808765
       _cons |    56.8295   1.170562     48.549   0.000       54.52099    59.13802
-------------+----------------------------------------------------------------
write        |
      female |   4.771211   1.181876      4.037   0.000       2.440385    7.102037
       prog1 |  -4.832929   1.482956     -3.259   0.001      -7.757528   -1.908331
       prog3 |  -9.438071   1.430021     -6.600   0.000      -12.25827   -6.617868
       _cons |   53.62162   1.042019     51.459   0.000       51.56661    55.67662
-------------+----------------------------------------------------------------
math         |
      female |  -.6737673   1.176059     -0.573   0.567      -2.993122    1.645587
       prog1 |  -6.723945   1.475657     -4.557   0.000      -9.634149    -3.81374
       prog3 |  -10.32168   1.422983     -7.254   0.000        -13.128   -7.515352
       _cons |   57.10551    1.03689     55.074   0.000       55.06062     59.1504
------------------------------------------------------------------------------
```

Now, let's **test female**. Note, that **female** was statistically significant in only one of the three equations. Using the test command after **mvreg** allows us to test **female** across all three equations simultaneously. And, guess what? It is significant. This is consistent with what we found using **sureg** (except that **sureg** did this test using a **Chi-Square** test).

```
test female

 ( 1)  [read]female = 0.0
 ( 2)  [write]female = 0.0
```

```
 ( 3)  [math]female = 0.0

       F(  3,   196) =   11.63
            Prob > F =    0.0000
```

We can also test **prog1** and **prog3**, both separately and combined. Remember these are multivariate tests.

```
test prog1

 ( 1)  [read]prog1 = 0.0
 ( 2)  [write]prog1 = 0.0
```

```
 ( 3)  [math]prog1 = 0.0

       F(  3,   196) =    7.72
            Prob > F =    0.0001
```

```
test prog3

 ( 1)  [read]prog3 = 0.0
 ( 2)  [write]prog3 = 0.0
 ( 3)  [math]prog3 = 0.0

       F(  3,   196) =   21.47
            Prob > F =    0.0000
```

```
test prog1 prog3

 ( 1)  [read]prog1 = 0.0
 ( 2)  [write]prog1 = 0.0
 ( 3)  [math]prog1 = 0.0
 ( 4)  [read]prog3 = 0.0
 ( 5)  [write]prog3 = 0.0
 ( 6)  [math]prog3 = 0.0

       F(  6,   196) =   11.83
            Prob > F =    0.0000
```

Many researchers familiar with traditional multivariate analysis may not recognize the tests above. They don't see Wilks' Lambda, Pillai's Trace or the Hotelling-Lawley Trace statistics, statistics that they are familiar with. It is possible to obtain these statistics using the **mvtest** command written by David E. Moore of the University of Cincinnati. **mvtest** , which UCLA updated to work with Stata 6 and above, can be downloaded over the internet like this.

```
net from https://stats.idre.ucla.edu/stat/stata/ado/analysis
net install mvtest
```

Now that we have downloaded it, we can use it like this.

```
mvtest female

                    MULTIVARIATE TESTS OF SIGNIFICANCE


Multivariate Test Criteria and Exact F Statistics for
the Hypothesis of no Overall "female" Effect(s)

                            S=1    M=.5    N=96

Test                         Value          F       Num DF     Den DF   Pr > F
Wilks' Lambda             0.84892448    11.5081           3   194.0000   0.0000
Pillai's Trace            0.15107552    11.5081           3   194.0000   0.0000
Hotelling-Lawley Trace    0.17796108    11.5081           3   194.0000   0.0000


mvtest prog1 prog3

                    MULTIVARIATE TESTS OF SIGNIFICANCE


Multivariate Test Criteria and Exact F Statistics for
the Hypothesis of no Overall "prog1 prog3" Effect(s)

                            S=2    M=0    N=96

Test                         Value          F       Num DF     Den DF   Pr > F
Wilks' Lambda             0.73294667    10.8676           6   388.0000   0.0000
Pillai's Trace            0.26859190    10.0834           6   390.0000   0.0000
Hotelling-Lawley Trace    0.36225660    11.6526           6   386.0000   0.0000
```

We will end with an **mvtest** including all of the predictor variables. This is an overall multivariate test of the model.

```
mvtest female prog1 prog3
```

MULTIVARIATE TESTS OF SIGNIFICANCE

Multivariate Test Criteria and Exact F Statistics for
the Hypothesis of no Overall "female prog1 prog3" Effect(s)

S=3 M=-.5 N=96

Test	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.62308940	11.2593	9	472.2956	0.0000
Pillai's Trace	0.41696769	10.5465	9	588.0000	0.0000
Hotelling-Lawley Trace	0.54062431	11.5734	9	578.0000	0.0000

The **sureg** and **mvreg** commands both allow you to test multi-equation models while taking into account the fact that the equations are not independent. The **sureg** command allows you to get estimates for each equation which adjust for the non-independence of the equations, and it allows you to estimate equations which don't necessarily have the same predictors. By contrast, **mvreg** is restricted to equations that have the same set of predictors, and the estimates it provides for the individual equations are the same as the OLS estimates. However, **mvreg** (especially when combined with **mvtest**) allows you to perform more traditional multivariate tests of predictors.

4.6 Summary

This chapter has covered a variety of topics that go beyond ordinary least squares regression, but there still remain a variety of topics we wish we could have covered, including the analysis of survey data, dealing with missing data, panel data analysis, and more. And, for the topics we did cover, we wish we could have gone into even more detail. One of our main goals for this chapter was to help you be aware of some of the techniques that are available in Stata for analyzing data that do not fit the assumptions of

OLS regression and some of the remedies that are possible. If you are a member of the UCLA research community, and you have further questions, we invite you to use our consulting services (https://stats.idre.ucla.edu/ucla/policies/) to discuss issues specific to your data analysis.

4.7 Self Assessment

1. Use the **crime** data file that was used in chapter 2 (use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/crime) and look at a regression model predicting **murder** from **pctmetro**, **poverty**, **pcths** and **single** using OLS and make a **avplots** and a **lvr2plot** following the regression. Are there any states that look worrisome? Repeat this analysis using regression with robust standard errors and show **avplots** for the analysis. Repeat the analysis using robust regression and make a manually created **lvr2plot**. Also run the results using **qreg**. Compare the results of the different analyses. Look at the weights from the robust regression and comment on the weights.

2. Using the elemapi2 data file (use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/elemapi2) pretend that 550 is the lowest score that a school could achieve on **api00**, i.e., create a new variable with the **api00** score and recode it such that any score of 550 or below becomes 550. Use **meals**, **ell** and **emer** to predict api scores using 1) OLS to predict the original api score (before recoding) 2) OLS to predict the recoded score where 550 was the lowest value, and 3) using **tobit** to predict the recoded api score indicating the lowest value is 550. Compare the results of these analyses.

3. Using the elemapi2 data file (use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/elemapi2) pretend that only schools with api scores of 550 or higher were included in the sample. Use **meals**, **ell** and **emer** to predict api scores using 1) OLS to predict api from the full set of observations, 2) OLS to predict api using just the observations with api scores of 550 or higher, and 3) using **truncreg** to predict api using just the observations where api is 550 or higher. Compare the results of these analyses.

4. Using the **hsb2** data file (use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/hsb2) predict **read** from **science**, **socst**, **math** and **write**. Use the **testparm** and **test** commands to test the equality of the coefficients for **science**, **socst** and **math**. Use **cnsreg** to estimate a model where these three parameters are equal.

5. Using the elemapi2 data file (use https://stats.idre.ucla.edu/stat/stata/webbooks/reg/elemapi2) consider the following 2 regression equations.

```
api00 = meals ell emer
api99 = meals ell emer
```

Estimate the coefficients for these predictors in predicting **api00** and **api99** taking into account the non-independence of the schools. Test the overall contribution of each of the predictors in jointly predicting api scores in these two years. Test whether the contribution of **emer** is the same for **api00** and **api99**.

Click here (/stata/webbooks/reg/chapter4/regressionwith-statachapter-4-answers-to-excersises/) for our answers to these self assessment questions.

4.8 For more information

- Stata Manuals
  - **[R] rreg**
  - **[R] qreg**
  - **[R] cnsreg**
  - **[R] tobit**
  - **[R] truncreg**
  - **[R] eivreg**
  - **[R] sureg**
  - **[R] mvreg**
  - **[U] 23 Estimation and post-estimation commands**
  - **[U] 29 Overview of model estimation in Stata**
- Web Links
  - How standard errors with cluster() can be smaller than those without (http://www.stata.com/support/faqs/stat/cluster.html)
  - Advantages of the robust variance estimator (http://www.stata.com/support/faqs/stat/robust_var.html)
  - How to obtain robust standard errors for tobit (http://www.stata.com/support/faqs/stat/tobit.html)
  - Pooling data in linear regression (http://www.stata.com/support/faqs/stat/awreg.html)

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