Special Relativity: Lorentz Transformations

Consider two observers S and S' moving relative to one another with velocity v with along the x-axis. Now suppose there is an event E that is measured by S to have the coordinates (x, t), whereas S' measures the event to have coordinates (x', t').

The Special Theory of Relativity states that the S and S' measurements are related by a **Lorentz Transformation**, which is given by

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{1}$$

$$x' = \gamma\left(x - vt\right) \tag{2}$$

$$y' = y$$

$$z' = z$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the so-called **Lorentz Factor**. Note that $\gamma \geq 1$, and that the closer that v gets to c the **larger** it becomes.

Example (Time Dilation): Suppose that S is holding a clock at its origin and measures the time between two ticks to be t. Now, since according to S the clock has not moved during that time, the spatial coordinate of the clock remains $x = 0$. **What does S' see**? Well, plugging our measurements into (1) we have that

$$t' = \gamma t$$

That is, S' will say that the time it takes for clock to make one tick is longer than t (since $\gamma \geq 1$). That is, S' **claims that the clock S is holding is running slow**. This result is known as time dilation.