Exponents and Logs - Summative Assessment

1) Here is a sequence:

\[ a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \]

8, 16, 32, 64,

a) Find the values of \( a_5 \) and \( a_6 \).

b) Explain the sequence verbally.

c) Write in symbols a rule for your sequence either explicitly or recursively.

d) State whether your rule above is either recursive or explicit.

2) a) Make a graph of the function \( f(x) = 2^x \), clearly listing at least four important points in a table and graphing those points.

b) Find \( f^{-1}(x) \), again make a table and graph with at least four points.

\[ f(x) = 2^x \]

\[ f^{-1}(x) = \quad \text{___________} \]
3. Rewrite each log equation as an exponential equation, and each exponential equation as a log equation.

a. \( \log_5 125 = 3 \)  
   c. \( 4^{-3} = \frac{1}{64} \)

b. \( W = \log_U L \)  
   d. \( 21^x = R \)

4. Evaluate or simplify each log expression.

a. \( \log_8 1 \)  
   b. \( \log_2 \left( \frac{1}{64} \right) \)  
   c. \( \log_3 9 \)

d. \( 3 \log_7 \left( \frac{1}{49} \right)^2 \)  
   e. \( \frac{\log_5 25^x}{2x} \)  
   f. \( \log_2 (\log_2 16) \)
5) Solve each of the following equations.

a) \[ 72 = 4(5)^x + 12 \]

b) \[ 3 - \frac{7}{x} = \frac{1}{2x} \]

c) \[ 7 = \sqrt{3x + 4} \]
6) For the table below, identify both a linear function $f(x)$ and an exponential function $g(x)$ that fits the two given values. Complete the table and graph for each function.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear $f(x)$</td>
<td></td>
<td></td>
<td>10</td>
<td>5</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Exponential $g(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Linear Function

$f(x) =$ __________________________

Find the average rate of change of $f(x)$ on the interval $0 \leq x \leq 1$.

Find the average rate of change of $f(x)$ on the interval $2 \leq x \leq 3$.

b) Exponential Function

$g(x) =$ __________________________

Find the average rate of change of $g(x)$ on the interval $0 \leq x \leq 1$.

Find the average rate of change of $g(x)$ on the interval $2 \leq x \leq 3$.

c) Compare and contrast the average rate of change of $f(x)$ and $g(x)$ on the intervals shown in the graph.

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________
1) Here is a sequence:

\[
\begin{array}{cccccc}
a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\
8, & 16, & 32, & 64, & 128, & 256 \\
\end{array}
\]

a) Find the values of \(a_5\) and \(a_6\).

b) Explain the sequence verbally.

The sequence begins with 8 and doubles each time. Or, each term is 2 to the term number times 4.

c) Write in symbols a rule for your sequence either explicitly or recursively.

\[a_1 = 8, a_{n+1} = 2 \cdot a_n \text{ or } a_n = 4 \cdot 2^n\]

d) State whether your rule above is either recursive or explicit.

_The first rule is recursive. The second rule is explicit._

2) a) Make a graph of the function \(f(x) = 2^x\), clearly listing at least four important points in a table and graphing those points.

b) Find \(f^{-1}(x)\), again make a table and graph with at least four points.

\[
f(x) = 2^x
\]

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-1 & \frac{1}{2} \\
0 & 1 \\
1 & 2 \\
2 & 4 \\
3 & 8 \\
4 & 16 \\
\hline
\end{array}
\]

\[
f^{-1}(x) = \_\_\_\_\_\_\log_2 x \_
\]

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
\frac{1}{2} & -1 \\
1 & 0 \\
2 & 1 \\
4 & 2 \\
8 & 3 \\
16 & 4 \\
\hline
\end{array}
\]
3. Rewrite each log equation as an exponential equation, and each exponential equation as a log equation.

   a. \( \log_5 125 = 3 \)
      \( 5^3 = 125 \)

   b. \( W = \log_U L \)
      \( U^W = L \)

   c. \( 4^{-3} = \frac{1}{64} \)
      \( \log_4 \frac{1}{64} = -3 \)

   d. \( 21^x = R \)
      \( \log_{21} R = x \)

4. Evaluate or simplify each log expression.

   a. \( \log_8 1 \)
      0

   b. \( \log_2 \left( \frac{1}{64} \right) \)
      -6

   c. \( \log_3 9 \)
      2

   d. \( 3 \log_7 \left( \frac{1}{49} \right)^2 \)
      -12

   e. \( \frac{\log_5 25^x}{2x} \)
      1

   f. \( \log_2 (\log_2 16) \)
      2
5) Solve each of the following equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
</table>
| **a)** $72 = 4(5)^x + 12$ | $60 = 4(5)^x$  
$15 = (5)^x$  
$log_{5} 15 = x$  
$x \approx 1.683$ |
| **b)** $3 - \frac{7}{x} = \frac{1}{2x}$ | $6x - 14 = 1$  
$6x = 15$  
$x = \frac{15}{6} = 2.5$ |
| **c)** $7 = \sqrt{3x + 4}$ | $49 = 3x + 4$  
$45 = 3x$  
$15 = x$ |
6) For the table below, identify both a linear function \( f(x) \) and an exponential function \( g(x) \) that fits the two given values. Complete the table and graph for each function.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear ( f(x) )</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>Exponential ( g(x) )</td>
<td>40</td>
<td>20</td>
<td></td>
<td>2.5</td>
<td>1.25</td>
<td></td>
</tr>
</tbody>
</table>

a) Linear Function

\[ f(x) = -5x + 20 \]

Find the average rate of change of \( f(x) \) on the interval \( 0 \leq x \leq 1 \).

-5

Find the average rate of change of \( f(x) \) on the interval \( 2 \leq x \leq 3 \).

-5

b) Exponential Function

\[ g(x) = \text{________________________} \]

Find the average rate of change of \( g(x) \) on the interval \( 0 \leq x \leq 1 \).

-20

Find the average rate of change of \( g(x) \) on the interval \( 2 \leq x \leq 3 \).

-5

c) Compare and contrast the average rate of change of \( f(x) \) and \( g(x) \) on the intervals shown in the graph.

The linear function is decreasing at a constant rate, -5, but the exponential function starts out decreasing very fast, -20, but later is decreasing slowly. On the interval \( 2 \leq x \leq 3 \) the two functions have the same average rate of change because they share the same function values at \( x=2 \) and \( x=3 \).
Scoring Rubric

1) 5 points
   a. Two points, one for each of the missing values.
   b. One point.
   c. One point for a correct rule.
   d. One point for correctly identifying their own rule.

2) 10 points
   a. Four points, one each for the four points listed in the table and graphed.
   b. Four points, one each for the four points listed in the table and graphed.
      Two points for the correct equation.

3) 4 points
   a. One point each for the correct answers.

4) 6 points
   a. One point each for the correct answers.

5) 9 points
   a. Three points each for correct solutions to the equations. Give one or two
      points for one or more correct steps toward a solution with a mistake along
      the way.

6) 16 points, Eight points for the missing values in the table
   a. Three points, one point for the correct function, one point each for the
      average rates of change.
   b. Three points, one point for the correct function, one point each for the
      average rates of change.
   c. Two points, one for mentioning that they have the same average rate of
      change on the interval $2 \leq x \leq 3$, one for some reference to the idea that
      the average rate of change of $g(x)$ is changing (decreasing) while the
      average rate of change of $f(x)$ is constant.