III — CHANGING THE ORDER

EX WRITE $\int_0^1 \int_y^1 \int_0^{\sqrt{z}} \sin(x^2)\, dx\, dz\, dy$

AS $\int\int\int \sin(x^2)\, dy\, dz\, dx$

WILL BE GIVEN

1) KNOW

$0 \le x \le \sqrt{z}$

No x → $y \le z \le 1$

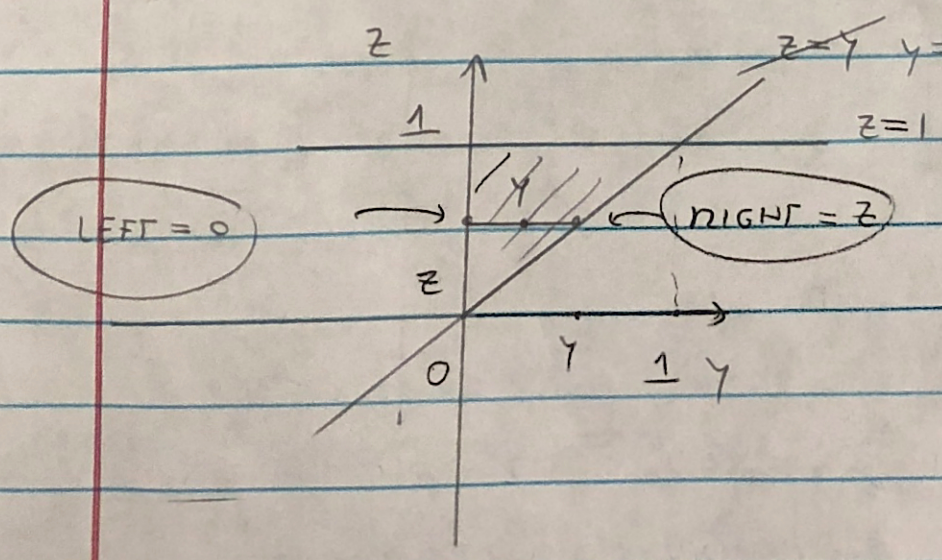
CONSTANTS → $0 \le y \le 1$ } (*)

WANT $? \le y \le ?$

No y → $? \le z \le ?$

CONSTANTS → $? \le x \le ?$

2) DRAW THE REGION GIVEN BY (*):



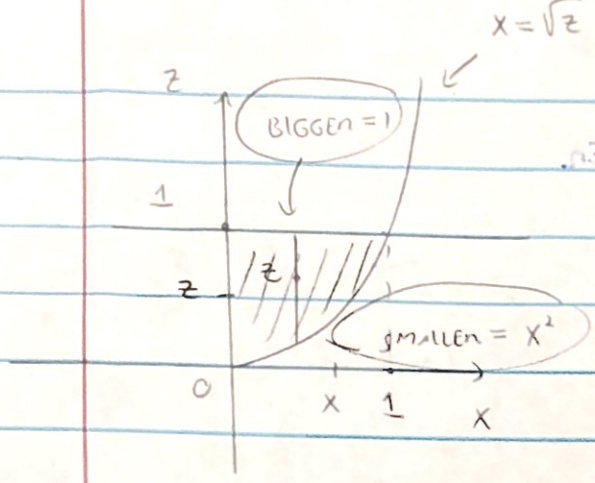
START W/ THE EASIEST EQUATION:

$0 \le y \le 1$

FIND THE EQUATION IN (*) THAT HAS y:

$y \le z \le 1$

SO DRAW { $0 \le y \le 1$ IN YZ-PLANE
$y \le z \le 1$



NOW DEAL WITH THE OTHER EQUATION:

$0 \le x \le \sqrt{z}$

(SO xz-PLANE),

BUT WE'RE STUCK!

BUT BASED ON PREVIOUS PICTURE

ALSO KNOW $0 \le z \le 1$

3) FIND y

BASED ON FIRST PICTURE (AS A HORIZONTAL REGION)

LEFT $\le y \le$ RIGHT

$$0 \le y \le z$$

FIND z (RECALL: NO y)

BASED ON SECOND PICTURE (VERTICAL REGION),

SMALLER $\le z \le$ BIGGER

$$x^2 \le z \le 1$$

GET 1 BY SETTING $z = 1$ IN $x = \sqrt{z}$

FIND x (RECALL: CONSTANTS)

BASED ON SECOND PICTURE, YOU SEE THAT $0 \le x \le 1$

4) ANSWER =

$$\int_0^1 \int_{x^2}^1 \int_0^z \sin(x^2)\, dy\, dz\, dx$$