6.1 Areas Between Curves

\[ \text{Area} = \int_a^b f(x) \, dx \]

\[ \text{Area} = \int_a^b f(x) \, dx - \int_a^b g(x) \, dx \]

\[ = \int_a^b (f(x) - g(x)) \, dx \]
EXAMPLE Find the area of the region bounded above by \( y = e^x \), bounded below by \( y = x \), and bounded on the sides by \( x = 1 \) and \( x = 2 \).

\[
\text{Area} = \int_1^2 e^x \, dx - \int_1^2 x \, dx \\
= \int_1^2 (e^x - x) \, dx \\
= \left[ e^x - \frac{x^2}{2} \right]_{x=1}^{x=2} \\
= \left( e^2 - \frac{2^2}{2} \right) - \left( e^1 - \frac{1^2}{2} \right) \\
= e^2 - e - \frac{3}{2}
\]
EXAMPLE  Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi/2$.

\[ A_1 = \int_0^{\pi/4} (\cos x - \sin x) \, dx \]
\[ = \left[ \sin x + \cos x \right]_{x=0}^{x=\pi/4} \]
\[ = \left( \sin \left( \frac{\pi}{4} \right) + \cos \left( \frac{\pi}{4} \right) \right) - \left( \sin(0) + \cos(0) \right) \]
\[ = \sqrt{2} - 1 \]

\[ A_2 = \int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx \]
\[ = \sqrt{2} - 1 \]

Area = $A_1 + A_2 = 2\sqrt{2} - 2$
Find the area of the shaded region.

\[
\int_1^8 \left( \frac{3\sqrt{x}}{x} - \frac{1}{x} \right) \, dx
\]
Find the area of the shaded region.

\[
\int_0^1 (e^x - xe^{x^2}) \, dx
\]
Find the area of the shaded region.

\[ \int_{-1}^{1} \left[ e^y - (y^2 - 2) \right] \, dy \]
Find the area of the shaded region.

\[
\int_0^3 [(2y - y^2) - (y^2 - 4y)] \, dy
\]