How much yellow paint do I need to mix with blue paint to get this shade of green paint?

Essential Lessons

- Proportion Lab
- What Does it Mean to be Proportional?
- Understanding Proportions
- Understanding Unit Rate
- Speed Racers & Revisited
- Proportional Lab Revisited
- Choose at least1: Art Class, Robot Racers, Amusement Park
- Four Methods to Solve Proportions

References:
1) “Progressions Documents for the Common Core Math Standards.” The University of Arizona. Brookhill Foundation, 2007.

2) “Mathematics Framework Chapters.” Curriculum Frameworks (CA Dept of Education). 2013.

Grade 7

Proportional Reasoning

Storybook

Standards Addressed

RP 1: Compute unit rates associated with ratios of fractions, including ratios of length, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

RP 2: Recognize and represent proportional relationships between two quantities

RP 2a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

RP 2b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Unit Story

THE REASONING BEHIND THE FLOW

The unit begins with the hook lesson, Making Lemonade. This lesson engages students in real world, hands on learning. By determining how many scoops of lemonade powder to add to water to make the best lemonade, students will show their understanding of ratio. This lesson serves as a formative assessment for the teacher to understand how much of ratio was understood in grade 6. The conclusion of this lesson asks students to use a tape diagram and also to turn a ratio comparing part to part into a ratio (fraction) comparing part to whole. These tasks allow the teacher to see what understadings and misconceptions the students have as they begin this unit.

Proportional Lab:The major concept of this unit, *what does it mean for data to be proportional,* is addressed through this lab. There are 5 stations in which students collect data two ways: method 1 is proportional and method 2 is not. By collecting data, recording it in a table, graphing the data and comparing/contrasting, students build the foundation for what it means for data to be proportional.

In the subsequent lessons, What Does It Mean to be Proportional and Understanding Proportions, students formalize their definition of proportional relationships as those whose data forms a straight line through the origin when graphed on the coordinate plane, in which each row or column is a multiple of the other piece of data, and whose data form equivlaent fractions. Students will also identify the constant of proportionality in proportiaonal data sets. This rounds out the understanding of the first major concept. From here, teacher may wish to do some “pre-unit” to review using equivalent fractions, cross-products or equations to solve basic proportions.

Formative Assessment

Big Idea #1: Understanding what it means for data to be proportional

- Lesson Understanding Proportions: for quick formative assessment and then diagram for students to fully demonstrate understanding.

Big Idea #2: Solving Proportions with a Double-Sided Number Line

- Lesson Four Methods to Solving a Proportion: Choose 1-2 problems to collect and grade.

Big Idea #3: Unit Rate

- Lesson Amusement Park: Use lesson as formative assessment, having students demonstrate orally or in writing which plan is better when and why. Alternate option: Which is the Better Deal - choose a set of problems and have students explain which is the better rate.

Big Idea #4: Slope/Rate on the Coordinate Plane

- Lesson Speed Racers and Speed Racers Revisited: Use questions embedded in lesson to measure student understanding of rate on the coordinate plane.

Big Idea #5: Comparing Methods to Understand/Solve problems with Proportions

- Lesson: 4 Methods to Solve Proportions: Choose 1 scenario to collect and analyze student work.

General Strategies For Formative Assessments

Use the following strategies throughout the unit

Thumbs up/down/ sideways

Ask students to rate their understanding. A thumbs up means they understand the topic, thumbs down means they don’t and in the middle means they get part of it but still need additional support.

Mini White Boards

Can be used in a variety of ways. Suggestion: Give students problems to solve on white boards and have them raise their boards to show you their answer.

Writing a Summary Statement

Ask students to write a summary of the day’s/week’s lesson.

Fist to Five

Similar to thumbs up/down, students rate their understanding on a scale of zero (fist) to 5, with 5 being fully understand.

Ticket Out The Door (Exit Ticket)

In the last couple minutes of class, give students a problem or two to complete on a slip of paper. Collect the paper as students are leaving the class.

Sentence Frames & Starters

Here are some options to provide to students throughout the activities.

- I agree with _____ because _____.
- I disagree with _____ because _____.
- I did not understand _____.
- I prefer _____method/ strategy because _____.
- I think that _____.
- What do you mean by _____?
- I think _____ means _____.

Tips For Lesson Planning

#1 Always work through the activity/lesson before doing the lesson with the class.

#2 Know how far into the activity you want to be at the end of the period and understand your main objective for the day. What is it that you want the students to walk away with as they leave class?

#3 Decide where and how you want to "chunk the activity". How much of the activity will groups/students do before sharing out with the class? How will you have them report out?

#4 Set time limits for the "chunks" and use a timer. This helps keep you and the students focused, on task, and moving forward through the lesson.

#5 Decide which pieces of the activities are the most vital to discuss or have students present to the class. Not everything in every activity needs to be shared out as a class.

#6 Use tools strategically and intentionally. Plan ahead for when you are going to the document camera, white boards, have students make posters, etc.

#7 Know what materials you need for the lesson and have them ready to go before class. Setup a station for commonly used materials like colored pencils, rulers, etc.

#8 Anticipate where students might struggle, have difficulties, or misconceptions within the lesson. Pre-plan good questions that will help them to continue to move forward

#9 Intentionally and continuously use formative assessment. Find out what your student understand before during and after an activity. Use that information to help guide your lessons and lesson planning.

& Flow

AND INTENTION OF EACH LESSON

The next major topic addressed is the idea of rate. In Speed Racers, students will take turns sprinting for different set amounts of time and compare their results, trying to determine who was faster and who would win in an actual race. Students work iwth ratio tables and interpret speed graphs in context. In Speed Racers Revisited, students connect the ideas of the constant of proportionaltiy, speed/ rate, and slope and begin to write distance/time equations.
Students will then work off of their understanding of rate and speed to develop formally the concept of unit rate,building upon work in grades 4-6 with the lesson Understanding Unit Rate. Students will use ratio tables, doublse-sided numberlines and graphs to reason about and derive a method to calculate unit rate. This will be followed by real world applications of calculating and comparing unit rates in the lesson Which is the Better Deal?

In the lesson Rate of Change on a Graph - Slope Triangles, students will use slope triangles to practice calculting the slope of graphed lines. Students will then use this knowledge to return to their workwith the Proportional Lab in Proportional Lab Revisited. Then they will find unit rates for each lab and compare them to the slopes of the graphs to discover that proportional relationships have a constant slope which is equal to the unit rate (constant of proportionality). Finally, students will practice methods for solving proportional word problems using a variety of streategies in Proportion Match Up and Four Methods to Solving Proportions. Students finish out the unit with problem solving tasks, in lessons such as Art Class, Robot Races, Leaky Faucets, or Amusements Parks.

An optional connection is made at the end of the unit where the geometry standards about scale figures and drawings can be taught as supporting the ideas of proportionality. If not taught here, these lessons can be done in the 2D/3D Geometry unit.

Standards Addressed

RP 2c: Represent proportional relationships by equations.

RP 2d: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

RP 3: Use proportional relationships to solve multi-step ratio problems.

G1: Solve problems involving scale drawings of geometric figures, including computing actual lengths from a scale drawing

SBAC Claims

- CLAIM #1: CONCEPTS AND PROCEDURES (40% STUDENT SCORE)
- CLAIM #2: PROBLEM SOLVING (20% STUDENT SCORE)
- CLAIM #3: COMMUNICATING AND REASONING (20% STUDENT SCORE)
- CLAIM #4: MODELING AND DATA ANALYSIS (20% STUDENT SCORE)

Types Of Knowledge

- **MEMORIZATION (QUICK RECALL)**
- **PROCEDURAL (FOLLOW STEPS/ DO SOMETHING)**
- **CONCEPTUAL (UNDERSTND BIG IDEA/ EXPLAIN/ DERIVE)**
- **RELATIONAL (APPLY/ ANALYZE/ EVALUATE)**

Unit

WHY PROPORTIONAL RELATIONSHIPS

Proportional reasoning is arguably the most important topic in grade 7. It marks a clear distinction between students comparing quantities absolutely (10 is greater than 8) versus relatively (10 out of 20 is proportionally less than 8 out of 10). An understanding of ratio is essential in the sciences to make sense of quantities that involve derived attributes such as speed, acceleration, density, surface tension, electric or magnetic field strength, and to understand percentages and ratios used in describing chemical solutions. Ratios and percentages are also useful

Where Does

Grades 3-5

In grade 3, students first begin to study multiplicative comparisons, e.g., the giraffe is three times as tall as the hippo. Before this, students compared numbers using addition (e.g., the giraffe is ___ feet ___ inches taller than the hippo). The study of multiplicative comparisons continues into grade 4. Students use multiplicative relationships demonstrated in tables to convert within measurement systems in grades 4 and 5. Students also look for patterns involving multiplication of a given input to result in the output. In grades 3-5, students master multi-digit multiplication, useful in solving for an unknown in a proportion. In addition, grades 3 & 4 have a strong emphasis on understand equivalent fractions.

Grade 6

Students understand and write ratios to describe the relationship between two quantities. Students understand and compute unit rates. Students solve problems involving ratios by using equivalent ratios, tape diagrams, double-sided number lines and equations. Grade 6 students graph points in all four quadrants on the coordinate grid (grade 5 was first quadrant only) and use coordinate graphing to model the data from tables of equivalent ratios.

The Unit! (Cont.)

Tape Diagrams

While a more common tool in grade 6, tape diagrams can be used to compare items having the same unit. For example, 3 cups of sugar to 2 cups of water can be represented in a tape diagram as shown below. The tape can be expanded to then find how many cups of sugar are needed to be mixed with 4 cups of water.

Ex Tape Diagram:

Sugar	Sugar	Sugar
Water	Water	

Misconceptions

Absolute vs. Relative Thinking

When students compare numbers using addition or subtraction or just compare numbers in isolation, they are using absolute thinking. Ex: 15 is more than 10. 15 is 5 more than 10. This is how students first learn to consider numbers. As we begin thinking proportionally, or relationally, we must consider more than just the numbers themselves. 15 is 1.5 times larger than 10. Can 10 even be "more" than 15? For example, if a class has 15 girls with a total of 30 students and another class has 10 girls in a class or 12 students, you could reason that the "10 girls" class had proportionately more girls than the class with 15, as 10:12 is a greater rate than 15:30. This is also true when comparing a teacher salary to a 12-month employee, as one person's check is divided over 9 months and another 12, so the figures themselves don't tell you who makes more money per hour.

Student Talk Strategies

- **Report to a partner**
  Each student reports his/ her own answer to a peer. The students listen to their partner's response. ("Turn to a partner on your left." "Now turn to a partner on your right" etc.)
- **Give one get one**
  After brainstorming ideas, students circulate among other students sharing one idea and getting one. Students fold paper lengthwise they label the left side "give one" and the right side "get one".
- **Think, Pair, Share**
  Students think about a topic suggested by the teacher. Pairs discuss the topic. Students individually share information from their discussion with the class.
- **Inside-outside circle**
  Two concentric circles of students stand or sit, facing one another. The teacher poses a question to the class, and the partner responds. At a signal, the outer of inner circle or outer circle rotates and the conversation continues.
- **Appointment clock**
  Partnering to make future discussion/work appointments.
- **Jigsaw**
  Group of students assigned a portion of a text; teach that portion to the remainder of the class.

Math Practice Standards

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

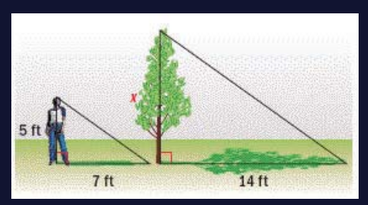
4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

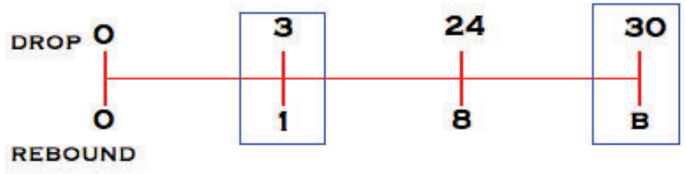
7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



The Math Behind

One row of the number line represents the scale of 1 item, while the other row represents the second item and the numbers do not line up numerically (meaning that 5 does not line up with 5), but the numbers line up to show the ratio of the items.
For example, a ball dropped from a height of 24 cm rebounds 8 cm. How high will the ball rebound when dropped from 30 cm?



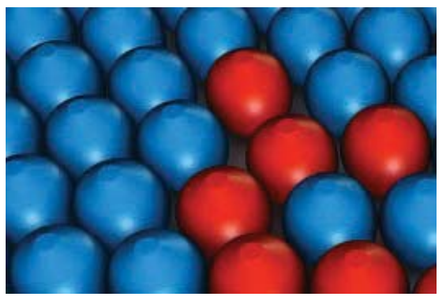
Common

Ratios vs. Fractions

Not all ratios are fractions. Ratios can compare part to part or part to whole. Fractions represent a part to whole relationship, so when a ratio represents part to whole, it can be written as a fraction using the same numbers.

Ex: 5 out of the 20 balls are red means $\frac{5}{20}$ of the balls are red.

5 balls are red and 20 balls are blue does NOT mean the balls are $\frac{5}{20}$ red, but in fact the balls would be $\frac{5}{25}$ red in this case (assuming there are only red and blue balls).



Overview

ARE IMPORTANT IN GRADE 7

in many situations in daily life, such as in cooking and in calculating tips, miles per gallon, taxes, and discounts. They also are also involved in a variety of descriptive statistics, including demographic, economic, medical, meteorological, and agricultural statistics (e.g., birth rate, per capita income, body mass index, rain fall, and crop yield) and underlie a variety of measures, for example, in finance (exchange rate), medicine (dose for a given body weight), and technology (kilobits per second).

This Topic Fit?

Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life

Grade 8

Grade 8 understanding of linear functions depends upon grade 7 work with rate/slope. Students will build upon graphing equations of proportional relationships by adding in the initial value in grade 8. Grade 8 work with similar figures also builds upon the proportional reasoning foundation laid in grade 7. Students will use similar triangles to prove that the slope of the line is constant between any two points.

High School

Students will use ratios when they study sine, cosine, tangent and other trigonometric ratios. High School students will also rely on grade 7 work with proportional relationships when studying functions with a constant rate of change. The grade 7 work lays a foundation for the eventual study of instantaneous rate of change in Calculus. Grade 7 work in this unit also supports HS work in the sciences and descriptive statistics.

Coherence

Connections to other Grade 7 Topics

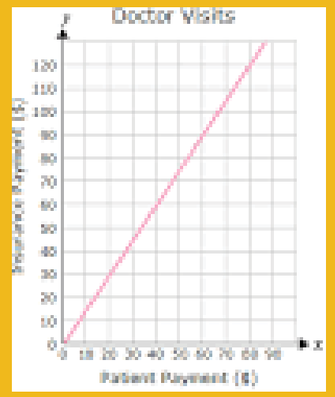
Proportions will be used in the percent unit, as percents represent a number out of 100.

Proportional relationships will be used in the probability unit when predicting how many times an event will occur.

Proportional relationships forms the basis for understanding scale drawings and later on, similar figures.

Real-World Application

- Recipe conversions
- Reading maps
- Speed and distance
- Comparisons when shopping
- Converting measurements
- Scale drawings
- Problems with rate



Essential Question # 1
What is a unit rate?

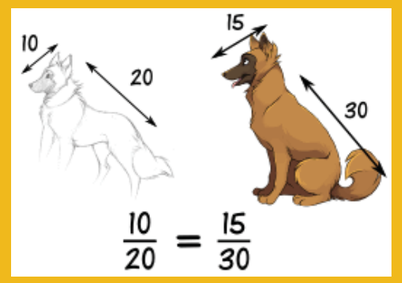
Essential Question # 2
Are these data proportional?

Essential Question # 3
Which method is most helpful to use to find missing data in each situation?

Essential Question # 4
What is the constant of proportionality?

Essential Question # 5
Which has the better unit rate?

Essential Question # 6
When do I use proportional comparisons?

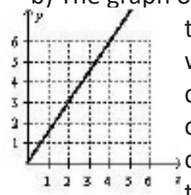


The Math Behind The Unit!

Data is proportional when the following factors are true:

a) When proportional quantities are represented in a table, pairs of entries represent equivalent ratios. Ex: $\frac{4}{6} = \frac{6}{12}$

b) The graph of a proportional relationship lies on a straight line that passes through the point (0,0), indicating that when one quantity is 0, so is the other.



c) Equations or proportional relationships in a ratio of a:b, always take the form $y=kx$, where k is the constant if the variables x and y are defined so that the ratio $x:y$ is equivalent to $a:b$.

Ex: Apples cost $3 for every 2 pounds. $c=1.5a$ where c represents cost and a represents number of pounds of apples.

Proportions can be solved using the following methods:

Example: 10 shirts cost $50, how much will 100 shirts cost.

a) Set cross products equal to one another and solve (traditional method, not necessarily supported by framework).

$$\frac{10}{50} = \frac{100}{x}$$

Solution: $10 \cdot x = 50 \cdot 100$
$10x = 5000$
$x = 500$

b) Determine what the unit rate, $\frac{a}{b}$, and apply to equivalent ratio.

$\frac{50}{10} = 5$, so $5 \cdot 100$ is 500.

c) Use equivalent fractions (Determine what factor to multiply "a" by to get "b", and do the same with "x").

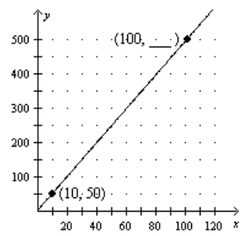
$50 \cdot \frac{1}{5} = 10$, so $x \cdot \frac{1}{5} = 100$; $x=500$

d) Write an equation. Equations of proportional relationships are typically written in the format $y=kx$, where k represents the constant of proportionality.

Let c represents total cost and s represents number of shirts.

$c = 5 \cdot s$
$c = 5 \cdot 100$
$c = 500$

e) Graph the data and extend the line to find the coordinate of the missing data piece.



Unit Rate

For a ratio, $a:b$ with both a and b not equal to 0, the unit rate is the number $\frac{a}{b}$.

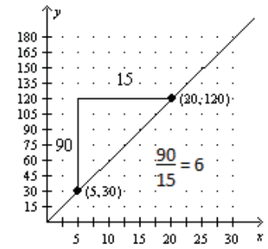
Ex: 8 gallons every $2\frac{1}{2}$ seconds. Unit rate is $\frac{8}{2\frac{1}{2}} = \frac{16}{5}$, meaning 16 gallons per 5 seconds. This unit rate can also be written in terms of "per second" by finding an equivalent ratio with a denominator of 1, in this case, 3.2 gallons per second. $\frac{8}{2\frac{1}{2}} = \frac{3.2}{1}$

Unit rates can be found using equivalent fractions, by dividing a/b or by reasoning with the quantities; e.g., if I run 8 miles in 42 minutes, then I can take half of each quantity to get 4 miles in 21 minutes, take half of that to get 2 miles in 11.5 minutes and then half again to get 1 mile in 5.75 minutes.

Rate on the Coordinate Plane

When proportional data is graphed on the coordinate plane, the slope of the line is the constant of proportionality.

Ex: Haley runs 5 miles every 30 minutes. At this rate, how far will she run in 2 hours?



Slope (constant of proportionality in this unit)

Slope is the rate of change of a function. In a linear function, and in particular in proportional relationships, the slope is constant and measures the rate at which y increases as x increases. Slope can be easily calculated from a table of equivalent ratios or by drawing and counting change in y over change in x, using a slope triangle on a graph.

EX:

x	y
5	30
20	120

+15 (x) +90 (y)

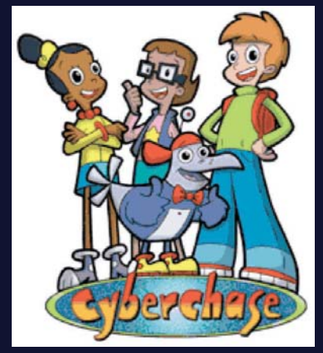
Slope $= \frac{90}{15} = 6$

Double-Sided Number Line

Students use double sided number lines as a tool to set up, visualize, estimate and then write a proportion from data. A double sided number line is used when the two items being measured have different units or represent different quantities.

Academic Language

- ABSOLUTE VALUE
- CROSS-PRODUCTS
- UNIT RATE
- RELATIVE
- SPEED
- SLOPE
- EQUIVALENT FRACTIONS
- MULTIPLICATIVE
- PROPORTION
- PER
- RATE
- CONSTANT OF PROPORTIONALITY



What is Inez's Unit Rate?