FINAL EXAM – STUDY GUIDE

The Final Exam takes place on **Wednesday, June 13, 2018, from 10:30 AM to 12:30 PM in 1100 Donald Bren Hall** (**not** the usual lecture room!!!) **NO** books/notes/calculators/cheat sheets will be allowed. **Please bring your ID, for verification purposes.** It counts for 50 $\%$ of your grade, and covers sections 10.1 – 10.3, 12.1 – 12.6, 13.1 – 13.4, 14.1 – 14.8, and 15.1 – 15.3 and 15.6. That said, $70-80\%$ of the exam will be on the material after the midterm (section 14.5 and onwards). I expect this exam to be much harder than the midterm, so please study hard for it. It will have about $9-12$ questions. This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, the practice exam, and the suggested homework.

Useful trig identities to know:

(1) $\sin^2(x)+\cos^2(x)=1$
(2) $1+\tan^2(x)=\sec^2(x)$
(3) $\cos(-x)=\cos(x), \sin(-x)=-\sin(x)$
(4) $\sin(2x)=2\sin(x)\cos(x)$, $\cos(2x)=\cos^2(x)-\sin^2(x)$

Know how to:

Section 10.1: Curves Defined by Parametric Equations

- Sketch a parametric curve in 2 dimensions by plotting some points
- Find the parametric equations of a circle, of a line segment (see Lecture 1), and of an ellipse (see 10.1.34)
- I will **NOT** ask you about qualitative graphs like I did in lecture
- Ignore the sections on graphing devices, the cycloid, and families of parametric curves

Section 10.2: Calculus with Parametric Curves

- Find the derivative $\frac{dy}{dx}$ of a parametric curve and use it to find the equation of a tangent line to the curve at a point
- Find where the tangent line is horizontal or vertical
- Find the second derivative $\frac{d^2y}{dx^2}$ of a parametric curve, and find where it is concave up/down.

- Find the area under a parametric curve, or between two parametric curves
- Remember the ellipse example from Lecture 2.
- Know how to find $\int_0^\pi \sin^2(t)dt$ and $\int_0^\pi \cos^2(t)dt$
- Find the arclength of a parametric curve
- Ignore the section on Surface Area

Section 10.3: Polar Coordinates

- Convert from polar coordinates to Cartesian coordinates; I will not ask you the opposite!
- Sketch polar curves; in my opinion it's easiest to plot some points. In case you don't know which θ to pick first, look at the first value where $r = 0$, so for, say, $\cos(2\theta)$ it should be $\theta = \frac{\pi}{4}$
- Find derivatives of polar curves. I don't recommend memorizing the formula, it's easier to start with $\frac{dy}{dx}$ and use the formulas for x and y.
- Find the slope and the equation of a tangent line at a given θ
- Find where a tangent line is horizontal or vertical. Remember your trig for that!
- I won't ask you for length of polar curves, but I could ask you how to derive it (see AP2 on HW2)
- Ignore the sections on 'Symmetry' and 'Graphing Polar Curves with Graphing Devices'

Section 12.1: Three-Dimensional Coordinate Systems

- Plot points in $3D$.
- **QUICKLY** sketch surfaces like $z = 3$, $x = 3$, $y = 3$ but also regions like $-1 \leq y \leq 2$ (see the trick in lecture)
- **QUICKLY** sketch cylinders like $x^2 + y^2 = 1$
- Know the equation of a sphere
- Know how to find equations by completing the square, like in 12.1.17

Section 12.2: Vectors

- Hopefully this section is not too crazy for you!
- Know what it means for vectors to be parallel
- Produce a unit vector in the same direction as another vector
- Ignore the section on Applications

SECTION 12.3: THE DOT PRODUCT

- Know the definition of the dot product, the Angle Formula, and the fact that two vectors are perpendicular if and only if their dot product is 0.
- Find the angle between vectors
- Know the formula for the $\hat{\mathbf{u}} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$, the projection of $\mathbf{u}$ on a vector $\mathbf{v}$. I will only ask you about vector projection, **NOT** scalar projection! The way to remember this is: First you want a multiple of $\mathbf{v}$, so $\hat{\mathbf{u}} = c\mathbf{v}$ and to find c, use the hugging analogy in lecture!
- Ignore the section on direction angles and direction cosines, and the section on work.

SECTION 12.4: THE CROSS PRODUCT

- Know the definition of the cross product of two vectors
- The only property about cross products you need to know is that $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$; you **DON'T** need to know any of the other properties, like the angle formula, or torque
- Know how to find the area of a parallelogram; you are **NOT** responsible for the parallelipiped
- Ignore the sections on Triple Products and on Torque
- Remember the Additional problem on HW 3

SECTION 12.5: LINES AND PLANES

- Remember the following:
  - To find the equation of a line, find a **point** on the line and a **direction vector**
  - To find the equation of a plane, find a **point** on the plane and a **normal vector**
- Here are some questions I could ask you:
  - Find the equations of a line through a point and with a given direction vector
  - Where does a line with a given equation intersect the xy-plane? (or $xz-$plane or $yz-$plane)
  - I will **NOT** ask you about symmetric equations
  - Find equations for a line through two points
  - Are two lines (with given equations) parallel, intersecting, or skew? If so, find the point of intersection
  - Sketch a plane with given equations
  - Find the equation of a plane going through a point and with a given normal vector
  - Find a plane containing three points

  - Find the angle between two planes
  - When does a line intersect a given plane (see Example 6 on page 828)
  - Find the line of intersection between two planes. To find the point, it's ok if you just wing it, as long as you give me the correct answer
  - Find a plane containing two lines (see mock midterm)
  - Find a plane containing a line and a point (see mock midterm)
- Remember that this section is also about the equation of a segment
- For the distance formula: Do **NOT** memorize it (I will provide it to you if necessary), but do know how to use it
- Find the distance between two parallel planes: See Example 10; I didn't have time to cover it in lecture, but you're responsible for it.

SECTION 12.6: CYLINDERS AND QUADRIC SURFACES

- **YOU ABSOLUTELY NEED TO MEMORIZE THE 6 FORMULAS ON TABLE 1 ON PAGE 837!!!!!!!** I accept the terms "Saddle" (for hyperbolic paraboloid), "Dress" (for hyperboloid of one sheet), and "Two cups" (for hyperboloid of two sheets), and for elliptic paraboloid you can just call it "Paraboloid." Know the names and how to draw them
- You don't have to use traces if you find that useless
- I could ask you a question about completing the square, like the question on the midterm.
- I could ask you those surfaces but in another direction (like $x^2 = y^2 + z^2$, which is a cone in the x direction)
- Know how to sketch the region between two surfaces
- Know how to draw cylinders (like $x^2 + y^2 = 1$ or $z = \sin(y)$ or $z = x^2$)

SECTION 13.1: VECTOR FUNCTIONS AND SPACE CURVES

- Find domains and limits of vector functions
- Draw a 3D parametric curve by plotting points
- The only qualitative thing I could ask you is the helix
- Find parametric equations for the intersection of two surfaces
- Skip the section "Using Computers to Draw Space Curves"

SECTION 13.2: DERIVATIVES AND INTEGRALS OF VECTOR FUNCTIONS

- Hopefully this section is easy peasy!

- Find derivatives of vector functions and parametric equations of tangent lines at a point
- Find the unit tangent vector to a curve
- Know the formula $(u \cdot v)' = u' \cdot v + u \cdot v'$ and similar for cross products
- Find Integrals of vector functions; remember to have different constants!

SECTION 13.3: ARCLENGTH AND CURVATURE

- In this section, I'll only ask about arclength

SECTION 13.4: MOTION IN SPACE: VELOCITY AND ACCELERATION

- Only know how to find velocity, acceleration, and speed, given a position vector; and how to find position given an acceleration (or velocity) vector; skip everything else.

SECTION 14.1: FUNCTIONS OF SEVERAL VARIABLES

- Section 14.1 is important but has a ton of useless information, so you can skip most of the reading, **but** know how to do Examples 1, 4 – 6, 8, 10 – 15.
- Find the domain of a function of two or three variables; I won't ask about range.
- Sketch the graph of a function of two variables; if I ask you that, it'll be one of the 7 surfaces you'll need to know, or just a plane.
- Sketch the level curves/surfaces of a function of two/three variables. I can either ask you to find specific level curves like $z = 1$, or to find all of them $z = k$. Sometimes you'll need to split it into cases depending on whether the right-hand-side is positive or negative.
- **You do not need to know how to draw the graph of a function given a level curve,** so for the $z = xy$ example in Lecture 10, you just need to know how to draw the contour plot, not the whole graph

SECTION 14.2: LIMITS AND CONTINUITY

- Know how to show that a limit does not exist. I will only ask you about 3 directions: Along the $x-$ axis, along the $y-$ axis, and along $y = x$. You're not responsible for parabolic directions (like Example 3)
- I will **NOT** ask you how to do an epsilon-delta argument, and you can skip the section on "Functions of Three or More Variables."
- Know how to find limits using the polar coordinate trick (like 14.2.39 and 14.2.40)

- Show a function is or is not continuous at a point
- Find where a function is continuous

Section 14.3: Partial Derivatives

- Calculate partial derivatves f_x and f_y (and f_z) of a function, either in general or at a point.
- You also need to know how to do that with implicit equations, like Example 5 on page 917
- You don't need to know the definition of partial derivatives as a limit (basically ignore the first 2-3 pages of section 14.3), and you don't need to know how to interpret them as slopes (page 915).
- Calculate higher-order partial derivatives like f_{xx} or f_{xy} or $f_{xxxxyyyyxxx}$ (hopefully not that complicated :P)
- Know Clairaut's theorem: $f_{xy} = f_{yx}$
- Show that a given function satisfies a partial differential equation
- Skip the section on The Cobb-Douglas Production Function

Section 14.4: Tangent Planes and Linear Approximations

- Find the equation of a tangent plane of a function at a point
- Find the linear approximation of a function at a point and use it to approximate values of f like $f(1.01, 0.99)$
- Use differentials to approximate an error (like Example 6 on page 934)
- There is a lot of useless information in this section; if you know how to do Examples 1, 2, 4, 5, 6, then you're fine! In particular, you can ignore Definition 7 and Theorem 8 and Example 3.

Section 14.5: The Chain Rule

- Make sure you are comfortable with the chain rule, to the point that you don't need the tree diagrams any more!
- Skip Example 2
- Use the chain rule to find $\frac{\partial z}{\partial t}$, where $z = z(x(t), y(t))$ (= one variable). I may ask you to do this at specific points (say $t = 0$).
- Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$, where $z = z(x(s,t), y(s,t))$.
- Of course all this works for functions of any number of variables
- Find higher-order partial derivatives, like z_{st}
- Find derivatives of implicit functions, like $\frac{\partial z}{\partial x}$ where $x^3 + y^3 + z^3 + 6xyz = 1$. Here it's up to you: You can either do it directly, using the methods in section 14.3, or you can use Formulas 6 and 7. So only memorize those formulas if you find them useful.

SECTION 14.6: THE DIRECTIONAL DERIVATIVE AND THE GRADIENT VECTOR

- Remember that to calculate directional derivatives, you need to have a **unit** vector!
- Find the gradient ∇f of a function, and evaluate it at a point P.
- **Know that the ∇f is perpendicular to level curves of f.**
- Find the directional derivative $D_{\mathbf{u}}f$, where $\mathbf{u}$ is given.
- Find in which direction f increases the fastest, and find the greatest rate of increase of f
- Find the equation of the tangent plane to a given surface at a given point. Don't worry about normal lines.
- Also make sure to look at problems 51, 54, and 61 in that section.

SECTION 14.7: MAXIMUM AND MINIMUM VALUES

- Don't worry about all the definitions and theorems in that section; as long as you know the computations, you're fine! Also note that what the book calls the Second Derivatives Test, I call the Saddle Point test and the Second Derivative test
- Here's my suggestion: First find the critical points, then apply the saddle point test. If the determinant is negative (you 'failed' the test), then it's a saddle. If it's positive, then move on and apply the Second Derivative test and look at f_{xx}.
- Find and classify the critical points of a function, that is, say if f has a local maximum, local minimum, or a saddle point at a critical point.
- Find the local max/min/saddle points of a function (same thing, but you calculate f at the point)
- Ignore problem 17
- Find the absolute max/min of a function on rectangle or a triangle. Remember that for **absolute** max/min, you do **NOT** have to use the second derivative test.
- Solve word problems using max/min. Some examples include, but are not limited to:
  - Find the point on the plane (or a surface) that is closest to a given point (problems 42, 43); remember to square things with square roots!
  - Find the biggest volume of a box inside a sphere (47)
  - Find the smallest surface area of a box with a given volume (48)
  - Find the biggest volume of a box with a given surface area (50)

- **Note:** In word problems, **UNLESS** I tell you not to use the second derivative test, you **HAVE** to use the second derivative test! The books skips it sometimes, but you'll have to do that!

SECTION 14.8: LAGRANGE MULTIPLIERS

- Remember that for Lagrange multipliers, you do **NOT** need to use the second derivative test!
- Find the absolute max/min of f with a given constraint g. Here are some examples of tricks to remember:
  - Do it by cases (first example in lecture) For example, if you have $x = \lambda x$, then $x(1-\lambda) = 0$, so $x = 0$ or $\lambda = 1$, and this gives you already 2 cases.
  - Solve for x, y, z in terms of λ (second example in lecture). For example, if $x - 3 = \lambda x$, then $x(1-\lambda) = 3$, and $x = \frac{3}{1-\lambda}$, and you do the same for y and z, and you plug it into the constraint to solve for λ.
  - Set everything equal to λ, like the other IKEA problem.
- Some Lagrange multiplier problems get quite ridiculous; I'll try to make it reasonable, so don't expect anything crazy like problems 6, 9, or 11.
- Remember to use the constraint **last**.
- Find the absolute max/min of f on a region, like problem 21. This just means find the critical points and use Lagrange multipliers. Make sure your critical points are in fact inside your region!
- Solve word problems using Lagrange multipliers. Again, some examples include, but are not limited to:
  - Find the point on the plane (or a surface) that is closest to a given point remember to square things with square roots!
  - Find the smallest surface area of a box with a given volume (38)
  - Find the biggest volume of a box with a given surface area (40)
- **Note:** Don't think that you can just skip Lagrange multipliers; I may ask you to do a max/min problem where you **HAVE** to use Lagrange multipliers!
- Also look at 49
- Ignore the section on "Two Constraints."

SECTION 15.1: DOUBLE INTEGRALS OVER RECTANGLES

- You do **NOT** need to know the full definition of the integral with double sums, so ignore the first 3 pages of 15.1. As long as you know how to do problem 1 in 15.1, you're all set with Riemann sums. ou also don't need to know the statement of Fubini's theorem and the part about slices on page 994, but know how to use it. Ignore formula 11 unless you find it useful, and ignore Example 9.

- Estimate the volume under a function by using upper right points, and by using midpoints (like Problem 1)
- Calculate a double integral over a rectangle (like 15 or 27)
- Remember that sometimes it's useful (or even necessary) to interchange dx and dy, like $\int_0^\pi \int_1^2 x\cos(xy)dxdy$
- Evaluate a double integral by interpreting it as a volume (like 9 or 12 or the problems in lecture about the sphere or the cylinder, see also 66 in 15.2)
- Find the average value of f over a given region (see also 62 in 15.2)

Section 15.2: Double Integrals over General Regions

- As long as you know how to do the examples, you'll be fine! You do not need to know the definition of type 1 and type 2 regions, and hopefully the properties of double integrals should be straightforward enough that you don't have to memorize them.
- Find the double integral of a function over a general region. Sometimes the region is a vertical region, so you'll have to do Smaller $\leq y \leq$ Bigger, and sometimes it's a horizontal region, in which case you have to solve for x in terms of y and do Left $\leq x \leq$ Right.
- Change the order of integration; for example, write $\int_0^1 \int_x^1 \sin(y^2)dydx$ as $dxdy$ and evaluate the integral
- Sometimes the region is composed of several pieces, like in 57. In this case you have to split up the integral over each piece.
- Sketch the solid whose volume is given by a given integral (39)
- Use the comparison property to estimate a given integral, like Example 6 or the example presented in lecture.

Section 15.3: Double Integrals in Polar Coordinates

- You only need to know Formula 2 on top of page 1012, as well as Examples 1 and 2; ignore Examples 3 and 4. The hardest trig integral I can ask you is $\int \sin^2(x)$ or $\int \cos^2(x)$, I won't ask you about $\int \sin^4(x)$ or stuff like that.
- Evaluate an integral by changing to polar coordinates. Remember that this is excellent if you see $x^2 + y^2$ or your region is a disk or a wedge or a ring (annulus). **Don't forget about the r!!!**
- Find the volume between two surfaces; in particular remember the ice cream cone problem I did in lecture, it's a good practice for 15.6

- Calculate $\int_{-\inf}^{\inf} e^{-x^2} dx$ (I'll do that in the very last lecture, or check out this YouTube video, but I recommend seeing it live ☺)

Section 15.6: Triple Integrals

- You **don't** need to know the definition of the integral or Fubini's theorem (but of course know how to use it) and you don't need to know the names of the regions (and personally I find the formulas on page 1031 confusing). Finally, the only applications you need to know are Formula 12 on page 1034 (volume) and the average value (exercise 53), you are **not** responsible for moments, center of mass, etc.
- Find a triple integral over a box
- Find a triple integral over a general $3D$ region; good examples of regions are either tetrahedrons, or regions between two surfaces (like in lecture); *usually* you have to do Smaller $\leq z \leq$ Bigger
- Find the average value of a function over a $3D$ region (like 54 or 54)
- Remember that sometimes you region faces the $y-$ direction (the book calls this type 3), in which case you have to do Left $\leq y \leq$ Right or the $x-$direction (type 2), in which case you have to do Back $\leq x \leq$ Front.
- Sketch the solid whose volume is given by a given integral.
- Change the order of integration, that is write five other integrals that are equal to a given integral (like 31, 33, 35). Remember that for this, you don't have to find the full region E, but just sketch a couple of $2D$ figures, like y in terms of x, or z in terms of x. The regions won't be very hard in this case.