6. For what value of b is the value of $\int_b^{b+1} (x^2 + x)\, dx$ a minimum?

(A) 0 (B) -1 (C) -2 (D) -3 (E) -4

7. In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

(A) One (B) Two (C) Three (D) Four (E) Eight

8. Suppose that the function f is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If f is continuous, which of the following intervals could be its domain?

(A) $\left(\frac{3\pi}{4}, \pi\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(-\frac{\pi}{4}, 0\right)$

(E) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$

9. $\int_0^1 \frac{x}{2 - x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\log 2 - e}{2}$ (D) $-\frac{\log 2}{2}$ (E) $\frac{\log 2}{2}$

12. $\lim_{x \to 0} \frac{\sin 2x}{(1 + x)\log(1 + x)} =$

(A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

---

13. $\lim_{n \to \infty} \int_1^n \frac{1}{x^n}\, dx =$

(A) 0 (B) 1 (C) e (D) π (E) $+\infty$

---

15. Let $f(x) = \int_1^x \frac{1}{1+t^2}\,dt$ for all real x. An equation of the line tangent to the graph of f at the point $(2, f(2))$ is

(A) $y - 1 = \frac{1}{5}(x - 2)$ (B) $y - \text{Arctan } 2 = \frac{1}{5}(x - 2)$ (C) $y - 1 = (\text{Arctan } 2)(x - 2)$

(D) $y - \text{Arctan } 2 + \frac{\pi}{4} = \frac{1}{5}(x - 2)$ (E) $y - \frac{\pi}{2} = (\text{Arctan } 2)(x - 2)$

---

16. Let $f(x) = e^{g(x)}h(x)$ and $h'(x) = -g'(x)h(x)$ for all real x. Which of the following must be true?

(A) f is a constant function.
(B) f is a linear nonconstant function.
(C) g is a constant function.
(D) g is a linear nonconstant function.
(E) None of the above

---

17. $1 - \sin^2\left(\text{Arccos } \frac{\pi}{12}\right) =$

(A) $\sqrt{\frac{1 - \cos\frac{\pi}{24}}{2}}$ (B) $\sqrt{\frac{1 - \cos\frac{\pi}{6}}{2}}$ (C) $\sqrt{\frac{1 + \cos\frac{\pi}{24}}{2}}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi^2}{144}$

---

18. If $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ for all $x \in (0, 1)$, then $f'(x) =$

(A) $\sin x$ (B) $\cos x$ (C) $\frac{1}{1 + x^2}$ (D) $\frac{-2x}{(1 + x^2)^2}$ (E) $\frac{2x}{(1 - 2x)^2}$

23. Let f be a real-valued function continuous on the closed interval $[0, 1]$ and differentiable on the open interval $(0, 1)$ with $f(0) = 1$ and $f(1) = 0$. Which of the following must be true?

I. There exists $x \in (0, 1)$ such that $f(x) = x$.
II. There exists $x \in (0, 1)$ such that $f'(x) = -1$.
III. $f(x) > 0$ for all $x \in [0, 1)$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

25. Let f be a real-valued function with domain $[0, 1]$. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all x and y in $[0, 1]$, which of the following must be true?

(A) f is discontinuous at each point of $(0, 1)$.
(B) f is not continuous on $(0, 1)$, but is discontinuous at only countably many points of $(0, 1)$.
(C) f is continuous on $(0, 1)$, but is differentiable at only countably many points of $(0, 1)$.
(D) f is continuous on $(0, 1)$, but may not be differentiable on $(0, 1)$.
(E) f is differentiable on $(0, 1)$.

29. $\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2^k} \right) =$

(A) 0 (B) 1 (C) 2 (D) 4 (E) $+\infty$

31. If $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } 0 \leq x \leq 1 \\ x - 1 & \text{for } 1 < x \leq 2, \end{cases}$

then $\int_0^2 f(x)\,dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2} + \frac{\pi}{4}$

(D) $\frac{1}{2} + \frac{\pi}{2}$

(E) undefined

34. $\frac{d}{dx} \int_0^{x^2} e^{-t^2}\, dt =$

(A) e^{-x^2} (B) $2e^{-x^2}$ (C) $2e^{-x^4}$ (D) $x^2e^{-x^2}$ (E) $2xe^{-x^4}$

38. $\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^2 - \left(\frac{3i}{n}\right)\right] =$

(A) $-\frac{1}{6}$ (B) 0 (C) 3 (D) $\frac{9}{2}$ (E) $\frac{31}{6}$

---

39. For a real number x, $\log(1 + \sin 2\pi x)$ is not a real number if and only if x is

(A) an integer

(B) nonpositive

(C) equal to $\frac{2n-1}{2}$ for some integer n

(D) equal to $\frac{4n-1}{4}$ for some integer n

(E) any real number

---

43. Let n be an integer greater than 1. Which of the following conditions guarantee that the equation

$x^n = \sum_{i=0}^{n-1} a_i x^i$ has at least one root in the interval (0, 1)?

I. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i < 1$

II. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i > 1$

III. $a_0 < 0$ and $\sum_{i=0}^{n-1} a_i > 1$

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III

---

44. If x is a real number and P is a polynomial function, then $\lim_{h \to 0} \dfrac{P(x+3h) + P(x-3h) - 2P(x)}{h^2} =$

(A) 0 (B) $6P'(x)$ (C) $3P''(x)$ (D) $9P''(x)$ (E) ∞