MIDTERM 1 – STUDY GUIDE

The Midterm takes place on **Friday, May 4, 2018** during our usual lecture time in our usual lecture room. **NO** books/notes/calculators/cheat sheets will be allowed. **Please bring your ID, for verification purposes.** It counts for 30 % of your grade, and covers sections 10.1 – 10.3, all of Chapter 12, bits and pieces of Chapter 13, and everything up to and including Section 14.4 (Tangent planes and Linear Approximations). This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, the practice exam, and the suggested homework.

The format of the exam will be as follows: There are 6 problems in total, 2 of which will be from Chapters 10/13, 2 of which from Chapter 12, and 2 of which from Chapter 14. Know how to:

SECTION 10.1: CURVES DEFINED BY PARAMETRIC EQUATIONS

- Sketch a parametric curve in 2 dimensions by plotting some points
- Find the parametric equations of a circle, of a line segment (see Lecture 1), and of an ellipse (see 10.1.34)
- I will **NOT** ask you about qualitative graphs like I did in lecture
- Ignore the sections on graphing devices, the cycloid, and families of parametric curves

SECTION 10.2: CALCULUS WITH PARAMETRIC CURVES

- Find the derivative $\frac{dy}{dx}$ of a parametric curve and use it to find the equation of a tangent line to the curve at a point
- Find where the tangent line is horizontal or vertical
- Find the second derivative $\frac{d^2y}{dx^2}$ of a parametric curve, and find where it is concave up/down.
- Find the area under a parametric curve, or between two parametric curves
- Remember the ellipse example from Lecture 2.
- The ellipse problem involved finding $\int_0^{\pi} \sin^2(t)dt$; know how to find it!
- Find the arclength of a parametric curve

- Ignore the section on Surface Area

Section 10.3: Polar Coordinates

- Convert from polar coordinates to Cartesian coordinates; I will not ask you the opposite!
- Sketch polar curves; in my opinion it's easiest to plot some points. In case you don't know which θ to pick first, look at the first value where $r = 0$, so for, say, $\cos(2\theta)$ it should be $\theta = \frac{\pi}{4}$
- Find derivatives of polar curves. I don't recommend memorizing the formula, it's easier to start with $\frac{dy}{dx}$ and use the formulas for x and y.
- Find the slope and the equation of a tangent line at a given θ
- Find where a tangent line is horizontal or vertical. Remember your trig for that!
- I won't ask you for length of polar curves, but I could ask you how to derive it (see AP2 on HW2)
- Ignore the sections on 'Symmetry' and 'Graphing Polar Curves with Graphing Devices'

Section 12.1: Three-Dimensional Coordinate Systems

- Plot points in $3D$.
- **QUICKLY** sketch surfaces like $z = 3$, $x = 3$, $y = 3$ but also regions like $-1 \leq y \leq 2$ (see the trick in lecture)
- **QUICKLY** sketch cylinders like $x^2 + y^2 = 1$
- Know the equation of a sphere
- Know how to find equations by completing the square, like in 12.1.17

Section 12.2: Vectors

- Hopefully this section is not too crazy for you!
- Know what it means for vectors to be parallel
- Produce a unit vector in the same direction as another vector
- Ignore the section on Applications

Section 12.3: The Dot Product

- Know the definition of the dot product, the Angle Formula, and the fact that two vectors are perpendicular if and only if their dot product is 0.
- Find the angle between vectors

- Know the formula for the $\hat{\mathbf{u}} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$, the projection of $\mathbf{u}$ on a vector $\mathbf{v}$. I will only ask you about vector projection, **NOT** scalar projection! The way to remember this is: First you want a multiple of $\mathbf{v}$, so $\hat{\mathbf{u}} = c\mathbf{v}$ and to find c, use the hugging analogy in lecture!
- Ignore the section on direction angles and direction cosines, and the section on work.

SECTION 12.4: THE CROSS PRODUCT

- Know the definition of the cross product of two vectors
- The only property about cross products you need to know is that $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$; you **DON'T** need to know any of the other properties, like the angle formula, or torque
- Know how to find the area of a parallelogram; you are **NOT** responsible for the parallelipiped
- Ignore the sections on Triple Products and on Torque
- Remember the Additional problem on HW 3

SECTION 12.5: LINES AND PLANES

- This is definitely the trickiest section for the midterm, because there are so many things that I could ask you! That said, please remember the following:
  - To find the equation of a line, find a **point** on the line and a **direction vector**
  - To find the equation of a plane, find a **point** on the plane and a **normal vector**
- Seriously, all the questions about lines and planes boil down to this!!!
- Here are some questions I could ask you:
  - Find the equations of a line through a point and with a given direction vector
  - Where does a line with a given equation intersect the xy-plane? (or $xz-$plane or $yz-$plane)
  - I will **NOT** ask you about symmetric equations
  - Find equations for a line through two points
  - Are two lines (with given equations) parallel, intersecting, or skew? If so, find the point of intersection
  - Sketch a plane with given equations
  - Find the equation of a plane going through a point and with a given normal vector
  - Find a plane containing three points
  - Find the angle between two planes

  - When does a line intersect a given plane (see Example 6 on page 828)
  - Find the line of intersection between two planes. To find the point, it's ok if you just wing it, as long as you give me the correct answer
  - Find a plane containing two lines (see mock exam)
  - Find a plane containing a line and a point (see mock exam)
- Remember that this section is also about the equation of a segment
- For the distance formula: Do **NOT** memorize it (I will provide it to you if necessary), but do know how to use it
- Find the distance between two parallel planes: See Example 10; I didn't have time to cover it in lecture, but you're responsible for it.

SECTION 12.6: CYLINDERS AND QUADRIC SURFACES

- **YOU ABSOLUTELY NEED TO MEMORIZE THE 6 FORMULAS ON TABLE 1 ON PAGE 837!!!!!!!** It is crucial for your survival for the rest of the course! I accept the terms "Saddle" (for hyperbolic paraboloid), "Dress" (for hyperboloid of one sheet), and "Two cups" (for hyperboloid of two sheets), and for elliptic paraboloid you can just call it "Paraboloid." Know the names and how to draw them
- You don't have to use traces if you find that useless
- I could ask you a question about completing the square
- I could ask you those surfaces but in another direction (like $x^2 = y^2 + z^2$, which is a cone in the x direction)
- Know how to sketch the region between two surfaces
- Know how to draw cylinders (like $x^2 + y^2 = 1$ or $z = \sin(y)$ or $z = x^2$)

SECTION 13.1: VECTOR FUNCTIONS AND SPACE CURVES

- Find domains and limits of vector functions
- Draw a 3D parametric curve by plotting points
- The only qualitative thing I could ask you is the helix
- Find parametric equations for the intersection of two surfaces
- Skip the section "Using Computers to Draw Space Curves"

SECTION 13.2: DERIVATIVES AND INTEGRALS OF VECTOR FUNCTIONS

- Hopefully this section is easy peasy!
- Find derivatives of vector functions and parametric equations of tangent lines at a point

- Find the unit tangent vector to a curve
- Know the formula $(u \cdot v)' = u' \cdot v + u \cdot v'$ and similar for cross products
- Find Integrals of vector functions; remember to have different constants!

SECTION 13.3: ARCLENGTH AND CURVATURE

- In this section, I'll only ask about arclength

SECTION 13.4: MOTION IN SPACE: VELOCITY AND ACCELERATION

- Only know how to find velocity, acceleration, and speed, given a position vector; and how to find position given an acceleration (or velocity) vector; skip everything else.

SECTION 14.1: FUNCTIONS OF SEVERAL VARIABLES

- Section 14.1 is important but has a ton of useless information, so you can skip most of the reading, **but** know how to do Examples 1, $4-6$, 8, $10-15$.
- Find the domain of a function of two or three variables; I won't ask about range.
- Sketch the graph of a function of two variables; if I ask you that, it'll be one of the 7 surfaces you'll need to know, or just a plane.
- Sketch the level curves/surfaces of a function of two/three variables. I can either ask you to find specific level curves like $z = 1$, or to find all of them $z = k$. Sometimes you'll need to split it into cases depending on whether the right-hand-side is positive or negative.
- **You do not need to know how to draw the graph of a function given a level curve,** so for the $z = xy$ example in Lecture 10, you just need to know how to draw the contour plot, not the whole graph

SECTION 14.2: LIMITS AND CONTINUITY

- Know how to show that a limit does not exist. I will only ask you about 3 directions: Along the $x-$ axis, along the $y-$ axis, and along $y = x$. You're not responsible for parabolic directions (like Example 3)
- I will **NOT** ask you how to do an epsilon-delta argument, and you can skip the section on "Functions of Three or More Variables."
- Know how to find limits using the polar coordinate trick (like 14.2.39 and 14.2.40)
- Show a function is or is not continuous at a point
- Find where a function is continuous

SECTION 14.3: PARTIAL DERIVATIVES

- Calculate partial derivatves f_x and f_y (and f_z) of a function, either in general or at a point.
- You also need to know how to do that with implicit equations, like Example 5 on page 917
- You don't need to know the definition of partial derivatives as a limit (basically ignore the first 2-3 pages of section 14.3), and you don't need to know how to interpret them as slopes (page 915).
- Calculate higher-order partial derivatives like f_{xx} or f_{xy} or $f_{xxxxyyyyxxx}$ (hopefully not that complicated :P)
- Know Clairaut's theorem: $f_{xy} = f_{yx}$
- Show that a given function satisfies a partial differential equation
- Skip the section on The Cobb-Douglas Production Function

SECTION 14.4: TANGENT PLANES AND LINEAR APPROXIMATIONS

- Find the equation of a tangent plane of a function at a point
- Find the linear approximation of a function at a point and use it to approximate values of f like $f(1.01, 0.99)$
- Use differentials to approximate an error (like Example 6 on page 934)
- There is a lot of useless information in this section; if you know how to do Examples 1, 2, 4, 5, 6, then you're fine! In particular, you can ignore Definition 7 and Theorem 8 and Example 3.