MATH 2E REVIEW FOR MIDTERM

The midterm is in class, 50 minutes, 5–6 problems, no notes.

Double Integral.

(1) $\iint_D xy\,dA$, where $D = \{(x, y) \mid 0 \le y \le 1, y^2 \le x \le y + 2\}$.

(2) $\iint_D \frac{y}{1+x^2} dA$, D is bounded by $y = \sqrt{x}$, $y = 0$, $x = 1$.

(3) $\iint_D x\,dA$, where D is the region in the first quadrant that lies between the circles $x^2+y^2 = 1$ and $x^2 + y^2 = 2$.

Triple Integral.

(1) $\int_E y^2 z^2 dV$, E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

(2) $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dz dx dy$.

(3) Find the volume of the solid given by the region above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

Line Integral of scalar functions.

(1) $\int_C x\,ds$, C is the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

(2) $\int_C yz \cos(x) ds$, $C : x = t, y = 3 \cos(t), z = 3 \sin(t)$, $0 \le t \le \pi$.

(3) $\int_C y dx + (x + y^2)dy$, C is the ellipse $4x^2 + 9y^2 = 36$, with counterclockwise orientation.

Line Integral of vector fields.

(1) $\int_C F \cdot dr$, where $F = \langle xy, x^2\rangle$ and C is given by $r(t) = \langle \sin 9t), (1 + t)\rangle$, with $0 \le t \le \pi$.

(2) $\int_C \langle xy, y^2, yz\rangle \cdot dr$ where C is the line segment from $(1, 0, -1)$ to $(3, 4, 2)$.

(3) $\int_C F \cdot dr$, where $F = \langle 4x^3y^2 - 2xy^3, 2x^4y - 3x^2y^2 + 4y^3\rangle$ with C given by $r(t) = \langle t + \sin(\pi t), 2t + \cos(\pi t)\rangle$, $0 \le t \le 1$.

(4) $\int_C \sqrt{1 + x^3}dx + 2xy dy$, with C given by the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.

Surfaces.

(1) Find the equation of the tangent plane to the surface $r(u, v) = \langle\sin(u), \cos(u) \sin(v), \sin(v)\rangle$ at the point $u = \frac{\pi}{6}$, $v = \frac{\pi}{6}$.

(2) Find the area of the part of the surface $x = z^2 + y$ that lies between the plane $y = 0$, $y = 2$, $z = 0$, and $z = 2$.

(3) Find the area of the part of the paraboloid $y = x^2 + z^2$ that lies within the cylinder $x^2 + z^2 = 16$.