NOTES.

(A) [See p. 80].

One of the favourite objections, brought against the Science of Logic by its detractors, is that a Syllogism has no real validity as an argument, since it involves the Fallacy of *Petitio Principii* (i.e. "Begging the Question", the essence of which is that the whole Conclusion is involved in *one* of the Premisses).

This formidable objection is refuted, with beautiful clearness and simplicity, by these three Diagrams, which show us that, in each of the three Figures, the Conclusion is really involved in the *two* Premisses taken together, each contributing its share.

Thus, in Fig. I., the Premiss xm_0 empties the *Inner* Cell of the N.W. Quarter, while the Premiss ym_0 empties its *Outer* Cell. Hence it needs the *two* Premisses to empty the *whole* of the N.W. Quarter, and thus to prove the Conclusion xy_0.

Again, in Fig. II., the Premiss xm_0 empties the Inner Cell of the N.W. Quarter. The Premiss ym_1 merely tells us that the Inner Portion of the W. Half is *occupied*, so that we may place a 'I' in it, *somewhere*; but, if this were the *whole* of our information, we should not know in *which* Cell to place it, so that it would have to 'sit on the fence': it is only when we learn, from the other Premiss, that the *upper* of these two Cells is *empty*, that we feel authorised to place the 'I' in the *lower* Cell, and thus to prove the Conclusion $x'y_1$.

Lastly, in Fig. III., the information, that *m exists*, merely authorises us to place a 'I' *somewhere* in the Inner Square——but it has large choice of fences to sit upon! It needs the Premiss xm_0 to drive it out of the N. Half of that Square; and it needs the Premiss ym_0 to drive it out of the W. Half. Hence it needs the *two* Premisses to drive it into the Inner Portion of the S.E. Quarter, and thus to prove the Conclusion $x'y'_1$.

APPENDIX,

ADDRESSED TO TEACHERS.

§ 1.

Introductory.

There are several matters, too hard to discuss with *Learners*, which nevertheless need to be explained to any *Teachers*, into whose hands this book may fall, in order that they may thoroughly understand what my Symbolic Method *is*, and in what respects it differs from the many other Methods already published.

These matters are as follows:—

The "Existential Import" of Propositions.
The use of "is-not" (or "are-not") as a Copula.
The theory "two Negative Premisses prove nothing."
Euler's Method of Diagrams.
Venn's Method of Diagrams.
My Method of Diagrams.
The Solution of a Syllogism by various Methods.
My Method of treating Syllogisms and Sorites.
Some account of Parts II, III.

§ 2.

The "Existential Import" of Propositions.

The writers, and editors, of the Logical text-books which run in the ordinary grooves——to whom I shall hereafter refer by the (I hope inoffensive) title "The Logicians"——take, on this subject, what seems to me to be a more humble position than is at all necessary. They speak of the Copula of a Proposition "with bated breath", almost as if it were a living, conscious Entity, capable of declaring for itself what it chose to mean, and that we, poor human creatures, had nothing to do but to ascertain *what* was its sovereign will and pleasure, and submit to it.

In opposition to this view, I maintain that any writer of a book is fully authorised in attaching any meaning he likes to any word or phrase he intends to use. If I find an author saying, at the beginning of his book, "Let it be understood that by the word '*black*' I shall always mean '*white*', and that by the word '*white*' I shall always mean '*black*'," I meekly accept his ruling, however injudicious I may think it.

And so, with regard to the question whether a Proposition is or is not to be understood as asserting the existence of its Subject, I maintain that every writer may adopt his own rule, provided of course that it is consistent with itself and with the accepted facts of Logic.

Let us consider certain views that may *logically* be held, and thus settle which of them may *conveniently* be held; after which I shall hold myself free to declare which of them *I* intend to hold.

The *kinds* of Proposition, to be considered, are those that begin with "some", with "no", and with "all". These are usually called Propositions "in *I*", "in *E*", and "in *A*".

First, then, a Proposition in *I* may be understood as asserting, or else as *not* asserting, the existence of its Subject. (By "existence" I mean of course whatever kind of existence suits its nature. The two Propositions, "*dreams* exist" and "*drums* exist", denote two totally different kinds of "existence". A *dream* is an aggregate of ideas, and exists only in the *mind of a dreamer*: whereas a *drum* is an aggregate of wood and parchment, and exists in the *hands of a drummer*.)

First, let us suppose that *I* "asserts" (i.e. "asserts the existence of its Subject").

Here, of course, we must regard a Proposition in *A* as making the *same* assertion, since it necessarily *contains* a Proposition in *I*.

We now have *I* and *A* "asserting". Does this leave us free to make what supposition we choose as to *E*? My answer is "No. We are tied down to the supposition that *E* does *not* assert." This can be proved as follows:—

If possible, let *E* "assert". Then (taking x, y, and z to represent Attributes) we see that, if the Proposition "No xy are z" be true, some things exist with the Attributes x and y: i.e. "Some x are y."

Also we know that, if the Proposition "Some xy are z" be true, the same result follows.

But these two Propositions are Contradictories, so that one or other of them *must* be true. Hence this result is *always* true: i.e. the Proposition "Some x are y" is *always* true!

Quod est absurdum. (See Note (A), p. 195).

We see, then, that the supposition "*I* asserts" necessarily leads to "*A* asserts, but *E* does not". And this is the *first* of the various views that may conceivably be held.

Next, let us suppose that *I* does *not* "assert." And, along with this, let us take the supposition that *E does* "assert."

Hence the Proposition "No x are y" means "Some x exist, and none of them are y": i.e. "*all* of them are *not*-y," which is a Proposition in *A*. We also know, of course, that the Proposition "All x are not-y" proves "No x are y." Now two Propositions, each of which proves the other, are *equivalent*. Hence every Proposition in *A* is equivalent to one in *E*, and therefore "*asserts*".

Hence our *second* conceivable view is "*E* and *A* assert, but *I* does not."

This view does not seem to involve any necessary contradiction with itself or with the accepted facts of Logic. But, when we come to *test* it, as applied to the actual *facts* of life, we shall find I think, that it fits in with them so badly that its adoption would be, to say the least of it, singularly inconvenient for ordinary folk.

Let me record a little dialogue I have just held with my friend Jones, who is trying to form a new Club, to be regulated on strictly *Logical* principles.

Author. "Well, Jones! Have you got your new Club started yet?"

Jones (*rubbing his hands*). "You'll be glad to hear that some of the Members (mind, I only say '*some*') are millionaires! Rolling in gold, my boy!"

Author. "That sounds well. And how many Members have entered?"

Jones (*staring*). "None at all. We haven't got it started yet. What makes you think we have?"

Author. "Why, I thought you said that some of the Members——"

Jones (contemptuously). "You don't seem to be aware that we're working on strictly *Logical* principles. A *Particular* Proposition does *not* assert the existence of its Subject. I merely meant to say that we've made a Rule not to admit *any* Members till we have at least *three* Candidates whose incomes are over ten thousand a year!"

Author. "Oh, *that's* what you meant, is it? Let's hear some more of your Rules."

Jones. "Another is, that no one, who has been convicted seven times of forgery, is admissible."

Author. "And here, again, I suppose you don't mean to assert there *are* any such convicts in existence?"

Jones. "Why, that's exactly what I *do* mean to assert! Don't you know that a Universal Negative *asserts* the existence of its Subject? *Of course* we didn't make that Rule till we had satisfied ourselves that there are several such convicts now living."

The Reader can now decide for himself how far this *second* conceivable view would fit in with the facts of life. He will, I think, agree with me that Jones' view, of the 'Existential Import' of Propositions, would lead to some inconvenience.

Thirdly, let us suppose that neither I nor E "asserts".

Now the supposition that the two Propositions, "Some x are y" and "No x are not-y", do *not* "assert", necessarily involves the supposition that "All x are y" does *not* "assert", since it would be absurd to suppose that they assert, when combined, more than they do when taken separately.

Hence the *third* (and last) of the conceivable views is that neither I, nor E, nor A, "asserts".

The advocates of this third view would interpret the Proposition "Some x are y" to mean "If there *were* any x in existence, some of them *would* be y"; and so with E and A.

It admits of proof that this view, as regards A, conflicts with the accepted facts of Logic.

Let us take the Syllogism *Darapti*, which is universally accepted as valid. Its form is

"All m are x;
All m are y.
∴ Some y are x".

This they would interpret as follows:—

"If there were any m in existence, all of them would be x;
If there were any m in existence, all of them would be y.
∴ If there were any y in existence, some of them would be x".

That this Conclusion does *not* follow has been so briefly and clearly explained by Mr. Keynes (in his "Formal Logic", dated 1894, pp. 356, 357), that I prefer to quote his words:—

"*Let no proposition imply the existence either of its subject or of its predicate.*

"Take, as an example, a syllogism in *Darapti*:—

'*All M is P,*
All M is S,
∴ *Some S is P.*'

"Taking S, M, P, as the minor, middle, and major terms respectively, the conclusion will imply that, if there is any S, there is some P. Will the premisses also imply this? If so, then the syllogism is valid; but not otherwise.

"The conclusion implies that if S exists P exists; but, consistently with the premisses, S may be existent while M and P are both non-existent. An implication is, therefore, contained in the conclusion, which is not justified by the premisses."

This seems to *me* entirely clear and convincing. Still, "to make sicker", I may as well throw the above (*soi-disant*) Syllogism into a concrete form, which will be within the grasp of even a *non*-logical Reader.

Let us suppose that a Boys' School has been set up, with the following system of Rules:—

"All boys in the First (the highest) Class are to do French, Greek, and Latin. All in the Second Class are to do Greek only. All in the Third Class are to do Latin only."

Suppose also that there *are* boys in the Third Class, and in the Second; but that no boy has yet risen into the First.

It is evident that there are no boys in the School doing French: still we know, by the Rules, what would happen if there *were* any.

We are authorised, then, by the *Data*, to assert the following two Propositions:—

"If there were any boys doing French, all of them would be doing Greek;
If there were any boys doing French, all of them would be doing Latin."

And the Conclusion, according to "The Logicians" would be

"If there were any boys doing Latin, some of them would be doing Greek."

Here, then, we have two *true* Premisses and a *false* Conclusion (since we know that there *are* boys doing Latin, and that *none* of them are doing Greek). Hence the argument is *invalid*.

Similarly it may be shown that this "non-existential" interpretation destroys the validity of *Disamis*, *Datisi*, *Felapton*, and *Fresison*.

Some of "The Logicians" will, no doubt, be ready to reply "But we are not *Aldrichians!* Why should *we* be responsible for the validity of the Syllogisms of so antiquated an author as Aldrich?"

Very good. Then, for the *special* benefit of these "friends" of mine (with what ominous emphasis that name is sometimes used! "I must have a private interview with *you*, my young *friend*," says the bland Dr. Birch, "in my library, at 9 a.m. tomorrow. And you will please to be *punctual!*"), for their *special* benefit, I say, I will produce *another* charge against this "non-existential" interpretation.

It actually invalidates the ordinary Process of "Conversion", as applied to Propositions in '*I*'.

Every logician, Aldrichian or otherwise, accepts it as an established fact that "Some x are y" may be legitimately converted into "Some y are x."

But is it equally clear that the Proposition "If there *were* any x, some of them *would* be y" may be legitimately converted into "If there *were* any y, some of them would be x"? I trow not.

The example I have already used——of a Boys' School with a non-existent First Class——will serve admirably to illustrate this new flaw in the theory of "The Logicians."

Let us suppose that there is yet *another* Rule in this School, viz. "In each Class, at the end of the Term, the head boy and the second boy shall receive prizes."

This Rule entirely authorises us to assert (in the sense in which "The Logicians" would use the words) "Some boys in the First Class will receive prizes", for this simply means (according to them) "If there *were* any boys in the First Class, some of them *would* receive prizes."

Now the Converse of this Proposition is, of course, "Some boys, who will receive prizes, are in the First Class", which means (according to "The Logicians") "If there *were* any boys about to receive prizes, some of them *would* be in the First Class" (which Class we know to be *empty*).

Of this Pair of Converse Propositions, the first is undoubtedly *true:* the second, *as* undoubtedly, *false*.

It is always sad to see a batsman knock down his own wicket: one pities him, as a man and a brother, but, as a *cricketer*, one can but pronounce him "Out!"

We see, then, that, among all the conceivable views we have here considered, there are only *two* which can *logically* be held, viz.

I and *A* "assert", but *E* does not.
E and *A* "assert", but *I* does not.

The *second* of these I have shown to involve great practical inconvenience.

The *first* is the one adopted in this book. (See p. 19.)

Some further remarks on this subject will be found in Note (B), at p. 196.

§ 3.

The use of "is-not" (or "are-not") as a Copula.

Is it better to say "John *is-not* in-the-house" or "John *is* not-in-the-house"? "Some of my acquaintances *are-not* men-I-should-like-to-be-seen-with" or "Some of my acquaintances *are* men-I-should-*not*-like-to-be-seen-with"? That is the sort of question we have now to discuss.

This is no question of Logical Right and Wrong: it is merely a matter of *taste*, since the two forms mean exactly the same thing. And here, again, "The Logicians" seem to me to take much too humble a position. When they are putting the final touches to the grouping of their Proposition, just before the curtain goes up, and when the Copula—always a rather fussy 'heavy father', asks them "Am *I* to have the 'not', or will you tack it on to the Predicate?" they are much too ready to answer, like the subtle cab-driver, "Leave it to *you*, Sir!" The result seems to be, that the grasping Copula constantly gets a "not" that had better have been merged in the Predicate, and that Propositions are differentiated which had better have been recognised as precisely similar. Surely it is simpler to treat "Some men are Jews" and "Some men are Gentiles" as being, both of them, *affirmative* Propositions, instead of translating the latter into "Some men are-not Jews", and regarding it as a *negative* Proposition?

The fact is, "The Logicians" have somehow acquired a perfectly *morbid* dread of negative Attributes, which makes them shut their eyes, like frightened children, when they come across such terrible Propositions as "All not-x are y"; and thus they exclude from their system many very useful forms of Syllogisms.

Under the influence of this unreasoning terror, they plead that, in Dichotomy by Contradiction, the *negative* part is too large to deal with, so that it is better to regard each Thing as either included in, or excluded from, the *positive* part. I see no force in this plea: and the facts often go the other way. As a personal question, dear Reader, if *you* were to group your acquaintances into the two Classes, men that you *would* like to be seen with, and men that you would *not* like to be seen with, do you think the latter group would be so *very* much the larger of the two?

For the purposes of Symbolic Logic, it is so *much* the most convenient plan to regard the two sub-divisions, produced by Dichotomy, on the *same* footing, and to say, of any Thing, either that it "is" in the one, or that it "is" in the other, that I do not think any Reader of this book is likely to demur to my adopting that course.

§ 4.

The theory that "two Negative Premisses prove nothing".

This I consider to be *another* craze of "The Logicians", fully as morbid as their dread of a negative Attribute.

It is, perhaps, best refuted by the method of *Instantia Contraria*.

Take the following Pairs of Premisses:—

"None of my boys are conceited;
None of my girls are greedy".

"None of my boys are clever;
None but a clever boy could solve this problem".

"None of my boys are learned;
Some of my boys are not choristers".

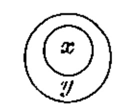
(This last Proposition is, in *my* system, an *affirmative* one, since *I* should read it "are not-choristers"; but, in dealing with "The Logicians," I may fairly treat it as a *negative* one, since *they* would read it "are-not choristers".)

If you, dear Reader, declare, after full consideration of these Pairs of Premisses, that you cannot deduce a Conclusion from *any* of them—why, all I can say is that, like the Duke in *Patience*, you "will have to be contented with our heart-felt sympathy"! [See Note (C), p. 196.]

§ 5.

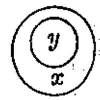
Euler's Method of Diagrams.

Diagrams seem to have been used, at first, to represent *Propositions* only. In Euler's well-known Circles, each was supposed to contain a Class, and the Diagram consisted of two Circles, which exhibited the relations, as to inclusion and exclusion, existing between the two Classes.

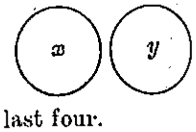


Thus, the Diagram, here given, exhibits the two Classes, whose respective Attributes are x and y, as so related to each other that the following Propositions are all simultaneously true:—"All x are y", "No x are not-y", "Some x are y", "Some y are not-x", "Some not-y are not-x", and, of course, the Converses of the last four.

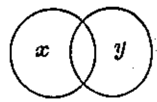
Similarly, with this Diagram, the following Propositions are true:—"All y are x", "No y are not-x", "Some y are x", "Some x are not-y", "Some not-x are not-y", and, of course, the Converses of the last four.



Similarly, with this Diagram, the following are true:—"All x are not-y", "All y are not-x", "No x are y", "Some x are not-y", "Some y are not-x", "Some not-x are not-y", and the Converses of the last four.



Similarly, with this Diagram, the following are true:—"Some x are y", "Some x are not-y", "Some not-x are y", "Some not-x are not-y", and, of course, their four Converses.



Note that *all* Euler's Diagrams assert "Some not-x are not-y." Apparently it never occurred to him that it might *sometimes* fail to be true!

Now, to represent "All x are y", the *first* of these Diagrams would suffice. Similarly, to represent "No x are y", the *third* would suffice. But to represent any *Particular* Proposition, at least *three* Diagrams would be needed (in order to include all the possible cases), and, for "Some not-x are not-y", all the *four*.

§ 6.

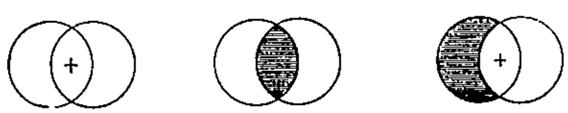
Venn's Method of Diagrams.

Let us represent "not-x" by "x'".

Mr. Venn's Method of Diagrams is a great advance on the above Method.

He uses the last of the above Diagrams to represent *any* desired relation between x and y, by simply shading a Compartment known to be *empty*, and placing a + in one known to be *occupied*.

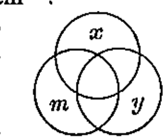
Thus, he would represent the three Propositions "Some x ar y", "No x are y", and "All x are y", as follows:—



It will be seen that, of the *four* Classes, whose peculiar Sets of Attributes are xy, xy', $x'y$, and $x'y'$, only *three* are here provided with closed Compartments, while the *fourth* is allowed the rest of the Infinite Plane to range about in!

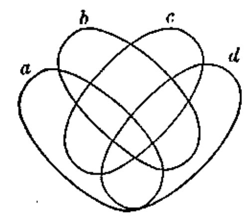
This arrangement would involve us in very serious trouble, if we ever attempted to represent "No x' are y'." Mr. Venn *once* (at p. 281) encounters this awful task; but evades it, in a quite masterly fashion, by the simple foot-note "We have not troubled to shade the outside of this diagram"!

To represent *two* Propositions (containing a common Term) *together*, a *three*-letter Diagram is needed. This is the one used by Mr. Venn.

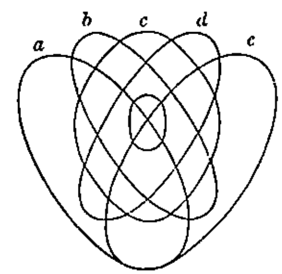


Here, again, we have only *seven* closed Compartments, to accommodate the *eight* Classes whose peculiar Sets of Attributes are xym, xym', &c.

"With four terms in request," Mr. Venn says, "the most simple and symmetrical diagram seems to me that produced by making four ellipses intersect one another in the desired manner". This, however, provides only *fifteen* closed compartments.



For *five* letters, "the simplest diagram I can suggest," Mr. Venn says, "is one like this (the small ellipse in the centre is to be regarded as a portion of the *outside* of c; i.e. its four component portions are inside b and d but are no part of c). It must be admitted that such a diagram is not quite so simple to draw as one might wish it to be; but then consider what the alternative is if one undertakes to deal with five terms and all their combinations—nothing short of the disagreeable task of writing out, or in some way putting before us, all the 32 combinations involved."



This Diagram gives us 31 closed compartments.

For *six* letters, Mr. Venn suggests that we might use *two* Diagrams, like the above, one for the *f*-part, and the other for the not-*f*-part, of all the other combinations. "This", he says, "would give the desired 64 subdivisions." This, however, would only give 62 closed Compartments, and *one* infinite area, which the two Classes, $a'b'c'd'e'f$ and $a'b'c'd'e'f'$, would have to share between them.

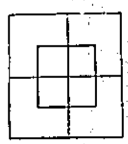
Beyond *six* letters Mr. Venn does not go.

§ 7.

My Method of Diagrams.

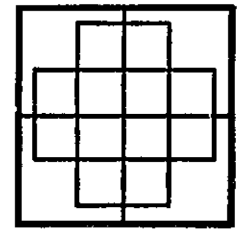
My Method of Diagrams *resembles* Mr. Venn's, in having separate Compartments assigned to the various Classes, and in marking these Compartments as *occupied* or as *empty*; but it *differs* from his Method, in assigning a *closed* area to the *Universe of Discourse*, so that the Class which, under Mr. Venn's liberal sway, has been ranging at will through Infinite Space, is suddenly dismayed to find itself "cabin'd, cribb'd, confined", in a limited Cell like any other Class! Also I use *rectilinear*, instead of *curvilinear*, Figures; and I mark an *occupied* Cell with a 'I' (meaning that there is at least *one* Thing in it), and an *empty* Cell with a 'O' (meaning that there is *no* Thing in it).

For *two* letters, I use this Diagram, in which the North Half is assigned to 'x', the South to 'not-x' (or 'x''), the West to y, and the East to y'. Thus the N.W. Cell contains the xy-Class, the N.E. Cell the xy'-Class, and so on.

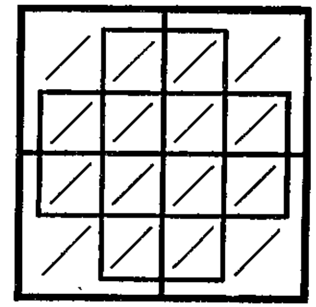


For *three* letters, I subdivide these four Cells, by drawing an *Inner* Square, which I assign to m, the *Outer* Border being assigned to m'. I thus get the *eight* Cells that are needed to accommodate the eight Classes, whose peculiar Sets of Attributes are xym, xym', &c.

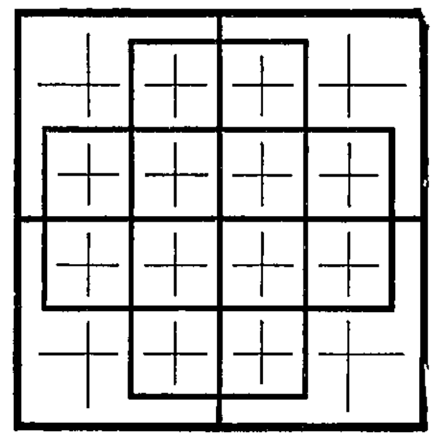
This last Diagram is the most complex that I use in the *Elementary* Part of my 'Symbolic Logic.' But I may as well take this opportunity of describing the more complex ones which will appear in Part II.



For *four* letters (which I call a, b, c, d) I use this Diagram; assigning the North Half to a (and of course the *rest* of the Diagram to a'), the West Half to b, the Horizontal Oblong to c, and the Upright Oblong to d. We have now got 16 Cells.

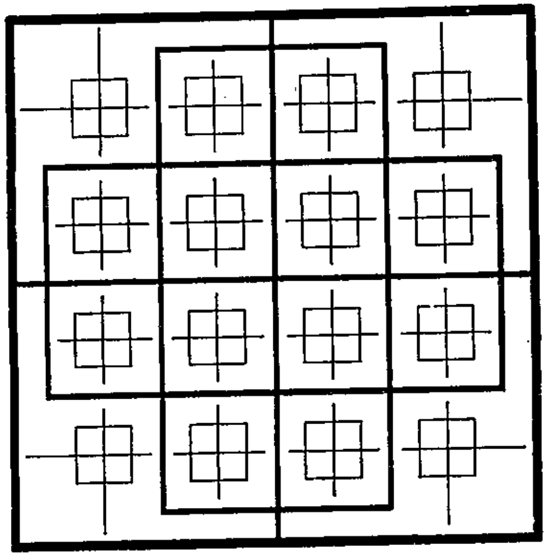


For *five* letters (adding e) I subdivide the 16 Cells of the previous Diagram by *oblique* partitions, assigning all the *upper* portions to e, and all the *lower* portions to e'. Here, I admit, we lose the advantage of having the e-Class all *together*, "in a ring-fence", like the other 4 Classes. Still, it is very easy to find; and the operation, of erasing it, is nearly as easy as that of erasing any other Class. We have now got 32 Cells.



For *six* letters (adding h, as I avoid *tailed* letters) I substitute upright crosses for the oblique partitions, assigning the 4 portions, into which each of the 16 Cells is thus divided, to the four Classes eh, eh', $e'h$, $e'h'$. We have now got 64 Cells.

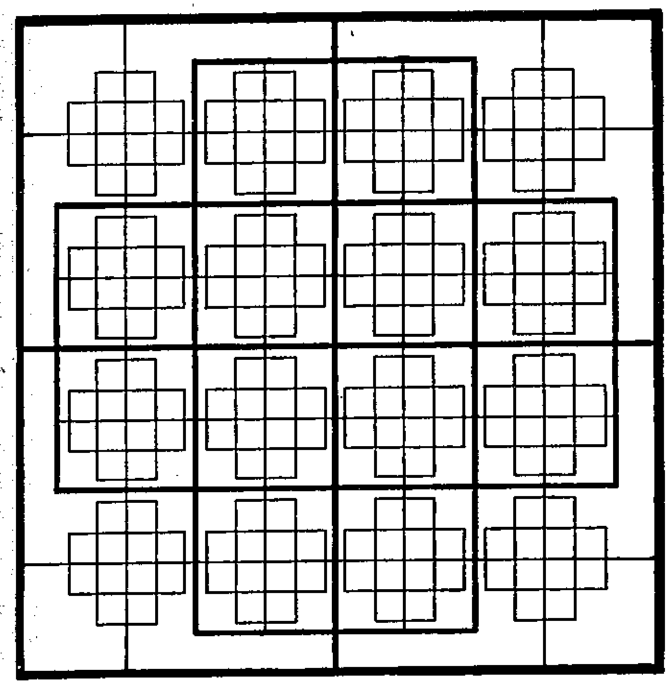
For *seven* letters (adding *k*) I add, to each upright cross, a little inner square. All these 16 little squares are assigned to



the *k*-Class, and all outside them to the *k'*-Class; so that the 8 little Cells (into which each of the 16 Cells is divided) are respectively assigned to the 8 Classes *ehk*, *ehk'*, &c. We have now got 128 Cells.

For *eight* letters (adding *l*) I place, in each of the 16 Cells, a *lattice*, which is a reduced copy of the whole Diagram; and, just as the 16 large Cells of the whole Diagram are assigned to the 16 Classes *abcd*, *abcd'*, &c., so the 16 little Cells of each lattice are assigned to the 16 Classes *ehkl*, *ehkl'*, &c. Thus, the lattice in the N.W. corner serves to accommodate the 16 Classes *abc'd'ehkl*, *abc'd'eh'kl'*, &c. This Octoliteral Diagram (see next page) contains 256 Cells.

For *nine* letters, I place 2 Octoliteral Diagrams side by side, assigning one of them to *m*, and the other to *m'*. We have now got 512 Cells.



Finally, for *ten* letters, I arrange 4 Octoliteral Diagrams, like the above, in a square, assigning them to the 4 Classes *mn*, *mn'*, *m'n*, *m'n'*. We have now got 1024 Cells.

§ 8.

Solution of a Syllogism by various Methods.

The best way, I think, to exhibit the differences between these various Methods of solving Syllogisms, will be to take a concrete example, and solve it by each Method in turn. Let us take, as our example, No. 29 (see p. 102).

"No philosophers are conceited;
Some conceited persons are not gamblers.
∴ Some persons, who are not gamblers, are not philosophers."

(1) *Solution by ordinary Method.*

These Premisses, as they stand, will give no Conclusion, as they are both negative.

If by 'Permutation' or 'Obversion', we write the Minor Premiss thus,

'Some conceited persons are not-gamblers,'

we can get a Conclusion in *Fresison*, viz.

"No philosophers are conceited;
Some conceited persons are not-gamblers.
∴ Some not-gamblers are not philosophers"

This can be proved by reduction to *Ferio*, thus:—

"No conceited persons are philosophers:
Some not-gamblers are conceited.
∴ Some not-gamblers are not philosophers".

The validity of *Ferio* follows directly from the Axiom '*De Omni et Nullo*'.

(2) *Symbolic Representation.*

Before proceeding to discuss other Methods of Solution, it is necessary to translate our Syllogism into an *abstract* form.

Let us take "persons" as our 'Universe of Discourse'; and let x = "philosophers", m = "conceited", and y = "gamblers."

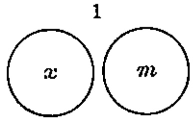
Then the Syllogism may be written thus:—

"No x are m;
Some m are y'.
∴ Some y' are x'."

(3) *Solution by Euler's Method of Diagrams.*

The Major Premiss requires only *one* Diagram, viz.

1



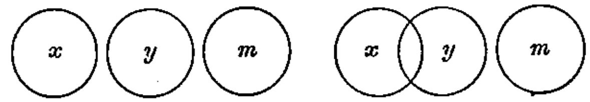
The Minor requires *three*, viz.

2 3 4

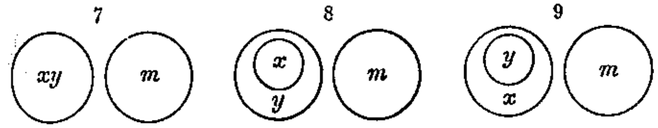
The combination of Major and Minor, in every possible way, requires *nine*, viz.

Figs. 1 and 2 give

5 6

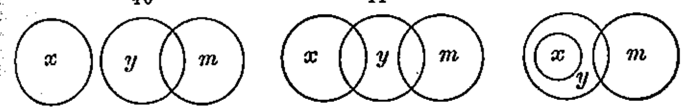


7 8 9



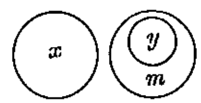
Figs. 1 and 3 give

10 11 12



Figs. 1 and 4 give

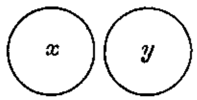
13



From this group (Figs. 5 to 13) we have, by disregarding m, to find the relation of x and y. On examination we find that Figs. 5, 10, 13 express the relation of entire mutual exclusion; that Figs. 6, 11 express partial inclusion and partial exclusion; that Fig. 7 expresses coincidence; that Figs. 8, 12 express entire inclusion of x in y; and that Fig. 9 expresses entire inclusion of y in x.

We thus get five Biliteral Diagrams for x and y, viz.

14



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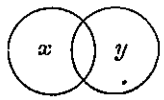

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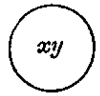

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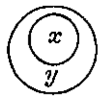

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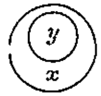
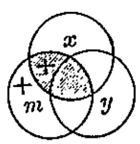


where the only Proposition, represented by them all, is "Some not-y are not-x," i.e. "Some persons, who are not gamblers, are not philosophers"——a result which Euler would hardly have regarded as a *valuable* one, since he seems to have assumed that a Proposition of this form is *always* true!

(4) *Solution by Venn's Method of Diagrams.*

The following Solution has been kindly supplied to me by Mr. Venn himself.

"The Minor Premiss declares that some of the constituents in my' must be saved: mark these constituents with a cross.



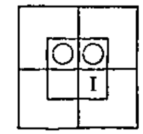
The Major declares that all xm must be destroyed; erase it.

Then, as some my' is to be saved, it must clearly be $my'x'$. That is, there must exist $my'x'$; or, eliminating m, $y'x'$. In common phraseology,

Some y' are x'', or, 'Some not-gamblers are not-philosophers.'"

(5) *Solution by my Method of Diagrams.*

The first Premiss asserts that no xm exist: so we mark the xm-Compartment as empty, by placing a 'O' in each of its Cells.



The second asserts that some my' exist: so we mark the my'-Compartment as occupied, by placing a 'I' in its only available Cell.

The only information, that this gives us as to x and y, is that the $x'y'$-Compartment is *occupied*, i.e. that some $x'y'$ exist.

Hence "Some x' are y'": i.e. "Some persons, who are not philosophers, are not gamblers".

(6) *Solution by my Method of Subscripts.*

$$xm_0 \dagger my'_1 \P x'y'_1$$

i.e. "Some persons, who are not philosophers, are not gamblers."

§ 9.

My Method of treating Syllogisms and Sorites.

Of all the strange things, that are to be met with in the ordinary text-books of Formal Logic, perhaps the strangest is the violent contrast one finds to exist between their ways of dealing with these two subjects. While they have elaborately discussed no less than *nineteen* different forms of *Syllogisms*——each with its own special and exasperating Rules, while the whole constitute an almost useless machine, for practical purposes, many of the Conclusions being incomplete, and many quite legitimate forms being ignored——they have limited *Sorites* to *two* forms only, of childish simplicity; and these they have dignified with special *names*, apparently under the impression that no other possible forms existed!

As to *Syllogisms*, I find that their nineteen forms, with about a score of others which they have ignored, can all be arranged under *three* forms, each with a very simple Rule of its own; and the only question the Reader has to settle, in working any one of the 101 Examples given at p. 101 of this book, is "Does it belong to Fig. I, II., or III.?"

As to *Sorites*, the only two forms, recognised by the text-books, are the *Aristotelian*, whose Premisses are a series of Propositions in A, so arranged that the Predicate of each is the Subject of the next, and the *Goclenian*, whose Premisses are the very same series, written backwards. Goclenius, it seems, was the first who noticed the startling fact that it does not affect the force of a Syllogism to invert the order of its Premisses, and who applied this discovery to a Sorites. If we assume (as surely we may?) that he is the *same* man as that transcendent genius who first noticed that 4 times 5 is the same thing as 5 times 4, we may apply to him what somebody (Edmund Yates, I think it was) has said of Tupper, viz., "here is a man who, beyond all others of his generation, has been favoured with Glimpses of the Obvious!"

These puerile——not to say infantine——forms of a Sorites I have, in this book, ignored from the very first, and have not only admitted freely Propositions in E, but have purposely stated the Premisses in random order, leaving to the Reader the useful task of arranging them, for himself, in an order which can be worked as a series of regular Syllogisms. In doing this, he can begin with *any one* of them he likes.

I have tabulated, for curiosity, the various orders in which the Premisses of the Aristotelian Sorites

1. All a are b;
2. All b are c;
3. All c are d;
4. All d are e;
5. All e are h.

∴ All a are h.

may be syllogistically arranged, and I find there are no less than *sixteen* such orders, viz., 12345, 21345, 23145, 23415, 23451, 32145, 32415, 32451, 34215, 34251, 34521, 43215, 43251, 43521, 45321, 54321. Of these the *first* and the *last* have been dignified with names; but the other *fourteen*——first enumerated by an obscure Writer on Logic, towards the end of the Nineteenth Century——remain without a name!

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§ 10.

Some account of Parts II, III.

In Part II. will be found some of the matters mentioned in this Appendix, viz., the "Existential Import" of Propositions, the use of a *negative* Copula, and the theory that "two negative Premisses prove nothing." I shall also extend the range of Syllogisms, by introducing Propositions containing *alternatives* (such as "Not-all x are y"), Propositions containing 3 or more Terms (such as "All ab are c", which, taken along with "Some bc' are d", would prove "Some d are a'"), &c. I shall also discuss Sorites containing Entities, and the *very* puzzling subjects of Hypotheticals and Dilemmas. I hope, in the course of Part II., to go over all the ground usually traversed in the text-books used in our Schools and Universities, and to enable my Readers to solve Problems of the same kind as, and far harder than, those that are at present set in their Examinations.

In Part III. I hope to deal with many curious and out-of-the-way subjects, some of which are not even alluded to in any of the treatises I have met with. In this Part will be found such matters as the Analysis of Propositions into their Elements (let the Reader, who has never gone into this branch of the subject, try to make out for himself what *additional* Proposition would be needed to convert "Some a are b" into "Some a are bc"), the treatment of Numerical and Geometrical Problems, the construction of Problems, and the solution of Syllogisms and Sorites containing Propositions more complex than any that I have used in Part II.

I will conclude with eight Problems, as a taste of what is coming in Part II. I shall be very glad to receive, from any Reader, who thinks he has solved any one of them (more especially if he has done so *without* using any Method of Symbols), what he conceives to be its complete Conclusion.

It may be well to explain what I mean by the *complete* Conclusion of a Syllogism or a Sorites. I distinguish their Terms as being of two kinds——those which *can* be eliminated (e.g. the Middle Term of a Syllogism), which I call the "Eliminands," and those which *cannot*, which I call the "Retinends"; and I do not call the Conclusion *complete*, unless it states *all* the relations, among the Retinends only, which can be deduced from the Premisses.

1.

All the boys, in a certain School, sit together in one large room every evening. They are of no less than *five* nationalities —English, Scotch, Welsh, Irish, and German. One of the Monitors (who is a great reader of Wilkie Collins' novels) is very observant, and takes MS. notes of almost everything that happens, with the view of being a good sensational witness, in case any conspiracy to commit a murder should be on foot. The following are some of his notes:—

(1) Whenever some of the English boys are singing "Rule Britannia", and some not, some of the Monitors are wide-awake;
(2) Whenever some of the Scotch are dancing reels, and some of the Irish fighting, some of the Welsh are eating toasted cheese;
(3) Whenever all the Germans are playing chess, some of the Eleven are *not* oiling their bats;
(4) Whenever some of the Monitors are asleep, and some not, some of the Irish are fighting;
(5) Whenever some of the Germans are playing chess, and none of the Scotch are dancing reels, some of the Welsh are *not* eating toasted cheese;
(6) Whenever some of the Scotch are *not* dancing reels, and some of the Irish *not* fighting, some of the Germans are playing chess;
(7) Whenever some of the Monitors are awake, and some of the Welsh are eating toasted cheese, none of the Scotch are dancing reels;
(8) Whenever some of the Germans are *not* playing chess, and some of the Welsh are *not* eating toasted cheese, none of the Irish are fighting;

(9) Whenever all the English are singing "Rule Britannia," and some of the Scotch are *not* dancing reels, none of the Germans are playing chess;
(10) Whenever some of the English are singing "Rule Britannia", and some of the Monitors are asleep, some of the Irish are *not* fighting;
(11) Whenever some of the Monitors are awake, and some of the Eleven are *not* oiling their bats, some of the Scotch are dancing reels;
(12) Whenever some of the English are singing "Rule Britannia", and some of the Scotch are *not* dancing reels, * * * *

Here the MS. breaks off suddenly. The Problem is to complete the sentence, if possible.

[N.B. In solving this Problem, it is necessary to remember that the Proposition "All x are y" is a *Double* Proposition, and is equivalent to "Some x are y, and none are y'." See p. 17.]

2.

(1) A logician, who eats pork-chops for supper, will probably lose money;
(2) A gambler, whose appetite is not ravenous, will probably lose money;
(3) A man who is depressed, having lost money and being likely to lose more, always rises at 5 a.m.;
(4) A man, who neither gambles nor eats pork-chops for supper, is sure to have a ravenous appetite;
(5) A lively man, who goes to bed before 4 a.m., had better take to cab-driving;
(6) A man with a ravenous appetite, who has not lost money and does not rise at 5 a.m., always eats pork-chops for supper;
(7) A logician, who is in danger of losing money, had better take to cab-driving;
(8) An earnest gambler, who is depressed though he has not lost money, is in no danger of losing any;
(9) A man, who does not gamble, and whose appetite is not ravenous, is always lively;