Accounting for Population Exposure to Pollutants in the Toll Design Problem

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Introduction

- Motor vehicle-generated emissions remain a threat to public health

- Road pricing is one approach for mitigating the negative externalities of vehicle emissions

- Here a toll design problem is proposed for determining toll locations and corresponding toll levels such that a population's exposure to motor vehicle-generated air pollutants is minimized

- In addition, a new surrogate-based solution algorithm is proposed to solve mixed-integer network design problems like the presented toll design problem

Problem Formulation

- A toll design problem (TDP) is proposed for determining toll locations and levels that minimize the expected population exposure to air pollutants subject to probabilistic constraints on pollutant concentration levels and implementation costs

- The TDP is formulated as bilevel optimization problem where meteorological uncertainty is accounted for by using a scenario-based modeling approach

Notation

$\boldsymbol{\tau}$	=	Vector of toll values (decision variables)
$\boldsymbol{y}$	=	Vector of toll locations (decision variables)
$\boldsymbol{v}(\cdot)$	=	Vector of link flows
s	=	Index for meteorological scenario
π^s	=	Probability of scenario s
I^s	=	Indicator of population exposure to pollutants in scenario s
r	=	Index for receptor location
C	=	Pollutant concentration
θ_r	=	Minimum probability for C_r not exceeding a threshold $C_{r,max}$
a	=	Link index (with $\bar{A}$ referring to the set of candidate tolling locations)
h_a	=	Cost of charging a toll in link a
H	=	Total budget

Upper Level Problem

$$\text{Minimize } \varphi(\boldsymbol{\tau}, \boldsymbol{y}) = \sum_{s \in S} \pi^s I^s(\boldsymbol{v}(\boldsymbol{\tau}, \boldsymbol{y})) \qquad (1)$$

subject to:

$$\sum_{s \in S} \pi^s G_r^s(\boldsymbol{v}(\boldsymbol{\tau}, \boldsymbol{y})) \geq \theta_r \qquad \forall\, r \qquad (1a)$$

$$\sum_{a \in \bar{A}} h_a y_a = H \qquad (1b)$$

$$0 \leq \tau_a \leq y_a \tau_{max,a} \qquad \forall\, a \in \bar{A} \quad (1c)$$

$$y_a \in \{0,1\} \qquad \forall\, a \in \bar{A} \quad (1d)$$

where:

$$G_r^s(\boldsymbol{v}(\boldsymbol{\tau}, \boldsymbol{y})) = \begin{cases} 1, if\ C_r(\boldsymbol{v}(\boldsymbol{\tau}, \boldsymbol{y})) \leq C_{r,max} \\ 0, if\ C_r(\boldsymbol{v}(\boldsymbol{\tau}, \boldsymbol{y})) > C_{r,max} \end{cases}$$

- A population's exposure to an air pollutant could be quantified using the following air pollution intake measure:

$$I = \sum_r \sum_g P_{gr} \times C_{gr} \times B_g \qquad (2)$$

where g is the index of a population group, P_{gr} is group g's population associated with receptor r, and B_g is group g's representative breathing rate

- Deterministic user equilibrium is assumed for the lower level problem.

A Surrogate-Based Solution Algorithm

- A surrogate-based solution algorithm is proposed for mixed integer network design problems where the continuous variables depend on the value of binary integer variables

- The algorithm is a variant of the Metric Stochastic Response Surface (MSRS) algorithm (Regis and Shoemaker, 2007), which was originally proposed for continuous problems but has been extended, for example, for problems with multiple objectives (Chow et al., 2014), and independent continuous and integer variables (Muller et al., 2013)

- The proposed algorithm is particularly useful for computationally expensive problems for which analytical expressions are not available
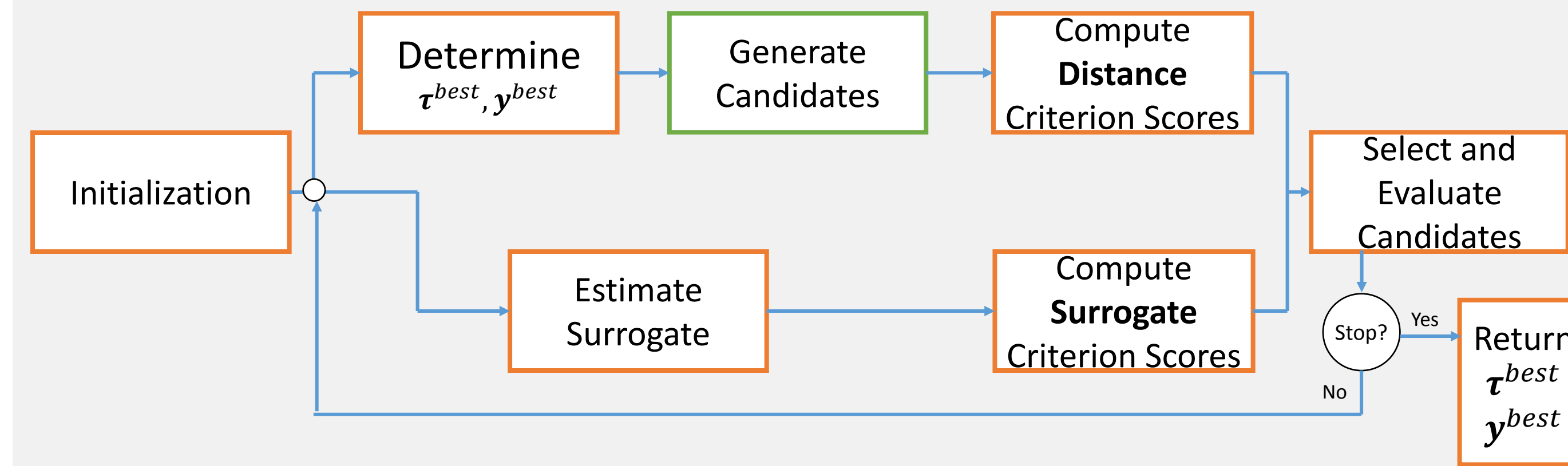

Figure 1. General steps of the MSRS algorithm

Basics of the MSRS Algorithm

- The basic idea behind the LMSRS methodology is to use a surrogate model (s) in the process of iteratively screening for the most promising solutions among a set of randomly generated candidate solutions

- The selected candidates are evaluated with the computationally expensive models to determine if they are better than the best known solution

- The most promising candidate solution is the one with the minimum weighted score $W = w^{RS} V^{RS} + w^{RS} V^{RS}$, where V^{RS} is the response surface (or surrogate) criterion score, V^D is the distance criterion score, and w^{RS} and w^D are their corresponding weights ($w^{RS} + w^D = 1$)

$$V^{RS}(\boldsymbol{\tau}) = \frac{s(\boldsymbol{\tau}) - s_{min}}{s_{max} - s_{min}} \quad (3)$$

$$V^D(\boldsymbol{\tau}) = \frac{\Delta_{max} - \Delta(\boldsymbol{\tau})}{\Delta_{max} - \Delta_{min}} \quad (4)$$

where:

$s(\boldsymbol{\tau})$	:	predicted objective function value for $\boldsymbol{\tau}$
s_{max}, s_{min}	:	max., min. $s(\boldsymbol{\tau})$ in pool of candidates

$\Delta(\boldsymbol{\tau})$	:	minimum distance between $\boldsymbol{\tau}$ and previous solutions
$\Delta_{max}, \Delta_{min}$	:	max., min. $\Delta(\boldsymbol{\tau})$ in pool of candidates

- The weights w^{RS} and w^D vary in each iteration in order to cycle between explorative and exploitative search

Rules for Generating Candidate Solutions

- Candidate solutions are generated by perturbing the "coordinates" (i.e., variables) of the best known solution according to a set of rules and variable parameters

- In the proposed algorithm, three groups of candidates are generated:

Group 1	:	candidates are generated by varying the continuous variables (the toll levels $\boldsymbol{\tau}$) in the locations indicated in $\boldsymbol{y}^{best}$ (i.e., fixed toll locations)
Group 2	:	candidates with toll location vectors that satisfy the condition $\sum_i \lvert y_i - y_i^{best} \rvert \leq \eta$, where η defines the neighborhood of $\boldsymbol{y}^{best}$
Group 3	:	candidates with toll location vectors that satisfy the condition $\sum_i \lvert y_i - y_i^{best} \rvert > \eta$; vectors are outside the neighborhood of $\boldsymbol{y}^{best}$

- Additional procedures are used to adjust the magnitude and frequency of the perturbations used in the process of generating candidate solutions for the three aforementioned groups

Numerical Tests

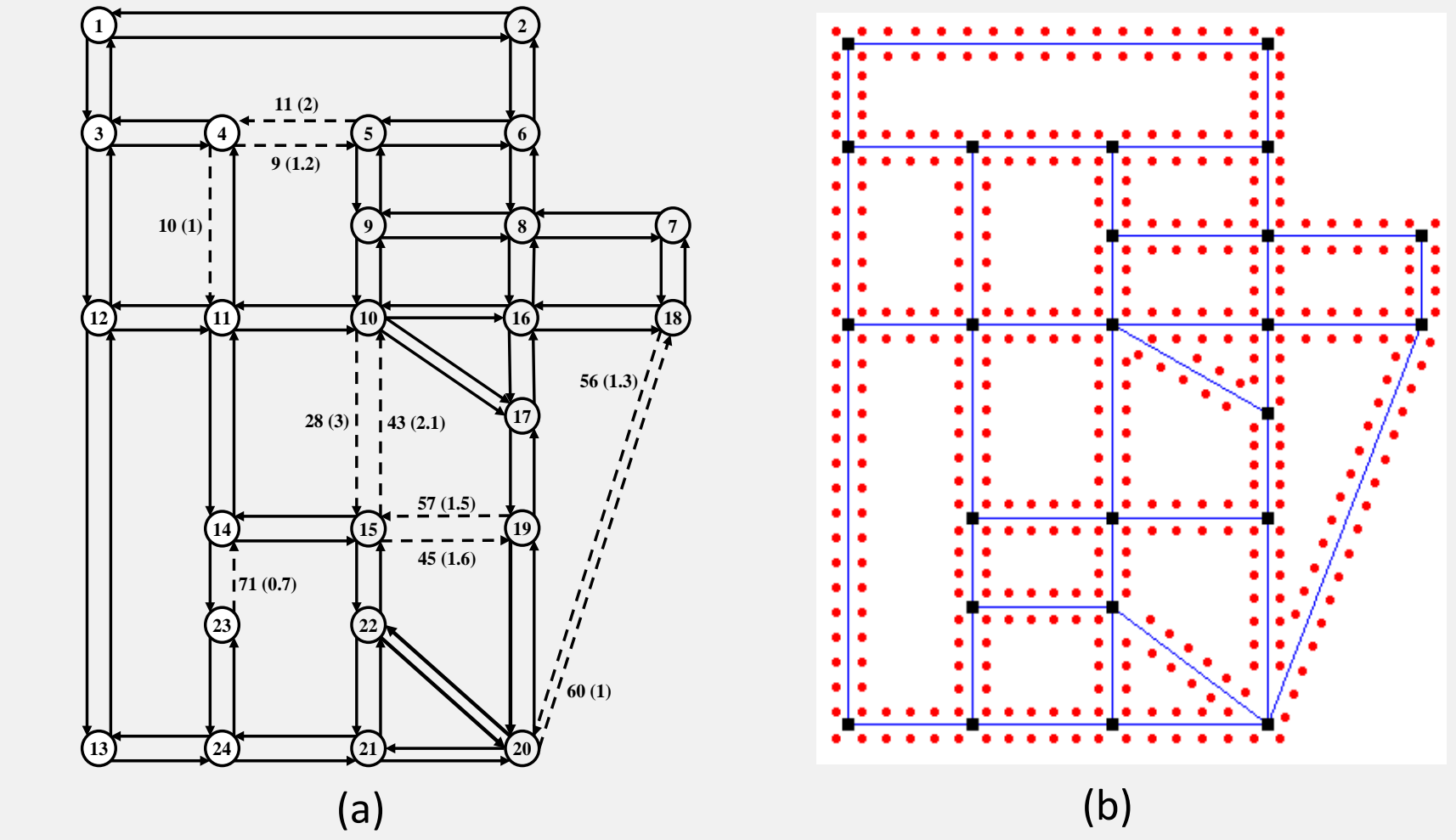- The Sioux Falls network was used in the numerical tests

Figure 2. The candidate toll links and their associated costs (a), and the receptor locations (b).

- A carbon monoxide emission function and a Gaussian plume model were used to compute emissions and concentrations, respectively

- The radial basis function (RBF) model with cubic functional form was selected as the surrogate model

- The proposed algorithm (MI-LMSRBF in Figures 3 and 4) was compared against two other algorithms developed for the toll design problem: a simulated annealing-genetic algorithm method (SA-GA; Yang and Zhang, 2002) and a genetic algorithm approach (Fan and Gurmu, 2014)
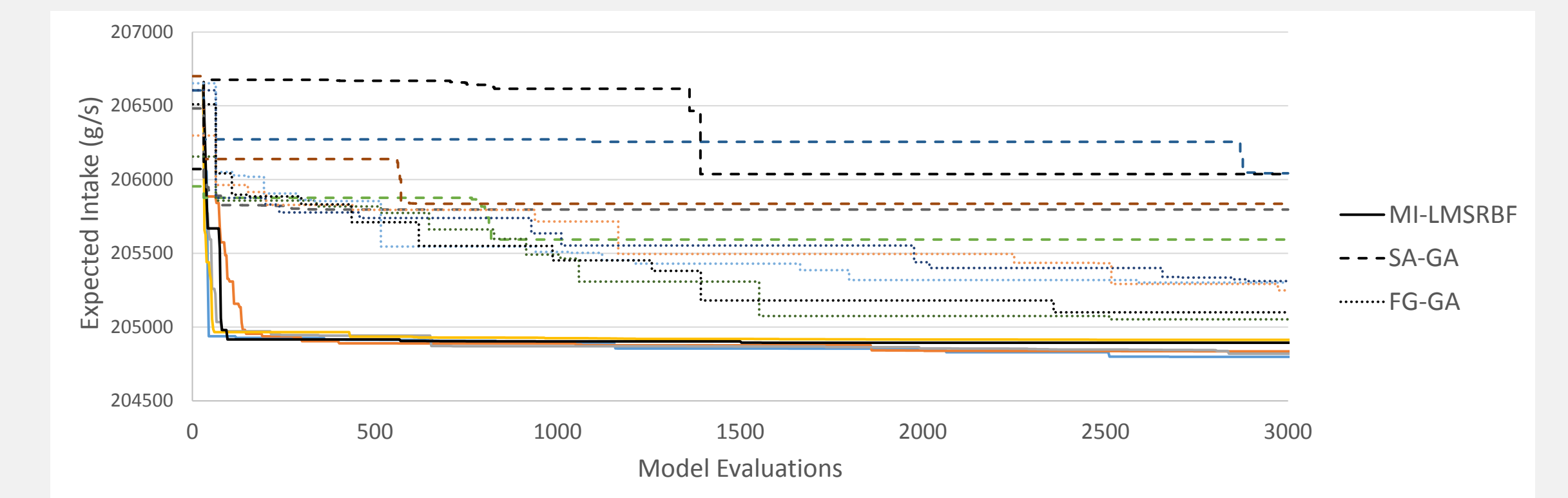

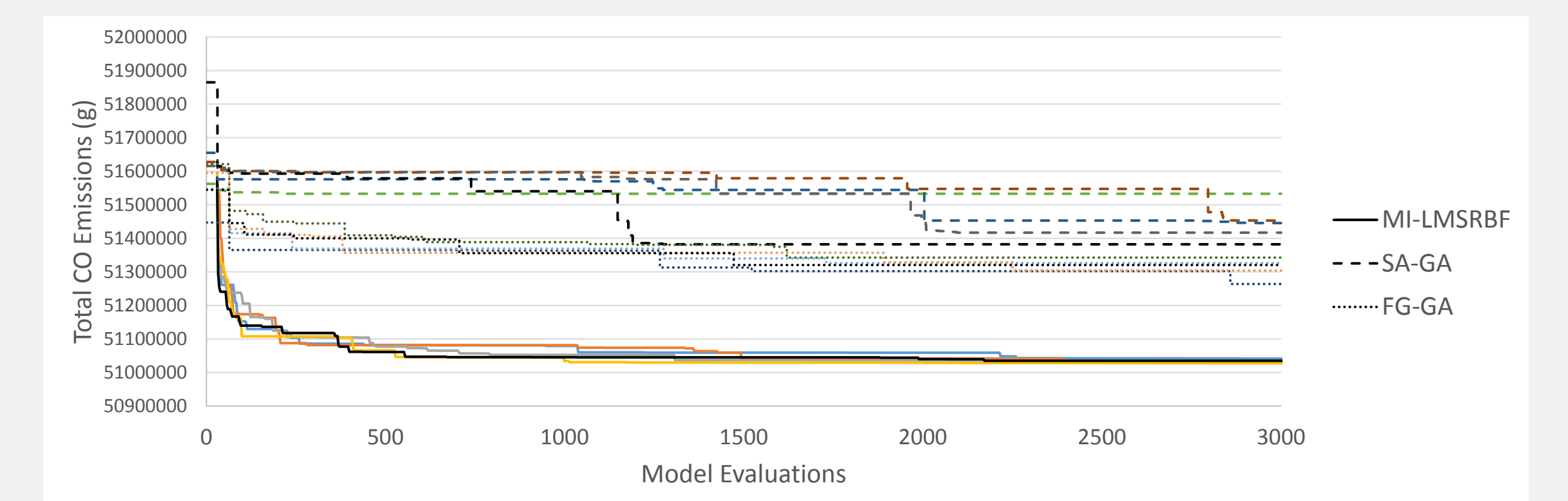Figure 3. Progression of lowest known objective function value for the population exposure TDP

Figure 4. Progression of lowest known objective function value for the total emissions TDP

Closing Remarks

- A toll design problem that accounts for population exposure to air pollutants was proposed along with a new surrogate-based algorithm that can be applied to mixed integer network design problems

- The numerical tests suggest that the new solution algorithm offers a promising approach for tackling mixed integer network design problems

- Future research could focus in incorporating in the TDP an explicit consideration of health costs and personal-level exposure metrics

- Additional tests will be conducted on the solution algorithm to study, among other things, how it performs with larger problems (both in terms of number of variables and network size) and what is the influence of the parameter η

Selected References
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