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Simultaneous State and Parameter Estimation in Newell's Simplified Kinematic Wave Model with Heterogeneous Data

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Abstract

Traffic state estimation provides valuable information for travelers and decision makers. To implement an estimation method, one needs to predetermine the initial state and model parameters. This paper presented a new method to estimate model parameters and initial states for Newell's simplified kinematic wave model based on heterogeneous data sources. Different from existing studies, the proposed method is a simultaneous framework which estimates parameters in the fundamental diagram together with traffic states. The problem is formulated in a least square optimization framework which is solved using the Gauss-Newton method. We verified the estimation method using NGSIM data and analyzed the impact of market penetration rate. The estimation result is consistent with observation.

Key words: Traffic State Estimation, Fundamental Diagram Estimation, Newell's Simplified Kinematic Wave Model, Vehicle Reidentification, Heterogeneous Data, NGSIM Dataset

• Introduction

Traffic information is essential to assist traffic operation decisions and to develop efficient traffic control/management strategies. Real-time traffic information is useful for various applications, such as travel time estimation, incident detection, traffic control and management. Traffic estimation is the process of estimating all traffic variables according to available data in a traffic network. More precisely, the estimation method should provide a complete picture of the traffic state based on available data.

• Newell's Simplified Kinematic Wave Model

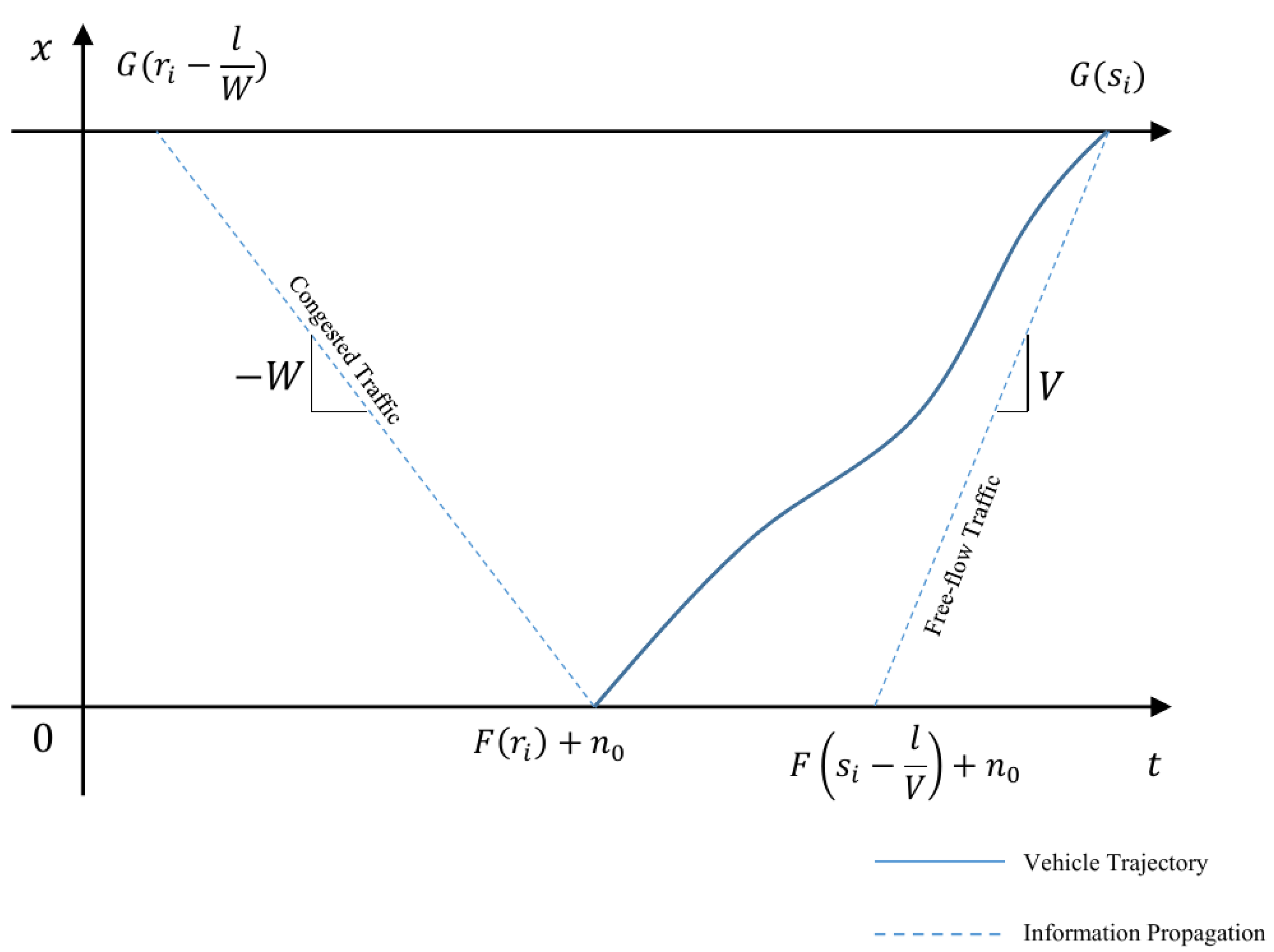
In Newell's simplified kinematic wave model, we have

$$n(t,x) = \min\left\{F\left(t-\frac{x}{V}\right)+n_0, G\left(t-\frac{l-x}{W}\right)+K(l-x)\right\}$$

This equation means that the cumulative flow is either determined by the upstream one (first term on the RHS) or the downstream one (second term on the RHS). From the information propagation point of view, the traffic state within a homogeneous link is determined by upstream and downstream combined. Newell's model is the variational theoretic formulation of the LWR model with a triangular fundamental diagram. In the original Newell's model, the road is assumed to be initially empty. However, from the Hopf-Lax formula of the Hamilton-Jacobi equation for the LWR model, the application domain of Newell's model can be extended for any initial conditions without a transonic rarefaction wave

• A Conceptual Framework

$F(t)$	The cumulative count at the upstream at t
$G(t)$	The cumulative count at the downstream at t
n_0	The initial number of vehicles within the segment
$n(t,x)$	The cumulative flow at location x and time t
V	Free-flow Speed
W	Shock-wave Speed
K	Jammed density
N	Total number of REID vehicles
ϵ_i, ξ_i	Random error in cumulative flow count
$X_i(t)$	Location of Vehicle i at time t
r_i	Entry time of vehicle i
s_i	Exit time of vehicle i



In the above plot, the solid line is the trajectory of vehicle i and the dash line is the backward traveling wave. The FIFO assumption leads to

$$G(s_i) + \epsilon_i = F(r_i) + n_0$$

According to Newell's simplified kinematic wave theory, when traffic congested, we have

$$G(s_i) = G\left(r_i - \frac{l}{W}\right) + Kl + \xi_i$$

The initial states and the parameters can be estimated by minimizing the sum of squared errors:

$$\min_{n_0,W,K} \sum_i \epsilon_i^2 + \xi_i^2$$

$$= \sum_i [G(s_i) - F(r_i) - n_0]^2 + \left[G\left(r_i - \frac{l}{W}\right) + Kl - G(s_i)\right]^2$$

• Solution Method

Notice that the objective function is consisted with two independent linear additive terms. Thus the objective function can be decomposed such that n_0 and W, K are estimated simultaneously (in parallel). This decomposition simplifies the optimization and allows the flexibility to use different methods respectively.

- LS estimator for n_0:

$$\hat{n}_0 = \sum_{i=1}^{N} \frac{[G(s_i) - F(r_i)]}{N}$$

- LS estimator for W, K:

$$\min_{\widehat{W},\widehat{K}} \sum_i \xi_i^2 = \left[G\left(r_i - \frac{l}{\widehat{W}}\right) + \widehat{K}l - G(s_i)\right]^2$$

Gauss-Newton Method:

- Starting from initial guess $\boldsymbol{\theta}^{(0)} = \left(W^{(0)}, K^{(0)}\right)$
- Update by iterating $\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)} - \left(\boldsymbol{J}^T\boldsymbol{J}\right)^{-1}\boldsymbol{J}^T\xi\left(\boldsymbol{\theta}^{(i)}\right)$

J-the Jacobian matrix of $\boldsymbol{\xi}\left(\boldsymbol{\theta}^i\right)$.
$\boldsymbol{\xi}$-the vector of $(\xi_1, \xi_2, \ldots, \xi_n)$

• A Real-World Example

The performance of the proposed method was evaluated in terms of its ability to estimate the initial traffic state (n_0), average density profile and upstream flow rate based on NGSIM I-101 data.

7:50 AM - 8:05 AM				
Case No.	MPR(%)	$\hat{n}(39)$	$\hat{W}$(mph)	$\hat{K}$(vpm)
1	100.00	38.31±0.00	20.00±0.00	156.51±0.00
2	50.00	38.33±0.23	23.51±2.37	144.67±7.10
3	20.00	38.25±0.49	23.19±3.29	145.99±9.08
4	5.00	38.32±1.02	23.39±5.11	146.78±12.98
8:05 AM - 8:20 AM				
Case No.	MPR(%)	$\hat{n}(39)$	$\hat{W}$(mph)	$\hat{K}$(vpm)
5	100.00	38.92±0.00	25.67±0.00	141.46±0.00
6	50.00	38.91±0.25	21.81±1.71	152.85±5.65
7	20.00	39.05±0.53	21.71±2.01	153.52±6.41
8	5.00	38.98±1.18	22.46±3.05	151.60±9.21
8:20 AM - 8:35 AM				
Case No.	MPR(%)	$\hat{n}(51)$	$\hat{W}$(mph)	$\hat{K}$(vpm)
9	100.00	51.32±0.00	21.15±0.00	157.98±0.00
10	50.00	51.34±0.33	20.90±0.69	158.97±2.36
11	20.00	51.36±0.62	20.98±1.14	158.80±3.80
12	5.00	51.32±1.34	21.45±2.15	157.52±6.97

• Conclusions and Discussions

- ❑ The algorithm is effective as validated using the NGSIM dataset. The method provides reasonable results even under low market penetration rate(5%)
- ❑ The initial number of vehicles cannot be determined from flow counting sensors alone.
- ❑ The proposed method only works for a linear freeway segment under congested traffic states. Extra Modelling work is needed to improve the current method.