MIDTERM 2 – REVIEW – SOLUTIONS

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1a. $Rank(A) = \dim(Col(A)) = \dim(Row(A)) = 5$
$\dim(Nul(A)) = 1$

1b. Basis for $Row(A)$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -13 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Basis for $Col(A)$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 6 \\ 0 \\ -1 \end{bmatrix} \right\}$$

2. $\begin{bmatrix} 11 \\ 16 \end{bmatrix}$

3. $\mathbf{v} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$ (or any other nonzero eigenvector corresponding to the eigenvalue $\lambda = -2 + \sqrt{2}$)

4.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

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5. $\hat{\mathbf{x}} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$; $\sqrt{13}/2$ (for the orthogonal projection-method, remember to apply Gram-Schmidt to the columns of A)

6. $\left(0, \frac{\pi}{2}\right)$

7. $y(t) = 54 + Be^{-2t}$

8. Because I didn't have time to go through this, you can find more detailed explanations below.

8a. $y_p(t) = (At^2 + Bt + C)t^2 e^t$

8b. $y_p(t) = (At + B)te^t \cos(2t) + (A't + B')te^t \sin(2t)$

8c. $y_p(t) = Ae^t \cos(3t) + Be^t \sin(3t)$

8d. $y_p(t) = Ae^{2t} \cos(2t) + Be^{2t} \sin(2t)$

8e. $y_p(t) = (At^2 + Bt + C) \cos(2t) + (A't^2 + B't + C') \sin(2t)$

8f. $y_p(t) = t^3(At + B)$

More detailed explanations for 8:

First, find the homogeneous solution $y_0(t)$.

The auxiliary equation is $r^3(r-1)^2(r^2-2r+5) = 0$, which gives you $r = 0$ (triple root), $r = 1$ (double root) and $r = 1 \pm 2i$, so the solution to the homogeneous equation is:

$$y_0(t) = A + Bt + Ct^2 + De^t + Ete^t + Fe^t \cos(2t) + Ge^t \sin(2t)$$

(a) $f(t) = t^2 e^t$

For t^2, guess $At^2 + Bt + C$ (**IMPORTANT:** This is OK, even though it coincides with y_0; this 'coinciding'-rule doesn't apply to polynomials, but see (f))

For e^t, guess e^t, but this coincides with y_0, so guess te^t, but this also coincides with y_0, so guess t^2e^t, which works.

Multiplying both terms, we get $\boxed{y_p(t) = (At^2 + Bt + C)t^2e^t}$

(b) $\underline{f(t) = te^t \cos(2t)}$

For t, guess $At + B$

For $e^t \cos(2t)$, guess $Ae^t \cos(2t) + Be^t \sin(2t)$, but this coincides with y_0, so guess $Ate^t \cos(2t) + Bte^t \sin(2t)$.

Multiplying both terms, we get: $\boxed{y_p(t) = (At + B)te^t \cos(2t) + (A't + B')te^t \sin(2t)}$

(c) $\underline{f(t) = e^t \cos(3t)}$

Here guess $\boxed{y_p(t) = Ae^t \cos(3t) + Be^t \sin(3t)}$, which doesn't coincide with y_0 (it's important to treat $e^t \cos(3t)$ as a *whole*; even though e^t coincides with y_0, $e^t \cos(3t)$ doesn't)

(d) $\underline{f(t) = e^{2t} \cos(2t)}$

Here guess $\boxed{y_p(t) = Ae^{2t} \cos(2t) + Be^{2t} \sin(2t)}$, which doesn't coincide with y_0.

(e) $\underline{f(t) = t^2 \sin(2t)}$

For t^2, guess $At^2 + Bt + C$ (again OK, even though it coincides with y_0)

For $\sin(2t)$, guess $A\cos(2t) + B\sin(2t)$, which doesn't coincide with y_0 (it's important to treat $e^t \sin(2t)$ as a *whole*; even though $e^t \sin(2t)$ coincides with y_0, $\sin(3t)$ alone doesn't)

Multiplying both terms, guess:

$$\boxed{y_p(t) = (At^2 + Bt + C)\cos(2t) + (A't^2 + B't + C')\sin(2t)}$$

(f) $\underline{f(t) = t}$

This is the only tricky-case! Here think of t as being te^{0t}

For t, guess $At + B$

For $e^{0t} = 1$, guess A, but this coincides with y_0, so guess At, which coincides with y_0, so guess At^2, which coincides with y_0, so guess At^3, which doesn't coincide with y_0.

Multiplying both terms, we get: $\boxed{y_p(t) = (At + B)t^3}$