MATH 2E REVIEW FOR FINAL

The final is in the usual classroom, Wed, December 12, 1:30pm – 3:30pm, 8–9 problems, covering Chapter 15 and 16 of Stewart calculus, no notes.

Chapter 15.

(1) Calculate $\int_{R} ye^{xy}dA$, where $R = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$.

(2) Calculate $\int_{0}^{1} \int_{\sqrt{y}}^{1} \frac{ye^{x^2}}{x^3}dxdy$.

(3) Calculate $\int_{E} z dV$, where $E$ is bounded by the planes $y = 0$, $z = 0$, $x + y = 2$ and the cylinder $y^2 + z^2 = 1$ in the first octant.

(4) Calculate $\int_{E} yz dV$ where $E$ lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

(5) Calculate $\int_{H} z^3 \sqrt{x^2 + y^2 + z^2} dV$, where $H$ is the solid hemisphere that lies above the $xy$-plane and has center the origin and radius 1.

(6) Evaluate $\int_{R} \frac{x - y}{x + y} dA$ where $R$ is the square with vertices $(0, 2), (1, 1), (2, 2)$ and $(1, 3)$.

(7) Find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes. Consider the transformation $x = u^2, y = v^2$, and $z = w^2$.

(8) Evaluate $\int_{R} xy dA$, where $R$ is the square with vertices $(0, 0), (1, 1), (2, 0), (1, -1)$.

(9) Given a curve $r(t) = \langle 1 + t, t^2, t^3 \rangle$, find the area of the triangle with vertices $r(-1), r(1)$ and $r(0)$.

Chapter 16.

(1) Evaluate $\int_{C} x ds$, where $C$ is the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

(2) Evaluate $\int_{C} y dx + (x + y^2) dy$, $C$ is the ellipse $4x^2 + 9y^2 = 36$ with counter clockwise orientation.

(3) Evaluate $\int_{C} F \cdot dr$, where $F = \langle \sqrt{xy}, e^y, xz \rangle$, $C$ is given by $r(t) = \langle t^4, t^2, t^3 \rangle$, $0 \leq t \leq 1$.

(4) Compute curl $F$ where $F = \langle e^y, xe^y + e^z, ye^z \rangle$. Then compute the line integral $\int_{C} F \cdot dr$ where $C$ is any curve from $(0, 2, 0)$ to $(4, 0, 3)$. Hint: fundamental theorem of line integrals.

(5) Verify Green’s theorem is true for the line integral $\int_{C} xy^2 dx - x^2 y dy$, where $C$ consists of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and the line segment from $(1, 1)$ to $(-1, 1)$.

(6) Find the area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0, 0), (1, 0)$ and $(1, 2)$.

(7) Find an equation of the tangent plane at the point $(4, -2, 1)$ to the parametric surface $S$ given by $r(u, v) = \langle v^2, -uv, u^2 \rangle$, $0 \leq u \leq 3, -3 \leq v \leq 3$. 


(8) Evaluate \( \iint_S zdS \) and \( \iint_S xdS \) where \( S \) is the part of the paraboloid \( z = x^2 + y^2 \) that lies under the plane \( z = 4 \).

(9) Evaluate \( \iint_S x^2z + y^2z dS \), where \( S \) is the part of the plane \( z = 4 + x + y \) that lies inside the cylinder \( x^2 + y^2 = 4 \).

(10) Evaluate \( \iint_S F \cdot dS \) where \( F = \langle xz, -2y, 3x \rangle \) and \( S \) is the sphere \( x^2 + y^2 + z^2 = 4 \) with outward orientation.

(11) Verify Stokes’ theorem is true for \( F = \langle x^2, y^2, z^2 \rangle \), where \( S \) is the part of the paraboloid \( z = 1 - x^2 - y^2 \) that lies above the \( xy \)-plane and \( S \) has upward orientation.

(12) Evaluate \( \int_C F \cdot dr \) where \( F = \langle xy, yz, zx \rangle \) and \( C \) is the triangle with vertices \((1, 0, 0), (0, 1, 0)\) and \((0, 0, 1)\), oriented counter clockwise as viewed from above.

(13) Calculate \( \iiint_S F \cdot dS \) where \( F = \langle x^3, y^3, z^3 \rangle \) and \( S \) is the surface of the solid bounded by the cylinder \( x^2 + y^2 = 1 \) and the planes \( z = 0 \) and \( z = 2 \).

(14) Compute the outward flux of \( F = \left( \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right) \) through the ellipsoid \( 4x^2 + 9y^2 + 6z^2 = 36 \).

(15) Compute \( \int_C F \cdot dr \) where \( F = \left( \frac{2x^3+2xy^2-2y}{x^2+y^2}, \frac{2y^3+2xy+2x}{x^2+y^2} \right) \) around any simple closed curve containing the origin \((0,0)\).

(16) Find the positively oriented simple closed curve \( C \) for which the value of the line integral \( \int_C (y^3 - y)dx - 2x^3 dy \) is a maximum.