Hypothesis Tests for a Proportion

For Proportions

- Recall: proportion is used for categorical (0/1) variables
    - Ex: What proportion of grad students watch Game of Thrones?
- Same procedure as the tests for a mean
- Slight variation in notation

1. Assumptions

- Independent random sample - trials are Bernoulli trials
    - $n*p > 5$ and $n*(1-p) > 5$
- Sampling distribution is normal

2. Hypotheses

- Null Hypothesis

$$H_0 : \pi = \pi_0$$

- Alternative Hypothesis

$$H_A : \pi \neq \pi_0 \text{ or } \pi < \pi_0 \text{ or } \pi > \pi_0$$

3. Test Statistic

$$z = \frac{\hat{\pi} - \pi_0}{\sigma_{\hat{\pi}}}$$

$$\sigma_{\hat{\pi}} = \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$$

Notice we use z instead of t. That's because even small proportion samples don't use the t-distribution

Notice we use the null value in the standard error calculation

Practice!

You flip a coin 50 times and come up heads 30 times. Is the coin a trick coin?

Use an alpha of 0.05

1. Assumptions

- Independent random sample
    - $n*p > 5$ and $n*(1-p) > 5$
- Sample distribution is normal

2. Hypotheses

- Null: the probability of getting heads with this coin is 0.5

$$H_0 : \pi = 0.5$$

- Alternative: the probability of getting heads with this coin is not 0.5

$$H_A : \pi \neq 0.5$$

- Alpha: 0.05

3. Test statistic

$$\pi_0 = 0.5$$

$$n = 50$$

$$x = 30$$

$$\hat{\pi} = \frac{30}{50} = 0.6$$

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$$= \frac{0.6 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{50}}}$$

$$= \frac{0.1}{0.0707}$$

$$z = 1.414$$

4. P-value

Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	z
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0

(This is a two-tailed test, so we need to double the p-value)

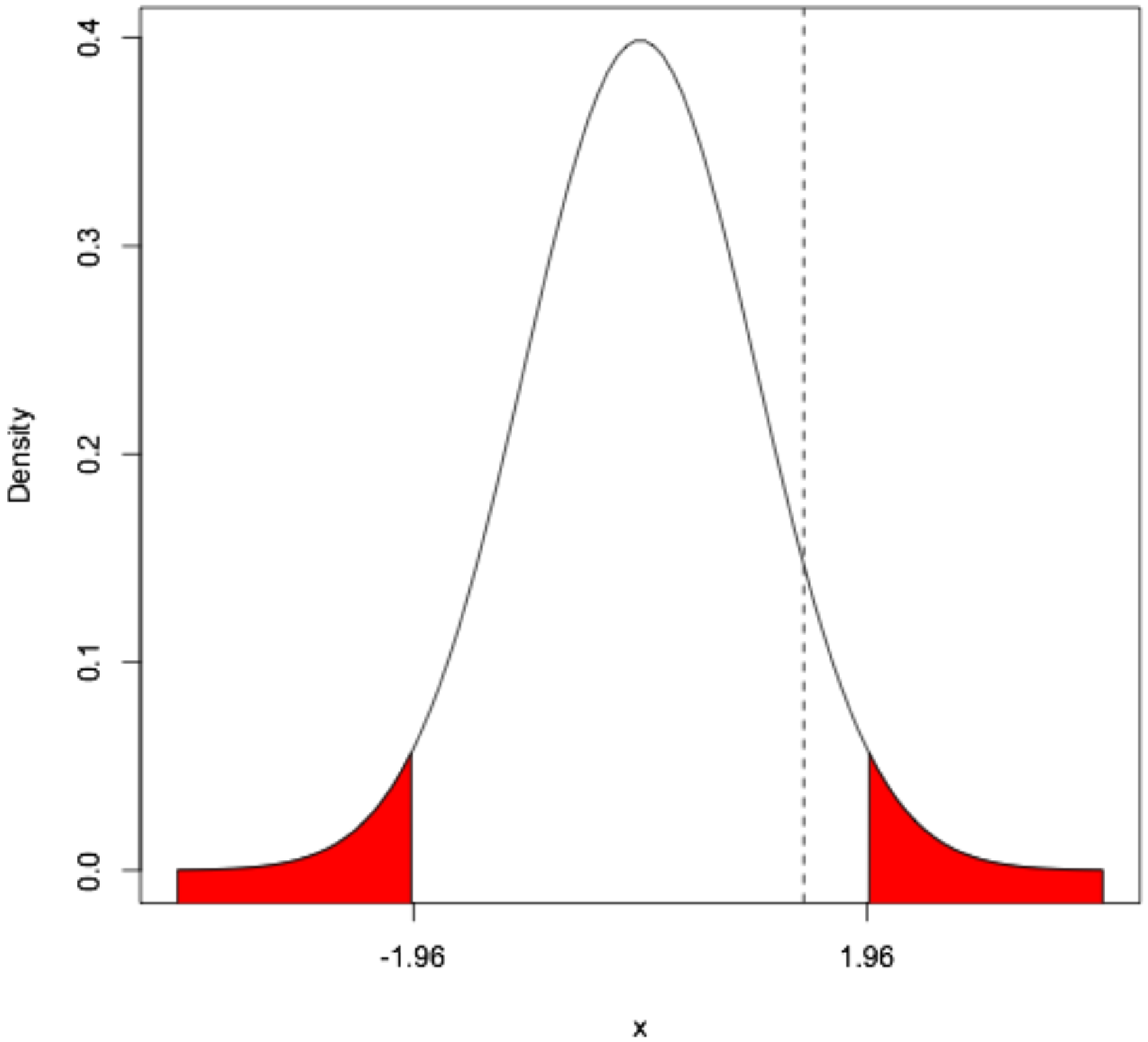
$$0.0793 * 2 = 0.1586$$

4. Critical Region

- The critical value for an alpha of 0.05 in a two-tailed test is 1.96

- Our test statistic is 1.414, which is less than the critical value

- Therefore, our test statistic does not lie within the critical region


Density
0.0
0.1
0.2
0.3
0.4
-1.96
1.96
x

5. Conclusion

- With a test statistic of 1.414 and a p-value of 0.1586, we fail to reject the null hypothesis

- There is not enough evidence to claim that the coin is a trick coin - 30 heads in 50 flips is a plausible outcome of a fair coin.

Practice!

A survey of 100 people find that 40 of them don’t like Beyonce. Test the hypothesis that less than half of all people don’t like Beyonce.

Use an alpha of 0.05

1. Assumptions

- Independent Random Sample
    - $n*p > 5$ and $n*(1-p) > 5$
- Sampling Distribution is Normal

2. Hypotheses

- Null: half of all people don't like Beyonce

$$H_0 : \pi = 0.5$$

- Alternative: less than half of all people don't like Beyonce

$$H_A : \pi < 0.5$$

- Alpha: 0.05

3. Test Statistic

$$\pi_0 = 0.5$$

$$n = 100$$

$$x = 40$$

$$\hat{\pi} = \frac{40}{100} = 0.4$$

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$$= \frac{0.4 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{100}}}$$

$$= \frac{-0.1}{0.05}$$

$$z = -2$$

4. P-value

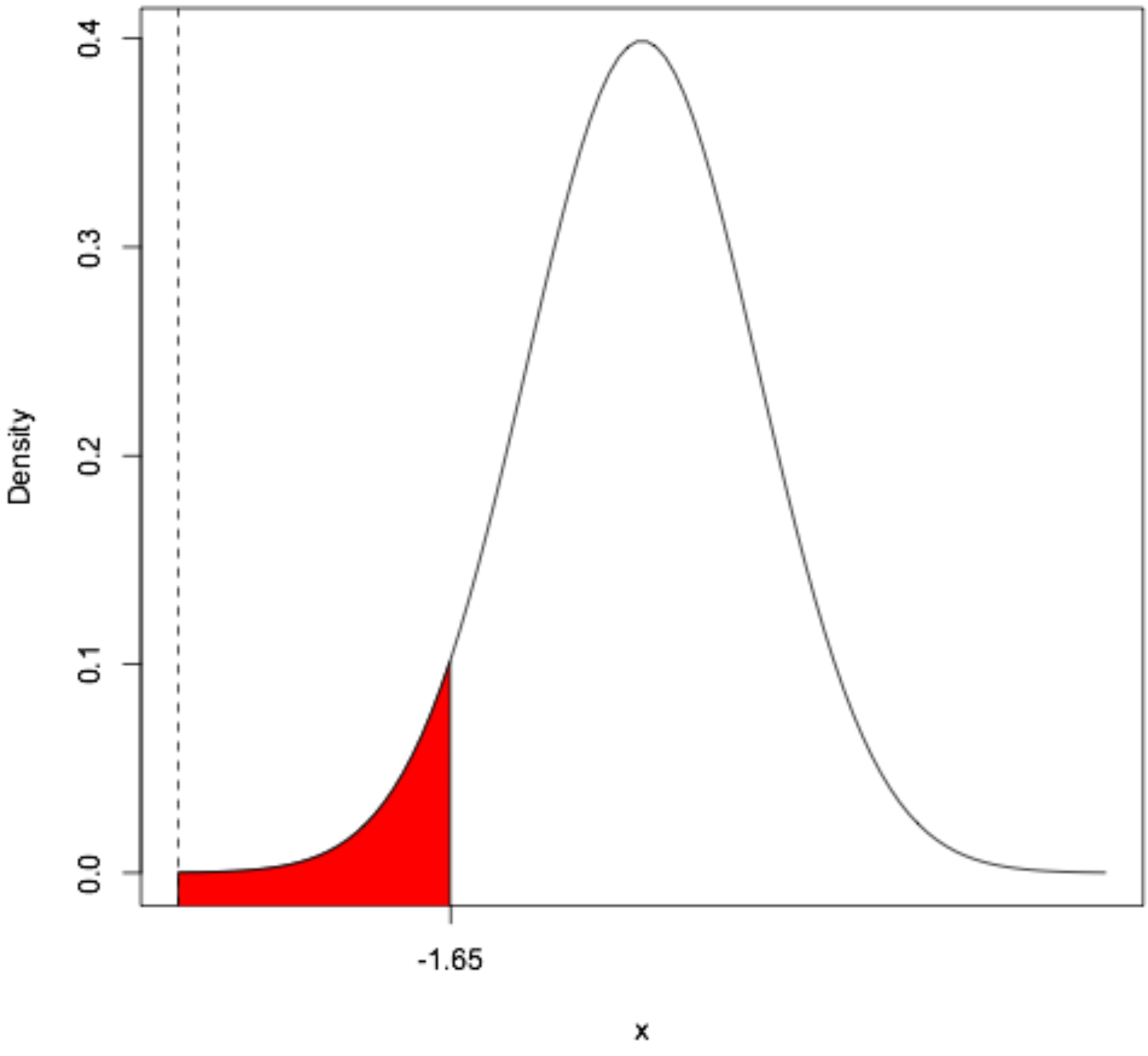
- The p-value is essentially 0

4. Critical Region

- The critical value for an alpha of 0.05 for a one tailed test is -1.65

- Our test statistic is -4, which is much lower than the critical value

- Therefore our test statistic is within the critical region


Density
0.0
0.1
0.2
0.3
0.4
-1.65
x

5. Conclusion

- With a test statistic of -2 and a p-value of approximately .0228, we reject the null hypothesis in favor of the alternative.

- There is enough evidence to say that the proportion of people who don’t like Beyonce is less than 0.5.

One more time!

Peter the Anteater claims that today's hottest night club is the AntHill Pub. It's so exclusive, only 2 out of every 5 people have even heard of the neighborhood it's in.

Practice

We interviewed 50 people to test that claim and found 25 people who had heard of the neighborhood.

Do more than 2 out of 5 people know where The AntHill Pub is located with an alpha of 0.05?

1. Assumptions

- Independent random sample
    - $n*p > 5$ and $n*(1-p) > 5$
- Sampling distribution is normal

2. Hypotheses

- Null: only 2 out of 5 people know where the hottest new club is

$$H_0 : \pi = 0.4$$

- Alternative: more than 2 out of 5 people know where the hottest new club is

$$H_A : \pi > 0.4$$

3. Test Statistic

$$\pi_0 = 0.4$$

$$x = 25$$

$$n = 50$$

$$\alpha = 0.05$$

$$\hat{\pi} = \frac{25}{50} = 0.5$$

$$Z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$$= \frac{0.5 - 0.4}{\sqrt{\frac{0.4(1-0.4)}{50}}}$$

$$= \frac{0.1}{\sqrt{\frac{0.24}{50}}}$$

$$= \frac{0.1}{0.0693}$$

$$Z = 1.443$$

4. P-Value

Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	z
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0

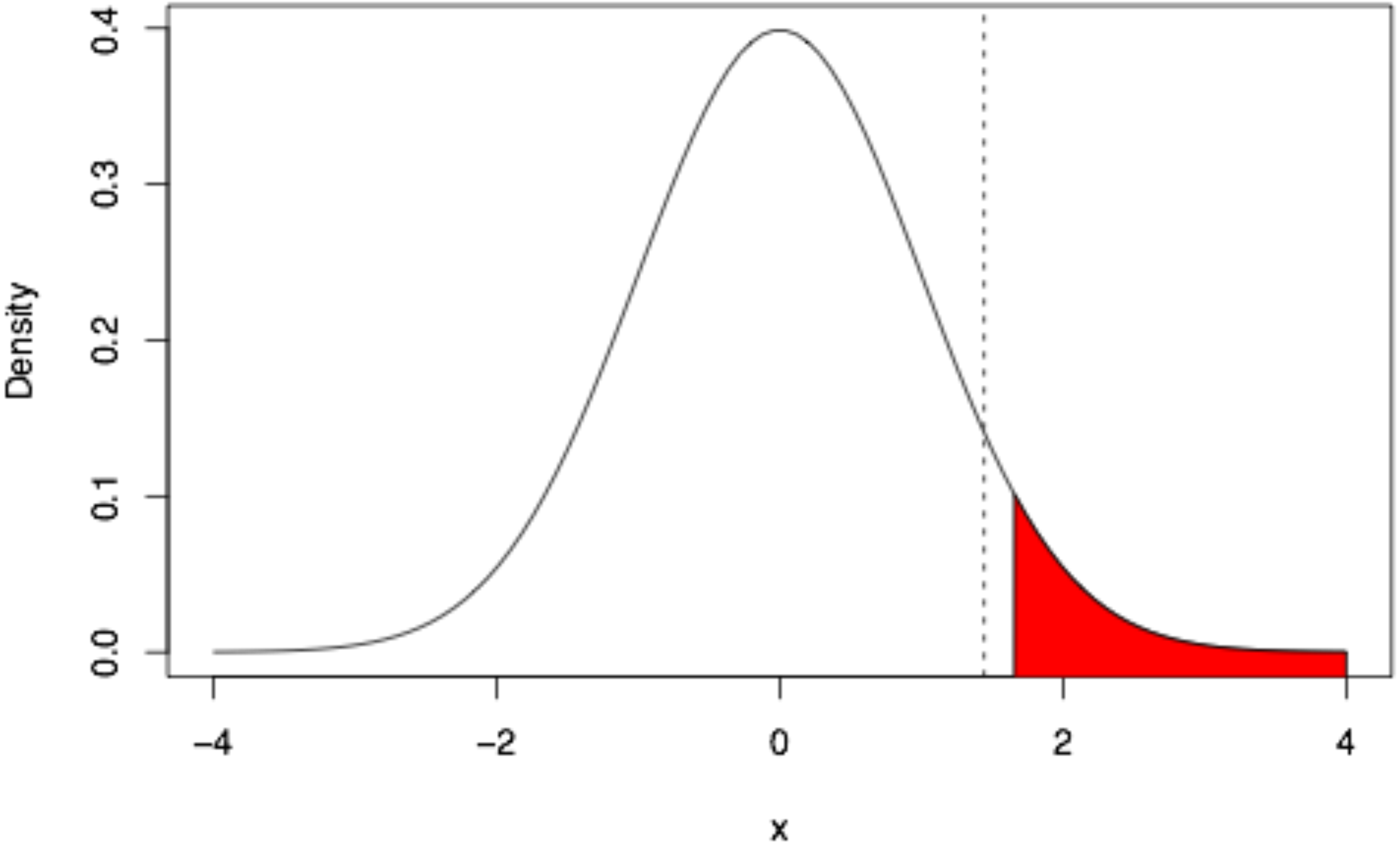
The p-value associated with our test statistic is 0.0749. Since this is a one tailed test, we do NOT need to double it.

4. Critical Region

- For a one tailed test with an alpha of 0.05, the critical value is 1.65

- Our test statistic is 1.443, which is less than the critical value

- Therefore, our test statistic does not lie in the critical region


Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
x

5. Conclusion

- With a p-value of 0.0749 and a test statistic of 1.443, we fail to reject the null hypothesis

- There is not enough evidence to claim that more than 2 out of every 5 people have heard of the neighborhood the AntHill Pub is located in.

Break