MIDTERM 1 – REVIEW – SOLUTIONS

PEYAM RYAN TABRIZIAN

1. $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{bmatrix}$
2. $$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$
3. Not invertible, because the columns of A are linearly dependent
4. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
5. Show $T(p+q) = T(p) + T(q)$ and $T(cp) = cT(p)$
   $Nul(T) = \{0\}$, so T is one-to-one, but $Ran(T) = Span\{t, t^2, t^3, t^4\} \neq P_4$ (calculate $T(a + bt + ct^2 + dt^3)$), so T is not onto P_4
6. -44
7. 0
8. $dim(Col(A)) = 3$, $dim(Nul(A)) = 2$

   Basis for $Col(A)$:

   $$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}$$

   Basis for $Nul(A)$:

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$$\mathcal{B} = \left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{9}{2} \\ 0 \\ -\frac{4}{3} \\ -3 \\ 1 \end{bmatrix} \right\}$$

9. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -39 \\ 22 \end{bmatrix}$
10. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -4 \\ -1 \\ 3 \end{bmatrix}$
11. See solutions to Problem 1 on Quiz 4.