MATH 54 – MOCK MIDTERM 3

PEYAM RYAN TABRIZIAN

Name: ____

Instructions: This is a mock midterm, designed to give you an idea of what the actual midterm will look like!

1		10
2		10
3		30
4		30
5		20
Total		100

Date: Friday, July 27th, 2012.

1. *(10 points, 2 points each)*

Label the following statements as **T** or **F**. **Write your answers in the box below!**

NOTE: In this question, you do **NOT** have to show your work! Don't spend *too* much time on each question!

(a) If A is similar to B, then $det(A) = det(B)$

(b) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 4, 0$, then A is diagonalizable

(c) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 4, 0$, then A is invertible

(d) If A is a 4×4 matrix with eigenvalues $\lambda = 1, 2, 2, 3$, then $det(A) = 12$

(e) If λ is an eigenvalue of A, then $Nul(\lambda I - A)$ *could* be $\{\mathbf{0}\}$.

(a)	
(b)	
(c)	
(d)	
(e)	

2. *(10 points)* Label the following statements as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer!!!

   This means:

   - If the answer is **TRUE**, you have to explain **WHY** it is true (possibly by citing a theorem)

   - If the answer is **FALSE**, you have to give a specific **COUNTEREXAMPLE**. You also have to explain why the counterexample is in fact a counterexample to the statement!

   (a) If A is diagonalizable, then A^2 is diagonalizable

   (b) If A has only one eigenvalue, then A is not diagonalizable

3. *(30 points)* Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$, where:

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 8 & -4 & 7 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

4. *(30 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

5. *(20 points, 10 points each)*

(a) *(10 points)* Use **undetermined coefficients** to find the general solution to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$, where:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \mathbf{f}(t) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Note: You may use the fact that the general solution to $\mathbf{x}' = A\mathbf{x}$ is: $\mathbf{x}_0(t) = Ae^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Be^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) *(10 points)* Use **variation of parameters** to find the general solution to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$, where:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \mathbf{f}(t) = \begin{bmatrix} e^{2t} \\ e^{4t} \end{bmatrix}$$

Note: You may use the fact that the general solution to $\mathbf{x}' = A\mathbf{x}$ is: $\mathbf{x}_0(t) = Ae^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Be^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$