Common Core Math Unit Plan

<table>
<thead>
<tr>
<th>Unit Name:</th>
<th>Exponential and Logarithmic Functions</th>
<th>Course:</th>
<th>Math III</th>
<th>Time Frame:</th>
<th>20-21 Class Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-Topics</td>
<td>Function features, function writing, function transformation, geometric series, inverse functions, log rules</td>
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<tr>
<td>Big Idea</td>
<td>An inverse function undoes the actions of an original function. Logarithmic functions are a new class of functions to understand as the inverse of exponential functions.</td>
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**Storyboard/Unit Flow**

- **Pre-Unit Hook Lesson:** Bacteria
- **Concept #1:** Geometric Sequences and Series
- **Practice:** Terms and Rules of Sequences, Sums of Series
- **Concept 2:** Inverse Functions
- **Concept 3:** Exponential and Log Graphs
- **Practice:** Finding Inverse Functions, Basic Exponent and Log Computation
- **Getting General:** Key Features of Graphs including Rational Functions
- **Problem Solving:** Understanding Visual Proofs
- **Concept 4:** Exponential & Logarithmic Rules
- **Concept 5:** Solving Exponential Equations Using Logs
- **Practice:** Exponent & Log Rules, Solving Equations
- **Modeling/Problem Solving**
- **Unit Review**
- **Summative Assessment**
Essential Questions
What is an exponential function? What are its characteristics compared to other functions?
What is a geometric sequence? series?
What are inverse functions?
What is a logarithmic function? What are its characteristics compared to other functions?

Key Vocabulary
exponential
logarithmic
sequence
series
inverse

Prior Knowledge
Linear Functions
Quadratic Functions
Domain
Range
Intercepts
Intervals
End behavior

Common Core Math Standards Taught and Assessed (M indicates Major standard, A/S indicates Additional or Supporting standard)

Seeing Structure in Expressions A-SSE
Interpret the structure of expressions. [Polynomial and rational]
1. Interpret expressions that represent a quantity in terms of its context. «
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it.
Write expressions in equivalent forms to solve problems.
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Functions- Interpreting Functions (F-IF)
Interpret functions that arise in applications in terms of the context. [Include rational, square root and cube root; emphasize selection of appropriate models.]
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Building Functions F-BF
Build a function that models a relationship between two quantities. [Include all types of functions studied.]
1. Write a function that describes a relationship between two quantities. 
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. [Include simple, radical, rational, and exponential functions; emphasize common effect of each transformation across function types.]

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.

Linear, Quadratic, and Exponential Models (F-LE)

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology. «Logarithms as solutions for exponentials»

4.1. Prove simple laws of logarithms. CA

4.2 Use the definition of logarithms to translate between logarithms in any base. CA

4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA
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<tr>
<th>Time</th>
<th>Lesson Title</th>
<th>Knowledge Type, Claims, &amp; Math Practices</th>
<th>Materials</th>
<th>Lesson Overview</th>
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</thead>
<tbody>
<tr>
<td>1 Period</td>
<td>Pre-Unit: Bacteria</td>
<td>KT: P, C</td>
<td>Calculators</td>
<td>Students will investigate the growth rate of bacteria and write an equation to model scenarios where the initial amount and the growth multiplier change to summarize and describe the effect of each on the overall growth.</td>
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<tr>
<td>1-2 Periods</td>
<td>Geometric Sequences</td>
<td>KT: P, C Claim: MP: 1, 8</td>
<td>None</td>
<td>Students develop a definition of a geometric sequence by examining examples and non-examples. They then describe sequences recursively and explicitly.</td>
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<tr>
<td>1 Period</td>
<td>Geometric Series</td>
<td>KT: P, C Claim: MP: 1, 3, 8</td>
<td>None</td>
<td>Students calculate the value of several series and then use the formula for the sum of a finite geometric series. They participate in the derivation/proof of the formula.</td>
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<tr>
<td>2 Periods</td>
<td>Practice: Teachers Find or Create</td>
<td>KT: P Claim: MP: 6, 8</td>
<td>None</td>
<td>Students develop fluency with geometric sequences and series.</td>
</tr>
<tr>
<td>1 Period</td>
<td>Inverse Functions</td>
<td>KT: C Claim: 1 MP: 2, 3</td>
<td>None</td>
<td>Students graph pairs of inverse functions. They use patty paper to identify the transformation that creates an inverse. Multiple representations (tables, graphs, words, equations) deepen the understanding of inverse. The need for a function (and its name) that is the inverse of an exponential function ends the lesson.</td>
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<tr>
<td>2 Periods</td>
<td>Exponential and Logarithmic Graphs</td>
<td>KT: C</td>
<td>None</td>
<td>Students analyze the features of pairs of exponential and logarithmic graphs. The detailed comparison and contrast of these features further develops the concept of inverse and the relationship between exponent and log functions.</td>
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<tr>
<td>2 Periods</td>
<td>Practice: Teachers Find or Create</td>
<td>KT: P</td>
<td>None</td>
<td>Students develop fluency with inverse functions and basic exponent and log computations.</td>
</tr>
<tr>
<td>2 Periods</td>
<td>Identifying Key Features of Graphs</td>
<td>KT: CP</td>
<td>Patty Paper (optional)</td>
<td>Students continue to identify key features of graphs. This selection of graphs now includes a variety of exponential and log graphs.</td>
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<tr>
<td>1 Period</td>
<td>Problem Solving: Teachers find or create.</td>
<td>KT: RK</td>
<td>None</td>
<td>Problem Solving Suggestion: The sums of many arithmetic and geometric series can be shown visually. These could be the basis of some good problems.</td>
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<tr>
<td>1 Period</td>
<td>Comparing Exponential and Logarithmic Rules</td>
<td>KT: C, RK</td>
<td>None</td>
<td>In teams, students examine specific examples of log computations to discover and describe two basic log rules. They also compare these to related exponential rules.</td>
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<tr>
<td>1 Period</td>
<td>Solving Exponential Equations Using Logs</td>
<td>KT: P, C</td>
<td>None</td>
<td>Students revisit the idea of using inverses to solve equations with various operations, leading to the use of logs to solve exponential equations.</td>
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<tr>
<td>2 Periods</td>
<td>Practice: Teachers Find or Create</td>
<td>KT: P</td>
<td>None</td>
<td>Students develop fluency in solving exponential equations.</td>
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<tr>
<td>1 Period</td>
<td>Rats!</td>
<td>KT: C, RK</td>
<td>Dice</td>
<td>Students model a population of rats by rolling dice and adding and removing dice. Students write a</td>
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<td>MP: 1, 2, 6</td>
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<td>function to model the situation and use logarithms to answer questions about the model.</td>
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<tr>
<td>1 Period</td>
<td>Unit Review:</td>
<td>ALL</td>
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<tr>
<td></td>
<td>Teachers Find or Create</td>
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<tr>
<td>1 Period</td>
<td>Summative Assessment</td>
<td>ALL</td>
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Legend:
- KT – Knowledge Type
- RK – Relational Knowledge
- M – Memorization
- P – Procedure
- C - Concept
- MP – Math Practice

SBAC Claims:
- Claim 1 – Concepts/Procedure
- Claim 2 – Problem Solving
- Claim 3 – Communicating & Reasoning
- Claim 4 – Modeling and Data Analysis