AP – SOLUTIONS (HW#2)

AP1 (HW#2)

(a) $x(t) = r\cos(t), \quad y(t) = r\sin(t)$

(b) $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} = \frac{(r\sin(t))'}{(r\cos(t))'}$$

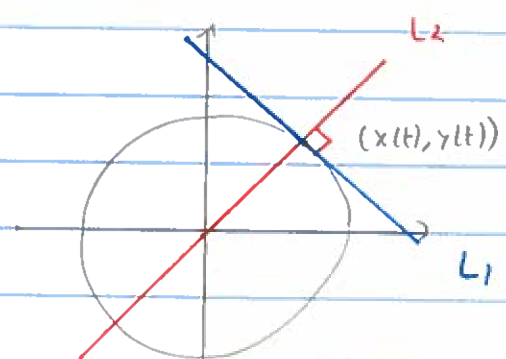
$$= \frac{\cancel{r}\cos(t)}{-\cancel{r}\sin(t)} = \frac{-\cos(t)}{\sin(t)}$$

(c) $$\text{SLOPE} = \frac{y(t) - 0}{x(t) - 0} = \frac{\cancel{r}\sin(t)}{\cancel{r}\cos(t)} = \frac{\sin(t)}{\cos(t)}$$

(d) $$\frac{-\cancel{\cos(t)}}{\cancel{\sin(t)}} \cdot \frac{\cancel{\sin(t)}}{\cancel{\cos(t)}} = -1$$

L_1 & L_2 ARE PERPENDICULAR

(e)



THE TANGENT LINE TO A CIRCLE IS PERPENDICULAR TO THE SECANT LINE.

AP 2
(HW #2)

$$\text{LENGTH} = \int_a^b \sqrt{(x'(\theta))^2 + (y'(\theta))^2}\, d\theta$$

$$x'(\theta) = (r\cos(\theta))' = r'\cos(\theta) - r\sin(\theta) \qquad \left(r' = \frac{dr}{d\theta}\right)$$

$$y'(\theta) = (r\sin(\theta))' = r'\sin(\theta) + r\cos(\theta)$$

$$(x'(\theta))^2 + (y'(\theta))^2 = (r'\cos(\theta) - r\sin(\theta))^2 + (r'\sin(\theta) + r\cos(\theta))^2$$
$$= (r')^2\cos^2(\theta) - \cancel{2r'\cos(\theta)r\sin(\theta)} + r^2\sin^2(\theta)$$
$$+ (r')^2\sin^2(\theta) + \cancel{2r'\sin(\theta)r\cos(\theta)} + r^2\cos^2(\theta)$$

$$= (r')^2(\cos^2(\theta) + \sin^2(\theta)) + r^2(\sin^2(\theta) + \cos^2(\theta))$$

$$= (r')^2 + r^2$$
$$= \left(\frac{dr}{d\theta}\right)^2 + r^2$$
$$= r^2 + \left(\frac{dr}{d\theta}\right)^2$$

HENCE THE FORMULA FOR THE LENGTH BECOMES:

$$\text{LENGTH} = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$