Incorporating Perceived Travel Time Reliability Into Transportation Planning and Simulation Models Using Information Entropy as the Measure

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OBJECTIVES

- **Measuring perceived travel time uncertainty using current consensus of category-based perception and (quantum) information entropy in cognitive science in order to set up theoretical foundations for modeling and analysis on uncertainty.**
- **Offering models with more flexibility especially comparing with variance-based measures.**
- **linearly embedding uncertainty into general cost function, which is critical in considering perceived uncertainty in the time-of-departure, mode choice, and traffic assignment modeling.**
- **Associating the input and output with economic/econometric concepts such as willingness-to-pay, time budget, time choice of departure, etc.**
- **Demonstrating the importance of considering the initial condition and order of interventions versus taking it as a model's shortcoming.**

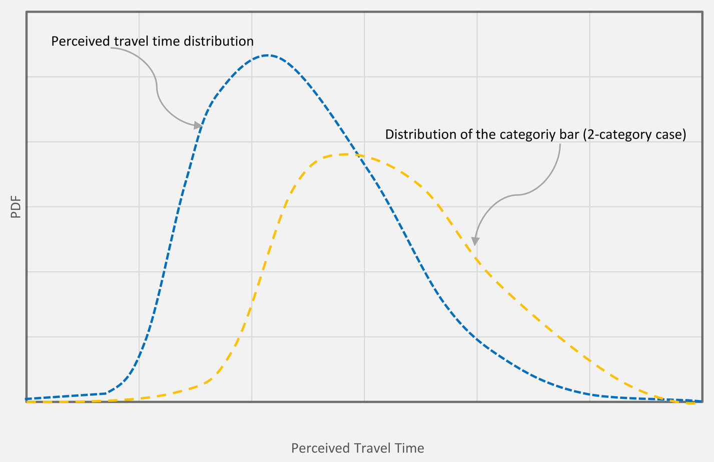
BACKGROUND

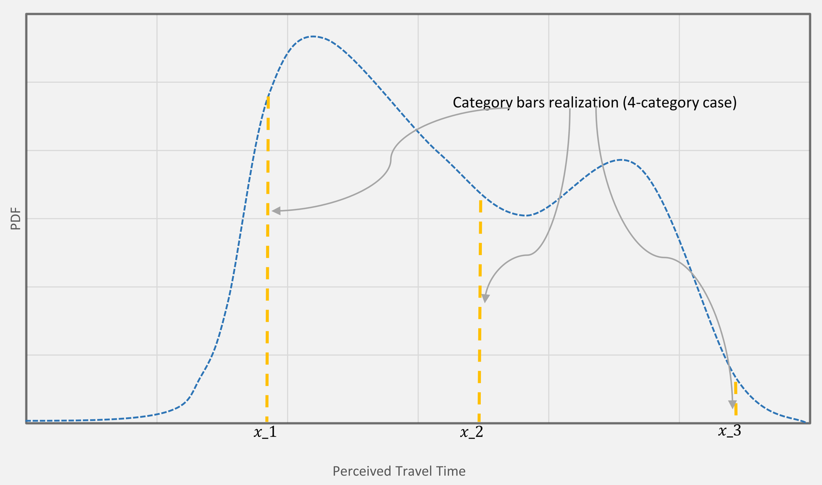
- **Cognitive Science**: offers theoretical support in modeling perception and behavior, category-based perception, Misperception
- **Information Theory**: Entropy is the natural candidate for measuring system uncertainty.
- **Bayesian Update**: Travelers/Shippers' perception is influenced by the historical events/information as well as the current observations (or traveler information with various degrees of beliefs). It has been considered a shortcomings that a model's results change with different initial conditions, but this is suggested to reconsider.
- **Quantum Cognitive Model**: Used in the field of cognitive science, finance, and decision making for modeling misperception ("fail of total probability theorem"). More general case: Quantum Markov Chain Model under Hilbert Space
- **Transportation Networks**: Path-based traffic assignment
- **Economics/Econometrics**: The methods proposed can be relatively easy to associate with Value of Time, Travel Time Budget, Willingness to Pay, and so on.
- **Examples**: When traveler information systems are improved, the actual travel time distribution might not vary, while the perceived entropy (uncertainty) for travelers/shippers might be reduced or increased – traveler information becomes easier to incorporate into the models ; Due to misperception, the entropy reduction might be systematically under-/over-estimated.

METHODOLOGY

- **Stochastic Segmentation of Perceptive Categories**

$$f^{q,\delta}_{X_1,X_2,\ldots,X_s}(x_1,x_2,\ldots,x_s|\bar{t}) = \begin{cases} b(\bar{t},x_1,x_2,\ldots,x_s) > 0, & 0 \le x_1 \le x_2 \le \cdots \le x_s \\ 0, & otherwise \end{cases}$$





- **Information Entropy Measure**

$$H(\mathcal{S}) = -\sum_{\forall \mathcal{S} \in \mathbb{S}} P(\mathcal{S}) \cdot \log\big(P(\mathcal{S})\big)$$

where, $\mathcal{S}$ is an event in the sample space $\mathbb{S}$. A continuous case makes the measure no more realistic due to the categorical perception nature of human. Function $P(\mathcal{S})$ is the (perceived) probability of event $\mathcal{S}$ to happen.

- **Combining Perceived Uncertainty with Stochastic Segmentation of Categories**

$$-\int_{\mathbb{R}^{S\times 1}} f(\boldsymbol{x}) \sum_{\forall \mathcal{S} \in \mathbb{S}} P(\mathcal{S}|\boldsymbol{x}) \cdot \log\big(P(\mathcal{S}|\boldsymbol{x})\big)\, d\boldsymbol{x}$$

$f(\boldsymbol{x})$ is the probability density for the market section $\boldsymbol{x} \in \boldsymbol{X}$. This also applies to discrete market segmentation by replacing integral with summation.

- **Bayesian Update for Sensed and Perceived Travel Time Distribution**

$$f_{\Theta|t}(\Theta_1,\Theta_2,\Theta_3\ldots|t,\bar{\xi}\ldots) \propto L(\Theta_1,\Theta_2,\Theta_3,\ldots;t)\cdot f_{\Theta}(\Theta_1,\Theta_2,\Theta_3,..|\underline{\xi})$$

Travel time distribution can be selected from gamma family (for non-negative support) and distribution for the hyper-parameters can be the corresponding conjugate priors in order to simplify the computation.

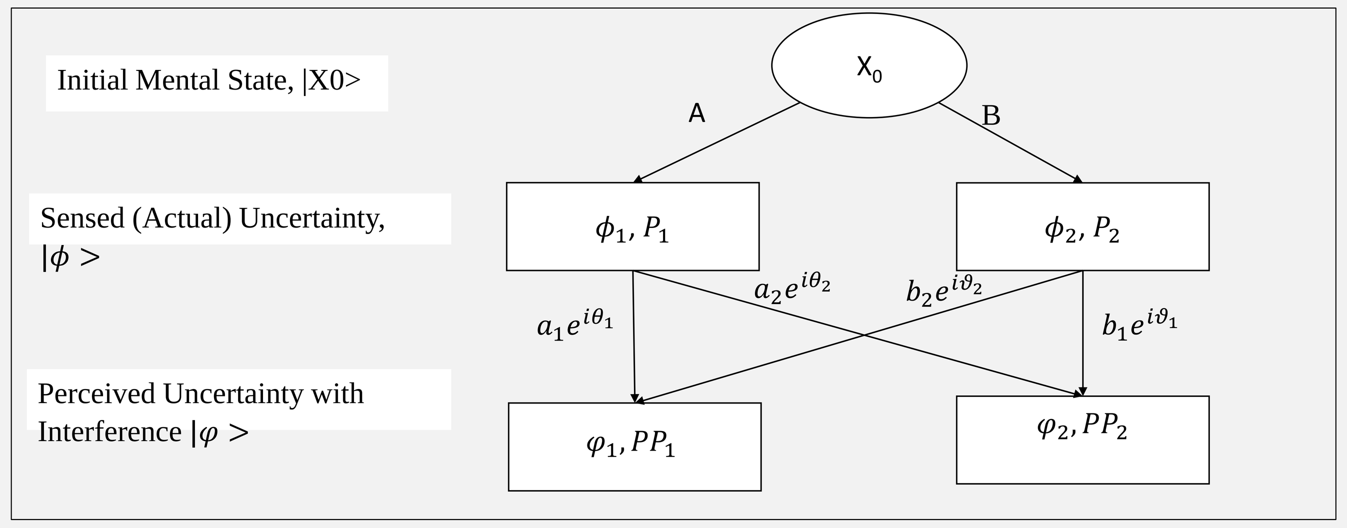
- **Misperception Using Quantum-like Cognitive Model in Hilbert Space**

Mental State under quantum superposition: $|\varphi_n\rangle = \sum_{\forall K_i \in \{k_n\}} \langle K_{in}|K_{in}\rangle \cdot \boldsymbol{e_i}$

For example, $|PR\rangle = a|On\ Time\rangle + b|Slightly\ Late\rangle + c|Significantly\ Late\rangle$

$$\Pr(\varphi_i|X_0) = |\langle\varphi_i|\phi_1\rangle\langle\phi_1|X_0\rangle + \langle\varphi_i|\phi_2\rangle\langle\phi_2|X_0\rangle|^2$$
$$= |\langle\varphi_i|\phi_1\rangle\langle\phi_1|X_0\rangle|^2 + |\langle\varphi_i|\phi_2\rangle\langle\phi_2|X_0\rangle|^2$$
$$+\,2(\langle\varphi_i|\phi_1\rangle\langle\phi_1|X_0\rangle)^* \langle\varphi_i|\phi_2\rangle\langle\phi_2|X_0\rangle \cdot \cos(\theta)$$

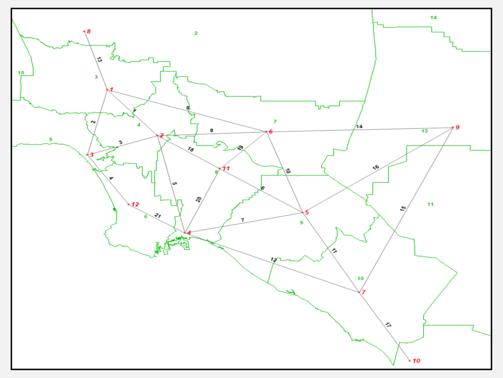
$S(\rho_n) = -Tr(\rho_n \cdot ln\rho_n)$ is the Von Neumam entropy, and $\rho_n = \sum_j p_{nj}|\varphi_{nj}\rangle\langle\varphi_{nj}|$ as the density matrix.

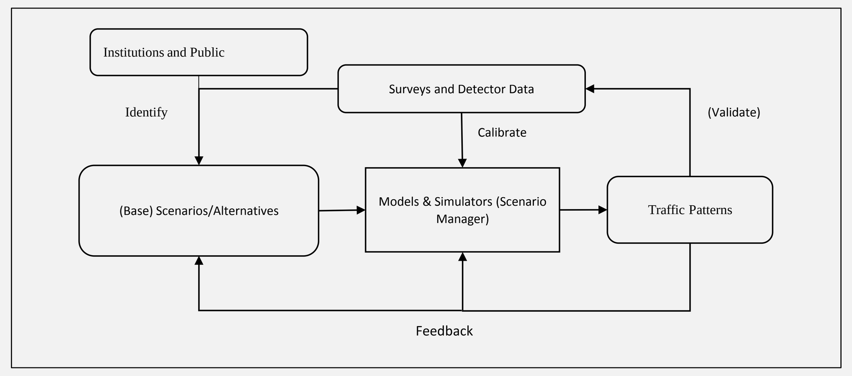


CASE TEST ON PATH-BASED TRAFFIC ASSIGNMENT

In order to test the methods and algorithms presented in this paper, a 24-link 12-node unimodal network (generally aggregated from that of Southern California Association of Government region) is used.

The coefficients for converting entropy to general cost are estimated through minimizing the weighted differences between the actual observed flow data and the modeled one. Due to non-linear space, here only selected three coefficients to demonstrate.





Algorithm Description:

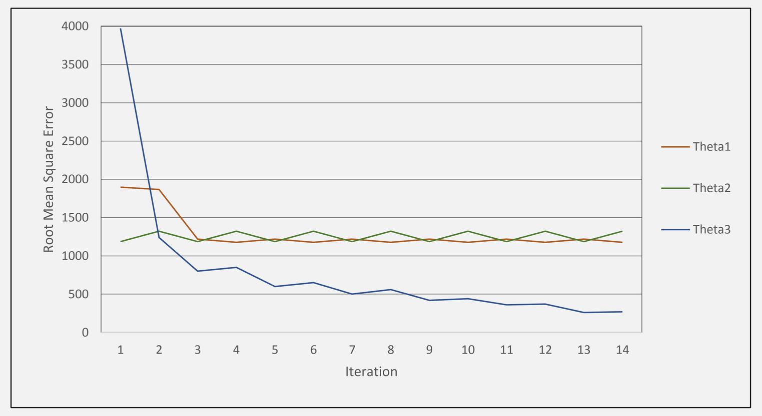
Step 0: Perform path-based traffic assignment in different scenarios in the scenario set to obtain the flow variation list (need to estimate the frequency for each scenario happening) so that *any* path-based travel cost distribution can be reconstructed later if need. (If need, each OD pair and user class has corresponding perceived prior travel time distribution for future update to obtain posterior distribution. To simplify the computation in this case study, we assumed "noninformative" prior to match the frequentists' results).

Step 1: Perform again the path-based traffic assignment in different scenarios in the scenario set, yet during which, add $\Theta \cdot q(y_k^{rs})$ obtained from previous step onto the path cost to obtain the total cost (if need, the entropy should be calculated based on posterior distribution of the travel time rather than just the sample itself obtained in this step) before determine the shortest one in the path set K from r to s.

Step 2: Compare both the link flows and variations with the previous assignment results to determine if it converges. If does, stop, or else, continue by going back to Step 1.

Results

Different coefficients for entropy-cost conversion might lead to change of astringency.



	T_o	Base, P=0.3	S1, P=0.25	S2, P=0.2	S3, P=0.15	S4, P=0.1	Weighted Average	Weighted Variance
Applying Assignment Model Without Considering Entropy								
Corridor 1, NB	99.0	106.3419	105.9213	106.4085	105.8283	106.3827	106.1771	0.077525
Corridor 1, SB	99.0	102.1285	101.9883	102.2552	102.2073	102.1794	102.1357	0.010456
Corridor 2, EB	60.0	61.8293	61.7735	61.6944	61.7091	61.7632	61.7632	0.003347
Corridor 2, WB	60.0	64.00508	64.11324	63.94818	63.9434	64.21629	64.03261	0.013822
Applying Assignment Model Considering Entropy (Theta3)								
Corridor 1, NB	99.0	107.1907	106.6498	107.3826	106.8813	107.0536	107.0337	0.079345
Corridor 1, SB	99.0	101.9435	101.7353	102.1017	101.9245	102.0056	101.9264	0.018143
Corridor 2, EB	60.0	61.09683	61.11214	61.08301	61.04685	61.06227	61.08694	0.000685
Corridor 2, WB	60.0	71.71383	71.56257	71.83199	71.55277	71.62671	71.66677	0.013637
Difference (%)								
Corridor 1, NB	--	0.79818	0.687775	0.915434	0.995008	0.630648	0.806765	2.34763
Corridor 1, SB	--	-0.18114	-0.24807	-0.15011	-0.27669	-0.17009	-0.20492	73.5176
Corridor 2, EB	--	-1.18466	-1.07062	-0.991	-1.07318	-1.13487	-1.09492	-79.5339
Corridor 2, WB	--	12.04397	11.61902	12.32844	11.90016	11.53978	11.9223	-1.33845

CONCLUSION & FUTURE RESEARCH

This paper presented a scheme to incorporate travel time reliability in transport network planning model, using Shannon's information entropy formulation and Von Neumann's quantum information entropy formulation. Future work expected includes economic interpretation, coefficients estimation methods, quantum Bayesian, choice of prior and initial conditions, path-based solution uniqueness issues, test of different transformations of entropy-converted cost, incorporating into dynamic and activity-based models, and so on.

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