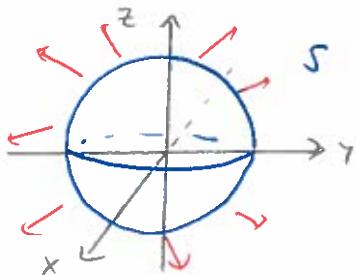


SOLUTIONS

1. (10 points) Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the sphere of radius 2 centered at $(0, 0, 0)$, oriented outwards, and $\mathbf{F} = \langle y^2x, z^2y, x^2z \rangle$. Include a picture of S and its orientation.

1) PICTURE

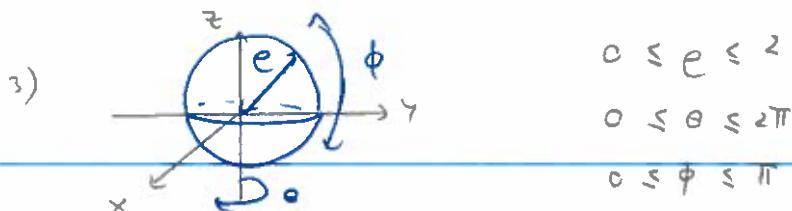


2) DIVERGENCE THEOREM (since S is closed)

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \operatorname{DIV}(F) dx dy dz \quad E = \text{BALL OF RADIUS } 2$$

$$\operatorname{DIV}(F) = (y^2x)_x + (z^2y)_y + (x^2z)_z = y^2 + z^2 + x^2 = x^2 + y^2 + z^2$$

$$\text{so } \iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E x^2 + y^2 + z^2 dx dy dz$$



$$0 \leq r \leq 2$$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq \phi \leq \pi$$

SPHERICAL COORDINATES

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_0^2 0^2 e^2 e^2 \sin(\phi) dr d\theta d\phi$$

$$= \left(\int_0^2 e^4 dr \right) \left(\int_0^{2\pi} d\theta \right) \left(\int_0^\pi \sin(\phi) d\phi \right)$$

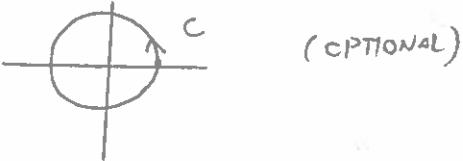
$$= \left[\frac{1}{5} e^4 \right]_0^2 (2\pi) \left[-\cos(\phi) \right]_0^\pi$$

$$= \frac{1}{5} (32) (2\pi) (2)$$

$$= \boxed{\frac{128\pi}{5}}$$

2. (10 points) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle x + y^3, 2x + x^3 \rangle$, where C is the circle centered at $(0, 0)$ and radius 2, oriented counterclockwise.

1) PICTURE



(OPTIONAL)

2) F conservative?

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = (2x + x^3)_x - (x - y^3)_y = 2 + 3x^2 - (-3y^2) \\ = 2 + 3x^2 + 3y^2 \neq 0$$

3) GREEN'S THEOREM

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dx dy$$

$$= \iint_D 2 + 3(x^2 + y^2) dx dy$$

$D = \underline{\text{DISK}}$ OF radius 2



$$= \int_0^{2\pi} \int_0^2 (2 + 3r^2) r dr d\theta$$

$$= 2\pi \left(\int_0^2 2r + 3r^3 dr \right)$$

$$= 2\pi \left[r^2 + \frac{3}{4}r^4 \right]_0^2$$

$$= 2\pi \left(4 + \frac{3}{4}(16) \right)$$

$$= 2\pi(4 + 12) = 2\pi(16)$$

$$= \underline{32\pi} \quad \left(\text{NOTE VERSION B: } \underline{40\pi} \right)$$

OTNEN SOL Do bINEGRY (BOO!)

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \langle t^2 t' e^{t^6}, t t' e^{t^6}, t t' e^{t^6} \rangle \cdot \langle 1, 2t, 3t^5 \rangle dt$$
$$= \int_1^2 t^3 e^{t^6} + 2t^2 e^{t^6} + 3t^5 e^{t^6} dt = \int_1^2 6t^5 e^{t^6} dt$$

MATH 2E - FINAL EXAM

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3. (10 points) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ with $1 \leq t \leq 2$, and $\mathbf{F} = \langle yze^{xyz}, xze^{xyz}, xyze^{xyz} \rangle$.

$$= [e^{t^6}] \Big|_{t=1}^{t=2} = e^{64} - e$$

1) F CONSERVATIVE?

$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yze^{xyz} & xze^{xyz} & xyze^{xyz} \end{vmatrix}$$

$$= \left\langle \frac{\partial}{\partial y} (xye^{xyz}) - \frac{\partial}{\partial z} (xze^{xyz}), - \frac{\partial}{\partial x} (xye^{xyz}) + \frac{\partial}{\partial z} (yze^{xyz}), \right.$$
$$\left. \frac{\partial}{\partial x} (xze^{xyz}) - \frac{\partial}{\partial y} (yze^{xyz}) \right\rangle$$
$$= \left\langle \cancel{xe^{xyz}} + \cancel{xyxze^{xyz}} - \cancel{xe^{xyz}} - \cancel{xze^{xyz}}, \right.$$
$$\left. -ye^{xyz} - \cancel{xyyze^{xyz}} + \cancel{ye^{xyz}} + \cancel{yze^{xyz}}, \right.$$
$$\left. \cancel{ze^{xyz}} + \cancel{xzyze^{xyz}} - \cancel{ze^{xyz}} - \cancel{yze^{xyz}} \right\rangle$$
$$= \langle 0, 0, 0 \rangle \quad (\text{GNEF SUCCES!}) \Rightarrow F \text{ is cons!}$$

2) FIND f $\nabla f = F \Rightarrow \langle f_x, f_y, f_z \rangle = \langle yze^{xyz}, xze^{xyz}, xyze^{xyz} \rangle$

$$f_x = yze^{xyz} \Rightarrow f = \int yze^{xyz} dx = ye^{xyz} \frac{1}{ye^{xyz}} e^{xyz} = e^{xyz} + \text{JUNK}$$

$$f_y = xze^{xyz} \Rightarrow f = \int xze^{xyz} dy = xe^{xyz} \frac{1}{xe^{xyz}} e^{xyz} = e^{xyz} + \text{JUNK}$$

$$f_z = xyze^{xyz} \Rightarrow f = \int xyze^{xyz} dz = ye^{xyz} \frac{1}{ye^{xyz}} e^{xyz} = e^{xyz} + \text{JUNK}$$

$$\Rightarrow f(x, y, z) = e^{xyz}$$

3) FTC For LINE INTEGRALS

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(r(2)) - f(r(1))$$
$$= f(2, 4, 8) - f(1, 1, 1)$$
$$= e^{(2)(4)(8)} - e^{(1)(1)(1)} = e^{64} - e$$

$$r(2) = \langle 2, 2^2, 2^3 \rangle = \langle 2, 4, 8 \rangle$$

$$r(1) = \langle 1, 1^2, 1^3 \rangle = \langle 1, 1, 1 \rangle$$

$$= e^{64} - e$$

(Vfns CONB: $e^{64} - 1$)

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OTHER SOL do directly: $\Gamma(t) = \langle 5\cos(t), 5\sin(t), 4 \rangle$

$$\int_C F \cdot d\Gamma = \int_0^{2\pi} \langle 5\cos(t) 5\sin(t), 5\sin(t) 4, 5\cos(t) 4 \rangle \cdot \langle -5\sin(t), 5\cos(t), 0 \rangle dt$$

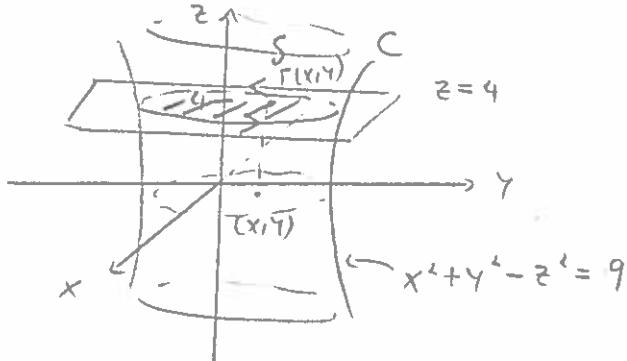
$$= \int_0^{2\pi} -125 \sin^2(t) \cos(t) + 100 \sin(t) \cos(t) dt = 0$$

MATH 2E - FINAL EXAM

4. (10 points) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xy, yz, xz \rangle$, and C (oriented counterclockwise) is the curve of intersection of the surfaces $x^2 + y^2 - z^2 = 9$ and $z = 4$. Include a picture of C and the surfaces.

HYPERBOLA OF
ONE SHEET (CONES)

1) PICUTURE



2) F cons?

$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix} = \left\langle \frac{\partial}{\partial y}(xz) - \frac{\partial}{\partial z}(yz), \right. \\ \left. - \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial z}(xy), \right. \\ \left. \frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xz) \right\rangle$$

$$= \langle -y, -z, -x \rangle \neq \langle 0, 0, 0 \rangle$$

3) STOKE'S THEOREM

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl}(F) \cdot d\mathbf{s} \quad S = \text{INSIDE OF } C$$

4) NOTE SETTING $z=4$ IN $x^2 + y^2 - z^2 = 9 \Rightarrow x^2 + y^2 - 4^2 = 9 \Rightarrow x^2 + y^2 = 25$
so C is a circle of radius 5 (and S is a disk of radius 5)

PARAMETRIZE S $\Gamma(x, y) = \langle x, y, 4 \rangle$

$$\left. \begin{array}{l} \Gamma_x = \langle 1, 0, 0 \rangle \\ \Gamma_y = \langle 0, 1, 0 \rangle \end{array} \right\} \hat{\mathbf{N}} = \Gamma_x \times \Gamma_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \langle 0, 0, 1 \rangle$$

so $\iint_S \text{curl}(F) \cdot d\mathbf{s} = \iint_D \langle -y, -z, -x \rangle \cdot \langle 0, 0, 1 \rangle dx dy$

$$= \iint_D -x dx dy = \int_0^5 \int_0^{2\pi} -5r \cos(\theta) r dr d\theta = \left(\int_0^5 -5r^2 dr \right) \left(\int_0^{2\pi} \cos(\theta) d\theta \right) = 0$$

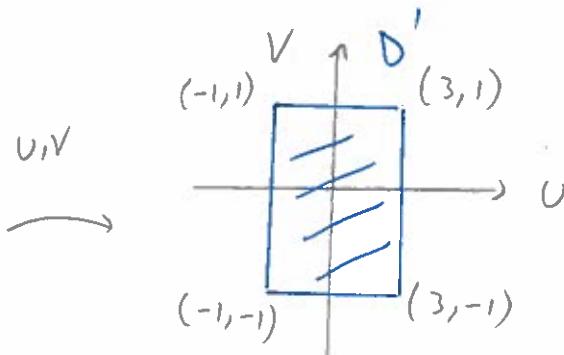
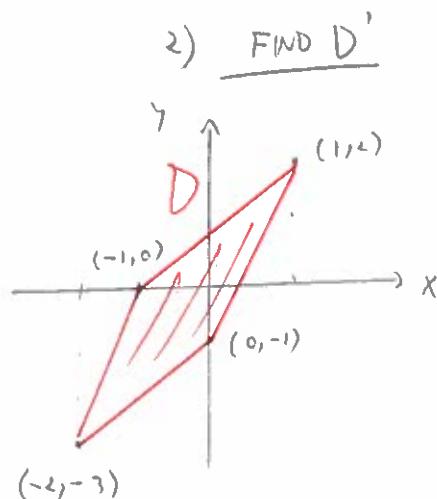
5. (10 points) Let D be the parallelogram with vertices $(-2, -3), (0, -1), (1, 2), (-1, 0)$.
 Draw a picture of D and calculate

$$\iint_D (\underline{\quad}) dxdy$$

$$(y-3x)^2 (y-x)^4$$

1) $U = y - 3x$

$V = y - x$



NOTE THE PIC SHOULD LOOK MORE LIKE 

$(-2, -3) \rightsquigarrow U = -3 - 3(-2) = 3, V = -3 + 2 = -1 \rightsquigarrow (3, -1)$

$(0, -1) \rightsquigarrow (-1, -1)$

$(1, 2) \rightsquigarrow (-1, 1)$

$(-1, 0) \rightsquigarrow (3, 1)$

3) $dUdV = \left| \frac{dUdV}{dx dy} \right| dx dy, \quad \frac{dUdV}{dx dy} = \begin{vmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ -1 & 1 \end{vmatrix} = -2$

$$= (-2) dx dy = 2 dx dy$$

$\Rightarrow dx dy = \frac{1}{2} dU dV$

4) $\iint_D (y-3x)^2 (y-x)^4 dxdy = \iint_{D'} U^2 V^4 \left(\frac{1}{2}\right) dU dV = \frac{1}{2} \int_{-1}^1 \int_{-1}^3 U^2 V^4 dU dV$

$$= \frac{1}{2} \left(\int_1^3 V^4 dV \right) \left(\int_{-1}^3 U^2 dU \right) = \frac{1}{2} \left(\frac{V^5}{5} \right) \left(\frac{1}{3} (U^3 + 1) \right) = \frac{1}{15} (28) = \boxed{\frac{28}{15}}$$

6. (15 points) Let S be the part of the cone with parametric equations $\mathbf{r}(u, v) = \langle u \cos(v), u \sin(v), u \rangle$, $0 \leq u \leq 2$, $0 \leq v \leq 2\pi$, oriented upwards (no need to draw a picture).

- (a) (5 points) Find the equation of the tangent plane to S at the point $(\sqrt{3}, 1, 2)$.

$$1) \quad \Gamma_u = \langle \cos(v), \sin(v), 1 \rangle$$

$$\Gamma_v = \langle -u \sin(v), u \cos(v), 0 \rangle$$

$$2) \quad \Gamma_u \times \Gamma_v = \begin{vmatrix} i & j & k \\ \cos(v) & \sin(v) & 1 \\ -u \sin(v) & u \cos(v) & 0 \end{vmatrix}$$

$$= \langle -u \cos(v), -u \sin(v), u \cos^2(v) + u \sin^2(v) \rangle$$

$$= \langle -u \cos(v), -u \sin(v), u \rangle$$

3) FIND U & V

$$\langle u \cos(v), u \sin(v), u \rangle = \langle \sqrt{3}, 1, 2 \rangle$$

$$\Rightarrow \underline{u=2} \quad \text{and} \quad \begin{aligned} u \cos(v) &= 2 \cos(v) = \sqrt{3} \Rightarrow \cos(v) = \frac{\sqrt{3}}{2} \\ u \sin(v) &= 2 \sin(v) = 1 \Rightarrow \sin(v) = \frac{1}{2} \end{aligned} \quad \left. \right\} \quad \begin{aligned} v &= \frac{\pi}{6} \end{aligned}$$

$$4) \quad \text{Normal Vector} \quad \Gamma_u \times \Gamma_v = \langle -u \cos(v), -u \sin(v), u \rangle \quad \left. \right\} \quad u=2, v=\pi/6$$

$$= \langle -2 \left(\frac{\sqrt{3}}{2} \right), -2 \left(\frac{1}{2} \right), 2 \rangle$$

$$= \langle -\sqrt{3}, -1, 2 \rangle$$

5) Pwr $(\sqrt{3}, 1, 2)$

Normal Vector $(-\sqrt{3}, -1, 2)$

Elevation

$$\boxed{-\sqrt{3}(x-\sqrt{3}) - 1(y-1) + 2(z-2) = 0}$$

Note Version is: $-(x-1) - \sqrt{3}(y-\sqrt{3}) + 2(z-2) = 0$

(b) (5 points) Calculate $\iint_S (x^2 + y^2)^2 dS$, where S is the surface in (a).

$$1) \quad r_u \times r_v = \langle -v \cos(v), -v \sin(v), v \rangle \quad (\text{From (a)})$$

$$2) \quad \|r_u \times r_v\| = \left(v^2 \cos^2(v) + v^2 \sin^2(v) + v^2 \right)^{\frac{1}{2}} \\ = (v^2 + v^2)^{\frac{1}{2}}$$

$$= \sqrt{2} v$$

$$= \sqrt{2} v \quad \boxed{2}$$

$$3) \quad \iint_S (x^2 + y^2)^2 dS = \iint_D f(r(u,v)) \underbrace{\|r_u \times r_v\| du dv}_{dS}$$

$$= \iint_D (v^2 \cos^2(v) + v^2 \sin^2(v))^2 \sqrt{2} v du dv$$

$$= \iint_0^{2\pi} (v^2)^2 \sqrt{2} v du dv \quad \boxed{1}$$

$$= 2\pi \sqrt{2} \left(\int_0^2 v^5 dv \right)$$

$$= 2\pi \sqrt{2} \left[\frac{v^6}{6} \right]_0^2$$

$$= 2\pi \sqrt{2} \frac{2^6}{6}$$

$$= \frac{64\pi\sqrt{2}}{3} \quad \boxed{2}$$

- (c) (5 points) Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle z, y, x \rangle$ and S is the surface in (a).

$$1) \text{ From (a), } \Gamma_{UV} \times \Gamma_{UV} = \langle -U \cos(V), -U \sin(V), U \rangle$$

$$\begin{aligned} 2) \quad \iint_S \mathbf{F} \cdot d\mathbf{S} &= \iint_D \mathbf{F}(\Gamma(U, V)) \cdot (\Gamma_{UV} \times \Gamma_{UV}) dU dV \\ &= \iint_D \langle U, U \sin(V), U \cos(V) \rangle \cdot \langle -U \cos(V), -U \sin(V), U \rangle dU dV \\ &= \iint_0^{2\pi} \left[-U^2 \cos^2(V) - U^2 \sin^2(V) + U^2 \cos(V) \right] dU dV \quad \boxed{3} \\ &= - \left(\int_0^2 U^2 dU \right) \left(\int_0^{2\pi} \sin^2(V) dV \right) \\ &= - \left[\frac{U^3}{3} \right]_0^2 \int_0^{2\pi} \frac{1}{2} - \frac{1}{2} \cos(2V) dV \\ &= - \frac{8}{3} \left[\frac{V}{2} - \frac{1}{4} \sin(2V) \right]_0^{2\pi} \\ &= - \frac{8}{3} \left(\frac{2\pi}{2} \right) \\ &= \boxed{-\frac{8\pi}{3}} \quad \boxed{2} \end{aligned}$$

7. (15 points)

- (a) (5 points) Find constants a and b such that $\mathbf{G} = \operatorname{curl}(\mathbf{F})$, where $\mathbf{G} = \langle x^2 e^z, 0, -2x e^z \rangle$ and $\mathbf{F} = \langle a x y e^z, 0, b x^2 y e^z \rangle$

$$\operatorname{curl}(\mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a x y e^z & 0 & b x^2 y e^z \end{vmatrix}$$

$$= \left\langle \frac{\partial}{\partial y} (b x^2 y e^z) - \frac{\partial}{\partial z} (0), -\frac{\partial}{\partial x} (b x^2 y e^z) + \frac{\partial}{\partial z} (a x y e^z), \right.$$

$$\left. \frac{\partial}{\partial x} (0) - \frac{\partial}{\partial y} (a x y e^z) \right\rangle$$

$$= \langle b x^2 e^z, -2 b x y e^z + a x y e^z, -a x e^z \rangle$$

$$\textcircled{WANT} = \langle x^2 e^z, 0, -2 x e^z \rangle \quad \textcolor{red}{\downarrow 3}$$

HENCE $\cancel{b x^2 e^z} = x^2 e^z \Rightarrow \underline{b = 1}$

$$-a x e^z = -2 x e^z \Rightarrow -a = -2 \Rightarrow \underline{a = 2} \quad \textcolor{red}{\downarrow 2}$$

ANSWER

$$a = 2, b = 1$$

(VERSION B : $c = 2, d = 1$)

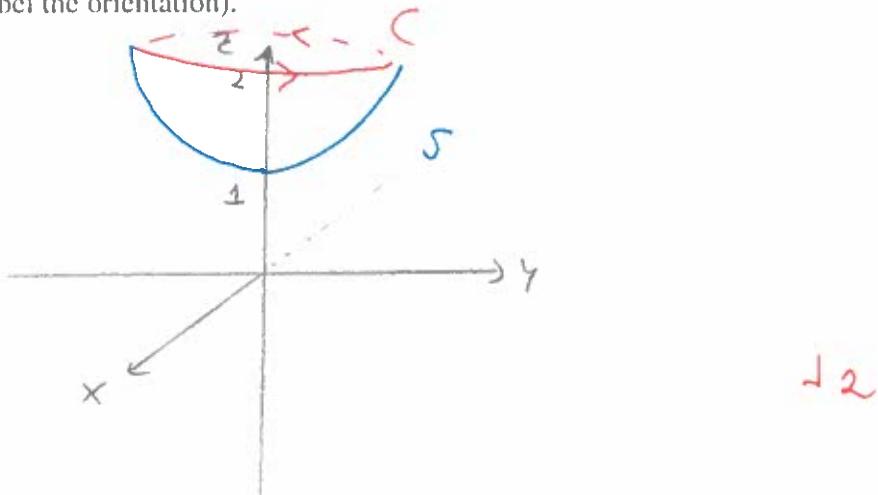
HYPERSOLOID OF TWO SHEETS
(TWO CUPS)

12

MATH 2E - FINAL EXAM

- (b) (10 points) Use your answer in (a) to calculate $\iint_S \mathbf{G} \cdot d\mathbf{S}$, where S is the part of the surface $x^2 + y^2 + z^2 = 1$ with $1 \leq z < 2$ (without the top) and $\mathbf{G} = \langle x^2 e^z, 0, -2x e^z \rangle$. Assume S is oriented in such a way that the boundary curve C is counterclockwise. Include a picture of S and C (but no need to label the orientation).

1) PICtURE



↓ 2

$$2) \quad \iint_S \mathbf{G} \cdot d\mathbf{S} = \iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$

↑
BY (a)
STOKES

$$\mathbf{F} = \langle 2xye^z, 0, x^2ye^z \rangle \quad ((a) \text{ WITH } a=2, b=1) \quad \downarrow 3$$

3) PARAMETRIZE C

$$\text{NOTE} \quad -x^2 - y^2 + \underbrace{z^2}_{z=2} = 1 \Rightarrow -x^2 - y^2 = -3 \Rightarrow x^2 + y^2 = 3$$

C : circle of radius $\sqrt{3}$ in the plane $z=2$, counterclockwise

$$\mathbf{r}(t) = \langle \sqrt{3} \cos(t), \sqrt{3} \sin(t), 2 \rangle \quad 0 \leq t \leq 2\pi$$

$$\left[-2\sqrt{3} \sin^2(t) e^z \right]_0^{2\pi} = \textcircled{0}$$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt \quad \downarrow 3$$

$$= \int_0^{2\pi} \langle 2(\sqrt{3} \cos(t))(\sqrt{3} \sin(t)) e^z, 0, 3 \cos^2(t) \sqrt{3} \sin(t) e^z \rangle \cdot \langle -\sqrt{3} \sin(t), \sqrt{3} \cos(t), 0 \rangle dt$$

$$= \int_0^{2\pi} -6\sqrt{3} \sin^2(t) \cos(t) e^z dt \quad \downarrow 2$$

8. (5 points) The Pre-Finale...

Let \mathbf{F} be a vector field, and let S be the sphere centered at $(0, 0, 0)$ and radius 2, oriented outwards. Calculate $\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$

SINCE S IS CLOSED, WE CAN USE THE DIVERGENCE THEOREM
↓₂
 TO CONCLUDE:

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = \iiint_E \underbrace{\operatorname{DIV}(\operatorname{curl}(\mathbf{F}))}_{O} dx dy dz = \iiint_E 0 = 0$$

WHERE E IS THE BALL OF RADIUS 2 CENTERED AT $(0, 0, 0)$
↓₃



9. (15 points) ...and the Grand Finale!

(a) (5 points)

Definition: If $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field and $f = f(x, y, z)$ is a function, then $f\mathbf{F}$ is the vector field $\langle fP, fQ, fR \rangle$.

Show that for any vector field \mathbf{F} and any function f ,

$$\operatorname{div}(f\mathbf{F}) = f(\operatorname{div}(\mathbf{F})) + (\nabla f) \cdot \mathbf{F}$$

$$\begin{aligned}\operatorname{div}(f\mathbf{F}) &= \operatorname{div}(\langle fP, fQ, fR \rangle) \\ &= (fP)_x + (fQ)_y + (fR)_z \\ &= f_x P + f_y Q + f_z R + fP_x + fQ_y + fR_z \quad \boxed{3} \\ &= fP_x + fQ_y + fR_z + f_x P + f_y Q + f_z R \\ &= f(P_x + Q_y + R_z) + \langle f_x, f_y, f_z \rangle \cdot \langle P, Q, R \rangle \\ &= f \operatorname{div}(\mathbf{F}) + \nabla f \cdot \mathbf{F} \quad \boxed{2}\end{aligned}$$

- (b) (10 points) Suppose that $f = f(x, y, z)$ solves Laplace's equation $\Delta f = 0$ in B (the book writes this as $\nabla^2 f = 0$), where B is the ball of radius R centered at $(0, 0, 0)$. Moreover, suppose that $f = 0$ on the sphere of radius R centered at $(0, 0, 0)$. Use (a) with a special choice of F to show that $f = 0$ in B .

)) USE (a) WITH $\underline{F = \nabla f}$ TO CONCLUDE:

$$\text{DIV}(f \nabla f) = f (\text{DIV}(\nabla f)) + \nabla f \cdot (\nabla f)$$

$$= f(\underbrace{\Delta f}_0) + |\nabla f|^2$$

$$\text{DIV}(f \nabla f) = |\nabla f|^2$$

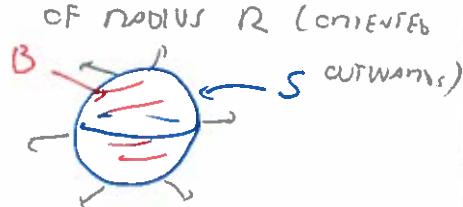
2) NOW INTEGRATE THIS IDENTITY OVER B TO GET:

$$\iiint_B \text{DIV}(f \nabla f) = \iiint_B |\nabla f|^2$$

HOWEVER, BY THE DIVERGENCE THEOREM,

$$\iiint_B \text{DIV}(f \nabla f) = \iint_S (f \nabla f) \cdot d\vec{s} \quad , \text{ WHERE } S \text{ IS THE SPHERE OF RADIUS } R \text{ CENTERED AT } (0, 0, 0)$$

$$= 0 \quad (\text{SINCE } f = 0 \text{ ON } S, \text{ BY ASSUMPTION})$$



THEFORE $\iiint_B |\nabla f|^2 = \iiint_B \text{DIV}(f \nabla f) = 0$, so $\iiint_B |\nabla f|^2 = 0$

BUT SINCE $|\nabla f|^2 \geq 0$, THIS IMPLIES $|\nabla f|^2 = 0$, so $\nabla f = 0$, i.e. $f = C$ IN B .

3) FINALLY, SINCE f IS CONSTANT AND $f = 0$ IN S , THIS IMPLIES $C = 0$, AND THEREFORE $f = 0$ IN B .

