MATH 54 – MIDTERM 3

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Name: ____

Instructions: This midterm counts for 20% of your grade. You officially have 90 minutes to take this exam. May your luck be diagonalizable! :)

1		10
2		15
3		30
4		25
5		20
Bonus		2
Total		100

Date: Friday, July 27th, 2012.

1. *(10 points, 2 points each)*

Label the following statements as **T** or **F**. **Write your answers in the box below!**

NOTE: In this question, you do **NOT** have to show your work! Don't spend *too* much time on each question!

(a) If A is diagonalizable, then A^3 is diagonalizable.

(b) If A is a 3×3 matrix with 3 (linearly independent) eigenvectors, then A is diagonalizable

(c) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 3$, then A is invertible

(d) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 3$, then A is (always) diagonalizable

(e) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 2$, then A is (always) not diagonalizable

(a)	
(b)	
(c)	
(d)	
(e)	

2. *(15 points)* Label the following statements as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer!!!

This means:

- If the answer is **TRUE**, you have to explain **WHY** it is true (possibly by citing a theorem)

- If the answer is **FALSE**, you have to give a specific **COUNTEREXAMPLE**. You also have to explain why the counterexample is in fact a counterexample to the statement!

IMPORTANT NOTE: If A is diagonalizable, explain why! And if A is not diagonalizable, *show* why it isn't!

(a) *(5 points)* If A is diagonalizable, then A is invertible.

(b) *(10 points, longer)* If A is invertible, then A is diagonalizable

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3. *(30 points)* Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$, where:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Note: Show *all* your work!

Helps: In case you're stuck:

– For 5 points, I can give you one root of the characteristic polynomial (ask me about it)
– For 10 points, I can give you the full characteristic polynomial! (ask me about it)

(Continuation)

4. *(25 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 0 & 5 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Note: Show *all* your work!

(Continuation)

5. *(20 points, 10 points each)*
Find the general solution to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$, where:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \mathbf{f}(t) = \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix}$$

Note: You may use the fact that the general solution to $\mathbf{x}' = A\mathbf{x}$ is: $\mathbf{x}_0(t) = Ae^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Be^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) *(10 points)* Using undetermined coefficients

(b) *(10 points)* Using variation of parameters

Note: Simplify your answer!

Bonus *(2 points)*

(a) *(0 points, but it'll help you for (b))* What is the general solution of $y'' = -b^2 y$

(b) *(2 points)* Use (a) and the *ideas* we talked about in lecture about the matrix exponential function to solve the following system $\mathbf{x}'' = A\mathbf{x}$ (note the double prime), where:

$$A = \begin{bmatrix} 2 & -3 \\ 6 & 7 \end{bmatrix}$$

Hint: You may use the fact that $A = -B^2$, where $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ as well as the fact that $B = PDP^{-1}$, where $P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Note: Converting this into a first-order system differential equations is a complete waste of time! Do it directly using the hint above!

Note: This problem is a tiny bit long, but you really need to write down the final answer to get full credit!

(Continuation)

(Scratch work)

Any comments about this exam? (too long? too hard?)