Name: ________________________________

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1. (10 points, 2 pts each)

Label the following statements as T or F.

**NOTE:** In this question, you do **NOT** have to show your work! Don’t spend *too* much time on each question!
(a) If the augmented matrix of the system \(Ax = b\) has a row of the form \([0 \ 0 \ 0 \ 1]\), then the corresponding system has no solutions.

(b) If \(A\) and \(B\) are two \(2 \times 2\) matrices, then \(det(AB) = det(BA)\)

(c) The equation \(Ax = 0\) always has either one or infinitely many solutions.

(d) If \(A\) is a \(3 \times 3\) matrix with two pivot positions, then the equation \(Ax = 0\) has a nontrivial solution.

(e) If \(A, B, C\) are square matrices with \(AB = AC\), then \(B = C\)
2. (10 points, 5 points each) Label the following statements as TRUE or FALSE. In this question, you HAVE to justify your answer!!!

This means:

- If the answer is TRUE, you have to explain WHY it is true (possibly by citing a theorem)

- If the answer is FALSE, you have to give a specific COUNTEREXAMPLE. You also have to explain why the counterexample is in fact a counterexample to the statement!

(a) If $A$ and $B$ are any $n \times n$ matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$

(b) If $A$ (not necessarily square) has a pivot in every row, then the system $Ax = b$ is always consistent.
3. (15 points) Solve the following system of equations (or say it has no solutions):

\[
\begin{align*}
2x + 2y + z &= 2 \\
x - y + 3z &= 3 \\
3x + 5y &= 1
\end{align*}
\]
4. (20 points) Solve the following system $Ax = b$, where:

$$
A = \begin{bmatrix}
1 & 2 & -3 & 9 \\
2 & -2 & 5 & -9 \\
1 & -1 & 0 & 3 \\
4 & 3 & -1 & 8
\end{bmatrix} \quad b = \begin{bmatrix}
5 \\
-2 \\
-1 \\
10
\end{bmatrix}
$$

Write your answer in (parametric) vector form
5. (15 points) Calculate $AB$, where $A$ and $B$ are given, or say that $AB$ is undefined.

(a)

$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$, \quad $B = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 4 & 1 & 3 \end{bmatrix}$

(b)

$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -3 & 1 \\ 0 & 2 & 1 \end{bmatrix}$, \quad $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}$
6. (15 points) Find $A^{-1}$ (or say ‘$A$ is not invertible’) where:

$$A = \begin{bmatrix}
1 & 2 & 1 \\
1 & 0 & 1 \\
3 & -1 & 2
\end{bmatrix}$$
7. (15 points) Find $\det(A)$, where:

\[
A = \begin{bmatrix}
1 & 0 & 1 & -1 \\
0 & 1 & 0 & 0 \\
2 & 0 & 3 & 1 \\
1 & 0 & 0 & 4
\end{bmatrix}
\]