SOLUTIONS

MATH 2D – MIDTERM

Name: ______________________

Student ID: ______________________

Discussion Section time: (please circle)

8 – 9 AM   9 – 10 AM

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 100 points. This is a closed book and closed notes exam and calculators and/or portable electronic devices are not allowed. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May the Fourth be with you!

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating will be subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: ______________________________

1		20
2		15
3		15
4		20
5		15
6		15
Total		100

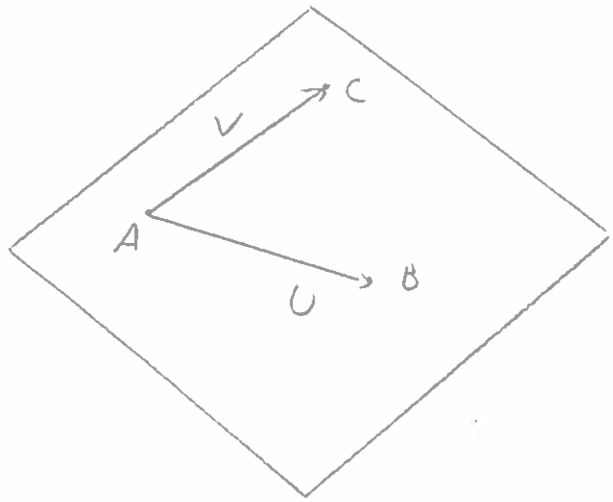
Date: Friday, May 4, 2018.

1. *(20 points)* Find an equation of the plane containing the points

$$A = (2,0,4), B = (-1,5,0), C = (0,1,3)$$

1) POINT $A = (2,0,4)$

2) NORMAL VECTOR



LET $U = \overrightarrow{AB} = \langle -1-2,\ 5-0,\ 0-4 \rangle = \langle -3, 5, -4 \rangle$

$V = \overrightarrow{AC} = \langle 0-2,\ 1-0,\ 3-4 \rangle = \langle -2, 1, -1 \rangle$

$$N = U \times V = \begin{vmatrix} i & j & k \\ -3 & 5 & -4 \\ -2 & 1 & -1 \end{vmatrix} = i(-5+4) - j(3-8) + k(-3+10)$$

$$= \langle -1, 5, 7 \rangle$$

3) EQUATION $A = (2,0,4)$ $N = \langle -1, 5, 7 \rangle$:

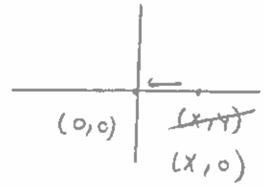
$$(-1)(x-2) + 5(y-0) + 7(z-4) = 0$$

$$\boxed{-(x-2) + 5y + 7(z-4) = 0}$$

3. *(15 points)* Does the following limit exist? Why or why not?

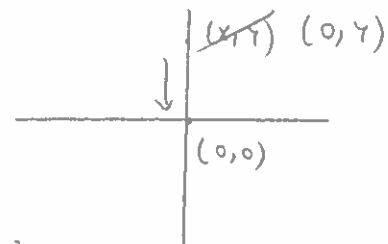
$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^3+y^3}$$

1) ALONG THE X-AXIS $y=0$



$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^3+y^3} \underset{y=0}{\uparrow =} \lim_{x\to 0} \frac{x^3-0^3}{x^3+0^3} = \lim_{x\to 0} \frac{\cancel{x^3}}{\cancel{x^3}} = 1$$

2) ALONG THE Y-AXIS $x=0$



$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^3+y^3} \underset{x=0}{\uparrow =} \lim_{y\to 0} \frac{0^3-y^3}{0^3+y^3} = \lim_{y\to 0} \frac{-\cancel{y^3}}{\cancel{y^3}} = -1$$

3) SINCE WE GET TWO DIFFERENT LIMITS, THE LIMIT DOESN'T EXIST

2. *(15 points)* At which point(s) is the tangent line to the curve vertical?

$$x(t) = t^2 + 1$$
$$y(t) = e^t - 1$$

1) $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} = \frac{e^t}{2t}$$

2) <u>WANT</u> $e^t \neq 0$ AND $2t = 0$

$\underbrace{e^t \neq 0}_{\text{ALWAYS TRUE}}$ AND $\underline{t = 0}$

3) <u>ANSWER</u> $(x(0), y(0)) = (0^2 + 1, e^0 - 1)$

$$= \boxed{(1, 0)}$$

5. *(15 points)* Find the length of the curve from $t = 0$ to $t = 3$

$$\mathbf{r}(t) = \left\langle \frac{1}{2}e^{2t}, 2e^{t}, 2t \right\rangle$$

$$\text{LENGTH} = \int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt \rightarrow +3$$

$$= \int_0^3 \sqrt{\left(\frac{1}{\cancel{2}}\cancel{2}e^{2t}\right)^2 + (2e^t)^2 + (2)^2}\, dt$$

+1 +1 +1.

$$= \int_0^3 \sqrt{e^{4t} + 4e^{2t} + 4}\, dt \quad +1$$

$$= \int_0^3 \sqrt{(e^{2t})^2 + (2)(2)e^{2t} + 2^2}\, dt$$

$$= \int_0^3 \sqrt{(e^{2t} + 2)^2}\, dt \quad +3$$

$$= \int_0^3 e^{2t} + 2\, dt \quad +2$$

$$= \left[\frac{1}{2}e^{2t} + 2t\right]_0^3$$

+3.

$$= \frac{1}{2}e^6 + 6 - \frac{1}{2} - 0$$

$$= \boxed{\frac{1}{2}e^6 + \frac{11}{2}}$$

4. *(20 points)* Name **and** sketch the following surface. Your sketch doesn't have to be 100 % accurate, but you clearly have to label the center of the surface and the direction it's facing in (x, y, or z–direction)

$$x^2 - y^2 + z^2 - 2x + 4y - 8z + 12 = 0$$

1) COMPLETE THE SQUARE

$$x^2 - 2x \quad - y^2 + 4y \quad + z^2 - 8z \quad + 12 = 0$$

$$(x-1)^2 - 1^2 - (y^2 - 4y) + (z-4)^2 - 4^2 + 12 = 0$$

$$(x-1)^2 - 1 - ((y-2)^2 - 4) + (z-4)^2 - 16 + 12 = 0$$

$$(x-1)^2 - 1 - (y-2)^2 \cancel{+4} + (z-4)^2 \cancel{-4} = 0$$

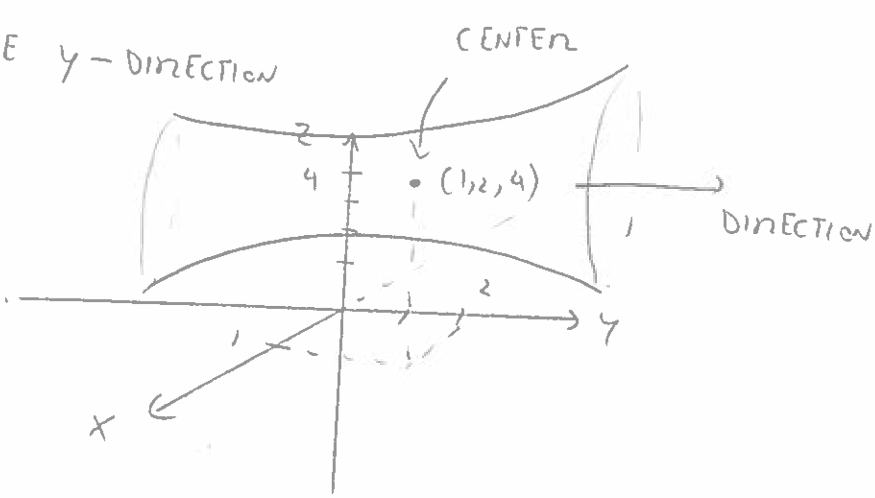
$$(x-1)^2 - (y-2)^2 + (z-4)^2 = 1$$

ONE MINUS

2) HYPERBOLOID OF ONE SHEET / DRESS,

CENTERED AT (1, 2, 4),

IN THE y – DIRECTION



6. *(15 points)* Find an equation of the tangent plane at $(3, 4)$ to the function

$$f(x, y) = \sqrt{x^2 + y^2}$$

1) EQN $\quad z - f(3,4) = f_x(3,4)(x-3) + f_y(3,4)(y-4)$

+3

2) $f(3,4) = \sqrt{3^2+4^2} = 5$ +2

$f_x(x,y) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}}$ +3

$f_x(3,4) = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$ +2

$f_y(x,y) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}y) = \frac{y}{\sqrt{x^2+y^2}}$ +3

$f_y(3,4) = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$ +2

3) ANSWER $\quad \boxed{z - 5 = \frac{3}{5}(x-3) + \frac{4}{5}(y-4)}$