MIDTERM 1 – REVIEW – PROBLEMS

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1. Linear equations

Problem 1: Solve the following system (or say it has no solutions):

$$\begin{cases} 2x + 2y + z = 2 \\ 3x + 4y + 2z = 3 \\ x + 2y + z = 1 \end{cases}$$

2. Inverses

Problem 2. Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 3. Is the following matrix invertible?

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Hint: This is a one-liner!

3. Linear Transformations

Problem 4. Assume $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points in the plane **clockwise** by $\frac{\pi}{2}$ radians. Find the matrix of T.

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Problem 5. Define $T : P_3 \to P_4$ by:

$$T(p) = \int_0^t p(x)dx$$

(Basically, $T(p)$ is the antiderivative of p without the constant)

(a) Show T is a linear transformation
(b) Find $Nul(T)$. Is T one-to-one?
(c) Find $Ran(T)$. Is T onto P_4?

4. Determinants

Problem 6. Find $\det(A)$, where:

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 0 & 0 & 3 \\ 3 & 1 & 4 & 7 \\ 1 & 1 & 0 & 3 \end{bmatrix}$$

Problem 7. Find $\det(A)$, where:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

5. $Nul(A)$, $Col(A)$

Problem 8.

(a) For the following matrix A, find a basis for $Col(A)$.
(b) Find a basis for $Nul(A)$
(c) Find $\dim(Col(A))$ and $\dim(Nul(A))$

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. Coordinates

Problem 9. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}$. Find $[\mathbf{x}]_{\mathcal{B}}$, where $\mathbf{x} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

Problem 10. Find the coordinates of $p = 1 + 2x - x^2$ with respect to $\mathcal{B} = \{x^2, -1 + x, x + x^2\}$

7. Vector spaces

Problem 11. Show that the set of $n \times n$ symmetric matrices forms a vector space.